Preregularization and the structure of loop-momentum ambiguities within quantum corrections to the supercurrent

A. M. Chowdhury, V. Elias, and D. G. C. McKeon
Department of Applied Mathematics, Univeristy of Western Ontario, London, Ontario, Canada N6A 5B9

R. B. Mann
Department of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7
(Received 21 January 1986)

The anomaly in the supercurrent amplitude $S_{\mu\nu}$ is analyzed to one-loop order for $N=1$ supersymmetry in the Wess-Zumino gauge within the framework of preregularization, in which loop-momentum-routing ambiguities percolate into shift-of-integration-variable surface terms peculiar to exactly four space-time dimensions. We find the supercurrent anomaly to be a consequence of the inability of such ambiguities (within a demonstrably finite set of quantum corrections) to absorb violations of gauge invariance ($q^{\nu}S_{\mu\nu}\neq 0$) and supersymmetry ($\partial^{\mu}S_{\mu\nu}\equiv\partial\cdot S\neq 0$) simultaneously, a feature quite similar to the inability of VVA-triangle ambiguities to absorb violations of gauge invariance and the axial-vector-current Ward identity simultaneously. We also find that if gauge invariance is preserved, the contribution to the supercurrent anomaly obtained from $O(g^2)$ quantum corrections to the supercurrent involves no infrared or ultraviolet infinities and resides in $\partial\cdot S$ rather than $\gamma\cdot S$. This last result is a consequence of maintaining exactly four space-time dimensions, as is necessary for momentum-routing ambiguities to appear at all in the quantum corrections. The connection between our results and similar results obtained from an Adler-Rosenberg symmetry argument is examined in detail.

Loop-momentum-routing ambiguities, which percolate into shift-of-integration-variable surface terms in 4 (but not $4-\epsilon$) dimensions,[1-3] may be used to uphold Ward identities (i.e., "preregularize") in non-dimensionally-continued perturbative calculations.[4] Anomalies manifest themselves in perturbation theory when such ambiguities prove insufficiently general to enforce the full set of Lagrangian symmetries. A textbook example is provided by the chiral anomaly, associated with the current divergences at vertices of the VVA-triangle graph and cross-graph.

Contractions of vertex momenta into these graphs yield finite shift-of-integration-variable surface terms from which the anomalous component of the axial-vector current may be obtained.[5] In his pedagogical review article of 1970, Adler avoids explicit parametrization of ultraviolet infinities while showing how an anomalous contribution to the axial-vector-current Ward identity would vanish (using standard four-dimensional Dirac algebra) if naive shifts of the integration variable were permitted in four-dimensional Feynman integrals.[6] However, the retention of shift-of-integration-variable surface terms in Adler's analysis does not automatically lead to the usual chiral anomaly. Rather, one finds that ambiguities in how one chooses to label the internal loop momenta percolate into vector and axial-vector current divergences. These ambiguities have been examined in an earlier paper,[7] in which the following results were obtained.

(1) Vector and axial-vector current divergences (i.e., $\partial\cdot V,\partial\cdot A$) do not require explicit parametrization of ultraviolet or infrared infinities. Rather, one-loop quantum corrections to $\partial\cdot V$ and $\partial\cdot A$ are manifestly finite, though ambiguous as a result of surface terms sensitive to the routing of internal momenta.

(2) VVA loop-momentum ambiguities in exactly four space-time dimensions are resolved fully through the imposition of gauge invariance (i.e., $\partial\cdot V=0$). The resulting expression for $\partial\cdot A$ then yields the usual axial-vector-current anomaly.[8]

(3) VVA loop-momentum-routing ambiguities in exactly four dimensions correspond fully to the ambiguities in dimensional regularization that 't Hooft and Veltman observed to be associated with the arbitrary choice within VVA γ-matrix traces for the initial location of the non-fully-anticommuting γ_5 matrix (appropriate for a Dirac-matrix algebra continued to more than four dimensions).[9]

(4) VVA loop-momentum-routing ambiguities in exactly four dimensions correspond fully to ambiguities in dimensional reduction associated with arbitrariness in choosing which triangle-graph vertex should initiate VVA γ-matrix traces, as trace cyclicity is no longer automatic when less-than-four-dimensional loop momenta are projected onto four-dimensional Dirac matrices.[10]

In supersymmetry, the supermultiplet structure of anomalies[11] would suggest a similar set of properties to those enumerated above for quantum corrections to the supercurrent. In particular, the old problem of regularization-procedure dependence of anomalous quantum corrections to either[10,12-14] $\gamma\cdot S$ or[15-17] $\partial\cdot S$ might be expected to correspond on a more fundamental level to a loop-momentum-routing ambiguity within manifestly finite corrections to the fermionic supercurrent.

To explore further the correspondence between chiral-anomaly and supercurrent-anomaly momentum-routing

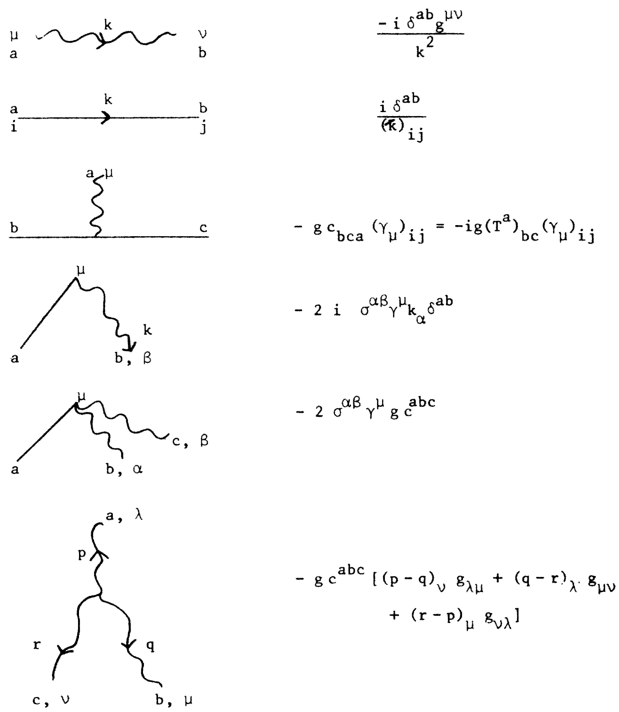

FIG. 1. Feynman rules.

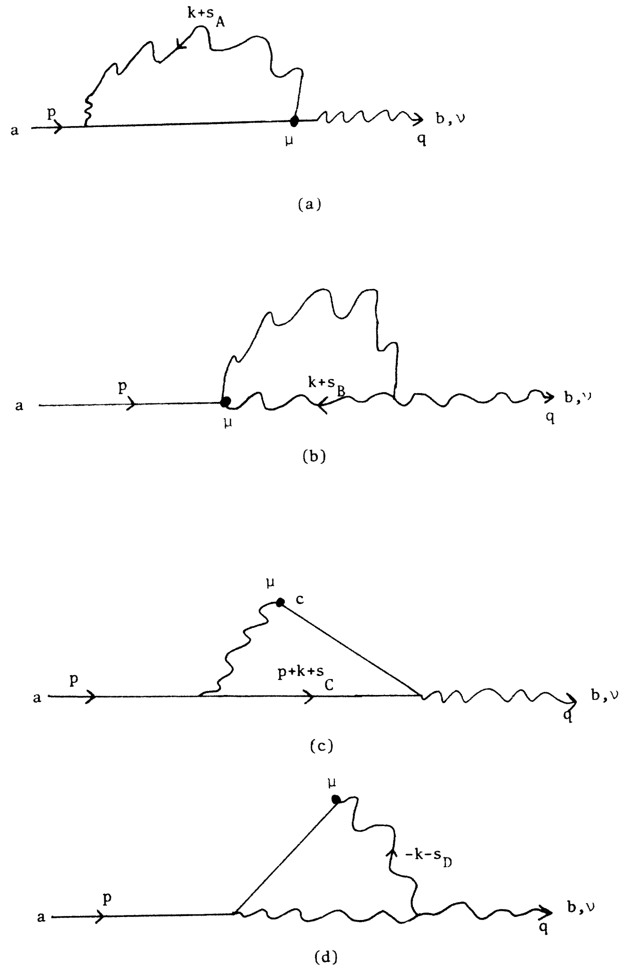

FIG. 2. One-loop diagrams.

ambiguities, we examine below how such ambiguities manifest themselves in quantum corrections to the fermionic supercurrent S^μ, evaluated to one-loop order for $N=1$ supersymmetry in the Wess-Zumino gauge. Appropriate Feynman rules[14] are listed in Fig. 1. We begin by considering the supercurrent amplitude $\epsilon_b^\nu(q)S_{\mu\nu}^{ab}(p,q)u_a(p)$ with p and q on mass shell. Thus all factors of $\not p$ within $S_{\mu\nu}^{ab}$ adjacent to $u_a(p)$ vanish, as well as factors of p^2, q^2, or q_ν within $S_{\mu\nu}^{ab}(p,q)$. The lowest-order contribution to the amplitude is mediated directly by the two-point supercurrent vertex appearing in Fig. 1:

$$(S_{\mu\nu}^{ab})^{\text{tree}} = -2i\sigma_{\beta\nu}\gamma_\mu q^\beta \delta^{ab} \,. \tag{1}$$

This contribution trivially satisfies gauge invariance ($q^\nu S_{\mu\nu}=0$), the spin-$\frac{3}{2}$ constraint $\gamma^\mu S_{\mu\nu}=0$, and on-shell supercurrent conservation $[(p-q)^\mu S_{\mu\nu}=0]$. The supercurrent anomaly manifests itself in violations of one of these relations when one-loop contributions to $S_{\mu\nu}^{ab}$, denoted henceforth by $\Sigma_{\mu\nu}^{ab}(p,q)$, are considered (Fig. 2).

The diagrams of Fig. 2 are subject to loop-momentum-routing ambiguities because they are more than logarithmically divergent. The contribution of each diagram to $\Sigma_{\mu\nu}^{ab}$, evaluated in *exactly* four dimensions,[3] will depend on how one chooses to identify internal-loop momenta in each diagram. We have chosen to parametrize this ambiguity by introducing arbitrary additive contributions s_A, s_B, s_C, s_D to the internal loop momenta of Figs. 2(a)–2(d), respectively, consistent with momentum conservation at each vertex. Each such s is an arbitrary linear combination of the external fermion and gauge-boson momenta p and q:

$$s_X^\alpha = m_X p^\alpha + n_X q^\alpha \;(X=\{A,B,C,D\}) \,. \tag{2}$$

Respective contributions of Figs. 2(a), 2(b), 2(c), and 2(d) to $\Sigma_{\mu\nu}^{ab}(p,q)$ are then given correspondingly by the expressions

$$\Sigma^A = N\int d^4k[(p+k+s_A)^{-2}(k+s_A)^{-2}]\sigma_{\rho\nu}\gamma_\mu(\not p+\not k+\not s_A)\gamma^\rho \,, \tag{3}$$

$$\Sigma^B = -(N/2)\int d^4k[(k+s_B)^{-2}(k+s_B+q)^{-2}]\sigma^{\alpha\beta}\gamma_\mu[g_{\beta\nu}(-2q-k-s_B)_\alpha + g_{\nu\alpha}(q-k-s_B)_\beta + g_{\alpha\beta}(2k+q+2s_B)_\nu] \,, \tag{4}$$

$$\Sigma^C = N\int d^4k[(p+k-q+s_C)^{-2}(p+k+s_C)^{-2}(k+s_C)^{-2}][\sigma^{\alpha\beta}\gamma_\mu(\not p+\not k-\not q+\not s_C)\gamma_\nu(\not p+\not k+\not s_C)\gamma_\alpha(k+s_C)_\beta] \,, \tag{5}$$

$$\Sigma^D = N\int d^4k[(k+s_D)^{-2}(p+k-q+s_D)^{-2}(k-q+s_D)^{-2}][\sigma^{\tau\beta}\gamma_\mu(\not p+\not k-\not q+\not s_D)\gamma^\alpha(k+s_D)_\tau]$$
$$\times[-(k+q+s_D)_\alpha g_{\beta\nu} + (2q-k-s_D)_\beta g_{\alpha\nu} + (2k-q+2s_D)_\nu g_{\alpha\beta}] \,, \tag{6}$$

where $N \equiv -ig^2 C_2(G)\delta^{ab}/(2\pi)^4$ $[C_2(G)\delta^{ab} \equiv c^{afg}c^{bfg}]$, and where Σ^{A-D} are understood to carry indices μ, ν, a, and b. (The sum of Σ^{A-D} is, of course, $\Sigma^{ab}_{\mu\nu}$.) Thus we see that eight arbitrary parameters (m_{A-D}, n_{A-D}) correspond to momentum-routing ambiguities which we wish to resolve through the imposition of appropriate conditions (i.e., gauge invariance) on the aggregate amplitude $\Sigma^{ab}_{\mu\nu}$.

To demonstrate how explicit parametrization of infinities can be avoided in the aggregate amplitude, we will present our Feynman-integral manipulations in some detail. Propagator denominators in Σ^{A-D} are combined in the standard way:

$$(p+k+s_A)^{-2}(k+s_A)^{-2} = \int_0^1 dx\{(k+s_A+px)^2-[p^2x(1-x)]\}^{-2}, \tag{7}$$

$$(k+s_B)^{-2}(k+s_B+q)^{-2} = \int_0^1 dx\{(k+s_B+qx)^2-[q^2x(1-x)]\}^{-2}, \tag{8}$$

$$(p+k-q+s_C)^{-2}(p+k+s_C)^{-2}(k+s_C)^{-2} = 2\int_0^1 dx \int_0^1 y\,dy\{(k+s_C+py-qyx)^2-2p\cdot qxy(1-y)+[y(p^2+q^2x)]\}^{-3}, \tag{9}$$

$$(k+s_D)^{-2}(p+k-q+s_D)^{-2}(k-q+s_D)^{-2} = 2\int_0^1 dx \int_0^1 y\,dy\{(k+s_D+pyx-qy)^2-2p\cdot qxy(1-y)+[y(p^2x+q^2)]\}^{-3}. \tag{10}$$

Quantities in square brackets on the right-hand side (RHS) in the above four equations vanish on the mass shell and can be disregarded. Surface terms proportional to s_{A-D} $(=m_{A-D}p+n_{A-D}q)$ arise when the origin of the integrals over k is shifted. For example, we can use the formulas of Ref. 3 to find that

$$\begin{aligned}\int d^4k \int_0^1 dx \int_0^1 y\,dy\, k^\rho k^\eta k^\tau[(k+s_D+pyx-qy)^2-2p\cdot qxy(1-y)]^{-3} \\ = \int d^4k \int_0^1 dx \int_0^1 y\,dy \frac{(k-s_D-pyx+qy)^\rho(k-s_D-pyx+qy)^\eta(k-s_D-pyx+qy)^\tau}{[k^2-2p\cdot qxy(1-y)]^3} \\ +\frac{i\pi^2}{72}\{[(3m_D+1)p^\rho+(3n_D-2)q^\rho]g^{\eta\tau}+(\rho\leftrightarrow\eta)+(\rho\leftrightarrow\tau)]\}.\end{aligned} \tag{11}$$

After utilizing the above procedures, we find after considerable Dirac algebra that one-loop quantum corrections to $q^\nu S_{\mu\nu}$ are given by

$$\begin{aligned} q^\nu\Sigma^{ab}_{\mu\nu} &= q^\nu(\Sigma^A+\Sigma^B+\Sigma^C+\Sigma^D) \\ &= N(2p_\mu \not{q} - p\cdot q\gamma_\mu)\Bigg[i\pi^2[-(2m_A+1)+\tfrac{2}{9}(3m_C+2)+\tfrac{5}{9}(3m_D+1)] \\ &\quad + \int d^4k \int_0^1 dx \Bigg[-\frac{2}{k^4} + k^2\int_0^1 y\,dy \frac{(10-4y-10xy)}{[k^2-2p\cdot qxy(1-y)]^3} \\ &\quad + p\cdot q\int_0^1 dy \frac{8xy^2[2(1-y)^2+(1-3y)(1-xy)]}{[k^2-2p\cdot qxy(1-y)]^3}\Bigg]\Bigg] \\ &\quad + Nq_\mu\not{q}\Bigg[i\pi^2[-4n_A-\tfrac{4}{9}(3n_C-1)+\tfrac{8}{9}(3n_D-2)] \\ &\quad + \int d^4k\int_0^1 dx\Bigg[-8k^2\int_0^1 \frac{dy\,y(1-2y+xy)}{[k^2-2p\cdot qxy(1-y)]^3} + 32p\cdot q\int_0^1 \frac{dy\,xy^3(1-y)}{[k^2-2p\cdot qxy(1-y)]^3}\Bigg]\Bigg]. \end{aligned} \tag{12}$$

Partial integration over the Feynman parameter y leads to the relations

$$k^2\int_0^1 \frac{dy\,y}{[k^2-2p\cdot qxy(1-y)]^3} = \frac{1}{2k^4} + 2p\cdot q\int_0^1 \frac{dy\,xy^3}{[k^2-2p\cdot qxy(1-y)]^3}, \tag{13}$$

$$k^2\int_0^1 \frac{dy\,y^2}{[k^2-2p\cdot qxy(1-y)]^3} = \frac{1}{3k^4} + \frac{2}{3}p\cdot q\int_0^1 \frac{dy\,xy^3(1+y)}{[k^2-2p\cdot qxy(1-y)]^3}. \tag{14}$$

Upon substitution of (13) and (14) we find that the integrands in large parentheses in (12), respectively, become

$$\left[\frac{1}{k^4}\left[\frac{5}{3}-\frac{10}{3}x\right] + p\cdot q\int_0^1 \frac{dy\,xy^2\left[8-\frac{20y}{3}-\frac{44xy}{3}-\frac{8y^2}{3}+\frac{52xy^2}{3}+16(1-y)^2\right]}{[k^2-2p\cdot qxy(1-y)]^3}\right] \tag{15}$$

and

$$\left[\frac{1}{k^4}\left[\frac{4}{3}-\frac{8x}{3}\right]+p\cdot q\int_0^1\frac{dy\,xy^3[-\frac{16}{3}+\frac{32}{3}y-\frac{16}{3}x-\frac{16}{3}xy-32y(1-y)]}{[k^2-2p\cdot qxy(1-y)]^3}\right]. \tag{16}$$

We see that the $1/k^4$ terms in both integrands disappear upon integration over x. (In the absence of explicit regularization, the order of integration for such potentially divergent terms must be rigidly respected,[1] thereby necessitating integration over x prior to integration over k.) The remaining portions of each integrand are finite upon integration over k, x, and y, thereby permitting a reversal of their order of integration. Using the finite Feynman integral

$$p\cdot q\int\frac{d^4k}{[k^2-2p\cdot qxy(1-y)]^3}=\frac{-i\pi^2}{4xy(1-y)} \tag{17}$$

we find that the multiple integration of (15) yields a finite result of

$$\int d^4k\int_0^1 dx(\cdots)=-i\pi^2\int_0^1\frac{y\,dy}{1-y}\int_0^1 dx\left[2-\frac{5y}{3}-\frac{11xy}{3}-\frac{2y^2}{3}+\frac{13xy^2}{3}+4(1-y)^2\right]$$

$$=-i\pi^2\int_0^1\frac{y\,dy}{(1-y)}[(1-y)(2-3y/2)+4(1-y)^2]=-7i\pi^2/6\,. \tag{18}$$

The y-parameter integral of (18) shows how potential manifestations of infrared divergences (poles at $y=1$) factor out to yield a finite result. In a similar manner, poles at $y=1$ factor out for the corresponding multiple integral over (16), which is found to equal $-2i\pi^2$. We thus find that violations of gauge invariance arising from quantum corrections to the supercurrent are subject to momentum-routing ambiguities:

$$q^\nu\Sigma_{\mu\nu}^{ab}=i\pi^2N(2p_\mu q\!\!\!/-p\cdot q\gamma_\mu)\{-\tfrac{7}{6}+[-(2m_A+1)+\tfrac{2}{9}(3m_C+2)+\tfrac{5}{9}(3m_D+1)]\}$$

$$+i\pi^2Nq_\mu q\!\!\!/\{-2+[-4n_A-\tfrac{4}{9}(3n_C-1)+\tfrac{8}{9}(3n_D-2)]\}\,. \tag{19}$$

Moreover, gauge invariance may be ensured by requiring that the six arbitrary parameters $m_{A,C,D}$, $n_{A,C,D}$ of such ambiguities satisfy the constraints

$$[-(2m_A+1)+\tfrac{2}{9}(3m_C+2)+\tfrac{5}{9}(3m_D+1)]=\tfrac{7}{6}\,, \tag{20}$$

$$[-4n_A-\tfrac{4}{9}(3n_C-1)+\tfrac{8}{9}(3n_D-2)]=2\,. \tag{21}$$

Precisely the same procedures may be employed to evaluate quantum corrections to $\partial\cdot S$ through contraction of $(p-q)^\mu$ into the sum of Eqs. (3)–(6). By combining propagators as in (7)–(10) and extracting surface terms as in (11), we find that

$$\begin{aligned}(p-q)^\mu\Sigma_{\mu\nu}^{ab}=Np_\nu q\!\!\!/\Bigg[&i\pi^2[-(2m_A+1)-2n_A+\tfrac{2}{9}(3m_C+2)-\tfrac{2}{9}(3n_C-1)+\tfrac{5}{9}(3m_D+1)+\tfrac{4}{9}(3n_D-2)]\\&+\int d^4k\int_0^1 dx\Bigg[-\frac{2}{k^4}+k^2\int_0^1\frac{dy\,y(6+4y-14xy)}{[k^2-2p\cdot qxy(1-y)]^3}\\&\qquad+8p\cdot q\int_0^1\frac{dy\,xy^2[-3+y+3xy-xy^2+(1-y)(-6+6y+2xy)]}{[k^2-2p\cdot qxy(1-y)]^3}\Bigg]\Bigg]\\+Np\cdot q\gamma_\nu\Bigg[&i\pi^2[2(2m_A+1)-2n_A-\tfrac{4}{9}(3m_C+2)-\tfrac{2}{9}(3n_C-1)-\tfrac{10}{9}(3m_D+1)+\tfrac{4}{9}(3n_D-2)]\\&+\int d^4k\int_0^1 dx\Bigg[\frac{4}{k^4}+8k^2\int_0^1 dy\frac{y[-3+2y+2xy]}{[k^2-2p\cdot qxy(1-y)]^3}\\&\qquad+16p\cdot q\int_0^1\frac{dy\,xy^2[1+y-xy-xy^2+(1-y)(2-yx)]}{[k^2-2p\cdot qxy(1-y)]^3}\Bigg]\Bigg]\,.\end{aligned} \tag{22}$$

Application of the partial integration formulas (13) and (14) within the two expressions in large parentheses in (22) yields coefficients of k^{-4} that vanish when integrated over x. [These coefficients are $(7-14x)/3$ and $(-8+16x)/3$, respectively.] The remaining integrals are evaluated precisely as in (18); poles at $y=1$ are seen to be removable singularities; and we eventually obtain a purely finite result:[18]

$$(p-q)^{\mu}\Sigma^{ab}_{\mu\nu}=i\pi^2Np_\nu q\!\!\!/\{\tfrac{23}{6}+[-(2m_A+1)+\tfrac{2}{9}(3m_C+2)+\tfrac{5}{9}(3m_D+1)]+\tfrac{1}{2}[-4n_A-\tfrac{4}{9}(3n_C-1)+\tfrac{8}{9}(3n_D-2)]\}$$
$$+i\pi^2Np\cdot q\gamma_\nu\{-\tfrac{14}{3}-2[-(2m_A+1)+\tfrac{2}{9}(3m_C+2)+\tfrac{5}{9}(3m_D+1)]+\tfrac{1}{2}[-4n_A-\tfrac{4}{9}(3n_C-1)+\tfrac{8}{9}(3n_D-2)]\}\;. \quad (23)$$

Remarkably enough, the same two linear combinations of m's and n's appear in (23) as appeared in (19). When the constraints (20) and (21) for gauge invariance of the quantum corrections are substituted into (23), we find that

$$(p-q)^{\mu}\Sigma^{ab}_{\mu\nu}=[3g^2C_2(G)\delta^{ab}/8\pi^2](p_\nu q\!\!\!/-p\cdot q\gamma_\nu)\;, \quad (24)$$

a result identical to that obtained in Ref. 16 from Adler-Rosenberg—type symmetry arguments.[19] The full correspondence between our procedure and Adler-Rosenberg—symmetry constraints on quantum corrections is explored in the Appendix to this paper.

We thus see that the imposition of gauge invariance removes all loop-momentum-routing ambiguities from quantum corrections to $\partial\cdot S$, which are seen to be anomalous. However, we also see that these ambiguities *do not* provide a mechanism for the anomaly to reside in quantum corrections to $\gamma\cdot S$. Indeed, our result differs from that expected from the supermultiplet structure of anomalies in that $\gamma^{\mu}\Sigma^{ab}_{\mu\nu}$ is necessarily zero from four-dimensional Dirac algebra; $\gamma^{\mu}\Sigma^{A-D}=0$ in (3)—(6), because $\gamma^{\mu}\sigma^{\alpha\beta}\gamma_{\mu}=0$.[20] We can, of course, construct a supercurrent whose anomaly resides in $\gamma\cdot S$ rather than $\partial\cdot S$ by adding to quantum corrections the gauge-invariant $O(g^2)$ structural contribution delineated in Eq. (4.2) of Ref. 16. In this regard, the nonzero result for quantum corrections to $\partial\cdot S$ following from (24) ultimately leads to the "correct" supercurrent anomaly, as is discussed in Ref. 16, if the additional $O(g^2)$ term is normalized so as to cancel the nonzero quantum corrections to $\partial\cdot S$ (Ref. 21).

We emphasize that the nonzero contribution to $\partial\cdot S$ which we obtain from the graphs of Fig. 2 is a direct consequence of our rigorous adherence to four-dimensional space-time and is *not* an artifact of our refusal to use a regulator in four dimensions. Indeed, we could have chosen initially to regulate Σ^{A-D} separately (the graphs are *individually* UV and IR divergent) using cutoffs, while still maintaining four space-time dimensions so as to allow momentum-routing ambiguities to appear in the amplitudes. To illustrate this point further, we outline below how the calculation proceeds with ultraviolet infinities in Σ^{A-D} regulated as naively as possible using cutoffs:

$$\int\frac{d^4k}{(k^2-\mu^2)^2}=i\pi^2\left[\ln\frac{\Lambda^2}{\mu^2}-1\right]\;. \quad (25)$$

We will now need to keep track of factors of p^2 appearing in the square brackets on the RHS of (7)—(10), factors which are zero on mass shell, so as to parametrize any infrared divergences with Σ^{A-D} as logarithms of p^2. Using the mass-shell conditions described earlier,[22] and defining $L\equiv\ln(\Lambda^2/2p\cdot q)-\frac{3}{2}, I\equiv\ln(-p^2/2p\cdot q)$, we find from (3)—(6) that

$$q^{\nu}\Sigma^{A}=i\pi^2N[(2p_\mu q\!\!\!/-p\cdot q\gamma_\mu)(-2L+2I-6-2m_A)+q_\mu q\!\!\!/(-4n_A)]\;, \quad (26)$$

$$q^{\nu}\Sigma^{B}=0\;, \quad (27)$$

$$q^{\nu}\Sigma^{C}=i\pi^2N\{(2p_\mu q\!\!\!/-p\cdot q\gamma_\mu)[2L/3+\tfrac{10}{9}+2(3m_C+2)/9]+q_\mu q\!\!\!/[-4L/3-\tfrac{32}{9}-4(3n_C-1)/9]\}\;, \quad (28)$$

$$q^{\nu}\Sigma^{D}=i\pi^2N\{(2p_\mu q\!\!\!/-p\cdot q\gamma_\mu)[4L/3-2I+\tfrac{23}{9}+5(3m_D+1)/9]+q_\mu q\!\!\!/[4L/3+\tfrac{20}{9}+8(3n_D-2)/9]\}\;, \quad (29)$$

$$(p-q)^{\mu}\Sigma^{A}=i\pi^2N[p_\nu q\!\!\!/(-2L+2I-6-2m_A-2n_A)+(p\cdot q)\gamma_\nu(4L-4I+12+4m_A-2n_A)]\;, \quad (30)$$

$$(p-q)^{\mu}\Sigma^{B}=0\;, \quad (31)$$

$$(p-q)^{\mu}\Sigma^{C}=i\pi^2N\{p_\nu q\!\!\!/[2+2(3m_C+2)/9-2(3n_C-1)/9]+(p\cdot q)\gamma_\nu[-2L-\tfrac{20}{3}-4(3m_C+2)/9-2(3n_C-1)/9]\}\;, \quad (32)$$

$$(p-q)^{\mu}\Sigma^{D}=i\pi^2N\{p_\nu q\!\!\!/[2L-2I+7+5(3m_D+1)/9+4(3n_D-2)/9]$$
$$+p\cdot q\gamma_\nu[-2L+4I-\tfrac{22}{3}-10(3m_D+1)/9+4(3n_D-2)/9]\}\;. \quad (33)$$

In summing (26)—(29) and (30)—(33) we see that the net coefficients of L and I in $q^{\nu}\Sigma^{ab}_{\mu\nu}$ and $(p-q)^{\mu}\Sigma^{ab}_{\mu\nu}$ are zero, obviating any need for infinite renormalizations.[14]

The imposition of gauge invariance alone is sufficient to remove all momentum-routing ambiguities from the divergence of the supercurrent on shell. The net coefficient of $(2p_\mu q\!\!\!/-p\cdot q\gamma_\mu)$ vanishes in $q^{\nu}\Sigma^{ab}_{\mu\nu}$ provided

$$[-2m_A+2(3m_C+2)/9+5(3m_D+1)/9]=\tfrac{7}{3}\;, \quad (34)$$

and the net coefficient of $q_\mu q\!\!\!/$ vanishes in $q^{\nu}\Sigma^{ab}_{\mu\nu}$ provided

$$[-4n_A-4(3n_C-1)/9+8(3n_D-2)/9]=\tfrac{4}{3}\;. \quad (35)$$

These particular linear combinations of m_{A-D}, n_{A-D} are the only ones that enter $(p-q)^{\mu}\Sigma^{ab}_{\mu\nu}$; if we substitute (34) and (35) into the sum of (30)–(33), we again obtain Eq. (24).[23] Indeed, this result could have been obtained entirely from Figs. 2(c) and 2(d) with no reference to the "bubble graphs" of Figs. 2(a) and 2(b), provided we allow m's and n's to contain regulated infinities. If $m_A=-L+I-3$ and $n_A=0$, we see from (26), (27), (30), and (31) that Σ^A and Σ^B make no contributions to $q^{\nu}\Sigma^{ab}_{\mu\nu}$ and $(p-q)^{\mu}\Sigma^{ab}_{\mu\nu}$ (Ref. 24). Moreover, the calculational route presented above [Eqs. (26)–(35)] is, in fact, a straightforward demonstration of preregularization,[4,5] the supplementing of an arbitrarily chosen regularization procedure with loop-momentum ambiguities, which, for this problem, are removed through the imposition of gauge invariance.

To conclude, we find that quantum corrections to the supercurrent evaluated in exactly four-dimensional space-time exhibit loop-momentum ambiguities which are resolved, as in the *VVA* triangle, through the imposition of gauge invariance. We also find that contractions of external on-shell photon and fermion momenta into these quantum corrections (corresponding to vector-current and supercurrent divergences) lead to finite expressions that may be evaluated without any use of regulators. Similar momentum contractions into the *VVA* triangle (corresponding to vector-current and axial-vector-current divergences) have also been shown to be finite and evaluable without explicit regularization.[7] However, we find that supersymmetric expectations for the quantum corrections of the supercurrent in four space-time dimensions, specifically the anomalous violation of the Rarita-Schwinger constraint $\gamma\cdot S=0$, cannot be accommodated through the use of loop-momentum ambiguities. This last feature is disturbing (despite the ameliorating redefinition of the supercurrent discussed in Ref. 16) because it suggests some clashing between supersymmetry and nondimensionally continued quantum field theory at even the one-loop level. Of course, discrepancies between supersymmetry and more standard dimensionally continued approaches to perturbative quantum field theory are also known to exist.[25]

We are grateful to H. Schnitzer and P. Majumdar for correspondence. This research was supported by the Natural Sciences Engineering and Research Council of Canada.

APPENDIX: ALDER-ROSENBERG SYMMETRY ARGUMENTS AND THE RESOLUTION OF LOOP-MOMENTUM AMBIGUITIES

In this Appendix we show how the imposition of gauge invariance on loop-momentum ambiguities in the supercurrent achieves the same results as the Adler-Rosenberg symmetry procedures presented in Ref. 16. The tensor structure of Σ^{A-D} is given by

$$\Sigma^A=-\int d^4k\,k^{-4}\Sigma^{(7)}_{\mu\nu}-i\pi^2[M_A\Sigma^{(7)}_{\mu\nu}+N_A(\Sigma^{(8)}_{\mu\nu}-\tfrac{1}{2}\Sigma^{(9)}_{\mu\nu})]\,, \quad \text{(A1)}$$

$$\Sigma^B=-\tfrac{3}{2}\int d^4k\,k^{-4}\Sigma^{(8)}_{\mu\nu}\,, \quad \text{(A2)}$$

$$\begin{aligned}\Sigma^C=4\int_0^1 dx\int_0^{1-x}dy\int &\frac{d^4k}{[k^2-2p\cdot qy(1-x-y)]^3}\\ &\times\{y(x+y)(1-x-y)\Sigma^{(1)}_{\mu\nu}-y^2(1-x-y)\Sigma^{(2)}_{\mu\nu}+y(1-x-y)\Sigma^{(3)}_{\mu\nu}+2y(1-x-y)\Sigma^{(4)}_{\mu\nu}\\ &+[y(x+y)(1-x-y)p\cdot q+(-1+2x+2y)k^2/4]\Sigma^{(5)}_{\mu\nu}\\ &+[-y^2(1-x-y)p\cdot q-yk^2/2]\Sigma^{(6)}_{\mu\nu}+[(1-x-y)k^2/2]\Sigma^{(7)}_{\mu\nu}+(yk^2/2)\Sigma^{(8)}_{\mu\nu}-(yk^2/4)\Sigma^{(9)}_{\mu\nu}\}\\ -(i\pi^2/6)[2M_C(\Sigma^{(5)}_{\mu\nu}-\Sigma^{(7)}_{\mu\nu})&+N_C(2\Sigma^{(6)}_{\mu\nu}-2\Sigma^{(8)}_{\mu\nu}+\Sigma^{(9)}_{\mu\nu})]\,,\end{aligned} \quad \text{(A3)}$$

$$\begin{aligned}\Sigma^D=4\int_0^1 dx\int_0^{1-x}dy\int &\frac{d^4k}{[k^2-2p\cdot qy(1-x-y)]^3}\\ &\times\{y^2(1-x-y)\Sigma^{(1)}_{\mu\nu}-y(x+y)(1-x-y)\Sigma^{(2)}_{\mu\nu}+y(1-x-y)\Sigma^{(3)}_{\mu\nu}-2y(-1+x+y)\Sigma^{(4)}_{\mu\nu}\\ &-(yk^2/4)\Sigma^{(5)}_{\mu\nu}+[(1+x+y)k^2/4]\Sigma^{(6)}_{\mu\nu}\\ &+[-y(1-y)(1+x+y)p\cdot q+(3-5y)k^2/4]\Sigma^{(7)}_{\mu\nu}\\ &+[(x+y)(1-y)(1+x+y)p\cdot q-(1-5x-5y)k^2/4]\Sigma^{(8)}_{\mu\nu}+[(2-x-y)k^2/4]\Sigma^{(9)}_{\mu\nu}\}\\ +(i\pi^2/6)[M_D(\Sigma^{(5)}_{\mu\nu}+5\Sigma^{(7)}_{\mu\nu})&+N_D(\Sigma^{(6)}_{\mu\nu}+5\Sigma^{(8)}_{\mu\nu}-\Sigma^{(9)}_{\mu\nu})]\,,\end{aligned} \quad \text{(A4)}$$

where $\Sigma^{(1)-(9)}$ are given by

$$\Sigma^{(1)}_{\mu\nu}\equiv(N/2)\sigma^{\beta\alpha}\gamma_\mu[p_\nu(p_\beta\gamma_\alpha-p_\alpha\gamma_\beta)\not q]\ , \tag{A5}$$

$$\Sigma^{(2)}_{\mu\nu}\equiv(N/2)\sigma^{\beta\alpha}\gamma_\mu[p_\nu(q_\beta\gamma_\alpha-q_\alpha\gamma_\beta)\not q]\ , \tag{A6}$$

$$\Sigma^{(3)}_{\mu\nu}\equiv(N/2)\sigma^{\beta\alpha}\gamma_\mu[\gamma_\nu(q_\beta p_\alpha-q_\alpha p_\beta)\not q]\ , \tag{A7}$$

$$\Sigma^{(4)}_{\mu\nu}\equiv(N/2)\sigma^{\beta\alpha}\gamma_\mu[p_\nu(q_\beta p_\alpha-q_\alpha p_\beta)]\ , \tag{A8}$$

$$\Sigma^{(5)}_{\mu\nu}\equiv(N/2)\sigma^{\beta\alpha}\gamma_\mu[\gamma_\nu(p_\beta\gamma_\alpha-p_\alpha\gamma_\beta)]\ , \tag{A9}$$

$$\Sigma^{(6)}_{\mu\nu}\equiv(N/2)\sigma^{\beta\alpha}\gamma_\mu[\gamma_\nu(q_\beta\gamma_\alpha-q_\alpha\gamma_\beta)]\ , \tag{A10}$$

$$\Sigma^{(7)}_{\mu\nu}\equiv(N/2)\sigma^{\beta\alpha}\gamma_\mu(g_{\beta\nu}p_\alpha-g_{\alpha\nu}p_\beta)\ , \tag{A11}$$

$$\Sigma^{(8)}_{\mu\nu}\equiv(N/2)\sigma^{\beta\alpha}\gamma_\mu(g_{\beta\nu}q_\alpha-g_{\alpha\nu}q_\beta)\ , \tag{A12}$$

$$\Sigma^{(9)}_{\mu\nu}\equiv(N/2)\sigma^{\beta\alpha}\gamma_\mu[(g_{\beta\nu}\gamma_\alpha-g_{\alpha\nu}\gamma_\beta)\not q]\ . \tag{A13}$$

The last square-bracketed expressions in (A1)–(A4) contain shift-of-integration-variable surface terms sensitive to the definition of the loop momentum whose arbitrariness we have reparametrized as

$$\int_0^1 dx(s_A+xp)\equiv M_A p+N_A q\ , \tag{A14}$$

$$2\int_0^1 dx\int_0^{1-x} dy[s_C+xp+y(p-q)]\equiv M_C p+N_C q\ , \tag{A15}$$

$$2\int_0^1 dx\int_0^{1-x} dy[s_D-xq+y(p-q)]\equiv M_D p+N_D q\ . \tag{A16}$$

We sum (A1)–(A4) to obtain $\Sigma^{ab}_{\mu\nu}$:

$$\Sigma^{ab}_{\mu\nu}=\sum_{i=1}^{9} Y^{(i)}\Sigma^{(i)}_{\mu\nu}\ . \tag{A17}$$

The set of invariants $\Sigma^{(1)}_{\mu\nu}$–$\Sigma^{(9)}_{\mu\nu}$ are equivalent to those of Ref. 16, and on mass shell this set is overcomplete:

$$\Sigma^{(1)}_{\mu\nu}+2\Sigma^{(4)}_{\mu\nu}=0\ , \tag{A18}$$

$$\Sigma^{(5)}_{\mu\nu}=0\ , \tag{A19}$$

$$\Sigma^{(6)}_{\mu\nu}+\Sigma^{(9)}_{\mu\nu}=0\ . \tag{A20}$$

These relations plus the requirement of gauge invariance ($q^\nu\Sigma^{ab}_{\mu\nu}=0$) impose the following constraints on the Y coefficients in (A17):[26]

$$p\cdot qY^{(2)}-Y^{(6)}+Y^{(9)}=0\ , \tag{A21}$$

$$2p\cdot qY^{(1)}-p\cdot qY^{(4)}-Y^{(7)}=0\ . \tag{A22}$$

The coefficients $Y^{(i)}$ may be gleaned from the sum of (A1)–(A4) and are generally ambiguous, because of the arbitrary last square-bracketed terms in (A1)–(A4) arising from loop-momentum-routing ambiguities. To resolve loop-momentum ambiguities, (A21) and (A22) represent conditions on M_{A-D}, N_{A-D} necessary for gauge invariance. Specifically, if we let each coefficient $Y^{(i)}$ be the sum of a Feynmann-integral contribution $Y^{(i)}_0$ not involving M's and N's and surface terms [i.e., $Y^{(i)}=Y^{(i)}_0+$ (terms linear in M_{A-D} or N_{A-D})], then (A21) and (A22) determine the value of two linear combinations of M's and N's:

$$(i\pi^2/6)(-3N_A-N_C+2N_D)=q\cdot pY^{(2)}_0-Y^{(6)}_0+Y^{(9)}_0\ , \tag{A23}$$

$$(i\pi^2/6)(-6M_A+2M_C+5M_D)=q\cdot p(2Y^{(1)}_0-Y^{(4)}_0)-Y^{(7)}_0\ . \tag{A24}$$

We see, therefore, that routings of loop momenta in Fig. 2 can always be found (or alternatively, appropriate choices for s_{A-D} can always be made) such that (A23) and (A24) are satisfied, *regardless* of how one chooses to parametrize the infinities occurring in the coefficients $Y^{(i)}$. If one chose to ignore routing ambiguities, one could perhaps still satisfy (A21) and (A22) with a sufficiently clever regularization of these infinities. Indeed, the thrust of an Adler-Rosenberg–type symmetry argument is *to assume* (A21) and (A22) are true. Loop-momentum-routing ambiguities provide a general means for substantiating such assumptions within any consistent regularizing procedure for Feynman integral infinities.

Contracting $(p-q)^\mu$ into (A17), we find that

$$\begin{aligned}(p-q)^\mu\Sigma^{ab}_{\mu\nu}=(N/2)\{&p_\nu\not q[-16p\cdot qY^{(2)}-8p\cdot qY^{(3)}+4p\cdot q(2Y^{(1)}-Y^{(4)})+4Y^{(7)}+8(Y^{(6)}-Y^{(9)})]\\ &+p\cdot q\gamma_\nu[8p\cdot qY^{(3)}-8Y^{(7)}+8(Y^{(6)}-Y^{(9)})]\}\\ =4p\cdot qN(&p_\nu\not q-p\cdot q\gamma_\nu)(2Y^{(1)}-Y^{(2)}-Y^{(3)}-Y^{(4)})\ .\end{aligned} \tag{A25}$$

The last line of (A25) is obtained from the preceding line by using (A21) and (A22) to eliminate $Y^{(6)}$, $Y^{(7)}$, and $Y^{(9)}$. The remaining coefficients $Y^{(1)-(4)}$ obtained from summing (A1)–(A4) are all finite and unambiguous:

$$Y^{(1)}=4\int d^4k\int_0^1 dx\int_0^{1-x} dy[k^2-2p\cdot qy(1-x-y)]^{-3}\times y(x+2y)(1-x-y)\ , \tag{A26}$$

$$Y^{(2)}=4\int d^4k\int_0^1 dx\int_0^{1-x} dy[k^2-2p\cdot qy(1-x-y)]^{-3}\times y(x+y-1)(x+2y)\ , \tag{A27}$$

$$Y^{(3)}=4\int d^4k\int_0^1 dx\int_0^{1-x} dy[k^2-2p\cdot qy(1-x-y)]^{-3}\times[2y(1-x-y)]\ , \tag{A28}$$

$$Y^{(4)}=4\int d^4k\int_0^1 dx\int_0^{1-x} dy[k^2-2p\cdot qy(1-x-y)]^{-3}\times[4y(1-x-y)]\ . \tag{A29}$$

Evaluation of these integrals and subsequent substitution into (A25) leads once again to Eq. (24) in the main body of our paper.

[1]J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Springer, Berlin, 1976), pp. 457–460.

[2]R. E. Pugh, Can. J. Phys. **47**, 1263 (1969).

[3]V. Elias, G. McKeon, and R. B. Mann, Phys. Rev. D **28**, 1978 (1983); Can. J. Phys. **63**, 1498 (1985).

[4]V. Elias, G. McKeon, R. B. Mann, and S. B. Phillips, Phys. Lett. **133B**, 83 (1983); A. M. Chowdhury, V. Elias, R. B. Mann, G. McKeon, S. B. Phillips, and S. Samant, Can. J. Phys. **63**, 1453 (1985).

[5]This approach was first utilized by J. Bell and R. Jackiw, Nuovo Cimento **60**, 47 (1969).

[6]S. Adler, in *Lectures on Elementary Particles and Quantum Field Theory,* edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, Mass., 1970), p. 1.

[7]V. Elias, G. McKeon, and R. B. Mann, Nucl. Phys. **B229**, 487 (1983).

[8]Consistency with the Adler-Bardeen theorem has been recently established within the context of utilizing gauge invariance to resolve loop-momentum ambiguities [A. M. Chowdhury and G. McKeon, Phys. Rev. D 33, 598 (1986)]. Such preregularization methods have been employed to obtain the axial anomaly in $2N$ dimensions [A. M. Chowdhury, G. McKeon, and R. B. Mann, *ibid.* 33, 3090 (1986)].

[9]G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972).

[10]H. Nicolai and P. K. Townsend, Phys. Lett. **93B**, 111 (1980).

[11]S. Ferrara and B. Zumino, Nucl. Phys. **B87**, 207 (1975).

[12]T. Curtright, Phys. Lett. **71B**, 85 (1977); H. Inagaki, *ibid.* **71B**, 403 (1977); J. Lukierski, *ibid.* **70B**, 183 (1977).

[13]T. Hagiwara, S. Pi, and H. Tsao, Ann. Phys. (N.Y.) **130**, 282 (1980).

[14]D. Espriu, Oxford University Report No. 57-84 (unpublished).

[15]H. Inagaki, Phys. Lett. **77B**, 56 (1977).

[16]L. Abbott, M. Grisaru, and H. Schnitzer, Phys. Rev. D **16**, 2995 (1977).

[17]P. Majumdar, E. Poggio, and H. Schnitzer, Phys. Rev. D **21**, 2203 (1980).

[18]The net absence of ultraviolet infinities in these radiative corrections was noted in Ref. 14; the net absence of infrared infinities was also noted in Ref. 17, in contradiction to a net infrared infinity claimed in Ref. 13.

[19]If in Eqs. (20) and (21) one chose to replace the right-hand side factors of $\frac{7}{6}$ and 2 with $-\frac{17}{6}$ and -2, respectively, one could then have quantum corrections respect conservation of the supercurrent; i.e., $(p-q)^\mu \Sigma^{ab}_{\mu\nu}$ in (23) would vanish. However, substitution into (19) would then lead to a quantum-correction violation of gauge invariance $[q^\nu \Sigma^{ab}_{\mu\nu} = (-g^2 C_2 \delta^{ab}/4\pi^2)(2p_\mu \not{q} - p\cdot q \gamma_\mu + q_\mu \not{q})]$. This tradeoff parallels the tradeoff observed in the *VVA* triangles between having a nonanomalous axial-vector-current Ward identity and having violations of gauge invariance, as in M. Chanowitz, M. Furman, and I. Hinchliffe, Nucl. Phys. **B159**, 225 (1979).

[20]Shift-of-integration-variable surface terms do not occur in Feynman integrals dimensionally continued away from four space-time dimensions (Ref. 3). Consequently, loop-momentum-routing ambiguities occur *only* in four space-time dimensions. Nevertheless, our insistence on standard Dirac algebra in evaluating quantum corrections is motivated by more than retaining methodological consistency with four-dimensional space-time. Indeed, supersymmetric Ward identities are known to be verifiable to two-loop order provided $\gamma_\mu \gamma^\mu = 4$ and $\{\gamma_\mu, \gamma_5\} = 0$ [P. K. Townsend and P. van Nieuwenhuizen, Phys. Rev. D **20**, 1932 (1979); T. Curtright and G. Ghandour, Ann. Phys. (N.Y.) **106**, 209 (1977); E. Sezgin, Nucl. Phys. **B162**, 1 (1980)]. These conditions (which explicitly rule out the dimensional continuation of Dirac algebra in Ref. 9) remain upheld when Dirac-algebra trace cyclicity is abandoned as a consequence of projecting less-than-four-dimensional loop momenta onto four-dimensional γ matrices (Ref. 10). Nevertheless, such an approach leads to contradictory results, depending on whether this projecting occurs before or after some γ matrices are multiplied together within γ-matrix traces [Ref. 7; see also F. B. Little, R. B. Mann, V. Elias, and G. McKeon, Phys. Rev. D **32**, 2707 (1985)]. Indeed this contradiction is responsible for the discrepancy between the dimensional-reduction calculations of the supercurrent presented in Refs. 10 and 17.

[21]A somewhat similar methodology can be employed to calculate the Schwinger mass term in two-dimensional QED, in which a surface-term-generated violation of gauge invariance is absorbed through a finite-order-e^2 mass counterterm [V. Elias, G. McKeon, and R. B. Mann, Phys. Rev. D **27**, 3027 (1983)]. The aggregate theory (radiative corrections plus counterterm) is gauge invariant and finite, and the resulting photon propagator has the usual e^2/π Schwinger mass pole.

[22]All IR-finite terms involving p^2 are evaluated with $p^2=0$.

[23]The finite discrepancy between (34) and (20) [or that between (35) and (21)] is a manifestation of a regularization-procedure dependence that is seen here to have no observable consequences, provided one does not attempt to substitute the constraints (34) and (35) on m's and n's obtained from a cutoff-regulator calculation of $q^\nu \Sigma_{\mu\nu}$ into the expression (23) for $(p-q)^\mu \Sigma_{\mu\nu}$ that was obtained *without* using regulator cutoffs.

[24]The null contribution of bubble graphs to anomalous quantum corrections in four dimensions is noted in the symmetry argument calculation of Ref. 16.

[25]W. Siegel, Phys. Lett. **94B**, 37 (1980); L. V. Avdeev and A. A. Vladimirov, Nucl. Phys. **B219**, 262 (1983).

[26]In Ref. 16, the initial omission of mass-shell identities (A18)–(A20) makes it appear as if there are three distinct conditions for gauge invariance. In terms of coefficients A_i in Ref. 16 [defined by Eq. (3.3b) of Ref. 16], our Eqs. (A21) and (A22) are equivalent to the relations $A_0 - A_3 = p\cdot k A_5$; $A_1 + 2A_2 = -p\cdot k(2A_6 + A_7)$.