Simple model for nuclear stopping power

James B. Bowlin and Alfred S. Goldhaber
Department of Physics and Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794-3800
(Received 14 March 1986)

A model is developed for the leading-baryon velocity distribution in the reaction $p+A\to p+X$. The model is compared to an existing theoretical model and to experimental data for five different target nuclei with incident-proton momentum of 100 GeV/c.

INTRODUCTION

The subject of nuclear stopping power is of great current interest as it pertains directly to the question of what types of quark-gluon plasma might be created in high-energy nucleus-nucleus collisions. In contrast with the case of a charged particle traversing an ordinary material medium, we are unable to make predictions of nuclear stopping power from first principles, and instead must use crude models based on experiment. The best known data so far come from the experimental work of Barton *et al.*[1] Several theoretical papers have already analyzed the data of Barton *et al.*, and some important points have come to light. Wong[2] showed the importance of realistic nuclear densities and proper normalizations in stopping-power models. Daté, Gyulassy, and Sumiyoshi[3] have stressed the importance of separating geometrical effects from dynamical effects. We shall use the same nuclear geometry as they, so that a direct comparison of our dynamical models is possible.

The major difference between this work and previous papers is in the treatment of successive collisions. We take concepts that have been used as input in some hydrodynamic calculations[4] and apply them to the stopping-power problem to test the validity of the assumptions used in an area where experimental data exist. In our model, as collisions occur the mass of the projectile increases, thus making successive collisions less and less effective in stopping the projectile. Furthermore, greater momentum loss in earlier collisions implies greater resistance to loss in later collisions. Previous models have assumed that momentum loss in each collision is dependent only on the number of previous collisions and is independent of the momentum lost in those collisions. The effects of these assumptions will be illustrated below.

THE MODEL

For the case of $p+p\to p+X$ we assume that there is a conservative collision between the two protons that happens on a time scale that is much smaller than the time needed to create on-shell secondary particles. Both protons emerge from the collision in excited states and eventually evaporate particles, leaving at least two baryons in their ground state. This process can be written schematically as $p+p\to p^*+p^*\to(p+X)+(p+X)$. By assuming strict conservation of energy and momentum in the collision and by assuming that the secondaries boil off isotropically with respect to each excited proton we have, in the laboratory frame,

$$m+(P_0{}^2+m^2)^{1/2}=(m_1{}^2+\Delta P^2)^{1/2}+[m_1{}^2+(P_0-\Delta P)^2]^{1/2} \qquad (1)$$

and

$$P_{\rm final}=\left[\frac{m}{m_1}\right](P_0-\Delta P)\ , \qquad (2)$$

where P_0 is the initial momentum of the projectile proton, m is the proton mass, m_1 is the mass of the excited proton, ΔP is the momentum exchanged in the collision, and $P_{\rm final}$ is the momentum of the proton that is detected. Since the final longitudinal momenta are 2 orders of magnitude greater than the transverse momenta, we solve the one-dimensional problem and let any contribution to the energy due to the transverse degrees of freedom enter through increases of the excited masses. From the experimental $P_{\rm final}$ distribution we can use the above equations to find the distribution of ΔP and hence m_1. This procedure is obviously self-consistent and reproduces the experimental $P_{\rm final}$ distribution.

In order to describe a collision between a proton and a nucleus we need to model the process of a series of n rapid nucleon-nucleon collisions creating excited nucleons, followed by a period of deexcitation and particle evaporation. Once again we assume the collisions are strictly conservative and that all of the collisions take place before the deexcitation begins. In the rest frame of the *projectile* we have for each of the n collisions

$$m_{n-1}+(m^2+P_n{}^2)^{1/2}=(m_n{}^2+\Delta P^2)^{1/2}+[\tilde{m}^2+(P_n-\Delta P)^2]^{1/2}\ , \qquad (3)$$

where m_{n-1} is the mass of the projectile before the nth collision, m_n is the mass of the projectile after the nth collision, P_n is the momentum of the nth target nucleon in the rest frame of the projectile, $\tilde{m}$ is the mass of the nth target nucleon after collision with the projectile, and ΔP is the three-momentum transferred. Since there are three unknown quantities in this equation, m_n, $\tilde{m}$, and ΔP, we need two more assumptions in order to solve it. First, we assume that the t distribution (t is the squared four-momentum transfer) is the same as in a collision of two particles of equal mass m_{sy} (not necessarily the proton mass), with the same s (squared total energy in the center-of-momentum frame) and y_0 (rapidity difference of incoming particles). The t distribution in this equivalent symmetric collision is found from the p-p data by scaling all of the energies and momenta in Eqs. (1) and (2) by a factor of m_{sy}/m where m is the proton mass and

$$m_{sy}{}^2 = \frac{s}{2(1+\cosh y_0)} \tag{4}$$

is the mass needed to have a symmetric collision with the same s and y_0 as in Eq. (3). Our second assumption is that there is no net energy transfer during the collision in the center-of-momentum frame. These two assumptions are of course somewhat arbitrary. For example, we could have taken the t distribution from a proton-proton collision with the same s and demanded that the energy transfer be zero in the center-of-rapidity frame. If we had assumed that the change in rapidity in each collision had the same distribution as in a proton-proton collision with the same y_0 our model would reproduce the results of the incoherent cascade model.[2] We have looked at six different variations of these assumptions and although they all give the same qualitative results and all fit the data of Barton *et al.* reasonably well, the quantitative results vary somewhat. We chose our particular set of assumptions because we found them most aesthetically pleasing and because they fit that data of Barton *et al.* at least as well as the other choices we explored. These assumptions are unnecessary in hydrodynamic models since they deal with average values and not distributions.

APPLYING THE MODEL

To apply this model to $p+A \to p+X$ reactions we must first find the probability distribution $Q_n(\Delta y)$ which gives the probability of the projectile losing rapidity Δy after n collisions. This was done up to $n=12$ using a Monte Carlo procedure. For each collision we picked a t at random from the distribution determined from the p-p data. By keeping track of the projectile mass and momentum one obtains the final proton momentum from

$$P_{\text{final}} = \frac{m}{m_n}\left[P_0 - \sum_i \Delta P_i\right]\Bigg|_{\text{lab}} . \tag{5}$$

In actually performing the Monte Carlo calculation we used the equivalent but simpler calculation

$$\Delta y_{\text{final}} = \sum_i \Delta y_i . \tag{6}$$

Thus we have a prescription for uniquely determining $Q_n(\Delta y)$ using the experimentally measured $Q_1(\Delta y)$ distribution as input. For simplicity we approximate this distribution as

$$Q_1(\Delta y) \sim e^{-\Delta y} , \tag{7}$$

which is known to agree quite well with the existing p-p data.[5]

In order to avoid ambiguities associated with target recoil nucleons we restrict the range of t for each collision so that the maximum allowed magnitude of the t corresponds to the projectile and target nucleons both coming to rest in their center-of-momentum frame (this does not violate energy conservation since we allow both masses to increase during the collision). In the case of a p-p collision this procedure results in zero probability of the leading baryon being slower than the center of momentum of the combined p-p system, and thus the $Q_1(\Delta y)$ distribution is identically zero for $y_{\text{beam}}/2 < \Delta y < y_{\text{beam}}$. For 100-GeV/$c$ incident protons this will cause a 7% change in the normalization in order to maintain a good fit to the p-p data. Since the change in normalization increases with decreasing incident-proton energy, the model may or may not be applicable to cases with significantly lower initial proton energies.

Figure 1 compares the results of our calculation with those of Daté, Gyulassy, and Sumiyoshi for n equal to 2, 4, 6, and 8 collisions. As the number of collisions increases, a rather dramatic qualitative difference in the models can be seen. Our distributions become more peaked with increasing n while their distributions become more spread out. This difference is mainly attributable to the "feedback" mechanism built into our model. Although the cutoff of Δy at the center-of-momentum rapidity in each collision certainly contributes to the peaking, we believe it is a very small effect, since adding the same cutoff to the model of Daté, Gyulassy, and Sumiyoshi does not cause peaking. After several collisions projectiles with low Δy's have had relatively small increases in mass and are more susceptible to increases in Δy in the next collision, while projectiles with higher Δy's have had larger increases in mass and are less susceptible to further increases in Δy. If there were some way to experimentally determine the number of nucleon-nucleon collisions in a given proton-nucleus collision then these two different dynamical models could be compared directly with experiment. Otherwise, the geometry of the nucleus must be considered, and an average over impact parameters must be performed before a direct comparison with experiment can be made.

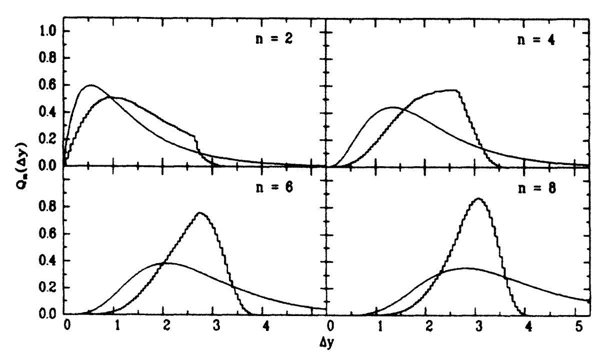

FIG. 1. Q_n is plotted as a function of Δy for $n=2$, 4, 6, and 8 collisions. $\Delta y = 0$ corresponds to the beam velocity. $\Delta y = 5.3$ corresponds to the rest frame of the target. The histograms are from our model and the smooth curves are from Daté, Gyulassy, and Sumiyoshi. All curves are normalized such that $\int Q_n d\Delta y = 1$.

GEOMETRICAL EFFECTS

The forward distribution of baryons in rapidity $y \equiv y_{\text{beam}} - \Delta y$ for a target nucleus A is taken to be

$$\frac{dN}{dy} = \sum P_A(n) Q_n(\Delta y) \ , \tag{8}$$

where $P_A(n)$ is the Glauber probability that the incident proton interacts with n nucleons in nucleus A, and $Q_n(\Delta y)$ is the rapidity distribution of the projectile after n collisions as calculated above. We have chosen the following normalization conventions:

$$\int \left[\frac{dN}{dy} \right] dy = 1 \ , \tag{9}$$

$$\int Q_n(\Delta y) dy = 1 \ , \tag{10}$$

$$\sum_n P_A(n) = 1 \ . \tag{11}$$

Our computation of the Glauber probabilities $P_A(n)$ is based on the same Woods-Saxon nuclear density distribution as used by Daté, Gyulassy, and Sumiyoshi:

$$\rho(r) = \frac{\rho_0}{1 + \exp[(r - R_0)/d]} \ , \tag{12}$$

where

$$R_0 = (1.19 A^{1/3} - 1.61 A^{-1/3}) \text{ fm} \ , \tag{13}$$

$$d = 0.54 \text{ fm} \ , \tag{14}$$

and ρ_0 is determined from the normalization

$$A = \int d^3 r \, \rho(r) \ . \tag{15}$$

We confirm that these densities in conjunction with a proton-nucleon inelastic cross section (σ_{inel}) of 32 mb reproduce within stated experimental errors the measured inelastic cross sections[6] for the five target nuclei which concern us here. However the σ_{inel} used above includes an inelastic-diffractive contribution, $\sigma_{\text{inel-diff}}$, of approximately 4 mb (Ref. 7), which corresponds to one of the incoming particles going to a low-lying excited state while the other incoming particle remains unexcited. One-half of $\sigma_{\text{inel-diff}}$ corresponds to the projectile becoming excited, and this half contributes to both the data of Barton *et al.* and the energy-loss mechanism we are modeling. The half of $\sigma_{\text{inel-diff}}$ that corresponds to target excitation results in negligible energy and momentum loss of the projectile, and was not within the rapidity range of the data of Barton *et al.* Likewise, our model only deals with collisions that result in projectile excitation, so we treat one-half of $\sigma_{\text{inel-diff}}$ as effectively elastic and define an effective nucleon-nucleon cross section σ_{eff} such that

$$\sigma_{\text{eff}} = \sigma_{\text{inel}} - \tfrac{1}{2}\sigma_{\text{inel-diff}} = 30 \pm 1 \text{ mb} \ , \tag{16}$$

and compute the Glauber probabilities from

$$P_A(n) = \frac{1}{\sigma_{\text{eff}}^A} \int d^2 b \begin{bmatrix} A \\ n \end{bmatrix} [N_A(b)/A]^n [1 - N_A(b)/A]^{A-n} \ , \tag{17}$$

$$N_A(b) = \sigma_{\text{eff}} \int dz \, \rho(z,b) \ , \tag{18}$$

$$\sigma_{\text{eff}}^A = \int d^2 b \{ 1 - [1 - N_A(b)/A]^A \} \ . \tag{19}$$

COMPARISON TO EXPERIMENT

The results of Barton *et al.* give the inclusive cross section, $E \, d^3\sigma^A/dp^3$ for protons emitted in the reaction $p + A \to p + X$, with incident-proton momentum fixed at 100 GeV/c and final transverse momentum fixed at 0.3 GeV/c. The speed of the protons can be measured by the variable x:

$$x = \frac{(E + P_z)_{\text{final}}}{(E + P_z)_{\text{initial}}} \approx \left. \frac{P_{\text{final}}}{P_{\text{initial}}} \right|_{\text{lab}} \ . \tag{20}$$

The target A ranged from H to Pb. Our model as well as the work mentioned above predicts inclusive leading-baryon distributions, which are not necessarily what were measured by Barton *et al.* In order to relate our predictions to the experimental data several assumptions are necessary. These assumptions are stated precisely in Ref. 5. In particular, we assume that the leading-proton distribution can be factorized as follows:

$$\frac{E \, d^3\sigma^A}{dp^3} = \sigma_{\text{eff}}^A r_p g(p_\perp) \frac{dN}{dy}(A,y) \ , \tag{21}$$

where r_p is the ratio of protons to baryons, $g(p_\perp)$ is the transverse-momentum distribution, and σ_{eff}^A is the effective inelastic cross section as defined above. In words, we assume that dN/dy is a function of A and Δy but r_p and $g(p_\perp)$ are not. In order to compare our model with experiment we transform the data of Barton *et al.* from $E \, d^3\sigma/dp^3$ vs x to dN/dy vs y by using the relations[5]

$$\frac{dN}{dy}(A,y) = \alpha \frac{E \, d^3\sigma^A(y)/dp^3}{\langle x^{-1} E \, d^3\sigma^{pp}(y)/dp^3 \rangle} \frac{\sigma_{\text{eff}}^{pp}}{\sigma_{\text{eff}}^A} \ , \tag{22}$$

$$\Delta y = -\ln(x) \ , \tag{23}$$

where α is defined as

$$\alpha = 1 + \exp(-y_{\text{beam}}/2) = 1.07 \tag{24}$$

to maintain proper normalization of the p-p data. Figure 2 shows the results of our model along with the data of Barton *et al.* and the results from Daté, Gyulassy, and Sumiyoshi. They used a normalization for the experimental data which is slightly different from ours, and we adjusted the normalization of their results to keep the same ratio of theory to experiment as in their paper. Their choice of σ_{eff} was 32 mb, the full inelastic cross section. Changing to 30 mb might improve the agreement of their model with experiment.

Considering the approximations involved, both models agree quite well with the existing experimental data for all of the targets, and neither model is clearly superior. The models have a rather dramatic disagreement just beyond the range of the experimental data, especially for the heavier targets. The difference is most striking for the lead target. Daté, Gyulassy, and Sumiyoshi predict a decrease of dN/dy for increasing Δy in this region, while our model predicts an increase. This qualitative differ-

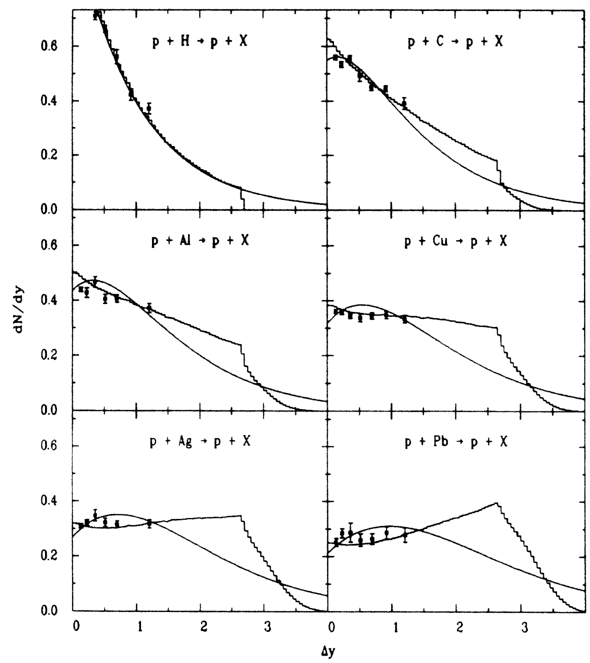

FIG. 2. dN/dy is plotted as a function of Δy for the six different targets of Barton *et al.* The hydrogen data are included for reference purposes only, since they were used as *input* for the models. The histograms are the results of our model, the smooth curve is from Daté, Gyulassy, and Sumiyoshi, and the data points are from Barton *et al.* with error bars indicating the stated experimental errors. See the text for an explanation of the normalizations.

ence is attributable mainly to the fact that our model, unlike theirs, implies a "feedback" mechanism which tends to make the net rapidity loss after many collisions approach a central value, hence the peaking shown in the figures. It may be possible to explore a larger region of rapidity experimentally with existing detection techniques by increasing the beam energy roughly by a factor of 10. It would also be quite interesting to see if another experiment reproduces the apparent structure in the experimental results in the region of $0.2 \leq \Delta y \leq 0.4$, a structure which neither model predicts.

We feel that the agreement between our model and experiment indicates that the assumptions used in the earlier and current hydrodynamic[4] models are reasonable. We hasten to note that mass increases predicted in our model are not to be taken literally. They are merely used as a bookkeeping technique to keep track of energy and momentum conservation, and should not be used lightly to make predictions of other features of these collisions. We do hope to make a better model of $p+A$ collisions that will predict more than just leading-baryon distributions. However, we feel that such a model must be based on more realistic and complicated dynamics than used here.

ACKNOWLEDGMENTS

We are grateful for conversations with W. Busza, M. Gyulassy, T. Ludlam, and D. Strottman. We also want to thank Peter M. Koch, who provided the computer facilities used during the initial phase of this project. This work was supported in part by the Department of Energy and the National Science Foundation.

APPENDIX

We have recently become aware of the experimental work of Bailey *et al.*[8] which describes measurements of inclusive proton distributions in the reaction $p+A \rightarrow p+X$, with incident proton energy of 120 GeV and A ranging from Be to U. They measured protons in the rapidity range of $0.5 \leq \Delta y \leq 2.23$. Since our model predicts leading-baryon distributions and they measured inclusive leading-*proton* cross sections, once again we need to make assumptions in order to make a comparison. We note that there is excellent agreement of our model with the data of Barton *et al.* extrapolated smoothly to $\Delta y = 0$. As mentioned by Daté, Gyulassy, and Sumiyoshi this is typical of most stopping-power models where $Q_n(0)=0$ for $n>1$, since dN/dy goes to $Q_1(\Delta y)P_A(1)$ as Δy goes to zero. Therefore we normalize the data of Bailey *et al.* by demanding that a smooth extrapolation of the data down to $\Delta y = 0$ agrees with our model predictions. The results of this procedure are shown in Fig. 3. If our assumption that r_p is independent of A and Δy is correct, then our normalization factors should all be identical and equal to r_p. Actually the normalization factor increases by about 50% from Be to U, and we conclude that if the Barton

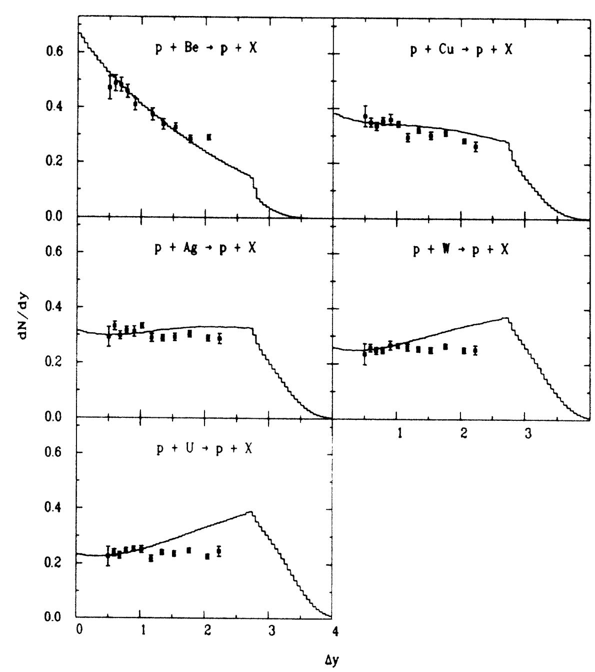

FIG. 3. dN/dy is plotted as a function of Δy for the five targets of Bailey *et al.* $\Delta y = 0$ corresponds to the beam velocity. $\Delta y = 5.48$ corresponds to the rest frame of the target. The histograms are the results of our model, the data points are from Bailey *et al.* with error bars indicating the stated experimental errors. See the Appendix for an explanation of the normalizations.

and Bailey experiments are consistent then there must be some error in our assumptions.

It is clear that the data do not show any signs of the upward slope predicted by our model, nor the downward slope predicted by Daté, Gyulassy, and Sumiyoshi, in the region $\Delta y > 1$. Note that an r_p decreasing for large values of Δy would imply a larger number of baryons in this region and a more upward slope in the total baryon distributions when compared with the measured proton distributions. Precise knowledge of r_p in this region may not be available until an experiment is performed that measures neutrons as well as protons.

[1]A. Barton *et al.*, Phys. Rev. D **27**, 2580 (1980).
[2]C. Y. Wong, Phys. Rev. D **30**, 972 (1984).
[3]S. Daté, M. Gyulassy, and H. Sumiyoshi, Phys. Rev. D **32**, 619 (1985).
[4]A. A. Amsden *et al.*, Phys. Rev. C **17**, 2080 (1978); A. S. Goldhaber and D. Strottman (unpublished).
[5]W. Busza and A. S. Goldhaber, Phys. Lett. **139B**, 235 (1984).
[6]A. S. Carroll *et al.*, Phys. Lett. **80B**, 319 (1979).
[7]R. D. Schamberger, Jr. *et al.*, Phys. Rev. D **17**, 1268 (1978).
[8]R. Bailey *et al.*, Z. Phys. C **29**, 1 (1985).