Observer-dependent quantum vacua in curved space

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(Received 7 November 1985)

An observer-dependent Hamiltonian is introduced. The vacuum state is defined by means of Hamiltonian diagonalization and minimization, which result to be equivalent criteria. This method encompasses a great number of known vacuum definitions, and works in an arbitrary geometry if the observer's field satisfies certain properties.

I. INTRODUCTION

Hamiltonian diagonalization is a criterion frequently used to define the vacuum state, when a field in a curved background is studied. Many quantities have been used for this purpose: they are always called "Hamiltonian" and they have different definitions,[1,2] but with a common feature: these definitions do not make any reference to the observers. Nevertheless it is well known that "vacuum" is a notion that strongly depends on the observers; the best studied case is the one of Rindler's observers: they perceive particles in a quantum state considered as the vacuum for Minkowski's observers.[3,4]

In this paper we introduce a new Hamiltonian, which is invariant by coordinate transformations (of course, the vacuum cannot depend on the used geometrical coordinates), but it is dependent on the observers (because each set of observers has its own vacuum). We show that it is the most appropriate Hamiltonian to define the vacuum. Also we show that diagonalization and minimization are equivalent criteria to define the vacuum state. Our method works in an arbitrary geometry if the observer's field is an irrotational one. In Sec. II we review the notion of observers or fluid of reference. In Sec. III we define the Hamiltonian. In Sec. IV we study the diagonalization and minimization. In Sec. V we show that our method encompasses the following definitions.

(1) All vacua that correspond to a Killing vector field (e.g., Minkowski, Rindler, Boulware, and some de Sitter vacua,[3] etc.).

(2) All "conformal vacua," that we shall define below (e.g., the vacua of papers,[5,6] Kruskal vacuum in a two-dimensional eternal black hole,[3] etc.).

(3) The Robertson-Walker and Bianchi type-I universe vacua (e.g., those of Ref. 7). In this case a local property, which is necessary in order to render satisfactory these vacua, may be unsatisfied (essentially that the renormalized vacuum expectation value of the energy-momentum tensor turns out to be finite, as is explained in Refs. 7 and 8).

In Sec. VI we draw our main conclusions.

Hamiltonian diagonalization has been criticized in the literature.[1] We believe some of the criticisms are overcome in Ref. 7, to which we refer. Aside from this, the present paper shows that, even if it is not completely satisfactory, Hamiltonian diagonalization works in a great number of cases.

II. REFERENCE SYSTEM

What is a reference system (or an observer's system) in general relativity? In classical physics a reference system is formed by a reference rigid body and a clock, and it is possible to choose different geometrical coordinate systems or charts (Cartesian, spherical, etc.) for the *same* reference system. To build an observer's system in general relativity it is necessary to replace the rigid body by a fluid. We shall use an irrotational fluid (see the Appendix).[9] On each matter point of this fluid there is a clock, which may not be a standard clock (it may not indicate proper time); it must be only a time measurement device that measures a continuous increasing arbitrary function of proper time. We assume that the measurements x^0 of two neighbor clocks differ infinitesimally. If each matter point of the fluid is labeled with three numbers x^1, x^2, x^3 (varying continuously), then (x^0, x^1, x^2, x^3) is a particular geometrical chart.

As soon as a chart has been built, we may determine the components of the metric tensor in this chart, its curvature scalar, etc., using metric rods and standard clocks.[10]

Because the fluid is irrotational, it has global spacelike hypersurfaces that are orthogonal to the universe lines of the matter points (see the Appendix). This fact introduces a natural notion of *time T*. The time T is a quantity that is constant on each of these hypersurfaces. In other words, a *natural time* (with respect to an observers system) is the one that is measured by clocks synchronized on the hypersurfaces orthogonal to the lines of fluid. Of course, there are several forms from which to choose the rate of the clocks; this rate will turn out to be irrelevant. Furthermore, generally it may occur that standard clocks do not measure T (proper time would not be equal to natural time).

Of course we can build other observer systems using other fluids. Each observer system could be considered as a different form to foliate the space-time into space and time. Space is the hypersurface orthogonal to the lines of fluid. Because the physical instruments of measurement distinguish these concepts (e.g., one only detects an electric field while the other one only detects a magnetic field), the measurements are affected by a change of the observer system. But of course they are not affected by a change of geometrical chart (a change of coordinates).

The initially proposed geometrical chart, with the additional condition that $x^0=T$ (i.e., x^0 is constant on a hypersurface orthogonal to lines of fluid) will be called the *adapted chart*. But of course we can use any other geometrical chart if we want to.

In order to characterize the observer system two vectors could be defined. The natural time T may be used to parametrize the lines of the fluid [a line is $x^\mu=x^\mu(T)$]. In this case, the vector $\mathbf{v}\equiv d/dT$ is defined in Ref. 11. Its components are $v^\mu=dx^\mu/dT$, where the derivative is taken along a line of the fluid. Vector $\mathbf{v}$ is tangent to the lines of fluid. Below we shall also use the unitary vector $\mathbf{u}$ tangent to the lines of fluid ($\mathbf{u}$ is the four-velocity of the matter points).

In an adapted chart the metric tensor is

$$g_{\mu\nu}=\begin{bmatrix} g_{00} & 0 \\ 0 & -\gamma_{ij}\end{bmatrix}, \quad \mu,\nu=0,1,2,3,\ldots,n-1\,, \quad i,j=1,2,3,\ldots,n-1\,. \tag{1}$$

(We shall work in an n-dimensional space-time.)

The vector $\mathbf{v}$ reads

$$v^\mu=(1,0,0,0,\ldots) \tag{2}$$

and the unitary vector $\mathbf{u}$ is

$$u^\mu=(g_{00}{}^{-1/2},0,0,0,\ldots)\,. \tag{3}$$

III. THE HAMILTONIAN

We use the convention $(---)$ of Ref. 12: i.e.,

$$g_{\mu\nu}=(+---\cdots)\,, \tag{4}$$

$$R_{\lambda\mu\nu\kappa}=\tfrac{1}{2}(\partial_{\kappa\mu}g_{\lambda\nu}-\partial_{\kappa\lambda}g_{\mu\nu}+\cdots)\,, \tag{5}$$

and

$$R_{\mu\kappa}=g^{\mu\nu}R_{\lambda\mu\nu\kappa}\,. \tag{6}$$

We study a neutral scalar field ϕ. The field equation is

$$(\Box+m^2+\xi R)\phi=0 \tag{7}$$

and the energy-momentum tensor reads[1]

$$\begin{aligned} T_{\mu\nu}&\equiv 2(-g)^{-1/2}\frac{\delta S}{\delta g^{\mu\nu}} \\ &=(\tfrac{1}{2}-\xi)\{\phi_{;\mu},\phi_{;\nu}\}+(\xi-\tfrac{1}{4})g_{\mu\nu}g^{\lambda\rho}\{\phi_{;\lambda},\phi_{;\rho}\}-\xi\{\phi,\phi_{;\mu\nu}\}+\xi g_{\mu\nu}\{\phi,\Box\phi\} \\ &\quad -\frac{1}{2}\left[\xi\left[R_{\mu\nu}-\frac{R}{2}g_{\mu\nu}\right]-\tfrac{1}{2}m^2g_{\mu\nu}\right]\{\phi,\phi\}\,, \end{aligned} \tag{8}$$

where

$$S\equiv\int d^nx\,\sqrt{-g}\,\tfrac{1}{2}(g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}-m^2\phi^2-\xi R\phi^2) \tag{9}$$

is the action.

We define the Hamiltonian as[11]

$$H_\Sigma\equiv\int_\Sigma T_{\mu\nu}v^\mu d\Sigma^\nu=\int_\Sigma T_{\mu\nu}v^\mu u^\nu d\Sigma\,, \tag{10}$$

where Σ is a hypersurface orthogonal to the fluid.

H_Σ is invariant under a geometrical coordinate change, but it is not under an observer system change. The inclusion of $\mathbf{v}$ is essential because two different reference fluids may have a common hypersurface Σ (this happens with a Minkowski fluid and a Rindler fluid at $T=0$ in plane space-time). On such Σ only it is possible to distinguish which observer belongs to each fluid by means of the rate of the clocks. This is the role of $\mathbf{v}$.

When $\mathbf{v}$ is a Killing vector in the region bounded by two hypersurfaces Σ and Σ', then $H_\Sigma=H_{\Sigma'}$ (Ref. 13). But in general H_Σ depends on Σ (i.e., H_Σ depends on T).

In the adapted chart the Hamiltonian and the field equation (7) read

$$H_\Sigma=\int_\Sigma T_{00}g_{00}{}^{-1/2}d\Sigma\,, \tag{11}$$

$$\frac{\phi_{,00}}{g_{00}}+\left[\frac{\gamma^{ij}\gamma_{ij,0}}{2g_{00}}-\frac{g_{00,0}}{2g_{00}{}^2}\right]\phi_{,0}-\frac{\gamma^{ij}g_{00,j}}{2g_{00}}\phi_{,i}-\Delta\phi+(m^2+\xi R)\phi=0\,, \tag{12}$$

where Δ is the Laplacian operator:

$$\Delta\phi\equiv\gamma^{ij}\phi_{\|ij}\,, \tag{13}$$

the covariant derivative $\|$ being built with the metric tensor γ_{ij} induced on Σ.

Then it is possible to write H_Σ as

$$H_\Sigma=\int_\Sigma d\Sigma\, g_{00}{}^{-1/2}\left[\tfrac{1}{2}(\phi_{,0})^2-\tfrac{1}{4}(\phi\phi_{,00}+\phi_{,00}\phi)+\frac{B}{2}(\phi\phi_{,0}+\phi_{,0}\phi)+C\phi^2\right], \tag{14}$$

where

$$B\equiv(\xi-\tfrac{1}{4})\gamma^{ij}\gamma_{ij,0}+\frac{g_{00,0}}{4g_{00}}\,, \tag{15}$$

$$C\equiv\xi\left[\frac{\gamma^{kl}\gamma^{ij}\gamma_{ik,0}\gamma_{lj,0}}{4}+\frac{\gamma^{ij}\gamma_{ij,0}g_{00,0}}{4g_{00}}-\frac{\gamma^{ij}\gamma_{ij,00}}{2}\right]. \tag{16}$$

To obtain this expression we have used the field equation (7) and Gauss's theorem on Σ:

$$\int_\Sigma d\Sigma(\gamma^{ij}g_{00}{}^{-1/2}\phi_{\|i}\phi)_{\|j}=0\ ,\qquad \int_\Sigma d\Sigma(\gamma^{ij}g_{00\|j}{}^{-1/2}\phi^2)_{\|i}=0\ , \tag{17}$$

assuming that the field goes to zero quickly enough at the infinite. [It may be seen that $\phi_{\|i}$ is a vector on Σ and g_{00} is a scalar on Σ (Ref. 13).]

Really, Eq. (14) is not an elegant one but is very advantageous because it only has ordinary time derivatives.

IV. THE VACUUM DEFINITION

Let $\{u_k,u_k^*\}$ be a basis of solutions of field equation (7). Then the field is

$$\phi(x)=\sum_k[a_ku_k(x)+a_k^\dagger u_k^*(x)] \tag{18}$$

and the Hamiltonian turns out to be

$$H_\Sigma=\int_\Sigma d\Sigma\, g_{00}{}^{-1/2}\sum_{kk'}a_ka_{k'}\left[\tfrac{1}{2}u_{k,0}u_{k',0}-\tfrac{1}{4}(u_ku_{k',00}+u_{k,00}u_{k'})+\frac{B}{2}(u_ku_{k',0}+u_{k,0}u_{k'})+Cu_ku_{k'}\right]$$
$$+\int_\Sigma d\Sigma\, g_{00}{}^{-1/2}\sum_{kk'}a_k^\dagger a_{k'}\left[\tfrac{1}{2}u_{k,0}u^*_{k',0}-\tfrac{1}{4}(u_ku^*_{k',00}+u_{k,00}u^*_{k'})+\frac{B}{2}(u_ku^*_{k',0}+u_{k,0}u_{k'})+Cu_ku^*_{k'}\right]+\text{H.c.} \tag{19}$$

$\{u_k,u_k^*\}$ is a set orthonormalized in the Klein-Gordon inner product:

$$\langle\psi,\varphi\rangle\equiv i\int_\Sigma(\psi\overleftrightarrow{\partial}_\mu\varphi^*)d\Sigma^\mu\ , \tag{20}$$

i.e.,

$$\langle u_k,u_{k'}\rangle=-\delta_{kk'}\ ,\qquad \langle u_k,u_{k'}^*\rangle=0\ ,\qquad \langle u_k^*,u_{k'}^*\rangle=\delta_{kk'}\ . \tag{21}$$

In the adapted chart the inner product reads

$$\langle\psi,\varphi\rangle=i\int_\Sigma(\psi\varphi^*_{,0}-\varphi^*\psi_{,0})g_{00}{}^{-1/2}d\Sigma\ . \tag{22}$$

Each basis of solutions has an associated vacuum state $|0\rangle$: the one defined by

$$a_k|0\rangle=0,\ \ \forall k\ . \tag{23}$$

Of course, if we change the basis, the operators $a_k,a_k^\dagger$ must change in order that ϕ should remain the same. Then $|0\rangle$ depends on the basis. Therefore to select a vacuum is equivalent to selecting the basis $\{u_k,u_k^*\}$, and a physical criterion is necessary in order to make this choice. We shall explore different criteria. In order to make the calculations easy, we shall apply the criteria in the cases when the natural time can be separated from the space variables in the field equation (7), at least in a neighborhood of Σ.

It is possible to separate variable x^0 in Eq. (7) when (1) $R=R(x^0)$, $g_{00}=g_{00}(x^0)$, $\gamma_{ij}=f(x^0)A_{ij}(x^k)$ [variable x^0 is separated multiplying Eq. (24) by $f(x^0)$]; (2) $R=R(x^k)$, $g_{00}=q(x^0)h(x^i)$, $\gamma_{ij}=A_{ij}(x^k)$ [variable x^0 is separated multiplying Eq. (24) by $h(x^i)$]; (3) $m=0$, $R=0$, $g_{00}=q(x^0)h(x^i)$, $\gamma_{ij}=f(x^0)A_{ij}(x^k)$ [variable x^0 is separated multiplying Eq. (24) by $f(x^0)h(x^i)$]; (4) when $g_{\mu\nu}=g_{\mu\nu}(x^0)$ there are solutions of Eq. (24) with the form

$$u_k=T_k(x^0)e^{i\mathbf{k}\cdot\mathbf{x}}\ .$$

Of course, in cases (1)–(3) the condition on R is the strongest one.

It is easy to see that in all the cases, the coefficients B and C in Eq. (14) are constant on Σ (they only depend on x^0). Besides, by taking a basis $\{u_k,u_k^*\}$ with the form

$$u_k(x)=T_k(x^0)E_k(x^i)\ , \tag{24}$$

Eq. (21) results

$$-\delta_{kk'}=i(T_kT^*_{k',0}-T^*_{k'}T_{k,0})\int_\Sigma E_kE^*_{k'}g_{00}{}^{-1/2}d\Sigma\ , \tag{25}$$

$$0=i(T_kT_{k',0}-T_{k'}T_{k,0})\int_\Sigma E_kE_{k'}g_{00}{}^{-1/2}d\Sigma\ . \tag{26}$$

The set $\{E_k\}$ is a basis of solutions of the spatial side of the field equation. It may be orthonormalized on Σ:

$$\int_\Sigma E_kE^*_{k'}g_{00}{}^{-1/2}d\Sigma=\delta_{kk'}\ . \tag{27}$$

Then, in Eq. (25) a condition for the Wronskian of T_k on Σ results:

$$(T_kT^*_{k,0}-T^*_kT_{k,0})|_\Sigma=i\ . \tag{28}$$

We note that, as the coefficients of the field equation are real, if E_k is a solution, E_k^* is also (for the same separation constant). Let us adopt the notation

$$E_{-k}\equiv E_k^*\ . \tag{29}$$

Then Eq. (27) reads

$$\int_\Sigma E_kE_{k'}g_{00}{}^{-1/2}d\Sigma=\delta_{k,-k'}\ . \tag{30}$$

Therefore given a value of the separation constant, we have two different modes for u_k: T_kE_k and $T_kE_k^*=T_kE_{-k}$; i.e.,

$$T_{-k}=T_k\ . \tag{31}$$

On the other hand, as the variable x^0 has been separated, the equation for $T_k(x^0)$ reads

$$T_{k,00}+bT_{k,0}+c_kT_k=0\ , \tag{32}$$

where b and c_k depend on the geometry and c_k also depends on the separation constant.

Then replacing Eq. (24) in (19), having in mind Eqs. (30) and (31), and putting $T_{k,00}$ as a function of $T_{k,0}$ and T_k [using Eq. (32)], H_Σ turns out to be

$$H_\Sigma=\sum_k a_k a_{-k}\left[\tfrac{1}{2}(T_{k,0})^2+\left[B+\frac{b}{2}\right]T_k T_{k,0}+\left[C+\frac{c_k}{2}\right]T_k{}^2\right]\Bigg|_\Sigma$$

$$+\sum_k a_k^\dagger a_k\left[\tfrac{1}{2}\,|T_{k,0}|^2+\left[B+\frac{b}{2}\right]\mathrm{Re}(T_{k,0}T_k^*)+\left[C+\frac{c_k}{2}\right]|T_k|^2\right]\Bigg|_\Sigma+\text{H.c}\ . \tag{33}$$

The functions E_k do not play any role in order to select the vacuum, because they do not appear in Eq. (33) (Ref. 14). They may be chosen taking into account some symmetry property of space or boundary conditions, etc.

To select a vacuum is equivalent to selecting a solution T_k of Eq. (32). But we may select a solution T_k giving values for T_k and $T_{k,0}$ on Σ. We study two criteria to fix the data $T_k|_\Sigma$ and $T_{k,0}|_\Sigma$.

(A) Hamiltonian diagonalization. The data are such that H_Σ has the form

$$H_\Sigma=\tfrac{1}{2}\sum_k h_k(a_k^\Sigma a_k^{\Sigma\dagger}+a_k^{\Sigma\dagger}a_k^\Sigma)\ . \tag{34}$$

We call $|0;\Sigma\rangle^D$ to the resulting vacuum. $a_k^\Sigma, a_k^{\Sigma\dagger}$ are the operators associated to the corresponding basis.

(B) Hamiltonian minimization. The data are such that ${}^M\langle 0;\Sigma|H_\Sigma|0;\Sigma\rangle^M$ turns out to be minimized, where $|0;\Sigma\rangle^M$ is the associated vacuum.

In order to obtain the data $T_k|_\Sigma$ and $T_{k,0}|_\Sigma$ that satisfy each criterion, we must take into account that they are not independent, because of the constraint (28). As a result, either the diagonalization or the minimization may be performed when

$$c_k>\left[B+\frac{b}{2}\right]^2-2C\ . \tag{35}$$

In this case both diagonalization and minimization are accomplished by the *same* data on Σ, which have the ratio

$$\frac{T_{k,0}}{T_k}\Bigg|_\Sigma=-\left[B+\frac{b}{2}\right]-i\left[2C+c_k-\left[B+\frac{b}{2}\right]^2\right]^{1/2}\Bigg|_\Sigma \tag{36}$$

as can easily be proved from Eq. (33).

Because of the normalization, $|T_k|^2|_\Sigma$ is

$$|T_k|^2|_\Sigma=\frac{1}{2}\left[2C+c_k-\left[B+\frac{b}{2}\right]^2\right]^{-1/2}\Bigg|_\Sigma\ . \tag{37}$$

Therefore $|0;\Sigma\rangle^D=|0;\Sigma\rangle^M$ and the Hamiltonian H_Σ turns out to be

$$H_\Sigma=\tfrac{1}{2}\sum_k\left[2C+c_k-\left[B+\frac{b}{2}\right]^2\right]^{1/2}\Bigg|_\Sigma\times(a_k^\Sigma a_k^{\Sigma\dagger}+a_k^{\Sigma\dagger}a_k^\Sigma)\ . \tag{38}$$

Of course, the solution $T_k(x^0)$ of Eq. (32) satisfying the required conditions on Σ, may not satisfy these conditions on other hypersurfaces Σ'. In this case $|0;\Sigma\rangle\neq|0;\Sigma'\rangle$ and there is particle creation.[3]

V. EXAMPLES

The formalism developed encompasses several well-known cases

A. v is a Killing vector in $\mathcal{N}_\Sigma$ (neighborhood of Σ)

Then the metric is independent on x^0 in $\mathcal{N}_\Sigma$ (Ref. 13). Therefore,

$$B=0,\quad C=0,\quad b=0,\quad c_k=\omega_k{}^2\ , \tag{39}$$

$$u_k(x)=\frac{e^{-i\omega_k x^0}}{(2\omega_k)^{1/2}}E_k(x^i)\ , \tag{40}$$

in $\mathcal{N}_\Sigma$, where $\omega_k{}^2$ is the separation constant. The Hamiltonian reads

$$H_\Sigma=\tfrac{1}{2}\sum_k\omega_k(a_k^\Sigma a_k^{\Sigma\dagger}+a_k^{\Sigma\dagger}a_k^\Sigma)\ . \tag{41}$$

Minkowski's observers and Rindler's observers in flat space-time, comoving observers in Schwarzschild's metric,[15] in the static Einstein metric,[16,17] and also the Killing observers in de Sitter space are particular examples of this case.

B. Conformal case

Let us consider two different space-times such that in each of them there is an observer system (both space-times may eventually be the same). Let us suppose that, when the adapted charts are used, the components of the metric tensors of both space-times $g_{\mu\nu}$ and $\bar g_{\mu\nu}$ satisfy the equation

$$\bar g_{\mu\nu}(x)=\Omega^2(x)g_{\mu\nu}(x)\ . \tag{42}$$

Then

$$d\bar\Sigma=\Omega^{n-1}d\Sigma\ , \tag{43}$$

$$\bar B=B+[2(\xi-\tfrac{1}{4})(n-1)+\tfrac{1}{2}]\frac{\Omega_{,0}}{\Omega}\ , \tag{44}$$

$$\frac{\bar C}{\xi}=\frac{C}{\xi}+(n-1)\left[\left[\frac{\Omega_{,0}}{\Omega}\right]^2-\frac{\Omega_{,00}}{\Omega}\right]-\frac{1}{2}\left[\gamma^{ij}\gamma_{ij,0}-(n-1)\frac{g_{00,0}}{g_{00}}\right]\frac{\Omega_{,0}}{\Omega}\ . \tag{45}$$

Let

$$\bar\phi\equiv\Omega^{(2-n)/2}\phi\ . \tag{46}$$

Thus, we obtain

$$\bar{H}_\Sigma[\bar{\phi}]=H_\Sigma[\phi]+\left[\xi(n-1)-\frac{n-2}{4}\right]$$
$$\times\int d\Sigma\, g_{00}{}^{-1/2}\left\{\left[\frac{\Omega_{,0}}{\Omega}\right](\phi_{,0}\phi+\phi\phi_{,0})-\left[\frac{\Omega_{,00}}{\Omega}+(n-3)\left[\frac{\Omega_{,0}}{\Omega}\right]^2+\frac{1}{2}\left[\frac{\Omega_{,0}}{\Omega}\right]\left[\gamma^{ij}\gamma_{ij,0}-\frac{g_{00,0}}{g_{00}}\right]\right]\phi^2\right\}, \tag{47}$$

where $\bar{H}_\Sigma[\bar{\phi}]$ is the Hamiltonian built with $\bar{\phi}$ in the metric $\bar{g}_{\mu\nu}$. When the field is massless and $\xi=(n-2)/4(n-1)$ (conformal coupling) it occurs that, if ϕ solves the field equation in metric $\mathbf{g}$, then $\bar{\phi}$ solves this equation in the metric $\bar{\mathbf{g}}$ (Ref. 3). Furthermore, because of Eq. (47), we get

$$\bar{H}_\Sigma[\bar{\phi}]=H_\Sigma[\phi]\ . \tag{48}$$

Therefore if we know the basis of solutions $\{u_k,u_k^*\}$ that perform the Hamiltonian diagonalization in metric $g_{\mu\nu}$, then we also know, via Eq. (48), that $\{\Omega^{(2-n)/2}u_k,\Omega^{(2-n)/2}u_k^*\}$ diagonalizes the Hamiltonian in metric $\bar{g}_{\mu\nu}$.

References 5, 6, and 18 are examples of this case. Also in Ref. 19 we have selected a vacuum for a massless field in a two-dimensional flat space-time bounded by two moving mirrors. According to the present paper, that vacuum is the one perceived by the observer whose lines of universe are drawn in Fig. 7 of Ref. 19. The Kruskal vacuum, in a two-dimensional eternal black hole, is another example.[3]

C. Geodesic fluid in a Robertson-Walker universe

In the adapted chart the line element is

$$ds^2=dt^2-a^2(t)[d\chi^2+f^2(\chi)(d\theta^2+\sin^2\theta\, d\phi^2)]\ , \tag{49}$$

where

$$f(\chi)=\begin{cases}\sin\chi,\ \ 0\le\chi\le2\pi\ \text{ spatially closed } (K=+1)\ ,\\ \chi,\ \ 0\le\chi<\infty\ \text{ spatially flat } (K=0)\ ,\\ \sinh\chi,\ \ 0\le\chi<\infty\ \text{ spatially hyperbolic } (K=-1)\ .\end{cases}$$

The curvature scalar is

$$R=\frac{6}{a^2}(a\ddot{a}+\dot{a}^2+K) \tag{50}$$

and coefficients B, C, b, c_k are

$$B=6(\xi-\tfrac{1}{4})\frac{\dot{a}}{a},\quad C=-3\xi\frac{\ddot{a}}{a}\ ,$$
$$b=3\frac{\dot{a}}{a},\quad c_k=m^2+\frac{k^2}{a^2}+\frac{6\xi}{a^2}(a\ddot{a}+\dot{a}^2+K)\ , \tag{51}$$

where k^2 is the separation constant. Condition (35) results

$$m^2+\frac{k^2}{a^2}>6\xi\left[(6\xi-1)\left[\frac{\dot{a}}{a}\right]^2-\frac{K}{a^2}\right]. \tag{52}$$

The data on Σ are

$$\left.\frac{T_{k,0}}{T_k}\right|_\Sigma=-6\xi\frac{\dot{a}}{a}-i\left[m^2+\frac{k^2+6\xi K}{a^2}+\left[\frac{\dot{a}}{a}\right]^2 6\xi(1-6\xi)\right]^{1/2}. \tag{53}$$

Then the Hamiltonian reads

$$H_\Sigma=\tfrac{1}{2}\sum_k\left[m^2+\frac{k^2+6\xi K}{a^2}+\left[\frac{\dot{a}}{a}\right]^2 6\xi(1-6\xi)\right]^{1/2}\times(a_k^\Sigma a_k^{\Sigma\dagger}+a_k^{\Sigma\dagger}a_k^\Sigma)\ . \tag{54}$$

Here the summation over k has a different character according to $K=+1$, 0, or -1, because the labels of u_k may be discrete or continuous depending on K.

D. For geodesic fluid in a Bianchi type-I universe (see Ref. 7)

In the last two cases the Hamiltonian diagonalization and minimization, even if they define univocally a quantum ground state, do not always yield a satisfactory vacuum endowed with the usual properties of the flat space-time vacuum. This phenomenon happens because a local condition is not satisfied for all the values of ξ and m, as it can be seen in Refs. 7 and 8. Essentially this condition, which we do not study in this paper, is to ask that the vacuum expectation value of the renormalized energy-momentum tensor turn out to be finite.

VI. CONCLUSION

We have reexamined the notion of a reference system and we have concluded that the role of an observer system is to establish what the time is and what the space is. This fact is important because a detector perceives the difference between time and space. Equal detectors in different reference systems do not obtain the same measurements at the same point because they do not agree on their definitions of time and space. Then it is a natural result that the notion of vacuum depends on the observer.

We have defined a Hamiltonian invariant by coordinate changes but observer dependent. In fact the integral in Eq. (10) is evaluated on a hypersurface Σ orthogonal to the lines of the reference fluid (the "space") and vector **v** appears in this integral, which is essential in order to distinguish observer systems on a given Σ. The aim of this dependence is to obtain quantum vacua dependent on the observer, as is commonly accepted in the literature.[3–5] But there is other evidence that support our definition for H_Σ. For example, if we would use

$$\mathscr{H}_\Sigma \equiv \int_\Sigma T_{\mu\nu} u^\mu d\Sigma^\nu$$

in an adapted chart $\mathscr{H}_\Sigma$ would be

$$\mathscr{H}_\Sigma = \int_\Sigma T_{00} g_{00}{}^{-1} d\Sigma \ .$$

Then a factor $g_{00}{}^{-1}$ appears at the place of the factor $g_{00}{}^{-1/2}$ in Eq. (11). But the factor $g_{00}{}^{-1/2}$ was essential for the diagonalization and minimization of H_Σ [see Eqs. (25)–(33)]. Furthermore, the presence of the factor $g_{00}{}^{-1/2}$ is also compelling in order that $\overline{H}_\Sigma$ would coincide with H_Σ in the conformal case. Therefore $\mathscr{H}_\Sigma$ and any other Hamiltonian must be discarded.

The separation of variables is not an essential condition for the Hamiltonian diagonalization. It only makes the calculations easier. For example, there are conformal cases where variables do not separate, but diagonalization still works. On the contrary the use of an irrotational fluid is certainly essential, because it makes possible the notion of time.

The above results do not depend on the natural time used. If we use $T'=T'(T)$ then vector $\mathbf{v}'=(dT/dT')\mathbf{v}$ appears in the definition of the Hamiltonian and we obtain $H'_\Sigma=(dT/dT')H_\Sigma$, which means that energy and time are conjugated variables. This physical property may be a good motivation in order to induce definition (10).

To conclude we wish to point out that a good vacuum must satisfy other requirements which were not studied in this paper: the vacuum must render the theory renormalizable. Thus the conjugation of a global condition (diagonalization and minimization of the Hamiltonian) and a local condition (renormalizability) is necessary in order to have a good vacuum.

ACKNOWLEDGMENT

One of us (R.F.) would like to thank the Consejo de Investigaciones Científicas y Tecnicas for financial support.

APPENDIX

We shall enumerate the tensorial quantities that characterize the behavior of a fluid.[9] Let **u** be the unitary vector tangent to the lines of fluid; then, we define the curvature vector

$$C_\mu \equiv u^\nu \nabla_\nu u_\mu \ ,$$

Killing tensor

$$K_{\mu\nu} \equiv \nabla_\mu u_\nu + \nabla_\nu u_\mu \ ,$$

and vortex tensor

$$\Omega_{\mu\nu} \equiv \nabla_\mu u_\nu - \nabla_\nu u_\mu = \partial_\mu u_\nu - \partial_\nu u_\mu \ .$$

In addition, we define

$$\gamma_{\mu\nu} \equiv u_\mu u_\nu - g_{\mu\nu} \ .$$

Then $u_\mu u^\nu$ and $-\gamma_\mu{}^\nu$ are projectors. Let us define

$$\tilde{A}_{\mu\nu} \equiv \gamma_\mu{}^a \gamma_\nu{}^\rho A_{\lambda\rho} \ .$$

Then, it is easy to see that

$$K_{\mu\nu} = \tilde{K}_{\mu\nu} + u_\mu C_\nu + u_\nu C_\mu \ ,$$

$$\Omega_{\mu\nu} = \tilde{\Omega}_{\mu\nu} + u_\mu C_\nu - u_\nu C_\mu \ .$$

When $C_\mu = 0$ the fluid is a geodesic one.

When $\tilde{\Omega}_{\mu\nu}=0$ the fluid is said to be irrotational or curl free. It can be proved that $\tilde{\Omega}_{\mu\nu}=0$ is a necessary and sufficient condition for the existence of global hypersurfaces orthogonal to the fluid lines.[9]

When $\Omega_{\mu\nu}=0$ the fluid is geodesic and irrotational.

When $\tilde{K}_{\mu\nu}=0$ the fluid is said to be rigid (according to Born[20]).

When $K_{\mu\nu}=0$ the fluid is geodesic and rigid. Besides **u** is a Killing vector field.

When $\tilde{\Omega}_{\mu\nu}=0$ an adapted chart system exists where $x^i=\text{const}$ $(i=1,2,3)$, on each line of fluid and $g_{0i}=0$. Then in this system

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & 0 \\ 0 & -\gamma_{ij} \end{bmatrix}, \quad u^\mu = (g_{00}{}^{-1/2}, 0,0,0) \ ,$$

$$u_\mu = (g_{00}{}^{1/2}, 0,0,0), \quad C_\mu = (0, \tfrac{1}{2}(\ln g_{00})_{,i}) \ ,$$

$$\Omega_{\mu\nu} = \tfrac{1}{2} g_{00}{}^{-1/2} \begin{bmatrix} 0 & -g_{00,i} \\ g_{00,i} & 0 \end{bmatrix} ,$$

$$K_{\mu\nu} = -g_{00}{}^{-1/2} \begin{bmatrix} 0 & g_{00,i} \\ g_{00,i} & \gamma_{ij,0} \end{bmatrix} .$$

Therefore, when $\tilde{K}_{\mu\nu}=0$ the distance between matter points of fluid do not change.

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