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Constraint on 't Hooft indices in preon models with complementarity

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We reexamine the SU(7) preon model of Dimopoulos, Raby, and Susskind that is the only model known to violate the conjecture that the 't Hooft indices for composite models satisfying complementarity are bounded in magnitude by 1. We show that the SU(7) model is in accord with this conjecture due to the hidden "family symmetry" SU(2).

In a previous paper with Gérard,[1] we conjectured that the 't Hooft indices l_i (Ref. 2) for preon models satisfying complementarity[3] are bounded in magnitude by 1, i.e.,

$$|\, l_i \,| \leq 1 \;. \tag{1}$$

We did not have a proof for this statement, but we argued as follows. When $|\, l_i \,| > 1$, we have degeneracies with massless fermions in the Higgs phase. However, n-fold degeneracies in the Higgs phase would imply the existence of a "new" SU(n) global symmetry which is not a subgroup of the original global color-flavor symmetry. Our view was thus that some "family symmetry," such as U(1) and SU(2), contained in the color-flavor symmetry, prevents degeneracies of surviving massless fermions in the Higgs phase, requiring condition (1). We remarked that except for the SU(7) model in Ref. 3, which seems to lead to $l=3$, all other models consistent with complementarity do give $|\, l_i \,| \leq 1$. In this paper we reexamine the SU(7) model and show that an extra "family symmetry" SU(2) does exist, supporting our conjecture (1).

The model[3] is based on the metacolor group $SU(7)_{MC}$ with three preons in the **35** representation of $SU(7)_{MC}$ and two presons in the $\overline{\mathbf{21}}$ representation of $SU(7)_{MC}$. The global color-flavor symmetry is thus $SU(3)_F \times SU(2)_F \times U(1)_F$, and the preons are given by[3]

$$\begin{aligned} \psi &= (\mathbf{35};\bar{\mathbf{3}},\mathbf{1},-\mathbf{1}) \\ \chi &= (\overline{\mathbf{21}};,\mathbf{1},\mathbf{2},\mathbf{3}) \;, \end{aligned} \tag{2}$$

under $SU(7)_{MC}\times SU(3)_F\times SU(2)_F\times U(1)_F$. The first most attractive channel (MAC) condensate transforms like the antisymmetric $\bar{\mathbf{7}}$, i.e.,

$$\mathbf{35}\times\mathbf{35}\rightarrow\bar{\mathbf{7}}_A \;, \tag{3}$$

under $SU(7)_{MC}$. By the meta-Pauli principle, this MAC condensate has the representation

$$\phi=(\bar{\mathbf{7}}_A;\mathbf{3}_A,\mathbf{1},-\mathbf{2}) \;, \tag{4}$$

under $SU(7)_{MC}\times SU(3)_F\times SU(2)_F\times U(1)_F$. This breaks the symmetry down to

$$SU(4)_{MC}\times SU(3)'_F\times SU(2)_F\times U(1)'_F \;, \tag{5}$$

where $SU(3)'_F$ is the diagonal subgroup of $SU(3)_F$ and an SU(3) subgroup $SU(3)_{MC}$ of $SU(7)_{MC}$. $U(1)'_F$ is a linear combination of $U(1)_F$ and $U(1)_{MC}$ coming from the breaking of $SU(7)_{MC}$ into $SU(4)_{MC}\times SU(3)_{MC}\times U(1)_{MC}$. The remaining massless fermions are, under (5),

$$(\mathbf{1};\bar{\mathbf{3}},\mathbf{1},-\mathbf{7})+(\mathbf{1};\mathbf{3},\mathbf{2},\mathbf{7})+(\mathbf{4};\bar{\mathbf{6}},\mathbf{1},-\tfrac{7}{2})+(\bar{\mathbf{4}};\bar{\mathbf{3}},\mathbf{2},\tfrac{7}{2}) \;. \tag{6}$$

Here we differ from Dimopoulos, Raby, and Susskind[3] in that we do not have $(\mathbf{6};\mathbf{1},\mathbf{2},\mathbf{0})$ as a massless fermion. Since $(\mathbf{6};\mathbf{1},\mathbf{2},\mathbf{0})$ is a real representation, we assume that it acquires a mass of the order of the MAC condensate in line with Georgi's survival hypothesis.[4]

The next MAC condensate is given by

$$\mathbf{4}\times\bar{\mathbf{4}}\rightarrow\mathbf{1} \;, \tag{7}$$

under $SU(4)_{MC}$. This condensate has the representation

$$\phi'=(\mathbf{1};\overline{\mathbf{10}},\mathbf{2},\mathbf{0}) \;\text{ or }\; \phi'=(\mathbf{1};\mathbf{8},\mathbf{2},\mathbf{0}) \;, \tag{8}$$

under (5). It is easy to check that both choices in (8) lead to exactly the same results. The condensate ϕ' does not break $SU(4)_C$ but breaks both $SU(3)'_F$ and $SU(2)_F$; however, we can save the diagonal subgroup $SU(2)'_F$ of $SU(2)_F$ and an SU(2) subgroup of $SU(3)'_F$. We are left with the symmetry

$$SU(4)_{MC}\times SU(2)'_F\times U(1)'_F \;. \tag{9}$$

Considering the branching of the fermions in (6), one finds that all but one fermion pair up to form massive Dirac fermions, leaving only one massless fermion,

$$(\mathbf{1};\mathbf{3},\mathbf{7}) \;, \tag{10}$$

under (9). Since we have only a metacolor singlet as the massless fermion, tumbling stops here. Note that Dimopoulos, Raby, and Susskind[3] end up with three copies of $(\mathbf{1};\mathbf{7})$ under $SU(4)_{MC}\times U(1)_F$, while in our case this threefold degeneracy is lifted as a triplet under the family symmetry $SU(2)'_F$.

Finally, we consider the SU(7) model in the confining phase. Following the Higgs phase, we assume that $SU(3)_F\times SU(2)_F$ in the color-flavor symmetry breaks into the diagonal $SU(2)'_F$. The global symmetry to which we should apply 't Hooft anomaly matching[2] is thus

$$SU(2)'_F \times U(1)_F . \tag{11}$$

The branching of the fermions in (2) will then give three preons:

$$\begin{aligned} \psi'_1 &= (35;1,-1) , \\ \psi'_2 &= (35;2,-1) , \\ \chi' &= (\overline{21};2,3) , \end{aligned} \tag{12}$$

under $SU(7)_{MC} \times SU(2)'_F \times U(1)_F$. We can form many $SU(7)_{MC}$-singlet fermion composites out of these fermions. If we take account of the meta-Pauli principle we have the composites listed in Table I. The 't Hooft anomaly-matching equations are

TABLE I. Composite fermions.

Composites	$SU(7)_{MC}$	$SU(2)'_F$	$U(1)_F$	Indices
$\bar\psi_1'^{\,4}\chi'$	1	2	7	l_1
$\bar\psi_1'^{\,3}\bar\psi_2'\chi'$	1	1,3	7	l_2, l_3
$\bar\psi_1'^{\,2}\bar\psi_2'^{\,2}\chi'$	1	2,4	7	l'_1, l_4
$\bar\psi_1'\bar\psi_2'^{\,3}\chi'$	1	1,3,5	7	l'_2, l'_3, l_5
$\bar\psi_2'^{\,4}\chi'$	1	2,4,6	7	l''_1, l'_4, l_6
$\psi'_1\bar\chi'\bar\chi'$	1	3	-7	l_7
$\psi'_2\bar\chi'\bar\chi'$	1	2,4	-7	l_8, l_9

$$\begin{aligned} [U(1)_F]^3:\ 1029 = {} & 686(l_1+l'_1+l''_1)+343(l_2+l'_2)+1029(l_3+l'_3)+1372(l_4+l'_4)+1715l_5 \\ & +2058l_6-1029l_7-686l_8-1372l_9 , \end{aligned} \tag{13a}$$

$$[SU(2)'_F]^2U(1)_F:\ 28 = 7(l_1+l'_1+l''_1)+28(l_3+l'_3)+70(l_4+l'_4)+140l_5+245l_6-28l_7-7l_8-70l_9 . \tag{13b}$$

One of the solutions is $(l_3+l'_3)=1$, all other $l_i=0$, corresponding to the same massless (composite) fermion as in the Higgs phase [see Eq. (10)]. Complementarity thus holds with the 't Hooft index not exceeding unity.

In this paper, we reexamined the SU(7) model of Ref. 3 and showed that it is consistent with our conjecture [Eq. (1)] about the constraint on the 't Hooft indices l_i. Therefore, we expect tumbling complementarity to always lead to 't Hooft anomaly-matching solutions with $l_i=0$ or 1. The corollary of this conclusion is that a preon-model solution of the generation problem satisfying complementarity will require the natural emergence of a "family" group.

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[1]J.-M. Gérard, Y. Okamoto, and R. E. Marshak, Phys. Lett. **169B**, 386 (1986); this conjecture had already been made by I. Bars [Nucl. Phys. **B207**, 77 (1982)] on the basis of family symmetry considerations.

[2]G. 't Hooft, in *Recent Development in Gauge Theories*, proceedings of the NATO Advanced Study Institute, Cargèse, 1979, edited by G. 't Hooft *et al.* (Plenum, New York, 1980).

[3]S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. **B173**, 208 (1980).

[4]H. Georgi, Nucl. Phys. **B156**, 126 (1979). Presumably, Dimopoulos, Raby, and Susskind retain $(\mathbf{6},\mathbf{1},\mathbf{2},\mathbf{0})$ as a massless fermion because the meta-Pauli principle requires the scalar condensate to be $(1_S;1,3_s,0)$ rather than $(1_s;1,1_A,0)$ under $SU(4)_{MC}\times SU(3)'_F\times SU(2)_F\times U(1)'_F$. We argue that the second possibility can be realized and the survival hypothesis maintained through a "higher-order" scalar condensate.