New parametrization of the Kobayashi-Maskawa matrix

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(Received 18 March 1986)

We propose a new perturbative parametrization of the quark mixing matrix (Kobayashi-Maskawa matrix) V, in terms of $\xi = |V_{us}V_{cb}|^{1/2} \simeq 0.11$. Generalization to a fourth generation is conjectured and its phenomenological implications on CP nonconservation in the neutral-K-meson system are discussed.

The mixing of quarks in the standard model which determines the flavor structure of the charged weak current is described by a 3×3 unitary matrix, the Kobayashi-Maskawa (KM) matrix:[1]

$$V=\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{1}$$

Experimentally the magnitudes of the KM matrix elements are constrained to be[2]

$$V=\begin{pmatrix} 0.9733\pm0.0024 & 0.225\pm0.005 & <0.009 \\ 0.24\pm0.03 & 0.82\pm0.13 & 0.058\pm0.009 \\ \cdots & \cdots & \cdots \end{pmatrix} . \tag{2}$$

It is apparent from (2) that V does not differ significantly from the unit matrix. Furthermore, the off-diagonal elements follow a hierarchical pattern:

$$\left|\frac{V_{us}}{V_{cb}}\right| \simeq 3.9, \quad \left|\frac{V_{us}}{V_{ub}}\right| \geq 25 . \tag{3}$$

Indeed, this observation has led Wolfenstein[3] to expand V in powers of a small parameter $\lambda = |V_{us}|$. In this Brief Report we wish to propose an alternate parametrization in powers of $\xi = |V_{us}V_{cb}|^{1/2} \simeq 0.11$.

Although a perturbative parametrization of the KM matrix is very handy, especially in calculations involving CP-nonconserving processes, the expansion parameter itself is by no means unique. Wolfenstein's choice of $\lambda = |V_{us}|$ was chiefly motivated by the observation that $|V_{cb}| \simeq |V_{us}|^2$. Here we take a rather orthogonal viewpoint and observe that the ratio $|V_{us}/V_{ub}| > 25$ asks for a hierarchy whereas $|V_{us}/V_{cb}| \simeq 3.9$ is still of order 1. Thus we choose $\xi = |V_{us}V_{cb}|^{1/2} \simeq 0.11$ to be our expansion parameter, so that the nearest-neighbor mixing $|V_{us}|$ and $|V_{cb}|$ are of order ξ, while $|V_{ub}|$ is of order ξ^2. Of course, expanding V in powers of ξ rather than λ does not change any physics, since it is merely a parametrization. There are some calculational simplifications, however, essentially because $\xi \simeq 0.11$ is a smaller parameter. For example, in order to calculate CP-violating processes,[3] one has to expand V to order λ^5 in Wolfenstein's form, while in our case order ξ^3 is sufficient. There is also a straightforward generalization to a fourth generation, which differs markedly from those already existing in the literature.[4]

Since $V_{ub} \sim \xi^2$, CP-violating phases will show up in the KM matrix only at order ξ^2. (All phases can be removed from V if V_{ub} is zero.) Unitarity then dictates the following form for V:

$$V=\begin{pmatrix} 1-\frac{1}{2}B^2\xi^2 & B\xi & \xi^2(\alpha-i\beta) \\ -B\xi & 1-\frac{1}{2}B^2\xi^2-\frac{1}{2}\frac{\xi^2}{B^2} & \frac{\xi}{B} \\ \xi^2(1-\alpha-i\beta) & -\frac{\xi}{B} & 1-\frac{1}{2}\frac{\xi^2}{B^2} \end{pmatrix} + O(\xi^3) . \tag{4}$$

Since CP-nonconserving effects in the neutral-K-meson system depend on $V_{td}V_{ts} = O(\xi^3)$, we expand V to order ξ^3 in the imaginary part. This is achieved by demanding that the imaginary part of the unitarity relation be satisfied to order ξ^3 in (4). Consequently,

$$V=\begin{pmatrix} 1-\frac{1}{2}B^2\xi^2 & B\xi & \xi^2(\alpha-i\beta) \\ -B\xi-\frac{i\beta}{B}\xi^3 & 1-\frac{1}{2}B^2\xi^2-\frac{1}{2}\frac{\xi^2}{B^2} & \frac{\xi}{B} \\ \xi^2(1-\alpha-i\beta) & -\frac{\xi}{B}-i\beta B\xi^3 & 1-\frac{\xi^2}{2B^2} \end{pmatrix} . \tag{5}$$

Using the central values of $|V_{ij}|$ given in Eq. (2) we obtain $B=1.97$ and $(\alpha^2+\beta^2)^{1/2} \leq 0.69$.

Several models which generalize V to include the mixing of a fourth generation of quarks $\binom{t'}{b'}$ already exist in the literature.[4] A common feature of such models seems to be that the mixing of the fourth generation to the first three generations is negligibly small. A weakly mixed fourth generation does not contribute significantly to the CP impurity parameters ε and ε' in the neutral-K-meson system unless the t' quark is very heavy. A straightforward generalization of Eq. (5), however, to a fourth generation could result in comparable contributions from t' to ε, although the contribution to ε' may still be negligible.

Equation (5) suggests that nearest-neighbor mixings are of order ξ, next-nearest-neighbor mixings of order ξ^2, etc. Generalizing this pattern we write

$$V_{ij} \sim \xi^{j-i} \text{ for } j \geq i \ . \quad (6)$$

Combined with the requirement of unitarity, we arrive at the following form for V for four generations:

$$V = \begin{pmatrix} 1-\frac{1}{2}B^2\xi^2 & B\xi & \xi^2(\alpha - i\beta) & C\xi^3(\gamma - i\delta) \\ -B\xi - \frac{i\beta}{B}\xi^3 & 1-\frac{1}{2}B^2\xi^2 - \frac{1\xi^2}{2B^2} & \frac{\xi}{B} & \frac{C}{B}\xi^2(\sigma - i\tau) \\ \xi^2(1-\alpha-i\beta) & -\frac{\xi}{B} - i\xi^3\left[B\beta + \frac{C^2}{B}\tau\right] & 1-\frac{1}{2}\frac{\xi^2}{B^2} - \frac{1}{2}C^2\xi^2 & C\xi \\ -C\xi^3(1-\alpha-i\beta+\gamma+i\delta-\sigma-i\tau) & \frac{C}{B}\xi^2(1-\sigma-i\tau) & -C\xi - \frac{i\tau C}{B^2}\xi^3 & 1-\frac{1}{2}C^2\xi^2 \end{pmatrix} . \quad (7)$$

There are nine parameters, as there should be, corresponding to six angles and three phases. I have kept terms of order ξ^2 in the real part and order ξ^3 in the imaginary part. Since $V_{ub'}$ and $V_{t'd}$ are of order ξ^3, I have shown their full expressions to order ξ^3.

What are the phenomenological implications of such a parametrization [Eq. (7)] on the CP impurity parameters ε and ε' in the K^0 system?[5] In particular, we wish to ask if the contributions from the fourth generation can be comparable to that from the first three generations. Following Ref. 6, we write

$$\varepsilon \propto \sum_{i,j=c,t,t'} \eta_{ij}\,\mathrm{Im}(V_{is}V_{id}^*V_{js}V_{jd}^*)E(\chi_i,\chi_j) \ , \quad (8)$$

where η_{ij} are QCD correction coefficients, roughly of order 1, and $E(\chi_i,\chi_j)$ with $\chi_i = m_i^2/m_W^2$ is the kinematic factor

$$E(\chi_i,\chi_j) = \chi_i\chi_j\left[\left(\frac{1}{4} + \frac{3}{2(1-\chi_i)} - \frac{3}{4(1-\chi_i)^2}\right)\frac{\ln\chi_i}{\chi_i - \chi_j} + (\chi_i \leftrightarrow \chi_j) - \frac{3}{4(1-\chi_i)(1-\chi_j)}\right] . \quad (9)$$

The ratio of the leading contribution to ε from the fourth generation to that from the first three is

$$R = \xi^2\left[\frac{\eta_{ct'}}{\eta_{ct}}\left(\frac{\beta+\delta+2\tau}{\beta}\right)\frac{E(\chi_c,\chi_{t'})}{E(\chi_c,\chi_t)}\right] . \quad (10)$$

Hence $R \sim \xi^2 m_{t'}^2/m_t^2$ and is of order one for $m_t/m_{t'} \sim \xi$. For $m_{t'} \sim 200-300$ GeV, a range of values allowed by the ρ-parameter constraint, the fourth-generation contribution to ε becomes comparable to other contributions. For ε', however, the t' contribution is insignificant, assuming that the penguin diagrams dominate, since the associated kinematic factor[7] does not increase very much with the mass of the intermediate heavy quark.

I thank Professor Sandip Pakvasa for discussions and a reading of the manuscript. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AM03-76SF00235.

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