Strangeness production in ultrarelativistic heavy-ion collisions. I. Chemical kinetics in the quark-gluon plasma

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(Received 26 November 1985)

We develop a kinetic theory of chemical reactions in a quark-gluon plasma in order to study the evolution of flavor composition in ultrarelativistic nucleus-nucleus collisions. The rates of production and annihilation of strange-quark pairs are computed in lowest order in perturbation theory assuming local equilibrium with respect to other, more frequent collision processes. Quantum-statistical effects are taken into account. The hydrodynamic equations coupled to the rate equation are derived and solved numerically in a homogeneous plasma, simulating the approach toward complete chemical equilibrium. The corresponding relaxation times are computed.

I. INTRODUCTION

Central collisions of heavy nuclei at ultrarelativistic energies provide a unique opportunity to explore the physics of matter at extremely high energy densities.[1] At sufficiently high beam energies ($E_{\rm c.m.} > 50$ GeV/nucleon) we expect that in the pp center-of-mass frame two highly Lorentz-contracted nuclei pass through each other and deposit their kinetic energy behind them gradually as heat.[2–5] Extrapolations from existing pp and pA data suggest that an energy density of one to three orders of magnitude greater than that of normal nuclear matter (0.15 GeV/fm^3) will be achieved in this region.[4–6] Some spectacular cosmic-ray events[7] support this conjecture. Under such circumstances it is most likely that the matter is formed initially as a dense plasma of unconfined quarks, antiquarks, and gluons. If the collisions among these excited quanta are very frequent, a local thermodynamic equilibrium will be established and the system will evolve subsequently according to (relativistic) hydrodynamics.[8] This hydrodynamic evolution may last for a long time on the scale of strong interactions, and thus the matter will undergo considerable processing before it disassembles into freely streaming particles. This rather complex development of the collision poses a very difficult question: What are the relics of the plasma which survive the later stages of evolution?

One of the characteristic properties of the quark-gluon plasma, as compared to a high-temperature hadron gas, is its large entropy content. This is largely due to the fact that color degrees of freedom are allowed to be thermally excited in the plasma phase, whereas they are essentially frozen in the confining phase. In actual observations, the large entropy produced by the plasma formation will be reflected in the high multiplicity of secondaries,[6] possibly accompanied by large fluctuations due to the instabilities which may arise in the course of the hadronization transition.[9] The quark content of this entropy generates thermal fluctuations in the electromagnetic current which provide a strong source of dileptons and photons. Since leptons and photons can penetrate the hadronic medium without suffering strong final-state interactions, they may serve as a clean probe of the early stages of matter evolution.[10]

Another characteristic property of the plasma which has potential importance as a diagnostic probe is its flavor composition.[11] It has been argued[12,13] that if a quark-gluon plasma is formed from compressed nuclear matter, as may happen in the nuclear fragmentation region[3] or in low-energy "stopping-regime" collisions,[12,14] then the abundance of s and $\bar{s}$ quarks in the system would be highly enhanced compared to that of $\bar{u}$ and $\bar{d}$ quarks since the Pauli exclusion principle strongly suppresses the creation of light-quark pairs; this asymmetry in the flavor composition of the baryon-rich quark-gluon plasma may be reflected in the final particle composition. In the baryon-free central rapidity region, however, one expects the formation of an equilibrium plasma at zero chemical potential; in such a plasma the u, d, and s quarks will be almost equally abundant if the initial temperature T_0 is large compared to the mass m_s of the strange quark. How will this symmetry in the initial flavor composition be reflected in the final observed particle abundance? Can we expect an anomalous enhancement in the K/π ratio?

Two extreme situations may be considered. If local chemical equilibrium is maintained throughout the expansion of the system, then information on the initial flavor composition will be washed out completely and the final relative particle abundances will merely reflect the freeze-out conditions. The entropy, which is conserved in an adiabatic expansion, would be perhaps the only hadronic observable to carry information about the initial conditions. At the other extreme, if the number of strange quarks and antiquarks is conserved, the initial strange-quark density is directly related to the final kaon density

dn_K/dy (where y is rapidity). In this case, the system inevitably deviates from chemical equilibrium as it expands. However, even here the observed K/π ratio will not necessarily become very large since a large number of pions ought to be produced in order to conserve the large initial entropy.[15] From these simple considerations, we see that to answer the above questions one needs to study two competing processes: how the system deviates from a (local) chemical equilibrium as it cools and hadronizes in a hydrodynamic expansion, and how the system reacts to return to equilibrium.

In this paper we develop a QCD kinetic theory which describes the approach to thermodynamic equilibrium in the quark-gluon plasma. It is not our purpose, however, to study the general nonequilibrium properties of this system. This would require a full development of the quantum transport theory of a quark-gluon plasma. We will, instead, focus on the nonequilibrium processes associated with the "chemical reactions" which change the strange-quark density in the plasma, and *assume* equilibrium (detailed balance) with respect to other, more frequent collision processes. The basic equation of interest is thus the rate equation which relates the rate of change of the strange-quark density to the rates of $s\bar{s}$ production and annihilation in collisions. The relativistic hydrodynamic equations coupled to the rate equation are introduced in Sec. II and the underlying physical assumptions are discussed. We estimate the reaction rates in lowest-order QCD perturbation theory in Sec. III.

Similar calculations have been done by other groups.[12,13] We note that in the earlier work only the gain term was computed in the absence of strange quarks, and hence with no Pauli blocking. The result was used in the classical relaxation time approximation to describe the evolution of the strange-quark density from zero to its equilibrium value. In the present work we calculate both gain and loss terms, taking into account the proper quantum-statistical effects. Hence, our calculation gives a more precise description of the chemical process for the entire range of strange-quark density that is likely to arise in the study of ultrarelativistic nucleus-nucleus collisions.

In Sec. IV we consider the relaxation of the strange-quark density in a nonexpanding plasma. The relaxation time is evaluated numerically and a comparison with the classical treatment (implicit in the previous studies) is made. We find that the classical relaxation time approximates the full quantum result to within 20% for the temperature regime considered. Previous work[13] employed quantum distribution functions in a classical relaxation formalism; their result disagrees with our fully classical result by a factor of 2.5, a discrepancy too large to be explained on physics grounds alone. See the *Note added* after Sec. IV for reconciliation of these results.

In a subsequent paper we will discuss the chemical reactions in the scaling hydrodynamic expansion[4] of a plasma created in the central rapidity region of a nucleus-nucleus collision.[16]

II. KINETICS OF CHEMICAL REACTIONS IN THE PLASMA

In this section we discuss the physical assumptions underlying the rate equation. Our discussion is based on a relativistic version of the semiclassical kinetic theory introduced by Uehling and Uhlenbeck. Here we explain the ideas in an intuitive way without referring to the original kinetic equation. A more extensive discussion of the formalism is given in the Appendix.

In the dense QCD plasma, various collision processes among the excited quanta (quarks and gluons) will be at work. Consider the following subset of processes:

$$q+g\rightarrow q+g\ , \tag{2.1}$$

$$g+g\rightarrow g+g\ , \tag{2.2}$$

$$q+g\leftrightarrow q+g+g\ , \tag{2.3}$$

$$g+g\leftrightarrow g+g+g\ . \tag{2.4}$$

The first two processes correspond to elastic scattering between a quark (q) and a gluon (g) and between two gluons; the others are inelastic collisions which produce one extra gluon. The semiclassical kinetic theory tells us that when the system is in local equilibrium with respect to elastic binary collisions like (2.1) or (2.2), the distribution functions in phase space of the various species take the form

$$f_{\pm}(p;x)=\frac{1}{\exp[\beta_\nu(p^\nu-\lambda^\nu)]\pm 1}\ , \tag{2.5}$$

where p_ν is the four-momentum of the particle. $\lambda_\nu(x)$ and $\beta_\nu(x)$ are four-vectors constructed from the local flow velocity $u_\nu(x)$, the local chemical potentials $\mu_i(x)$ (one for each species), and the inverse local temperature $\beta(x)=1/T(x)$:

$$\lambda_\nu(x)=\mu(x)u_\nu(x)\ , \tag{2.6}$$

$$\beta_\nu(x)=\beta(x)u_\nu(x)\ . \tag{2.7}$$

In (2.5) the upper sign refers to fermions (quarks) and the lower sign to bosons (gluons). As far as the inelastic processes (2.3) and (2.4) are concerned, detailed balance is achieved when the gluon chemical potential vanishes (see the Appendix). In the following discussion we assume that this equilibrium has already been established in the system.

In principle, light quarks and gluons on the one hand and heavy quarks on the other may possess different temperatures, as happens in the classical plasma composed of electrons and very heavy ions; if so, more careful treatment of the approach to complete kinetic equilibrium would be required. We expect, however, that in our case this is a minor effect since the mass difference between light quarks and s quarks is small compared to the temperature. Also we neglect here possible diffusion processes which may arise due to the different, finite mean free paths for the different species. Our formalism can be extended to incorporate these effects within the framework of kinetic theory.

Consider now some other reaction processes, which change the number of quarks:

$$g+g\leftrightarrow q+\bar{q}\ , \tag{2.8}$$

$$g+g\leftrightarrow s+\bar{s}\ , \tag{2.9}$$

$$q+\bar{q}\leftrightarrow s+\bar{s}\ . \tag{2.10}$$

Here we have used the symbols $q,\bar{q}$ only for the light quarks and antiquarks while the strange quarks and antiquarks are denoted explicitly by $s,\bar{s}$. If there is local equilibrium with respect to these "chemical reactions," the sum of the chemical potentials of the particles on the left-hand side equals the sum on the right-hand side for each process. Since the gluon chemical potential is zero, this leads to $\mu_q=-\mu_{\bar{q}}$. In the baryon-free plasma on which we shall concentrate, particle-antiparticle symmetry imposes $\mu_q=\mu_{\bar{q}}$. Thus, the light-quark chemical potentials vanish. The same would be true for the strange quarks if they were in chemical equilibrium; if equilibrium with respect to reactions such as (2.9) and (2.10) is not yet established, the system is characterized by nonvanishing μ_s and $\mu_{\bar{s}}$. Here also, particle-antiparticle symmetry means $\mu_s=\mu_{\bar{s}}$. As we shall see later, the relaxation time for equilibrium with respect to processes such as (2.8) is much shorter than that for the processes which create or annihilate strange quarks.

Hence, in the following we consider a system slightly out of equilibrium, with distribution functions for the gluons, light quarks, and light antiquarks given by

$$f_g(p;x)=\frac{1}{\exp[\beta_\mu(x)p^\mu]-1}\ , \tag{2.11}$$

$$f_q(p;x)=f_{\bar{q}}(p;x)=\frac{1}{\exp[\beta_\mu(x)p^\mu]+1}\ , \tag{2.12}$$

with $p^2=0$, while for the strange quarks and antiquarks,

$$f_s(p;x)=f_{\bar{s}}(p;x)=\frac{1}{\exp\{\beta_\mu(x)[p^\mu-\lambda^\mu(x)]\}+1}\ , \tag{2.13}$$

with $p^2=m^2$. Here m is the bare mass of the strange quarks; the small masses (<10 MeV) of the light quarks are safely neglected.[17] The nonvanishing strange-quark chemical potential μ_s (henceforth denoted simply as μ) is the parameter which measures the distance from complete chemical equilibrium.

Under these physical assumptions, the evolution of $\beta(x)$, $\mu(x)$, and $u_\nu(x)$ from given initial conditions is governed by the hydrodynamic equations. These include energy-momentum conservation

$$\partial_\mu T^{\mu\nu}(x)=0 \tag{2.14}$$

and the rate equation

$$\partial_\nu n_s^\nu(x)=\partial_\nu n_{\bar{s}}^\nu(x)=R_{\text{gain}}-R_{\text{loss}}\ . \tag{2.15}$$

The energy-momentum tensor is given by

$$T^{\mu\nu}(x)=\int\frac{d^3p}{(2\pi)^3E}p^\mu p^\nu \times\{\gamma_g f_g(p;x)+\gamma_q[f_q(p;x)+f_{\bar{q}}(p;x)] +\gamma_s[f_s(p;x)+f_{\bar{s}}(p;x)]\} \tag{2.16}$$

and the strange-quark current (equal to the strange-antiquark current) by

$$n_s^\nu(x)=n_{\bar{s}}^\nu(x)=\gamma_s\int\frac{d^3p}{(2\pi)^3E}p^\nu f_s(p;x)\ , \tag{2.17}$$

where $\gamma_g=2\times8\times1$, $\gamma_q=2\times3\times2$, and $\gamma_s=2\times3\times1$ are the products of the spin, color, and isospin degeneracy factors for the gluons, light quarks, and strange quarks, respectively. Equations (2.16) and (2.17) may be rewritten in the form

$$T^{\mu\nu}=g^{\mu\nu}p(x)+[\epsilon(x)+p(x)]u^\mu(x)u^\nu(x)\ , \tag{2.18}$$

$$n_s^\nu=n_s(x)u^\nu(x)\ , \tag{2.19}$$

where $\epsilon(x)$, $p(x)$, and $n_s(x)$ are the energy density, the pressure, and the strange-quark density in the local comoving frame. The production rate R_{gain} and the annihilation rate R_{loss} of strange quarks are (in lowest order in α_s) a sum of contributions from the processes (2.9) and (2.10). The gluon process gives

$$R_{\text{gain}}^{gg}=\frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4) \times\sum|\mathcal{M}_{gg\to s\bar{s}}|^2 f_g(p_1)f_g(p_2)[1-f_s(p_3)][1-f_{\bar{s}}(p_4)]\ , \tag{2.20a}$$

$$R_{\text{loss}}^{gg}=\frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4) \times\sum|\mathcal{M}_{s\bar{s}\to gg}|^2 f_s(p_3)f_{\bar{s}}(p_4)[1+f_g(p_1)][1+f_g(p_2)]\ . \tag{2.20b}$$

In (2.20a), the squared matrix element, summed over spin and color, is weighted by two gluon distribution functions f_g for the initial states, and the factor $(1-f_s)(1-f_{\bar{s}})$ indicates Pauli blocking. In the reverse process (2.20b) the rate is weighted by the distribution functions of s and $\bar{s}$ quarks for the initial states and the gluon final states each gain an enhancement factor $1+f_g$ from Bose-Einstein statistics. The factor of $\frac{1}{2}$ accounts for the identity of the two gluons. Similarly, the $q\bar{q}$ process (2.10) gives

$$R^{q\bar{q}}_{\text{gain}}=\int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$$
$$\times\sum|\mathcal{M}_{q\bar{q}\to s\bar{s}}|^2 f_q(p_1)f_{\bar{q}}(p_2)[1-f_s(p_3)][1-f_{\bar{s}}(p_4)]\;, \tag{2.21a}$$

$$R^{q\bar{q}}_{\text{loss}}=\int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$$
$$\times\sum|\mathcal{M}_{s\bar{s}\to q\bar{q}}|^2 f_s(p_3)f_{\bar{s}}(p_4)[1-f_q(p_1)][1-f_{\bar{q}}(p_2)]\;. \tag{2.21b}$$

Since the rates are scalars, they can only be functions of the local temperature $T(x)$ and the chemical potential $\mu(x)$; they do not depend on the local flow velocity $u_\nu(x)$. Our calculation proceeds in the comoving frame, where $u_\nu=(1,0,0,0)$.

In the next section we shall compute the invariant matrix element in perturbation theory and carry out the phase-space integrals. A first step is to use the identity

$$1\mp f_\pm=e^{\beta(E-\mu)}f_\pm \tag{2.22}$$

and the unitarity relation $|\mathcal{M}_{12}|^2=|\mathcal{M}_{21}|^2$ to combine the gain and loss terms, yielding

$$R^{gg}_{\text{gain}}-R^{gg}_{\text{loss}}=(e^{-2\beta\mu}-1)\tfrac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$$
$$\times|\mathcal{M}_{gg\to s\bar{s}}|^2 f_g(p_1)f_g(p_2)f_s(p_3)f_{\bar{s}}(p_4)$$
$$\times\exp[\beta(E_1+E_2)]$$
$$\equiv(e^{-2\beta\mu}-1)I_{\text{gluon}} \tag{2.23}$$

for the gluon processes, and

$$R^{q\bar{q}}_{\text{gain}}-R^{q\bar{q}}_{\text{loss}}=(e^{-2\beta\mu}-1)\int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^2p_3}{(2\pi)^3 2E_3}\int\frac{d^2p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$$
$$\times|\mathcal{M}_{q\bar{q}\to s\bar{s}}|^2 f_q(p_1)f_{\bar{q}}(p_2)f_s(p_3)f_{\bar{s}}(p_4)$$
$$\times\exp[\beta(E_1+E_2)]$$
$$\equiv(e^{-2\beta\mu}-1)I_{\text{quark}} \tag{2.24}$$

for the $q\bar{q}$ processes. From these expressions it is clear that the right-hand side of the rate equation (2.15) vanishes when $\mu=0$.

III. PERTURBATIVE CALCULATION OF THE COLLISION INTEGRAL

We shall calculate the collision integrals (2.23) and (2.24) in lowest-order QCD perturbation theory. The application of perturbation theory should be legitimate when the temperature of the system is large compared to the QCD energy scale $\Lambda\approx150$ MeV; it might appear to be invalid near the transition region where our main interest lies. We note, however, that low momentum transfer is forbidden in the strange-quark production and annihilation processes by mass thresholds, and therefore the strange-quark mass sets the energy scale when the temperature is low. Thus we expect a wider range of applicability of perturbation theory in the present problem. Of course, this conjecture will have to be tested by more reliable calculations, incorporating nonperturbative effects in the transport process.

The matrix elements we need are just those which appear in parton-model calculations[18] of heavy-quark production in hard pp collision processes. The difference between those calculations and ours is in the weight factors for the phase space integral: In the hard-scattering process one uses parton distribution functions obtained from deep-inelastic lepton scattering, while here we use thermal distribution functions. The lowest-order Feynman diagrams are shown in Fig. 1. There are three topologically distinct diagrams [Figs. 1(a)—1(c)] for the gluon process (2.9) and one [Fig. 1(d)] for the quark process (2.10). The matrix elements for these diagrams are given by

$$\mathcal{M}_{\rm a}=-g^2\epsilon_\mu(1)\epsilon_\nu(2)f_{abc}[g^{\mu\nu}(-p_1+p_2)^\rho+g^{\nu\rho}(-p_1-2p_2)^\mu+g^{\rho\mu}(2p_1+p_2)^\nu]\frac{1}{(p_1+p_2)^2}\bar{u}_s(3)\gamma_\rho\lambda_c v_s(4)\ , \tag{3.1}$$

$$\mathcal{M}_{\rm b}=-ig^2\epsilon_\mu(1)\epsilon_\nu(2)\bar{u}_s(3)\gamma^\mu\lambda_a\frac{\not{p}_3-\not{p}_1+m}{(p_3-p_1)^2-m^2}\gamma^\nu\lambda_b v_s(4)\ , \tag{3.2}$$

$$\mathcal{M}_{\rm c}=-ig^2\epsilon_\mu(1)\epsilon_\nu(2)\bar{u}_s(3)\gamma^\nu\lambda_b\frac{\not{p}_3-\not{p}_2+m}{(p_3-p_2)^2-m^2}\gamma^\mu\lambda_a v_s(4)\ , \tag{3.3}$$

$$\mathcal{M}_{\rm d}=ig^2\bar{v}_q(2)\gamma^\mu\lambda_a u_q(1)\frac{1}{(p_1+p_2)^2}\bar{u}_s(3)\gamma_\mu\lambda_a v_s(4)\ , \tag{3.4}$$

respectively, where we have used the abbreviated notations $\epsilon_\mu(1)=\epsilon_\mu(p_1,\zeta_1)$ for gluon polarization vectors and $u(1)=u(p_1,s_1)$, $v(1)=v(p_1,s_1)$ for quark spinors. λ_a are the SU(3) matrices, normalized by $\mathrm{tr}\lambda_a\lambda_b=\frac{1}{2}\delta_{ab}$, and f_{abc} are the structure constants, $[\lambda_a,\lambda_b]=if_{abc}\lambda_c$. As usual, the summation of the squared matrix elements over the initial- and final-quark spin states converts the quark wave functions $u(p,s)$, $v(p,s)$ into projection operators according to

$$\sum_{s=1,2}\bar{u}^{\,i}_{\alpha}(p,s)u^{i}_{\beta}(p,s)=(\not{p}+m_i)_{\alpha\beta}\ , \tag{3.5}$$

$$\sum_{s=1,2}v^{i}_{\alpha}(p,s)\bar{v}^{\,i}_{\beta}(p,s)=-(\not{p}+m_i)_{\alpha\beta}\ . \tag{3.6}$$

In summing over gluon polarizations ζ_i, care has to be taken in excluding the contributions of unphysical states.

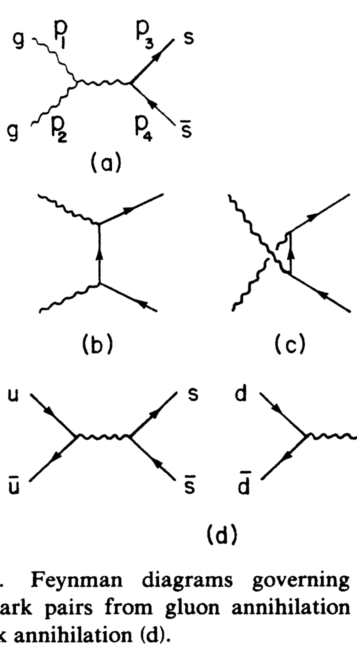

FIG. 1. Feynman diagrams governing the creation of strange-quark pairs from gluon annihilation (a)—(c) and from light-quark annihilation (d).

A trick which accomplishes this is to delete from (3.1) the terms containing $\epsilon_\mu(p,\zeta)p^\mu$, leaving

$$\begin{aligned}\mathcal{M}_{\rm a}=&-g^2\epsilon_\mu(1)\epsilon_\nu(2)f_{abc}\\&\times[g^{\mu\nu}(-p_1+p_2)^\rho-2g^{\nu\rho}p_2^\mu+2g^{\rho\mu}p_1^\nu]\\&\times\frac{1}{(p_1+p_2)^2}\bar{u}^{\,s}(3)\gamma_\rho\lambda_c v^s(4)\ .\end{aligned} \tag{3.7}$$

One can then safely perform the trace over Lorentz indices, using the formula

$$\sum_{\zeta}\epsilon_\mu(p,\zeta)\epsilon_\nu(p,\zeta)=-g_{\mu\nu}\ . \tag{3.8}$$

The results are most conveniently written in terms of the three Mandelstam variables, $s=(p_1+p_2)^2$, $t=(p_3-p_1)^2$, and $u=(p_3-p_2)^2$, which satisfy $s+t+u=2m^2$:

$$\sum|\mathcal{M}_{\rm a}|^2=\gamma_g{}^2\gamma_s{}^2\pi^2\alpha_s{}^2\frac{(m^2-t)(m^2-u)}{3s^2}\ , \tag{3.9}$$

$$\sum|\mathcal{M}_{\rm b}|^2=\gamma_g{}^2\gamma_s{}^2\pi^2\alpha_s{}^2\times\frac{2}{27}\frac{(m^2-t)(m^2-u)-2m^2(m^2+t)}{(m^2-t)^2}\ , \tag{3.10}$$

$$\sum|\mathcal{M}_{\rm c}|^2=\gamma_g{}^2\gamma_s{}^2\pi^2\alpha_s{}^2\times\frac{2}{27}\frac{(m^2-u)(m^2-t)-2m^2(m^2+u)}{(m^2-u)^2}\ , \tag{3.11}$$

$$\begin{aligned}\sum\mathcal{M}_{\rm a}\mathcal{M}_{\rm b}^*=&\sum\mathcal{M}_{\rm a}^*\mathcal{M}_{\rm b}\\=&\gamma_g{}^2\gamma_s{}^2\pi^2\alpha_s{}^2\\&\times\frac{1}{12}\frac{(m^2-t)(m^2-u)+m^2(u-t)}{s(m^2-t)}\ ,\end{aligned} \tag{3.12}$$

$$\begin{aligned}\sum\mathcal{M}_{\rm a}\mathcal{M}_{\rm c}^*=&\sum\mathcal{M}_{\rm a}^*\mathcal{M}_{\rm c}\\=&\gamma_g{}^2\gamma_s{}^2\pi^2\alpha_s{}^2\\&\times\frac{1}{12}\frac{(m^2-u)(m^2-t)+m^2(t-u)}{s(m^2-u)}\ ,\end{aligned} \tag{3.13}$$

$$\sum \mathcal{M}_b \mathcal{M}_c^* = \sum \mathcal{M}_b^* \mathcal{M}_c = \gamma_g^2 \gamma_s^2 \pi^2 \alpha_s^2 \frac{1}{108} \frac{m^2(s-4m^2)}{(m^2-u)(m^2-t)}, \quad (3.14)$$

$$\sum |\mathcal{M}_d|^2 = \tfrac{1}{2}\gamma_q^2 \gamma_s^2 \pi^2 \alpha_s^2 \times \frac{16}{81} \frac{(m^2-t)^2+(m^2-u)^2+2m^2 s}{s^2}, \quad (3.15)$$

where we have shown the contributions from the various diagrams and their interference terms separately. The sum of (3.9)–(3.14) gives $\sum |\mathcal{M}_{gg\to s\bar{s}}|^2$. Our result agrees with that obtained by Georgi, Glashow, Machacek, and Nanopoulos and by Combridge.[18]

To proceed further one needs to perform phase-space integrals over the initial and final particle momenta p_i. These twelve-dimensional integrals can be reduced to four-dimensional integrals analytically. To do that we first replace

$$\int \frac{d^3p}{2E} = \int d^4p\, \delta(p^2-m^2)\theta(p_0) \quad (3.16)$$

and change variables to

$$\begin{aligned} q &= p_1+p_2\,, \\ p &= \tfrac{1}{2}(p_1-p_2)\,, \\ q' &= p_3+p_4\,, \\ p' &= \tfrac{1}{2}(p_3-p_4)\,. \end{aligned} \quad (3.17)$$

Then one can immediately eliminate the energy-momentum-conserving δ function by carrying out the integral over q', giving for the $gg \leftrightarrow s\bar{s}$ process

$$I_{\text{gluon}} = \frac{1}{2}\frac{1}{(2\pi)^8}\int d^4q \int d^4p \int d^4p'\, \delta\left[\left(\frac{q}{2}+p\right)^2\right]\delta\left[\left(\frac{q}{2}-p\right)^2\right]\delta\left[\left(\frac{q}{2}+p'\right)^2-m^2\right]\delta\left[\left(\frac{q}{2}-p'\right)^2-m^2\right]$$
$$\times\theta\left[\frac{q_0}{2}+p_0\right]\theta\left[\frac{q_0}{2}-p_0\right]\theta\left[\frac{q_0}{2}+p_0'\right]\theta\left[\frac{q_0}{2}-p_0'\right]$$
$$\times\sum|\mathcal{M}_{gg\to s\bar{s}}|^2 f_g\left[\frac{q_0}{2}+p_0\right]f_g\left[\frac{q_0}{2}-p_0\right]f_s\left[\frac{q_0}{2}+p_0'\right]f_{\bar{s}}\left[\frac{q_0}{2}-p_0'\right]. \quad (3.18)$$

We now adopt spherical coordinates with $\mathbf{q}$ defining the z axis, i.e.,

$$q_\mu = (q_0,0,0,q), \quad p_\mu = (p_0, p\sin\theta, 0, p\cos\theta), \quad p'_\mu = (p_0', p'\sin\phi\sin\chi, p'\sin\phi\cos\chi, p'\cos\phi)\,, \quad (3.19)$$

in terms of which

$$I_{\text{gluon}} = \frac{1}{2}\frac{(4\pi)(2\pi)}{(2\pi)^8}\frac{1}{16}\int_0^\infty dq_0 \int_0^\infty dq \int_{-q_0/2}^{q_0/2} dp_0 \int_{-q_0/2}^{q_0/2} dp_0' \int_0^\infty dp \int_0^\infty dp' \int_{-1}^{1} d(\cos\theta)\int_{-1}^{1} d(\cos\phi)\int_0^{2\pi} d\chi$$
$$\times\delta\left[p-\left[p_0^2+\frac{s}{4}\right]^{1/2}\right]\delta\left[p'-\left[p_0'^2-m^2+\frac{s}{4}\right]^{1/2}\right]\delta\left[\cos\theta-\frac{q_0p_0}{qp}\right]\delta\left[\cos\phi-\frac{q_0p_0'}{qp'}\right]$$
$$\times\sum|\mathcal{M}_{gg\to s\bar{s}}|^2 f_g\left[\frac{q_0}{2}+p_0\right]f_g\left[\frac{q_0}{2}-p_0\right]f_s\left[\frac{q_0}{2}+p_0'\right]f_{\bar{s}}\left[\frac{q_0}{2}-p_0'\right] \quad (3.20)$$

with $s=q_0^2-q^2$. The integral over the azimuthal angle χ is elementary, while the integrals over p, p', θ, and ϕ must be performed carefully because of the kinematical constraints

$$q_0 \ge 2m, \quad s=q_0^2-q^2 \ge 4m^2, \quad p_0^2 \le \frac{q^2}{4}, \quad p_0'^2 \le \frac{q^2}{4}\left[1-\frac{4m^2}{s}\right]. \quad (3.21)$$

Upon defining four dimensionless variables u, v, x, and y, related to the original variables of integration by

$$q_0 = -T\ln v + 2m, \quad q=(q_0^2-4m^2)^{1/2}u, \quad p_0 = \frac{q}{2}\left[1-\frac{4m^2}{s}\right]^{1/2} x, \quad p_0' = \frac{q}{2}y\,, \quad (3.22)$$

we arrive at the following expression:

$$I_{\rm gluon}=\frac{\alpha_s^2}{2\pi^3}e^{2\beta m}T\int_0^1 du\int_0^1 dv\int_0^1 dx\int_0^1 dy\frac{u^2}{v^2}\left[1-\frac{4m^2}{s}\right]^{1/2}(q_0^2-4m^2)^{3/2}$$
$$\times f_g\left[\frac{q_0}{2}+p_0\right]f_g\left[\frac{q_0}{2}-p_0\right]f_s\left[\frac{q_0}{2}+p_0'\right]f_{\bar s}\left[\frac{q_0}{2}-p_0'\right]$$
$$\times\left[A+B\left[\frac{1}{K_+}+\frac{1}{K_-}\right]+C\left[\frac{\Delta_+}{K_+^3}+\frac{\Delta_-}{K_-^3}\right]\right], \tag{3.23}$$

where the functions A, B, C, $K_\pm$, $\Delta_\pm$ are defined by

$$A=3\left[1-\left[1-\frac{4m^2}{s}\right]\left[\frac{(1-x^2)(1-y^2)}{2}+x^2y^2\right]\right]-\frac{34}{3}-24\frac{m^2}{s},\quad B=\frac{16}{3}\left[1+\frac{4m^2}{s}+\frac{m^4}{s^2}\right],\quad C=-\frac{128}{3}\frac{m^4}{s^2},$$
$$K_\pm=\left[1-\left[1-\frac{4m^2}{s}\right](1-x^2-y^2)\pm 2\left[1-\frac{4m^2}{s}\right]^{1/2}xy\right]^{1/2},\quad \Delta_\pm=1\pm\left[1-\frac{4m^2}{s}\right]^{1/2}xy\ . \tag{3.24}$$

Similarly for the $q\bar{q}\leftrightarrow s\bar{s}$ process we obtain

$$I_{\rm quark}=\frac{\alpha_s^2}{2\pi^3}e^{2\beta m}T\int_0^1 du\int_0^1 dv\int_0^1 dx\int_0^1 dy\frac{u^2}{v^2}\left[1-\frac{4m^2}{s}\right]^{1/2}(q_0^2-4m^2)^{3/2}$$
$$\times f_q\left[\frac{q_0}{2}+p_0\right]f_{\bar q}\left[\frac{q_0}{2}-p_0\right]f_s\left[\frac{q_0}{2}+p_0'\right]f_{\bar s}\left[\frac{q_0}{2}-p_0'\right]D(u,v,x,y)\ , \tag{3.25}$$

where

$$D=1+\left[1-\frac{4m^2}{s}\right]\left[\frac{(1-x^2)(1-y^2)}{2}+x^2y^2\right]+\frac{4m^2}{s}\ . \tag{3.26}$$

The remaining four-dimensional integrals must be carried out numerically.

IV. APPROACH TO CHEMICAL EQUILIBRIUM IN A HOMOGENEOUS PLASMA

In this section we discuss the relaxation to equilibrium of the strange-quark density in a static (as opposed to an expanding) quark-gluon plasma. A quark-gluon plasma formed in the course of an ultrarelativistic heavy-ion collision would of course undergo a very rapid expansion because of large velocity and pressure gradients at its formation. We defer detailed study of the full dynamical problem to a subsequent paper. We focus here, instead, on the simpler case where all thermodynamic variables depend only on time and are uniform in space.

Since there is no pressure gradient, the system remains homogeneous for all time. Thus one can find a frame where $u^\mu(x)=(1,0,0,0)$ in which (2.14) and (2.15) are reduced to

$$\frac{d\epsilon(t)}{dt}=0 \tag{4.1}$$

and

$$\frac{dn_s(t)}{dt}=(e^{-2\beta\mu}-1)I(\mu,T)\ , \tag{4.2}$$

respectively, with $I\equiv I_{\rm gluon}+I_{\rm quark}$. Thus energy-momentum conservation imposes just constancy of the energy density. Any variations in other thermodynamic variables can be due only to variations in the strange-quark density induced by (4.2).

We first calculate the approximate time scale for relaxation of the strange-quark density to equilibrium. This relaxation time is defined from the behavior near complete equilibrium, i.e., $\mu\approx 0$. Applying the chain rule to (4.2) gives

$$\left[\frac{\partial n_s}{\partial\mu}\right]_\epsilon\frac{d\mu}{dt}=-2\beta\mu I(0,T)\ , \tag{4.3}$$

where on the right-hand side we have kept only the leading term in $\beta\mu\ll 1$. Hence the chemical potential approaches zero exponentially:

$$\mu(t)\propto\exp(-t/\tau_\epsilon)\ ,$$

with the relaxation time at constant ϵ given by

$$\tau_\epsilon=\frac{\left[\frac{\partial n_s}{\partial\mu}\right]_\epsilon}{2\beta I(T)}\ . \tag{4.4}$$

Here all variables are to be evaluated at $\mu=0$.

If the temperature of the system, rather than the energy density, is kept constant, then the isothermal relaxation time τ_T is given by the same expression with $(\partial n_s/\partial\mu)_\epsilon$ replaced by $(\partial n_s/\partial\mu)_T$. The two derivatives are related via

$$\left[\frac{\partial n_s}{\partial\mu}\right]_\epsilon=\left[\frac{\partial n_s}{\partial\mu}\right]_T\left[1-\frac{(\partial\epsilon/\partial n)_T}{(\partial\epsilon/\partial n)_\mu}\right]\ . \tag{4.5}$$

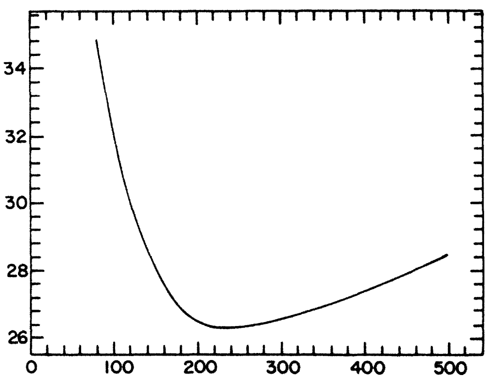

FIG. 2. Dimensionless collision integral $\tilde{I}(\beta m)$, defined at $\mu=0$ [see (4.6)].

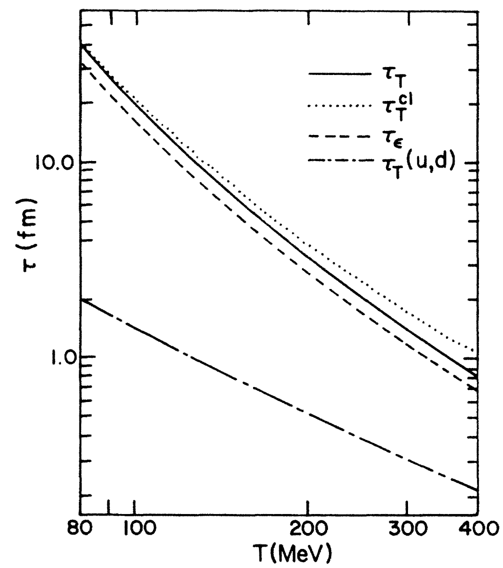

FIG. 3. Relaxation times for the strange-quark density: τ_T for relaxation at constant temperature and τ_ϵ for relaxation at constant energy density, along with the classical approximation τ_T^{cl} to τ_T. Also shown is the relaxation time for the light-quark densities, $\tau_T(u,d)$.

The difference between the two relaxation times should be small since $(\partial\epsilon/\partial n)_\mu$ is dominated by the nonstrange degrees of freedom.

We can write I in terms of a dimensionless integral $\tilde{I}$ [see (3.23) and (3.25)] as

$$I(0,T)=\frac{{\alpha_s}^2}{2\pi^3}e^{-2\beta m}T^4\tilde{I}(\beta m)\ . \tag{4.6}$$

In Fig. 2 we plot $\tilde{I}$ against T, for a strange-quark mass of 150 MeV. (The four-dimensional integrals in the reduced collision integral were evaluated numerically with VEGAS.[19]) The rapid decrease as T is raised past 100 MeV is due to the lowered density: the kinematical threshold effect expressed by (3.21) strongly limits the phase space of the particles available to the process. As the temperature increases, $\tilde{I}$ increases slowly: asymptotically,

$$\tilde{I}=c_1\ln\beta m+c_2+O(\beta m\ln\beta m)\ ,$$

with $c_1\simeq-32.5$ and $c_2\simeq-16.0$. The collision integral is logarithmically divergent in the limit $\beta m\to 0$ because of poles in the matrix element $\mathscr{M}_b$ at $t=m^2$.

The relaxation times τ_T and τ_ϵ are plotted in Fig. 3. Again, we set $m=150$ MeV; to vary m, note that dimensional analysis gives $\tau(\lambda m,\beta)=\lambda\tau(m,\lambda\beta)$. The relaxation time is proportional to ${\alpha_s}^{-2}$, and we have fixed $\alpha_s=0.6$ for convenience. It is evident from Fig. 2 that in the range of temperature $200<T<400$ MeV the reduced integral $\tilde{I}$ varies by less than 4%, so that most of the T dependence of τ comes from the other factors in (4.4)–(4.6). At large temperatures τ is proportional to β divided by a factor of $\ln\beta m$ from $\tilde{I}$.

We also display in Fig. 3 the relaxation time τ_T for light quarks; their density relaxes via the gluon process (2.8) alone, and we use $m_{u,d}=10$ MeV. As expected, the relaxation time for the saturation of the light-quark density is much shorter than that for the strange quarks at low temperatures. This is simply due to the fact that there is no kinematical suppression for the light-quark production and annihilation processes.

Having found the relaxation time for small deviations from equilibrium, we now examine the evolution of the thermodynamic variables from the initial condition $n_s(t=0)=0$. The results of numerical integration of (4.1) and (4.2) are shown in Fig. 4. It is seen that the strange-quark density in the system saturates at $t\simeq 3\tau_T$ as its chemical potential relaxes to zero. The energy needed to make the strange quarks comes from a small drop in the temperature, since the system is constrained to evolve at constant energy density ϵ. The relaxation process generates entropy, but we find that this entropy production is likewise not a very large effect. The reason is that the light species dominate both the energy density and the entropy of the system and hence behave as a large thermal reservoir.

The QCD coupling α_s should in principle be allowed to run with the temperature, $\alpha_s=\alpha_s(T)$. In the case at hand, however, the temperature varies very little: the energy density ϵ is dominated by the gluons and the light quarks, so that constant ϵ implies almost constant T. With α_s taken to be constant, it merely represents a multiplicative factor in the scale of time [see (4.2)]; it drops out entirely when t is measured in units of τ_T, which scales similarly. Thus the curves in Fig. 4 are invariant under changes in α_s.

Our results can be compared to previous work by studying the classical approximation. In this approximation one replaces the Fermi-Dirac and Bose-Einstein distribution functions by the exponential Boltzmann distribution, and eliminates the Pauli blocking and Bose enhancement factors from the final states in (2.20) and (2.21). The rate equation (4.2) becomes

$$\frac{dn_s}{dt}=\left[1-\left[\frac{n_s}{n_s^{\rm eq}}\right]^2\right]I'(T)\ , \tag{4.7}$$

where we have used the classical relation between the s-quark density and the chemical potential to set

$$n_s(\mu)=e^{\beta\mu}n_s(0)\ ,$$

and we have defined $n_s^{\rm eq}\equiv n_s(0)$ to be the equilibrium density. I' is $I(\mu=0)$ with classical distributions used for f_g, f_s, etc.; it can be rewritten as the sum of the two classical expressions:

$$I'_{\rm gluon}=\int\frac{d^3p_1}{(2\pi)^3}\int\frac{d^3p_2}{(2\pi)^3}\sum_{\rm initial}\sigma_{gg\to s\bar{s}}v_{12}e^{-\beta p_1-\beta p_2}\ , \tag{4.8}$$

$$I'_{\rm quark}=\int\frac{d^3p_1}{(2\pi)^3}\int\frac{d^3p_2}{(2\pi)^3}\sum_{\rm initial}\sigma_{q\bar{q}\to s\bar{s}}v_{12}e^{-\beta p_1-\beta p_2}\ , \tag{4.9}$$

which represent the free-space cross sections for the forward process (2.9) and (2.10) folded with Boltzmann distributions for the initial particles. $\sigma_{gg\to s\bar{s}}$ and $\sigma_{q\bar{q}\to s\bar{s}}$ are the spin- and color-averaged total cross sections:

$$\sigma_{gg\to s\bar{s}}=\frac{1}{2\gamma_g{}^2v_{12}2E_1 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\sum|\mathcal{M}_{gg\to s\bar{s}}|^2\ , \tag{4.10}$$

$$\sigma_{q\bar{q}\to s\bar{s}}=\frac{1}{\gamma_q{}^2v_{12}2E_1 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\sum|\mathcal{M}_{q\bar{q}\to s\bar{s}}|^2\ . \tag{4.11}$$

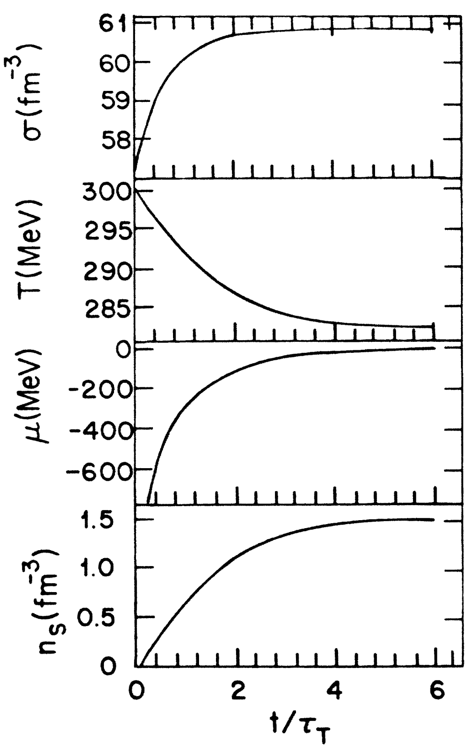

FIG. 4. Relaxation at constant energy density of the flavor distribution in the quark-gluon plasma. Shown are the density n_s of strange quarks, the corresponding chemical potential μ, the temperature T, and the entropy density σ. Initial conditions are $T=300$ MeV, $n_s=0$ ($\mu=-\infty$). Strange-quark pairs are produced until n_s reaches its equilibrium value, as signaled by $\mu\to 0$. The energy needed to make the strange quarks is supplied by a drop in T, and the entropy increases during the evolution.

The classical relation (4.7) was used in Refs. 12 and 13 to extrapolate the reaction rate at $n_s=0$ to nonzero n_s.

For fixed temperature, the solution to (4.7) is

$$n_s(t)=n_s^{\rm eq}\tanh\left[\frac{t}{2\tau_T^{\rm cl}}+{\rm const}\right] \tag{4.12}$$

with the classical relaxation time given by

$$\tau_T^{\rm cl}=\frac{n_s^{\rm eq}}{2I'}\ . \tag{4.13}$$

In Fig. 3 we compare the numerical result for $\tau_T^{\rm cl}$ to the quantum result. It is seen that the classical approximation is fairly good at low temperatures ($T\lesssim m$) and the difference grows as the temperature increases. This may seem peculiar at first glance since in the nonrelativistic case the classical approximation usually works better at higher temperature. The reason is that at low temperatures the mass thresholds for the strange-quark production and annihilation processes prohibit the participation of light quarks and gluons in the regions of phase space where the classical and quantum distributions differ significantly. As the temperature is raised, the distribution functions, which depend on p/T, begin to differ even in the allowed kinematical regime. We found that the classical calculation overestimates the relaxation time by about 20% at $T\approx 2m$. This is simply because the Boltzmann distribution function underestimates the Bose-Einstein distribution function for gluons.

It is amusing to note that (4.12) nevertheless describes the evolution of n_s shown in Fig. 4 extremely well, as long as one uses the values of $n_s^{\rm eq}$ and τ_T given by the full quantum theory.

Note added. The discrepancy between our classical result (4.13) and the result of Ref. 13 has been explained. First, note that the relaxation time plotted in Fig. 2 in the paper of Rafelski and Müller[13] is defined as *twice* our relaxation time [cf. their Eq. (9b) and our Eq. (4.12)]. The value of τ (300 MeV) plotted in their figure disagrees with the value of τ_T^{cl} plotted in our Fig. 3 by an apparent factor of $\sim\frac{5}{4}$, which is thus actually a factor of 2.5 when the differing definitions of τ are taken into account. Upon receipt of our result, the authors of Ref. 13 discovered an error in their calculation, namely, that they omitted the factor of $\frac{1}{2}$ appearing in our Eq. (2.20a) for the rate of the gluon process [cf. their Eq. (3); this factor was also omitted in Ref. 12]. Since the gluon process dominates the overall reaction rate, this accounts for almost a factor of 2 in the discrepancy.[20] The remainder of the disagreement is accounted for[21] by the fact that the authors of Ref. 1 used quantum distribution functions in evaluating the equations corresponding to our Eqs. (4.10) and (4.11).

ACKNOWLEDGMENTS

We thank S. Kahana and the Nuclear Theory Group at Brookhaven National Laboratory for their hospitality, and B. Müller, K. Kajantie, and J. Rafelski for timely correspondence. This work was supported by the U.S. Department of Energy under Grant No. DE-ACO2-76ERO3069.

APPENDIX

We here present a derivation of the hydrodynamic equations (2.14)–(2.17) from a semiclassical transport equation which is a relativistically invariant form[22] of the Uehling-Uhlenbeck equation.

The semiclassical transport equation[23] for the single-particle distribution function for particles of type (i.e., spin, color, flavor, ...) a is

$$\left[\frac{\partial}{\partial t}-\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right]f_a(\mathbf{p},\mathbf{x})=\sum_{\text{processes}}{}' C(\mathbf{p},\mathbf{x}) , \tag{A1}$$

where $\mathbf{v}=\mathbf{p}/E$ is the velocity with $E^2=\mathbf{p}^2+m_a{}^2$. The right-hand side is a sum of collision integrals for processes which contain at least one particle of type a in the initial or final state. A term corresponding to a particular n-body process is given by an integral over $(n-1)$-particle phase space:

$$C(\mathbf{p}_i,\mathbf{x}_i)=(\pm)\frac{1}{2E_iS_i}\int\prod{}' d\Gamma\,|\mathcal{M}|^2(2\pi)^4 \times\delta^4(P_{\text{in}}-P_{\text{out}})F(\{f(j)\}) , \tag{A2}$$

where the sign in front is $-$ $(+)$ for the case where particle i is incoming (outgoing) in the process, and the distribution factor F is

$$F(\{f(j)\})=\prod_{\text{in}} f(j)\prod_{\text{out}}[1\pm f(j)] - \prod_{\text{in}}[1\pm f(j)]\prod_{\text{out}} f(j) . \tag{A3}$$

We have used a shorthand notation for the $(n-1)$-particle invariant phase-space element,

$$\prod{}' d\Gamma=\prod_{j\neq i} d^3p_j/[(2\pi)^3(2E_j)] ,$$

and for the distribution function of the ith particle, $f(i)=f_{a_i}(p_i,x_i)$. P_{in} and P_{out} are the sums of the four momenta of the incoming and the outgoing particles, respectively. The statistical factor S_i is given by $S_i=\prod_a' m_a^{\text{in}}!m_a^{\text{out}}!$ when there are $m_a^{\text{in,out}}$ identical particles of species a in the initial or in the final state, *excluding the ith particle.*

We can write the transport equation in a manifestly covariant form by multiplying both sides by E:

$$p^\mu\partial_\mu f=\sum I , \tag{A4}$$

where the Lorentz-invariant collision integrals are given by $I=EC$.

In order to derive (2.14), we multiply (A1) by p^ν, integrate over $\mathbf{p}$, and take a sum over all particle species a, yielding on the left-hand side $\partial_\mu T^{\mu\nu}(x)$, where

$$T^{\mu\nu}(x)\equiv\sum_a\int\frac{d^3p}{(2\pi)^3E}p^\nu p^\mu f_a(\mathbf{p},\mathbf{x}) \tag{A5}$$

is the energy-momentum tensor of noninteracting particles. The right-hand side becomes a sum over *all* processes:

$$\sum_{a_i}\sum_{\text{processes}}{}'\frac{\pm1}{2S_i}\int\prod d\Gamma\,|\mathcal{M}|^2(2\pi)^4\delta^4(P_{\text{in}}-P_{\text{out}}) \times p_i^\nu F(\{f(j)\}) , \tag{A6}$$

where the integrals for n-body collision terms are to be performed over n-particle phase space:

$$\prod d\Gamma=\prod_{j=1,\ldots,n} d^3p_j/[(2\pi)^3(2E_j)] .$$

To see that it vanishes it is sufficient to note that we can modify

$$\begin{aligned}\sum_{a_i(i\in\text{in})}\frac{p_i^\nu}{S_i}&\to\frac{1}{S}\sum p_i^\nu=\frac{P_{\text{in}}^\nu}{S} ,\\ \sum_{a_i(i\in\text{out})}\frac{p_i^\nu}{S_i}&\to\frac{1}{S}\sum p_i^\nu=\frac{P_{\text{out}}^\nu}{S} ,\end{aligned} \tag{A7}$$

in (A6), where

$$S=\prod_a m_a^{\text{in}}!m_a^{\text{out}}!$$

is the statistical factor counted without distinguishing any particular particle. It is important to note that (2.14) is just the consequence of energy-momentum conservation and does not depend on the particular form of the single-particle distribution functions.

When the distribution functions of all species take the form of (2.5) with the same temperature and the same flow velocity, the following relation holds by virtue of the identity (2.22):

$$F(\{f(j)\})=\exp(\beta_\mu P^\mu_{\text{in}}) \times \left[\exp\left[-\sum_{\text{out}}\beta\mu_i\right] -\exp\left[-\sum_{\text{in}}\beta\mu_i\right]\right]\prod_{i=1}^{n} f(i)\,, \tag{A8}$$

where μ_i is the chemical potential of the ith particle. Hence we see that the detailed balance condition for the many-body collision process is given by

$$\sum_{i=1}^{m}\mu_i=\sum_{i=m+1}^{n}\mu_i\,. \tag{A9}$$

It follows immediately that detailed balance with respect to the processes (2.3) and (2.4) is attained when $\mu_g=0$ as we noted in Sec. II.

The rate equation (2.15) is derived by integrating (A1) over **p**, giving $\partial_\mu n^\mu$ on the left-hand side. The right-hand side becomes the collision integral, which is nonvanishing when (A9) is not satisfied, as in the case of strange-quark production and annihilation. In calculating the right-hand side of (2.15) via (2.23) and (2.24), we retained only the four-body collision terms corresponding to (2.9) and (2.10) since they are the lowest-order QCD processes.

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