PHYSICAL REVIEW D VOLUME 34, NUMBER 12 15 DECEMBER 1986

Brief Reports

Brief Reports are short papers which report on completed research which, while meeting the usual **Physical Review** *standards of scientific quality, does not warrant a regular article. (Addenda to papers previously published in the* **Physical Review** *by the same authors are included in Brief Reports.) A Brief Report may be no longer than $3\frac{1}{2}$ printed pages and must be accompanied by an abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Supersymmetric effective action in three dimensions

Nivedita Deo
Purdue University, West Lafayette, Indiana 47907
(Received 18 August 1986)

A perturbative calculation of the parity-violating part of the supersymmetric effective action for a scalar superfield coupled to a background-gauge superfield in three dimensions is presented using the massless version of the Bogolubov-Parasiuk-Hepp-Zimmermann renormalization procedure.

I. INTRODUCTION

Classical symmetries do not always carry over to quantum symmetries. Examples occur for the class of theories which involve fermions coupled to gauge and gravitational fields. The case can arise on quantizing such theories, in even dimensions, when the divergence of the classically conserved currents corresponding to chiral, Lorentz, and Einstein symmetries acquire anomalous terms. Hence the gauge invariance is broken and the theory must be modified for a consistent quantum-mechanical definition.[1] Analogously, in odd dimensions the effective action is not invariant under gauge, general coordinate, and parity transformations even though the classical action from which it is derived is constructed to be invariant. But in contrast with the even-dimensional case, these quantum effects do not violate the consistent definition of the odd-dimensional theories though they may put restrictions on the parameters of the theories.[2,3]

The purpose of this paper is to consistently define a perturbatively renormalized effective action describing the interaction of massless scalar superfields with gauge superfields in three-dimensional space-time.[4] For this purpose we will use a manifestly supersymmetric version of Bogolubov-Parasiuk-Hepp-Zimmermann- (BPHZ) renormalized perturbation theory generalized to include massless fields.[5] Although in this paper the gauge superfields are background fields, the BPHZ algorithm provides a means for integrating over these fields in a perturbatively well-defined manner. Within this context we determine the parity-violating part of the supersymmetric effective action, explicitly showing that it is given by the supersymmetric extension of the Chern-Simons secondary topological invariant in three dimensions. The nonperturbative evaluation of the parity-violating part of the effective action, as shown in Ref. 4, results, in addition, in an even-integer-valued nonlocal functional of the potential as well as the local Chern-Simons term for the case of boundary conditions which allow compactification of R^D to S^D. The Chern-Simons term cannot be canceled by local counterterms [in the $U(n>1)$, odd number of fermion case] since it is needed to cancel the gauge variation of the discrete topological term.

II. THE EFFECTIVE ACTION

The three-dimensional supersymmetric action for a massless scalar superfield $(\phi,\bar{\phi})$ interacting with external vector superfield (V_a) from which the effective action will be obtained is

$$S=\int \bar{\nabla}\bar{\phi}\nabla\phi\, d^3x\, d^2\theta \;, \tag{2.1}$$

where the gauge ∇ and supersymmetric D covariant derivatives are

$$\nabla_a\phi=(D_a-iV_a)\phi \;, \tag{2.2}$$

$$D_a=\frac{\partial}{\partial\bar{\theta}_a}-i(\gamma^\mu\theta)_a\partial_\mu \;, \tag{2.3}$$

with the conjugate derivatives given by

$$\bar{\nabla}_a\bar{\phi}=(\bar{D}_a+i\bar{V}_a)\bar{\phi} \;, \tag{2.4}$$

$$\bar{D}_a=\gamma^0_{ab}D_b \;, \tag{2.5}$$

and

$$\bar{V}_a=\gamma^0_{ab}V_b \;. \tag{2.6}$$

The notations and conventions employed here are those of Ref. 6.

The action is invariant under supersymmetric gauge transformations:

$$\phi'=\exp(i\Lambda)\phi=U\phi \;, \tag{2.7}$$

$$\bar{\phi}'=\bar{\phi}\exp(-i\Lambda)=\bar{\phi}U^{-1} \;, \tag{2.8}$$

$$V'_a = U(V_a + iD_a)U^{-1}\ , \tag{2.9}$$

where Λ is the scalar superfield gauge parameter. These as well as the gauge superfields are understood to be Lie-algebra valued:

$$\Lambda = \Lambda^i T_i\ , \tag{2.10}$$

$$V_a = V_a^i T_i\ , \tag{2.11}$$

with T_i the scalar superfield representation matrices. The infinitesimal transformation for the spinor superfields is

$$V'_a = V_a + \nabla_a \Lambda = V_a + D_a \Lambda - i[V_a,\Lambda]\ . \tag{2.12}$$

The action is also invariant under parity transformations:

$$\phi(x,\theta) \xrightarrow{P} \phi^P = \eta_\phi \phi(x_p,\theta_p)\ , \tag{2.13}$$

$$V_a(x,\theta) \xrightarrow{P} V_a^P = -\eta_V i\gamma^0_{ab} V_b(x_p,\theta_p)\ , \tag{2.14}$$

where $\eta_{\phi,V}$ are the intrinsic parities of ϕ and V. In three dimensions the parity transformation inverts only one space coordinate so that

$$x_p^\mu = (x^0, -x^1, x^2)\ , \tag{2.15}$$

$$\theta_{pa} = i\gamma^0_{ab}\theta_b\ , \tag{2.16}$$

$$\bar\theta_{pa} = -i\bar\theta_b \gamma^0_{ba}\ . \tag{2.17}$$

The vacuum persistence amplitude in the presence of the background supergauge field is given as

$$Z[V] = \int [d\phi][d\bar\phi]\exp(iS)\ . \tag{2.18}$$

It is not well defined because of UV divergences. But we cannot simply subtract at zero momentum since the scalar superfields are massless and we would induce spurious IR divergences. The generating functional can be consistently defined by means of Zimmermann's massless version of the BPHZ renormalization scheme extended to superspace. This amounts to adding an auxiliary mass term

$$\int d^3x\, d^2\theta(-i(1-s)m\bar\phi\phi)\ , \tag{2.19}$$

to the action which is parity violating but gauge invariant. The UV divergences are removed by subtracting the Taylor series in not only the external momenta but also the auxiliary mass parameter s, and the Grassmann coordinates θ of the integrand. The mass subtractions are at $s=0$ (i.e., at nonzero mass), so as not to introduce IR divergences, with a final post subtraction at $s=1$ (i.e., at zero mass) to guarantee that the massless limit is smooth. See Ref. 5 for the details of the subtraction procedure.

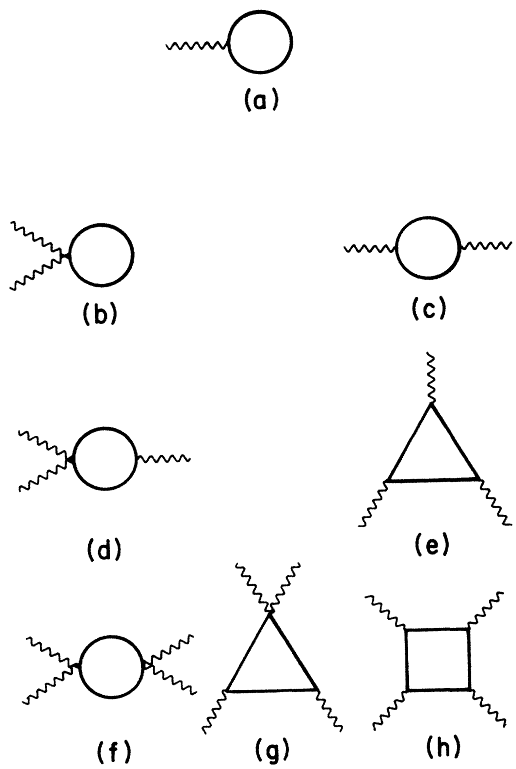

FIG. 1. The divergent supergraphs.

The renormalized one-particle-irreducible (1PI) generating functional is

$$\Gamma[V] = \ln Z^c = \left\langle 0 \left| T\exp\left[i\int d^3x\, d^2\theta N_2^2(-i\bar D\bar\phi V\phi + i\bar V\bar\phi D\phi + \bar V\bar\phi V\phi)\right] \right| 0\right\rangle^{1\mathrm{PI}}\Bigg|_{s=1}\ . \tag{2.20}$$

The normal-product symbol N_2^2 indicates the number of subtractions needed to render the 1PI generating functional finite. The only divergent integrals are the one-, two-, three-, and four-point functions; they are shown in Fig. 1.

The propagator in momentum space for the scalar superfield is found from the free generating functional

$$Z^0[V] = \int [d\phi][d\bar\phi]\exp\left[i\int [\bar D_a\bar\phi D_a\phi - (1-s)m\bar\phi\phi] d^3x\, d^2\theta \right] \tag{2.21}$$

to be

$$\langle 0|T\tilde{\bar\phi}(p_1,\theta_1)\tilde\phi(p_2,\theta_2)|0\rangle = \frac{\bar D_1 D_1 + (1-s)m}{p^2-(1-s)^2m^2}\delta^2(\theta_1-\theta_2)\delta^3(p_1-p_2)\ . \tag{2.22}$$

D_1 is the momentum-space supercovariant derivative at point (p_1,θ_1):

$$D_{1a}=\left[\frac{\partial}{\partial\bar{\theta}_{1a}}+\gamma^{\mu}_{ab}\theta_{1b}p_{1\mu}\right] . \tag{2.23}$$

In general the effective action will consist of a nonanalytic piece that preserves parity, Γ_+, and an anomalous parity-violating piece Γ_- (Refs. 3 and 4):

$$\Gamma[V]=\Gamma_+[V]+\Gamma_-[V] . \tag{2.24}$$

The parity-violating terms arise from the auxiliary mass subtractions needed to avoid spurious IR divergences. These are given by the parity-odd parts of the divergent diagrams shown in Fig. 1. The tadpole diagrams Figs. 1(a) and 1(b) vanish for BPHZ massless subtractions. The calculations involved can be illustrated by focusing on the V_a two-point-function contribution to the effective action, $\Gamma^{(2)}[V]$, i.e., Fig. 1(c):

$$\begin{aligned}\Gamma^{(2)}[V]=\frac{i^2}{2!}\mathrm{Tr}\int\frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\delta^3(q_1+q_2)d^2\theta_1 d^2\theta_2 V_{1a}(q_1)V_{2b}(q_2)(1-\tau^1_{q,s,\theta})\\
\times\int\frac{d^3k_1}{(2\pi)^3}\Bigg[i\bar{D}_{1a}\frac{\bar{D}_1D_1+(1-s)m}{4k_1{}^2-(1-s)^2m^2}\delta^2(\theta_1-\theta_2)(-i)\bar{D}_{2b}\frac{\bar{D}_2D_2+(1-s)m}{4(k_1+q_2)^2-(1-s)^2m^2}\delta^2(\theta_2-\theta_1)\\
+(-i)\bar{D}_{1a}(-i)\bar{D}_{2b}\frac{\bar{D}_1D_1+(1-s)m}{4k_1{}^2-(1-s)^2m^2}\delta^2(\theta_1-\theta_2)\frac{\bar{D}_2D_2+(1-s)m}{4(k_1+q_2)^2-(1-s)^2m^2}\delta^2(\theta_2-\theta_1)\\
+\frac{\bar{D}_1D_1+(1-s)m}{4k_1{}^2-(1-s)^2m^2}\delta^2(\theta_1-\theta_2)(-i)\bar{D}_{2b}i\bar{D}_{1a}\frac{\bar{D}_2D_2+(1-s)m}{4(k_1+q_2)^2-(1-s)^2m^2}\delta^2(\theta_2-\theta_1)\\
+(-i)\bar{D}_{2b}\frac{\bar{D}_1D_1+(1-s)m}{4k_1{}^2-(1-s)^2m^2}\delta^2(\theta_1-\theta_2)(-i)\bar{D}_{1a}\frac{\bar{D}_2D_2+(1-s)m}{4(k_1+q_2)^2-(1-s)^2m^2}\delta^2(\theta_2-\theta_1)\Bigg] ,\end{aligned} \tag{2.25}$$

where $\tau^1_{q,s,\theta}$ indicates the first-order supersymmetric, auxiliary mass BPHZ Taylor-series operator in q,s,θ. The terms with numerator linear in the mass (parity-violating terms) are isolated and the D algebra performed using the identities

$$\bar{D}_aD_b=\tfrac{1}{2}\delta_{ab}\bar{D}D+i\gamma^{\mu}_{ba}\partial_{\mu} , \tag{2.26}$$

$$\bar{D}DD_a=-2i(\gamma^{\mu}\partial_{\mu}D)_a=-D_a\bar{D}D , \tag{2.27}$$

$$-\tfrac{1}{4}\bar{D}D\bar{D}D=\Box , \tag{2.28}$$

$$\delta^2(\theta_2-\theta_1)\delta^2(\theta_1-\theta_2)=0 , \tag{2.29}$$

$$\delta^2(\theta_2-\theta_1)\bar{D}_{1a}\delta^2(\theta_1-\theta_2)=0 , \tag{2.30}$$

$$\delta^2(\theta_2-\theta_1)\bar{D}D\delta^2(\theta_1-\theta_2)=-\delta^2(\theta_1-\theta_2) , \tag{2.31}$$

$$\delta^2(\theta_2-\theta_1)\bar{D}D\bar{D}_b(k)\delta^2(\theta_1-\theta_2)=0 , \tag{2.32}$$

$$\delta^2(\theta_2-\theta_1)\bar{D}_a\bar{D}D\bar{D}_b(k)\delta^2(\theta_1-\theta_2)=-\gamma^0_{bd}\gamma^{\mu}_{da}k_{\mu}\delta^2(\theta_1-\theta_2) . \tag{2.33}$$

Bringing the Grassmann coordinates through the Taylor-series operator, the θ integration can be easily done so that the integral reduces to a point in θ space. Only the VV and $DVDV$ terms survive and the contribution to Γ_- is

$$\begin{aligned}\Gamma^{(2)}_-[V]=\frac{(i)^2}{2!}\mathrm{Tr}\left[\int\frac{d^3q_1}{(2\pi)^3}d^2\theta_1V_{1a}(q_1)V_{1b}(-q_1)\gamma^0_{bd}\gamma^{\mu}_{da}(-q_1)_{\mu}2+\int\frac{d^3q_1}{(2\pi)^3}d^2\theta_1\bar{V}_{1a}(q_1)D_{1a}\bar{D}_{1b}V_{1b}(-q_1)\right]\\
\times\int\frac{d^3k}{(2\pi)^3}(1-t^0_{q_1,s})\frac{2m(1-s)}{[4k_1{}^2-m^2(1-s)^2][4(k_1-q_1)^2-m^2(1-s)^2]}\Bigg|_{s=1}\end{aligned} \tag{2.34}$$

with $t^0_{q_1,s}$ the remaining zeroth-order momentum space and auxiliary mass subtraction. The momentum integral is evaluated to yield

$$\begin{aligned}\int\frac{d^3k_1}{(2\pi)^3}(1-t^0_{q_1,s})\frac{2m(1-s)}{[4k_1{}^2-m^2(1-s)^2][4(k_1-q_1)^2-m^2(1-s)^2]}\Bigg|_{s=1}&=\int\frac{d^3k_1}{(2\pi)^3}\frac{-2m}{(4k_1{}^2-m^2)^2}\\
&=\frac{(-m)}{32\pi i\sqrt{m}} .\end{aligned} \tag{2.35}$$

Thus we secure the parity-violating two-point-function contribution to the effective action

$$\Gamma_{-}^{(2)}[V]=\mathrm{Tr}\int d^3x\, d^2\theta_1 \partial_\mu V_{1a} V_{1b}\gamma^0_{bd}\gamma^\mu_{da}\frac{(-m)}{32\pi(m^2)^{1/2}}+\frac{im}{64\pi(m^2)^{1/2}}\mathrm{Tr}\int d^3x\, d^2\theta_1 \bar{V}_{1a}\bar{D}_{1b}D_{1a}V_{1b}(x)$$
$$+\frac{m}{32\pi(m^2)^{1/2}}\mathrm{Tr}\int d^3x\, d^2\theta_1\partial_\mu V_{1a}V_{1b}\gamma^0_{bd}\gamma^\mu_{da}$$
$$=\frac{im}{32\pi(m^2)^{1/2}}\int d^3x\, d^2\theta_1\,\mathrm{Tr}[\tfrac{1}{2}\bar{V}_{1a}\bar{D}_{1b}D_{1a}V_{1b}(x)]\ , \tag{2.36}$$

where we have used the anticommutation relation

$$\{D_a,\bar{D}_b\}=2i\gamma^\mu_{ab}\partial_\mu \tag{2.37}$$

in the last step.

Combining the contributions from Figs. 1(d) and 1(e) and 1(f), 1(g), and 1(h) or from the invariance under topologically trivial gauge transformations of the BPHZ scheme in one loop we find the parity-odd supersymmetric effective action to be, in agreement with the perturbative part of Ref. 4,

$$\Gamma^{\mathrm{PV}}[V]=\frac{im}{32\pi(m^2)^{1/2}}\int d^3x\, d^2\theta\,\mathrm{Tr}\left[\bar{V}_aF_a+\frac{i}{6}\{\bar{V}_a,\bar{V}_b\}D_bV_a+\tfrac{1}{12}\{\bar{V}_a,\bar{V}_b\}\{V_a,V_b\}\right]$$
$$=\frac{i\pi m}{(m^2)^{1/2}}W^{\mathrm{CHS}}_{\mathrm{SUSY}}[V]\ . \tag{2.38}$$

F_a and $W^{\mathrm{CHS}}_{\mathrm{SUSY}}$ are the supersymmetric field strength tensor and supersymmetric Chern-Simons term, respectively:

$$F_a=\tfrac{1}{2}\bar{D}_bD_aV_b-\frac{i}{2}[\bar{V}_b,D_bV_a]-\tfrac{1}{6}[\bar{V}_b,\{V_b,V_a\}]\ . \tag{2.39}$$

Under large gauge (which lie outside the domain of this present work, see Ref. 4) and parity transformations the supersymmetric Chern-Simons term transforms as

$$W^{\mathrm{CHS}}_{\mathrm{SUSY}}[V']=W^{\mathrm{CHS}}_{\mathrm{SUSY}}[V]+n[U]\ , \tag{2.40}$$

where

$$n[U]=\frac{-i}{96\pi^2}\int d^3x\, d^2\theta \times\mathrm{Tr}(U\bar{D}_aU^{-1}UD_bU^{-1}U\gamma^\mu_{ab}\partial_\mu U^{-1}) \tag{2.41}$$

is the winding number for supergauge transformation U and

$$W^{\mathrm{CHS}}_{\mathrm{SUSY}}[V^P]=-W^{\mathrm{CHS}}_{\mathrm{SUSY}}[V]\ . \tag{2.42}$$

In conclusion we have consistently defined the perturbative renormalized effective action for massless scalar superfields coupled to gauge superfields using the manifestly supersymmetric version of the BPHZ renormalization procedure. We then extract and explicitly calculate the parity-odd part of the effective action and show it to be the supersymmetric Chern-Simons secondary characteristic class, $W^{\mathrm{CHS}}_{\mathrm{SUSY}}$.

ACKNOWLEDGMENTS

I would like to thank Professor T. E. Clark for his assistance and encouragement during the course of this work. I also thank Svetlana Pacheva and Emil Nissimov for an enlightening correspondence of their nonperturbative results.

[1]L. Alvarez-Gaume and E. Witten, Nucl. Phys. **B234**, 269 (1984).

[2]S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) **140**, 372 (1982); A. J. Niemi and G. W. Semenoff, Phys. Rev. Lett. **51**, 2077 (1983).

[3]A. N. Redlich, Phys. Rev. Lett. **52**, 18 (1984).

[4]E. R. Nissimov and S. J. Pacheva, Phys. Lett. **155B**, 76 (1985); **157B**, 407 (1985).

[5]J. H. Lowenstein and W. Zimmermann, Commun. Math. Phys. **44**, 73 (1975); J. H. Lowenstein, *ibid.* **47**, 53 (1976); T. E. Clark and J. H. Lowenstein, Nucl. Phys. **B113**, 109 (1976); T. E. Clark, O. Piguet, and K. Sibold, Ann. Phys. (N.Y.) **109**, 418 (1977).

[6]T. E. Clark and S. T. Love, Nucl. Phys. **B186**, 347 (1981); S. J. Gates, M. Grisaru, M. Rocek, and W. Siegel, *Superspace* (Benjamin-Cummings, Reading, MA, 1983), Chap. 1.