Electromagnetism in a gauged chiral model

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It is shown that the unique addition of electromagnetism to a recently proposed gauged chiral Lagrangian of pseudoscalars and vectors results in the same model as that obtained by the "hidden symmetry" approach. The gauge-invariant treatment of "vector-meson dominance" is essentially equivalent to the old model of Kroll, Lee, and Zumino but we argue for the advantages of working in the "unusual basis" where the photon has zero mass. There is a possible indication that a small calculable deviation from complete vector-meson dominance can close the present gap between experiment and theory for the pion charge radius.

I. INTRODUCTION

The nonlinear σ model constructed out of, say, the nonet pseudoscalars describes the Nambu-Goldstone bosons of spontaneously broken chiral symmetry (for zero quark masses) and is now considered the "standard" low-energy approximation to QCD. However, in the quark model the vectors as well as the pseudoscalars are singled out for a special low-energy role by virtue of their being s-wave bound states. It is thus very appealing to construct a low-energy chiral Lagrangian of both pseudoscalars and vectors. This is hardly a new idea. The method for adding any particle to the chiral Lagrangian of pseudoscalars was formulated[1] a long time ago and an explicit pseudoscalar-vector Lagrangian was written down[2] for the two-flavor case. But the story does not end there. Because vector dominance (formulated usually by treating the vectors as massive Yang-Mills particles) works so well at low energies it was generally felt that the vectors should be added to the nonlinear σ model directly in this way rather than by using the general theory of nonlinear realizations. This involves, in order to maintain chiral symmetry, introducing the axial-vector mesons as gauge particles too. Both theoretically, because the axial-vector mesons are p-wave bound states in the quark model, and experimentally, because the mass and the width of the $I=1$ axial-vector meson A_1 have been elusive, this latter feature was a bit of an embarrassment. A possible way to eliminate the axial-vector mesons—by imposing a constraint analogous to the constraint eliminating the scalars in going from the linear to the nonlinear σ model—was tried and found to result in severe contradiction with experiment. (See Ref. 3 for a review of this problem.) The problem remained for a while (without much attention being paid to it) until recently when Kaymakcalan and Schechter wrote down[4] a nonlinear Lagrangian for pseudoscalars and vectors which contained an extra generalized mass term.

At about the same time, an interesting approach to this problem—based on noting the equivalence of the $[\mathrm{U}(3)_L\times\mathrm{U}(3)_R/\mathrm{U}(3)_V]_{\mathrm{global}}$ coset space structure of the nonlinear σ model to a $[\mathrm{U}(3)_L\times\mathrm{U}(3)_R]_{\mathrm{global}}\times[\mathrm{U}(3)_V]_{\mathrm{local}}$ structure and gauging the $[\mathrm{U}(3)_V]_{\mathrm{local}}$—was proposed[5] and intensively investigated.[6,7] It is claimed[5] that the underlying physics corresponds to the vector mesons behaving, in some sense, as composites of the pseudoscalars. The evidence for this claim is supposed to be first the agreement of the pure hadronic Lagrangian's predictions with experiment and especially the unique way in which electromagnetic interactions fit into the model.

In this paper we will first remark that the pure hadronic Lagrangian of the "composite vector" model is actually identical to the model of Ref. 4 (see also Ref. 7). Second, we will show that electromagnetism may be uniquely added to the model of Ref. 4 in a very straightforward way and one ends up again with a model identical to that in the composite vector approach. Since the present approach is, in some sense, based on a more conventional picture of the vector mesons (as quark, antiquark composites modified by a pion cloud), it seems that the claims of a new picture for the vector mesons may be premature. A more detailed dynamical argument[8] is really required in order to justify the claim. We also point out the advantages of doing the "classical" calculations of vector-meson dominance in a basis where the photon and vector mesons are decoupled from each other.

A final purpose of this paper is to study possible deviations from *exact* vector dominance in the present scheme. The ρ^0 leptonic widths and the pion "charge radius" turn out to be in slightly better agreement with experiment if one allows for such a deviation. This would be very exciting if the experimental uncertainties could be reduced so that the small deviation might be unambiguously confirmed.

II. HADRONIC LAGRANGIAN

The Lagrangian of Ref. 4 was constructed from the linearly transforming gauge fields A_μ^L, A_μ^R, and the pseudoscalar matrix field $U=\exp(2i\phi/F_\pi)$. The nonlinear gauge nonet field ρ_μ is obtained by the following substitutions for A_μ^L and A_μ^R:

$$A_\mu^L=\xi\rho_\mu\xi^\dagger+\frac{i}{g}\xi\partial_\mu\xi^\dagger\ , \tag{1a}$$

$$A_\mu^R=\xi^\dagger \rho_\mu \xi+\frac{i}{g}\xi^\dagger \partial_\mu \xi \ , \quad (1b)$$

where $\xi=U^{1/2}$. Apart from a gauge field kinetic term the Lagrangian simply consists of a mass and a generalized mass term:

$$-m_0{}^2\mathrm{Tr}(A_\mu^L A_\mu^L+A_\mu^R A_\mu^R)+B\,\mathrm{Tr}(A_\mu^L U A_\mu^R U^\dagger) \ . \quad (2)$$

Using Eqs. (2.12), (2.13), and (2.15) of Ref. 4 we reexpress the quantities g, $m_0{}^2$, and B above as $g=g_{\rho\pi\pi}/k$, $m_0{}^2=m_V{}^2(1+k)/8k$, and $B=m_V{}^2(1-k)/4k$, where

$$k\equiv g_{\rho\pi\pi}{}^2F_\pi{}^2/m_V{}^2 \ , \quad (3)$$

m_V is the vector-meson mass and the $\rho\pi\pi$ coupling constant is

$$g_{\rho\pi\pi}=8.65\pm 0.16 \ . \quad (4)$$

For this determination we used[9]

$$m_\rho=769\pm 3 \text{ MeV}, \quad \Gamma(\rho)=154\pm 5 \text{ MeV} \ . \quad (5)$$

The Lagrangian [Eqs. (2.10) and (2.11) of Ref. 4] may now be written as

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}\mathrm{Tr}[F_{\mu\nu}(\rho)F_{\mu\nu}(\rho)]-\tfrac{1}{2}m_V{}^2\mathrm{Tr}(\rho_\mu\rho_\mu)\\&-\frac{iF_\pi{}^2 g_{\rho\pi\pi}}{2}[\mathrm{Tr}\rho_\mu(\partial_\mu\xi\xi^\dagger+\partial_\mu\xi^\dagger\xi)]\\&-\frac{F_\pi{}^2}{4}(1+k)\mathrm{Tr}(\partial_\mu\xi\partial_\mu\xi^\dagger)\\&-\frac{F_\pi{}^2}{4}(1-k)\mathrm{Tr}(\xi^\dagger\partial_\mu\xi^\dagger\xi\partial_\mu\xi) \ , \end{aligned}\quad (6)$$

with

$$F_{\mu\nu}(\rho)=\partial_\mu\rho_\nu-\partial_\nu\rho_\mu-ig[\rho_\mu,\rho_\nu] \ .$$

Mass-breaking terms and terms proportional to $\epsilon_{\mu\nu\alpha\beta}$ have not been included above. It is now easy to see that (6) is the same as the "hidden symmetry" Lagrangian. The hidden symmetry is implemented by writing $U=\xi_L\xi_R^\dagger$ and requiring the theory to be invariant under

$$\begin{aligned}&\xi_{L,R}\to\xi_{L,R}h(x) \ ,\\&\rho_\mu\to h^\dagger\rho_\mu h+\frac{i}{g}h^\dagger\partial_\mu h \ ,\end{aligned}\quad (7)$$

with $h=h^\dagger$. The $[\mathrm{U}_L(3)\times\mathrm{U}_R(3)]_{\text{global}}\times[\mathrm{U}_V(3)]_{\text{local}}$-invariant Lagrangian is then

$$\mathcal{L}=-\tfrac{1}{4}\mathrm{Tr}[F_{\mu\nu}(\rho)F_{\mu\nu}(\rho)]+\mathcal{L}_A+k\mathcal{L}_V \ , \quad (8a)$$

$$\mathcal{L}_{V,A}=\frac{F_\pi{}^2}{8}\mathrm{Tr}[(\xi_L^\dagger D_\mu\xi_L\pm\xi_R^\dagger D_\mu\xi_R)^2] \ , \quad (8b)$$

$$D_\mu\xi_{L,R}\equiv\partial_\mu\xi_{L,R}+ig\xi_{L,R}\rho_\mu \ . \quad (8c)$$

Finally, to get a theory without extraneous fields one must choose a gauge for the $[\mathrm{U}_V(3)]_{\text{local}}$ by setting $\xi_L=\xi_R^\dagger\equiv\xi$. Inserting this into (8) and expanding results in (6) again.

III. ADDING ELECTROMAGNETISM

It is straightforward to add electromagnetism to (6) without using the hidden symmetry. Our job is made simple by knowing the $\mathrm{U_{EM}}(1)$ transformation properties of the linear fields in (2):

$$\begin{aligned}&\delta a_\mu=\frac{1}{e}\partial_\mu\epsilon \ ,\\&\delta U=i\epsilon[Q,U] \ ,\\&\delta A_\mu^{L,R}=i\epsilon[Q,A_\mu^{L,R}]+\frac{1}{g}Q\partial_\mu\epsilon \ ,\end{aligned}\quad (9)$$

where a_μ is the U(1) gauge field, e is the unit of electric charge, $\epsilon(x)$ is the U(1) gauge parameter and the quark charge matrix $Q=\mathrm{diag}(\frac{2}{3},-\frac{1}{3},-\frac{1}{3})$. We may apply a usual iterative[10] procedure to gauge (2); its variation under (9) is

$$\frac{-2m_0{}^2}{g}\partial_\mu\epsilon\,\mathrm{Tr}[Q(A_\mu^L+A_\mu^R)] +\frac{B}{g}\partial_\mu\epsilon\,\mathrm{Tr}[Q(UA_\mu^R U^\dagger+U^\dagger A_\mu^L U)] \ .$$

This expression is canceled by the variation of a_μ in the additional term:

$$\begin{aligned}\Delta^{(1)}=&\frac{2em_0{}^2}{g}a_\mu\mathrm{Tr}[Q(A_\mu^L+A_\mu^R)]\\&-\frac{eB}{g}a_\mu\mathrm{Tr}[Q(UA_\mu^R U^\dagger+U^\dagger A_\mu^L U)] \ .\end{aligned}\quad (10)$$

Finally the variations of A_μ^L and A_μ^R in (10) are canceled if one adds

$$\Delta^{(2)}=-\frac{2e^2m_0{}^2}{g^2}a_\mu{}^2\mathrm{Tr}(Q^2)+\frac{Be^2}{g^2}a_\mu{}^2\mathrm{Tr}(QUQU^\dagger) \ , \quad (11)$$

and the process ends. Expressing (10) and (11) in terms of the nonlinear field ρ_μ with (1) yields the total expression $\Delta=\Delta^{(1)}+\Delta^{(2)}+(a_\mu$ kinetic term) to be added to (6) in order to include electromagnetism in the effective chiral Lagrangian:

$$\begin{aligned}\Delta=&\frac{ieF_\pi{}^2}{4}a_\mu\mathrm{Tr}\{Q[(1+k)(\xi\partial_\mu\xi^\dagger+\xi^\dagger\partial_\mu\xi)+(k-1)(\xi\partial_\mu\xi\xi^{\dagger 2}+\xi^\dagger\partial_\mu\xi^\dagger\xi^2)-2ikg(\xi\rho_\mu\xi^\dagger+\xi^\dagger\rho_\mu\xi)]\}\\&-\frac{e^2F_\pi{}^2}{4}a_\mu{}^2\mathrm{Tr}[(1+k)Q^2+(k-1)(Q\xi^2Q\xi^{\dagger 2})]-\tfrac{1}{4}(\partial_\mu a_\nu-\partial_\nu a_\mu)^2 \ .\end{aligned}\quad (12)$$

In the hidden-symmetry approach, Eq. (12) may be obtained by modifying the covariant derivative in (8c) to

$$D_\mu\xi_{L,R}=\partial_\mu\xi_{L,R}+ig\xi_{L,R}\rho_\mu-ieQa_\mu\xi_{L,R} \ , \quad (13)$$

substituting back into (8b) and (8a) and choosing the gauge $\xi_L=\xi_R^\dagger=\xi$. From the present derivation one sees that the result (12) is due to gauge invariance alone, rather than any detailed dynamical mechanism.

It is instructive to expand the terms in (12) which are linear in a_μ to leading order:

$$\Delta = \cdots + e a_\mu \left[kgF_\pi{}^2 \mathrm{Tr}(Q\rho_\mu) + i \left[1 - \frac{k}{2} \right] \mathrm{Tr}[Q(\phi \overleftrightarrow{\partial}_\mu \phi)] \right] + \cdots . \tag{14}$$

From this we identify the electromagnetic current as

$$J_\mu^{\mathrm{EM}} = (\tfrac{2}{3})^{1/2} kgF_\pi{}^2 \rho_\mu^E + i \left[1 - \frac{k}{2} \right] \mathrm{Tr}[Q(\phi \overleftrightarrow{\partial}_\mu \phi)] + \cdots , \tag{15}$$

where we have defined, for convenience (working in the degenerate nonet limit),

$$\rho_\mu^E = \frac{1}{\sqrt{6}} (2\rho_{11\mu} - \rho_{22\mu} - \rho_{33\mu}) . \tag{16}$$

In the case when we set $k=2$ [which just corresponds to assuming the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[11] to hold], Eq. (15) expresses Sakurai's[12] vector-meson dominance of the electromagnetic current. For an arbitrary value of k the charged pion couples to the photon partially through ρ_μ^E and partly by the direct term in (14). We shall return to this point in the next section.

In Sakurai's original approach only terms linear in the photon field were included and the model was not gauge invariant. Kroll, Lee, and Zumino[13] showed how to make the model gauge invariant (by adding extra terms) without changing the main results. The present model is gauge invariant by construction and can be seen to be mathematically equivalent[14] to the Kroll-Lee-Zumino model in the $k=2$ limit insofar as the treatment of the a_μ and ρ_μ^E fields are concerned.

Rather than keeping a kinetic-type mixing term between a_μ and ρ_μ^E as advocated in Ref. 13 it seems neater, although equivalent, to completely diagonalize the quadratic terms in the Lagrangian (6) plus (12):

$$-\tfrac{1}{2} m_V{}^2 \left[\rho_\mu^E - \left[\frac{2}{3} \right]^{1/2} \frac{e}{g} a_\mu \right]^2 .$$

This diagonalization is achieved by the transformation[15] to physical fields (subscript p):

$$\begin{bmatrix} a_\mu \\ \rho_\mu^E \end{bmatrix} = \left[1 + \frac{2}{3} \left[\frac{e}{g} \right]^2 \right]^{-1/2} \begin{bmatrix} 1 & -\left[\frac{2}{3} \right]^{1/2} \frac{e}{g} \\ \left[\frac{2}{3} \right]^{1/2} \frac{e}{g} & 1 \end{bmatrix} \begin{bmatrix} a_{\mu p} \\ \rho_{\mu p}^E \end{bmatrix} . \tag{17}$$

$a_{\mu p}$ is the true zero-mass photon. The coupling of $a_{\mu p}$ to the charged pions receives a contribution from a direct $a\pi^2$ piece in (12),

$$ie(1-k/2) a_{\mu p} \pi^+ \overleftrightarrow{\partial}_\mu \pi^- ,$$

as well as from a $\rho\pi^2$ piece in (6),

$$\frac{iek}{2} a_{\mu p} \pi^+ \overleftrightarrow{\partial}_\mu \pi^- .$$

Adding these two gives the term

$$iea_{\mu p} \pi^+ \overleftrightarrow{\partial}_\mu \pi^- + O(a_{\mu p}{}^2) . \tag{18}$$

This looks like a simple minimal coupling of the physical photon to the pion. What has happened to vector-meson dominance of the electromagnetic current, which is now evidently just $i\pi^+ \overleftrightarrow{\partial}_\mu \pi^- + \cdots$? Furthermore, where are the ρ-meson form-factor effects?

The answers to these questions are rather amusing. First, it seems fair to say that in the physical basis the ρ meson does *not* dominate the electromagnetic current. But it does have an effect on the form factor in the following way. In order to find out what the pion form factor is, we should probe it by scattering a lepton against it. By gauge invariance the lepton field ψ must couple to a_μ rather than $a_{\mu p}$ [see (9)]. Using (17) we then find the lepton interaction term:

$$ie\bar{\psi}\gamma_\mu a_\mu \psi \simeq ie\bar{\psi}\gamma_\mu \left[a_{\mu p} - \left[\frac{2}{3} \right]^{1/2} \frac{e}{g} \rho_{\mu p}^E \right] \psi . \tag{19}$$

The lepton has a direct coupling to ρ, which, however, does not affect the lepton-lepton scattering at zero-momentum transfer (no charge renormalization). The lepton-pion scattering will contain both a_p exchange and ρ_p^E exchange contributions. This results in an effective propagator

$$\frac{1}{q^2} \left[1 - \frac{k}{2} \frac{q^2}{q^2 + M_V{}^2} \right] , \tag{20}$$

where $q^2 \equiv \mathbf{q}^2 - (q^0)^2$ is the squared momentum transfer and we have used the strong-interaction term

$$\frac{3ig_{\rho\pi\pi}}{2\sqrt{6}} \rho_{\mu p}^E \pi^+ \overleftrightarrow{\partial}_\mu \pi^- + \cdots .$$

Clearly the usual vector-meson-dominance prediction for the form factor is obtained from (20) when $k=2$.

Notice that the difference between the formula (20) for the pion form factor in the arbitrary k case and the complete vector-meson-dominance case $k=2$ is not an artifact of working in the physical photon basis. Equation (20) can also be derived in the usual way[12] starting from (14).

Finally, we would like to advocate the use of the physical photon basis for doing vector-meson-dominance calculations. In addition to being manifestly gauge invariant it directly displays [in (20)] the effect of the vector meson as giving the q^2 correction to the pion form factor rather than giving the whole thing. A further advantage for an effective Lagrangian (which at first sight might seem as a disadvantage) is that it directly [in (19)] gives a coupling of vector mesons to lepton pairs. This is in agreement with experiment.

IV. INCOMPLETE VECTOR-MESON DOMINANCE

In the present model exact vector-meson dominance need not be assumed but may emerge as a dynamical accident if $k=2$. How good is this? Using (4) and (5) we find

$$k=\left[\frac{g_{\rho\pi\pi}F_\pi}{m_\rho}\right]^2=\begin{cases}2.20\pm0.10, & F_\pi=132 \text{ MeV} ,\\ 2.14\pm0.09, & F_\pi=130 \text{ MeV} .\end{cases} \tag{21}$$

$F_\pi=132$ MeV is usually cited but the lower value is favored by a recent analysis.[16] This agreement is quite reasonable with perhaps an indication that the true value of k is slightly larger than 2. Since there is no fundamental theoretical reason for complete vector-meson dominance[17] it is interesting to see what experiment has to say about other quantities computed in this model. For this purpose we should restrict our attention to the two-flavor case since the effects of the nonzero strange-quark mass may easily mask the small deviations from $k=2$ that we would like to uncover.

First, consider the lepton decays of the ρ^0. A straightforward calculation using (19) (and remembering that $g=g_{\rho\pi\pi}/k$) gives

$$\Gamma(\rho^0\to e^+e^-)=\Gamma(\rho^0\to\mu^+\mu^-)=\frac{2\pi\alpha^2 g_{\rho\pi\pi}{}^2F_\pi{}^4}{3m_\rho{}^3} , \tag{22}$$

where $\alpha=e^2/4\pi$. This results in a prediction of 5.58 (5.24) keV for $F_\pi=132$ (130) MeV. Experimentally[9] $\Gamma(\rho^0\to e^+e^-)=7.08\pm0.53$ keV and $\Gamma(\rho^0\to\mu^+\mu^-)=10.3\pm2.2$ keV. Since the experimental data violate μ-e universality we should not take their exact values too seriously. Nevertheless they indicate that the predicted value may be a little too low. If we want to reach the lowest experimental limit 6.55 keV we would have to require (say) $F_\pi=137$ MeV which would raise k to 2.37. Alternatively one could increase $g_{\rho\pi\pi}$ which would also increase k.

A more direct test of k comes from the pion charge radius r_π, which is defined by the following expansion of the pion form factor $F(q^2)$ [the coefficient of $1/q^2$ in (20)]:

$$F(q^2)=1-\frac{r_\pi{}^2}{6}q^2+O(q^4) . \tag{23}$$

From (20) we see that the prediction of exact vector dominance ($k=2$) is

$$r_\pi=\frac{\sqrt{6}}{m_V}=0.629\pm0.002 \text{ fm} , \tag{24}$$

where (5) was used. This is to be compared with the recent measurement[18]

$$r_\pi=0.66\pm0.02 \text{ fm} . \tag{25}$$

There appears to be a small discrepancy which could be overcome if we consider $g_{\rho\pi\pi}$, F_π, and m_ρ to take their experimental values; we then have

$$r_\pi=\frac{\sqrt{3k}}{m_\rho}=\begin{cases}0.66\pm0.01 \text{ fm}, & F_\pi=132 \text{ MeV} ,\\ 0.65\pm0.01 \text{ fm}, & F_\pi=130 \text{ MeV} .\end{cases} \tag{26}$$

To sum up, this model fits experimental data slightly better with a small calculable deviation from exact vector-meson dominance. Of course, this is a rather small effect and it is important to tighten up the experimental and theoretical uncertainties on the various parameters to convince ourselves that the effect is real. One effect outside this model which may be roughly estimated is that of the $\rho'(1600)$ (the radially excited ρ) on r_π. If the form factor is written as the sum of ρ and ρ' pole terms, the relative ρ' contribution to r_π compared to the ρ contribution is about

$$\frac{1}{2}\left[\frac{m_\rho}{m_{\rho'}}\right]^3\left[\frac{\Gamma(\rho'\to\pi\pi)}{\Gamma(\rho\to\pi\pi)}\frac{\Gamma(\rho'\to e^+e^-)}{\Gamma(\rho\to e^+e^-)}\right]^{1/2}\approx0.037 . \tag{27}$$

This is probably an overestimate since the ρ'-photon transition amplitude is likely to be reduced on continuing from $-q^2=1600$ MeV to $-q^2=0$. (This could also help reconcile the $\rho^0\to e^+e^-$ predictions in this model.) Thus we might expect a contribution on the order of 0.01—0.02 fm in magnitude to r_π from $\rho'(1600)$.

Finally, it may be helpful to place the model discussed here in the context of QCD. It is clear that QCD requires the presence in $\mathcal{L}_{\text{eff}}$ of higher-derivative couplings and additional particles (a recent review of the attempts to systematically derive a low-energy effective theory from QCD is given in Ref. 19). Our point of view about which particles to include is to proceed in stages. Here we have included the s-wave $\bar{q}q$ states which are the ones which lie lowest experimentally. A reasonable next step would be to include the p-wave $\bar{q}q$ states as well as the radial excitations and gluonic states in the same energy range. As far as higher-derivative interactions are concerned we take the point of view that the minimal number of derivatives should be retained which will allow agreement with experiment. It should be evident that we do not consider the model above to be exact in any sense. Nevertheless it is such a simple one and appears so close to experiment in several respects that one would like to see just how accurate it really is. That has been the main goal of this section.

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