Non-Abelian Debye screening. II. The singlet potential

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The problems encountered in an earlier attempt to compute the non-Abelian Debye screening mass are rectified by considering the correlation of the eigenvalues of the untraced Polyakov loop. This gauge-invariant function is free from color averaging and is shown to yield the singlet potential of a static quark-antiquark pair at finite temperature. A perturbative computation requires that the electrostatic potential A_4 be endowed with a gauge-symmetry-breaking vacuum expectation value, the occurrence of which would explain the breakdown of perturbation theory encountered in the earlier calculation. While the symmetry breakdown cannot reliably be seen perturbatively, its effects on perturbative calculations can be assessed. It is found that most perturbative results, even the supposedly well-established low-order ones, would be affected. This would take the study of the quark-gluon plasma well out of the reach of perturbation theory and make nonperturbative techniques indispensable even at very high temperatures.

I. INTRODUCTION

In an attempt to define a gauge-invariant Debye screening mass in high-temperature quantum chromodynamics ("hot QCD"), we examined in an earlier paper[1] the color-averaged static quark-antiquark potential, given by the Polyakov loop correlation (PLC) function. The computation revealed two problems. (1) The color averaging, which results in PLC being dominated by two-gluon exchange, tends to wash out the Debye screening effect, leaving the mass gap of the magnetostatic (MS) sector as the dominant decay mass of the correlation. (2) Higher orders overwhelm the leading order at distances beyond the Debye screening length, making the PLC calculation invalid in the deep infrared region and implying an unexpectedly early breakdown of perturbation theory in the electrostatic (ES) sector. In order to obtain an acceptable definition of the non-Abelian Debye screening mass, these problems must be dealt with.

The first one could be solved by replacing the PLC function with a gauge-invariant ES correlation function dominated by *single*-gluon exchange, which would provide a cleaner manifestation of the non-Abelian Debye screening effect. Such a correlation function indeed exists: it is the correlation of the gauge-invariant eigenvalues of the Polykov loop operator and yields the potential energy of a static quark-antiquark pair in the color-singlet state. A simple approach to the second problem would be to imagine that the infrared divergences of hot QCD are so severe that they lead to a nonzero vacuum expectation value (VEV) for the color ES potential A_4. This would explain the PLC problem in the previous calculations as being due to an expansion about the wrong vacuum. It might also shed some light on the so-called "infrared problem" of hot QCD.

The infrared problem[2] has to do with the inevitable failure of a perturbative approach to hot QCD due to the infrared divergences in its MS sector, despite the possibility that these divergences might cure themselves by generating a "magnetic screening mass." The scenario goes as follows: At distances larger than $O(1/g^2T)$ (the "deep infrared") the ES field of hot QCD, having acquired an $O(gT)$ Debye mass, ought to decouple, leaving behind a pure MS sector described by three-dimensional (3D) Yang-Mills theory (QCD_3) (Refs. 2–4). The failure of perturbation theory at finite temperature arises essentially from this super-renormalizable three-dimensional MS sector. It was shown[5] that super-renormalizable theories cure their infrared divergences by generating nonanalytic terms (such as $g^2\ln g$) in their g^2 expansion, whether or not the propagators acquire "magnetic masses." On the basis of a toy model, it was subsequently argued[6] that such masses need not necessarily be generated. On the other hand, Monte Carlo simulations of finite-temperature four-dimensional lattice gauge theory showed that MS fields are indeed screened at high temperatures.[7] Thus, we may take it that the infrared divergences of perturbation theory cure themselves by generating a MS screening mass. Its leading value could then be obtained within QCD_3 alone, and on dimensional grounds would have to be $O(g^2T)$ (Ref. 4). This spells doom for perturbation theory, because the effective infrared expansion parameter is $g^2T/$(infrared cutoff). Since the cutoff is the magnetic mass, the expansion parameter is of order unity in the coupling constant and all orders of Feynman graphs could contribute to a given order in perturbation theory: this is the infrared problem.[2] Whether and how such a mass manifests itself in terms of a single-gluon propagator has remained unclear. We shall see below that the suggested symmetry breakdown provides a mechanism to give masses to the MS gluons, and implies that the decoupling of the ES field may not be as clean as has been believed.

There is already evidence, albeit inconclusive, of a nonzero VEV for A_4. In Ref. 8 the effective potential for A_4 was calculated in SU(2) and found to be minimized at two loops by a nonvanishing VEV, $v=g\sqrt{T}/4\pi$. Because of the infrared problem, this result might be affected by higher-order diagrams and therefore explicitly nonperturbative evidence is necessary. To this end, an SU(3) Monte Carlo simulation was performed;[9] though the data indicate a nonzero VEV, the statistics are somewhat sparse.

Combining the results of Refs. 8–10, the following scenario emerges: Infrared divergences in the static sector of hot QCD cause the ES potential A_4 to develop a vacuum expectation value $v \sim g\sqrt{T}$, thereby freezing the color degrees of freedom and breaking SU(N) down to $U(1)^{N-1}$; the tachyon pole corresponding to this symmetry breaking cannot be observed in perturbation theory. There are then $N(N-1)$ "off-diagonal" (broken) massive MS gluons coupled to the $(N-1)$ "diagonal" (unbroken) massive ES gluons ("Higgs particles"). The remaining $(N-1)$ diagonal massless MS gluons also acquire masses,[10] due to the magnetic screening caused by the monopole plasma which results from the broken SU(N) symmetry, thereby eliminating any residual infrared divergences. Various estimates of the magnetic screening mass (Monte Carlo, 3D mass gap, and monopole gas) all give approximately the same value:[10] $m_M \sim g^2T/4$. The deconfinement transition may be viewed as the condensation of the Higgs field and the liberation of monopoles at the critical temperature T_c; since there is no classical Higgs effect, these monopoles are stable only on the quantum level.

Finally, we should also note that the infrared divergences might imply the confinement (in the 3D sense) of the static modes of the theory into color singlets, a possibility which is fully compatible with and probably related to the occurrence of magnetic screening. Such "dynamical confinement" has already been suggested[11] on the basis of the area-law behavior of spacelike Wilson loops in hot QCD.

In this paper we shall continue our perturbative exploration of non-Abelian Debye screening by trying to compute the singlet potential via the Polyakov loop eigenvalue correlation (PLEC). This will be done in the "unitarity gauge,"[12] familiar from the electroweak model; in perturbation theory, this *requires* the introduction of a symmetry-breaking VEV for A_4. Thus the separate considerations of the singlet potential and symmetry breakdown are in fact closely connected. Since the latter is related through the Higgs mechanism to the magnetic screening mass, which first shows up at two loops and is perturbatively incalculable,[2–4] we do not expect to see the generation of a VEV below two loops, and our calculations bear this out. Such a VEV is therefore not calculable in standard perturbation theory either. We shall show that the VEV affects even leading orders of the Debye mass, and thereby low orders of the thermodynamic potential, making them also perturbatively incalculable. In fact, most perturbative results in high-temperature Yang-Mills theory appear to be seriously affected.

In Sec. II we discuss the relationship between the singlet potential and the eigenvalues of the Polyakov loop operator, with some formal derivations being relegated to an Appendix. Section III develops the unitarity gauge formulation of hot QCD (more precisely, of the infrared effective theory[1,4] $EQCD_3$). An explicit computation of the singlet potential for SU(2) is performed in Sec. IV. Our conclusions are presented in Sec. V.

II. INVARIANT INTERQUARK POTENTIALS AT FINITE TEMPERATURE

At $T=0$, the only gauge-invariant interquark potential we can define is through the Wilson loop,[13] which describes a quark and an antiquark interacting via the color-singlet channel. There is no gauge-invariant correlation function corresponding to a $\bar{q}q$ pair in the adjoint channel (nor, for that matter, for two quarks interacting via the symmetric or antisymmetric channels). At $T\neq 0$, there appears to be again only one gauge-invariant correlation function characterizing the interaction of a $\bar{q}q$ pair, viz., PLC, which yields a weighted average of the singlet (1) and adjoint (adj) potentials [or, for two quarks, of the symmetric (sym) and antisymmetric (ant) potentials]. However, by taking into account the additional gauge-invariant degrees of freedom created by the periodic boundary conditions at finite temperature, we can get a handle on the singlet and adjoint potentials separately.

The imaginary-time propagation of a quark in the finite-temperature formalism is described[14] by the untraced Polyakov loop operator

$$\Omega(\mathbf{x}) \equiv P \exp\left[-ig\int_0^\beta d\tau\, A_4(\mathbf{x},\tau)\right],$$

while for an antiquark, the corresponding operator is the matrix transpose of $\Omega^\dagger$. Since Ω transforms adjointly, it can be written in the form

$$\Omega(\mathbf{x}) = \omega(x)\hat{\Omega}(\mathbf{x})\omega^\dagger(x),$$

where $\hat{\Omega}$ is the diagonal matrix of the gauge-invariant eigenvalues of Ω; under a gauge transformation $U(\mathbf{x},\tau)$, only the ω's transform:

$$\omega(\mathbf{x}) \xrightarrow{U(\mathbf{x},\tau)} U(\mathbf{x},0)\omega(\mathbf{x}).$$

Since $\mathrm{Tr}\hat{\Omega} = \mathrm{Tr}\Omega$, the n-point functions of $\mathrm{Tr}\Omega$ are a subset of those of $\hat{\Omega}$, and the fact that $\hat{\Omega}$ contains more information than $\mathrm{Tr}\Omega$ will be made use of below.

In the Appendix, we have derived expressions for SU(N) interquark potentials in the zero-temperature Minkowski theory, in terms of the Wilson path-ordered exponential operators which describe the propagation of static quarks. Since the SU(N) algebra is independent of temperature, all we need do here is to replace the Wilson operators by their finite-temperature Euclidean counterparts. A naive substitution of the operators Ω in place of the Wilson ones would lead to PLC as being the only gauge-invariant correlation available. Substitution of the diagonal form $\hat{\Omega}$, on the other hand, leads to the following expressions for the SU(N) interquark potentials at finite T:

$$\exp(-\beta V_{\mathrm{sym}}) = \frac{N}{N+1}\langle \widetilde{\mathrm{Tr}}\Omega(\mathbf{R})\widetilde{\mathrm{Tr}}\Omega(\mathbf{0})\rangle + \frac{1}{N+1}\langle \widetilde{\mathrm{Tr}}\hat{\Omega}(\mathbf{R})\hat{\Omega}(\mathbf{0})\rangle,$$

$$\exp(-\beta V_{\mathrm{ant}}) = \frac{N}{N-1}\langle \widetilde{\mathrm{Tr}}\Omega(\mathbf{R})\widetilde{\mathrm{Tr}}\Omega(\mathbf{0})\rangle - \frac{1}{N-1}\langle \widetilde{\mathrm{Tr}}\hat{\Omega}(\mathbf{R})\hat{\Omega}(\mathbf{0})\rangle,$$

$$\exp(-\beta V_{\rm adj}) = \frac{N^2}{N^2-1}\langle \widetilde{\rm Tr}\Omega^\dagger(\mathbf{R})\widetilde{\rm Tr}\Omega(\mathbf{0})\rangle - \frac{1}{N^2-1}\langle \widetilde{\rm Tr}\hat{\Omega}^\dagger(\mathbf{R})\hat{\Omega}(\mathbf{0})\rangle ,$$

$$\exp(-\beta V_1) = \langle \widetilde{\rm Tr}\hat{\Omega}^\dagger(\mathbf{R})\hat{\Omega}(\mathbf{0})\rangle \equiv C_{\rm PLE}(R) .$$

Note that the last two equations imply for PLC the well-known result

$$C_{\rm PL}(R) \equiv \langle \widetilde{\rm Tr}\Omega^\dagger(\mathbf{R})\widetilde{\rm Tr}\Omega(\mathbf{0})\rangle = \frac{1}{N^2}\exp[-\beta V_1(R)] + \frac{N^2-1}{N^2}\exp[-\beta V_{\rm adj}(R)] .$$

Our formalism thus contains what is already known and goes beyond it to give separate expressions for the potentials in the individual channels. Since the Polyakov loop eigenvalue correlation (PLEC) gives us all the extra gauge-invariant information there is, it is uniquely the correlation function we were seeking, in the following sense: Any other gauge-invariant ES correlation function dominated by single-gluon exchange would have to be a linear combination of PLEC and PLC.

We end this section by noting that the diagonal form of the Polyakov loop operator $\hat{\Omega}$ is gauge invariant only up to permutations of its eigenvalues and therefore the above expressions are valid only in the continuum theory. On the lattice, additional measures would be necessary to take care of the possibility of being able to perform independent permutations at each site.

III. EQCD$_3$ IN DIAGONAL GAUGE

The infrared dynamics of the MS and ES potentials A_i,ϕ in hot QCD is governed by the effective theory[1,4] EQCD$_3$, described by the three-dimensional Euclidean Lagrangian density

$$\mathcal{L}_{\rm EQCD_3} = \tfrac{1}{2}{\rm Tr}F_{ij}{}^2(A) + {\rm Tr}(\partial_i\phi + iG[A_i,\phi])^2 + m_0{}^2{\rm Tr}\phi^2 ,$$

where $G = T^{1/2}g(T)$ and

$$m_0{}^2 = m_E{}^2 + \delta m^2 \equiv NG^2T/3 + \left[-2NG^2\int d^3\mathbf{q}/(2\pi)^3\mathbf{q}^2\right] .$$

The electrostatic field ϕ is an $N\times N$ traceless Hermitian matrix which gauge transforms as an adjoint scalar field, and may therefore be diagonalized by a suitably chosen gauge transformation. Let $\hat{\phi}$ be the $N\times N$ traceless diagonal matrix whose entries are the N eigenvalues of ϕ (of which only $N-1$ are independent). Write

$$\phi = f\hat{\phi}f^\dagger = f\lambda^{\hat{a}}T^{\hat{a}}f^\dagger ,$$

where $T^{\hat{a}}$ are the $(N-1)$ diagonal generators of SU(N). The physics is contained in the $(N-1)$ gauge-invariant eigenvalues $\lambda^{\hat{a}}$ while the "off-diagonal" component f is a gauge artifact and may be eliminated by a gauge transformation $U=f^\dagger$. This leaves the form of the EQCD$_3$ Lagrangian unchanged, except that the ϕ field may now be regarded as diagonal, and there is also a ghost term in the effective action corresponding to the change in the measure for the ϕ field.

One usually encounters the unitarity gauge in theories with broken gauge symmetries. In the present case, there is no *a priori* reason to consider symmetry breaking, and there is certainly no Higgs effect at the classical level. Nevertheless, we suspect on the grounds of our previous attempts to compute the Debye screening mass that such symmetry breaking may indeed be the result of radiative corrections. We shall therefore introduce a symmetry breaking parameter v, which would serve to "hold down" the fields in the above Lagrangian to their diagonal values, and write the eigenvalues of ϕ as

$$\hat{\phi} = \lambda^{\hat{a}}T^{\hat{a}} = \hat{v} + \epsilon^{\hat{a}}T^{\hat{a}} ,$$

where v is the vacuum expectation value of ϕ and ϵ is the perturbation away from the vacuum. We have thereby broken our gauge group $G={\rm SU}(N)$ down to the little group H generated by the $N-1$ diagonal generators $T^{\hat{a}}$. In our formalism, v is a parameter to be determined self-consistently.

With the above parametrization, we have

$${\rm Tr}(D\phi)^2 = {\rm Tr}(\partial\hat{\epsilon})^2 - G^2{\rm Tr}[\tilde{A},\hat{v}+\hat{\epsilon}]^2 ,$$

and we see that v gives mass to the off-diagonal gauge bosons $\tilde{A}\equiv A - A^{\hat{a}}T^{\hat{a}}$. We also notice the absence of a coupling of two ES and one MS fields. It is precisely such a coupling that is responsible for the $O(g^3)$ term in the naive Debye mass;[1] we see that such a term is in fact a gauge artifact.

In terms of $\hat{\phi}$, the diagonal Polyakov loop operator is given by

$$\hat{\Omega} = \exp(-ig\sqrt{\beta}\lambda^{\hat{a}}T^{\hat{a}}) = \exp(-ig\sqrt{\beta}\hat{v})\exp(-ig\sqrt{\beta}\epsilon^{\hat{a}}T^{\hat{a}}) .$$

The singlet potential is then given by

$$\exp[-\beta V_1(\mathbf{R})] = \langle \widetilde{\rm Tr}\exp\{ig\sqrt{\beta}T^{\hat{a}}[\epsilon^{\hat{a}}(\mathbf{R}) - \epsilon^{\hat{a}}(\mathbf{0})]\}\rangle .$$

At high temperature, the exponentials may be expanded to give

$$V_1(R) = -\frac{g^2}{2N}\langle [\epsilon^{\hat{a}}(\mathbf{R})\epsilon^{\hat{a}}(\mathbf{0}) - \epsilon^{\hat{a}}(\mathbf{0})^2]\rangle .$$

In the next section, we shall calculate $V_1(R)$ for SU(2).

IV. THE SU(2)-SINGLET POTENTIAL

We shall now specialize to SU(2) for simplicity. The generalization to SU(N) is straightforward if in places a bit tedious. The Polyakov loop operator in static gauge is given by

$$\Omega(\mathbf{x}) = \exp[-iG\beta\phi(x)] .$$

Let us parametrize ϕ as

$$\phi = \frac{\lambda}{2}\Theta \equiv \frac{\lambda}{2}\begin{bmatrix}\cos\theta & \sin\theta e^{-i\psi} \\ \sin\theta e^{i\psi} & -\cos\theta\end{bmatrix} ,$$

where

$$\lambda^2 \equiv 2\,{\rm Tr}\phi^2 = \phi^i\phi^i \quad (i=1,2,3) .$$

The gauge-invariant eigenvalues of ϕ are $\pm\lambda/2$, while those of Θ are ± 1. Let the corresponding eigenvectors be denoted $|\pm\rangle$. We easily find

$$|+\rangle = \begin{bmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\psi} \end{bmatrix},$$

$$|-\rangle = \begin{bmatrix} \sin\frac{\theta}{2}e^{-i\psi} \\ -\cos\frac{\theta}{2} \end{bmatrix}.$$

The matrix which diagonalizes Θ is then

$$f = (\,|+\rangle\,|-\rangle\,) = \begin{bmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2}e^{-i\psi} \\ \sin\frac{\theta}{2}e^{i\psi} & -\cos\frac{\theta}{2} \end{bmatrix},$$

with

$$ff^\dagger = f^2 = \mathrm{diag}(1,1)$$

and

$$f\sigma^3 f^\dagger = f\sigma^3 f = \Theta .$$

Thus,

$$\phi = \frac{\lambda}{2} f\sigma^3 f .$$

In terms of λ, the Polyakov loop operator is given by

$$\widetilde{\mathrm{Tr}}\Omega(\mathbf{x}) = \cos\left[\frac{G}{2T}\lambda(\mathbf{x})\right] = \cos\left[\frac{g\sqrt{\beta}}{2}\lambda(\mathbf{x})\right].$$

If we let

$$\lambda \to \lambda + 2\pi T/G = \lambda + 2\pi/g\sqrt{\beta} ,$$

then

$$\widetilde{\mathrm{Tr}}\Omega(\mathbf{x}) \to -\widetilde{\mathrm{Tr}}\Omega(\mathbf{x}) .$$

Thus λ may be taken to range from $-2\pi T/G$ to $2\pi T/G$.

Above the deconfinement transition, the one-loop effective potential for λ reflects the broken Z_2 symmetry of the theory;[15] see Fig. 1(a). For finite volumes, the system tunnels between the degenerate Z_2 vacua. For infinite volume, the Z_2 symmetry is spontaneously broken and the system sits in one of the zeros of the effective potential. We shall measure λ from whichever zero the system happens to have chosen. Since we are interested here in perturbative computations at high temperature, we may take $\pi/g\sqrt{\beta} \to \infty$ and the one-loop effective potential then looks as shown in Fig. 1(b).

We transform to diagonal (unitarity) gauge via the replacements

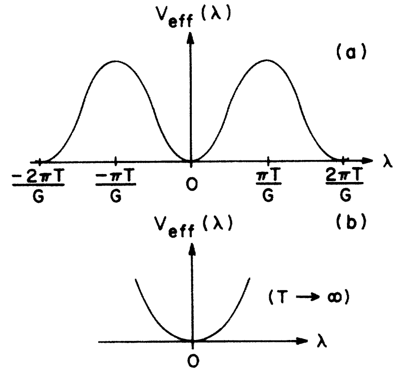

FIG. 1. One-loop effective potential (schematic) for A_4 in SU(2) diagonal gauge: (a) above the deconfinement transition and (b) in the thermodynamic limit at very high temperatures.

$$A \to f\left[A - \frac{i}{G}\partial\right]f , \quad \Theta \to f\sigma^3 f ,$$

whereby the f's disappear and we are left with

$$L_{\mathrm{EQCD}_3} = \tfrac{1}{2}\mathrm{Tr}F^2(A) + \tfrac{1}{2}(\partial\lambda)^2 + \tfrac{1}{2}m_0{}^2\lambda^2 + \tfrac{1}{2}G^2\lambda^2[(A^1)^2 + (A^2)^2] .$$

In anticipation of the possible generation of a vacuum expectation value for λ, let us now write

$$\lambda(\mathbf{x}) = v + \epsilon(\mathbf{x}) ,$$

where the VEV v is to be determined dynamically; note that the off-diagonal MS gluons will acquire screening masses $\mu = Gv$, and thus a nonzero VEV softens the infrared divergences of hot QCD.

We still have the freedom to make gauge transformations generated by σ^3, which leave $[(A^1)^2+(A^2)^2]$ invariant but change A^3. We must therefore add a gauge-fixing term for A^3. We choose the covariant gauge

$$\Delta L_{\mathrm{gf}3} = -\frac{1}{2\xi}(\partial_i A_i{}^3)^2 .$$

The ϕ measure for SU(2) is of the form

$$\int [\lambda^2 d\lambda][d\Theta(\theta,\psi)] .$$

Since we have eliminated $\Theta(\theta,\psi)$ from the action, we can drop the factor $\int[d\Theta]$. The measure for λ can be converted into a ghost term in the effective action in unitarity gauge; in terms of the parameter v, it is given by

$$\Delta L_{\mathrm{ghost},\lambda} = -\left[\frac{2}{v}\epsilon(\mathbf{x}) - \frac{1}{v^2}\epsilon^2(\mathbf{x})\right]\int d_3\mathbf{k} \qquad [d_3\mathbf{k} \equiv d^3\mathbf{k}/(2\pi)^3] .$$

The resultant effective Lagrangian density for EQCD_3 in unitarity gauge is then (after subtracting an irrelevant constant $\frac{1}{2}m_0{}^2v^2$)

$$\mathscr{L}^{\text{eff}}_{\text{EQCD}_3}=\tfrac{1}{2}\text{Tr}F^2(A)+\tfrac{1}{2}G^2v^2[(A^1)^2+(A^2)^2]-\frac{1}{2\xi}(\partial_i A_i{}^3)^2$$
$$+\tfrac{1}{2}[(\partial\epsilon)^2+m_E{}^2\epsilon^2]+\left[m_0{}^2v-\frac{2}{v}\int d_3\mathbf{k}\right]\epsilon+\left[\frac{2}{v^2}\int d_3\mathbf{k}-4G^2\int\frac{d_3\mathbf{k}}{\mathbf{k}^2}\right]\frac{\epsilon^2}{2}$$
$$+2G^2v\epsilon\frac{(A^1)^2+(A^2)^2}{2}+2G^2\frac{\epsilon^2}{2}\frac{(A^1)^2+(A^2)^2}{2}\ ,$$

from which the Feynman rules may be read off.

One-loop calculations

We now calculate the one-loop self-energy of ϵ, the relevant graphs for which are shown in Fig. 2(a). These lead to the following result for the one-loop ES vacuum polarization tensor in the presence of a nonzero VEV:

$$\hat{\Pi}_{44}=-m_E{}^2+\frac{2G^3v}{\pi}+\frac{11G}{12\pi v}\mathbf{k}^2+O(\mathbf{k}^4/G^2v^2)\ .$$

We determine v self-consistently by setting tadpole graphs to zero. At the one-loop level we have, from Fig. 2(b),

$$(\text{tadpole})=iv\left[\frac{G^3v}{\pi}-m_E{}^2\right]\ ,$$

which vanishes for $v=0,2\pi T/3G$. The second solution is an artifact of the Z_2-transformed vacuum well in the effective potential for λ, and may be discarded here. Thus at one loop we find, as expected, that $v=0$, which is consistent with the well-known fact that no magnetic screening mass arises at the one-loop level.

To obtain a nonvanishing v, we must clearly go to two loops and beyond. The unitarity gauge we are using is unsuitable for calculations beyond the one-loop level. More importantly, at two loops the infrared problem would begin to make its presence felt, regardless of what gauge we might choose. However, as mentioned in Sec. I, there is some evidence for a nonvanishing VEV. On dimensional grounds it is safe to say that, to leading order,

$$v=cG\ ,$$

where c is a nonperturbatively determined dimensionless constant.

In terms of c, we obtain, for $\hat{\Pi}_{44}$,

$$\hat{\Pi}_{44}=-m_E{}^2+\frac{2c}{\pi}g^4T^2+\frac{11}{12\pi c}\mathbf{k}^2+O(\mathbf{k}^4/cG^4)\ .$$

The long-distance behavior of the ϵ propagator is then

$$\langle\epsilon\epsilon\rangle\sim\frac{1}{\left[1-\frac{11}{12\pi c}\right]\mathbf{k}^2+\left[m_E{}^2-\frac{2cg^4T^2}{\pi}\right]}=\frac{1}{1-\frac{11}{12\pi c}}\frac{1}{\mathbf{k}^2+\frac{m_E{}^2-2cg^4T^2/\pi}{1-11/12\pi c}}\ .$$

Since c is a number of order unity, we see quite clearly that the Debye screening mass m_D is not calculable in perturbation theory.

The singlet potential for SU(2) at high temperature, which is essentially the Fourier transform of the ϵ propagator, is then given by

$$V_1(R)=-\frac{g^2}{4}\frac{1}{1-11/12\pi c}\frac{e^{-m_DR}}{4\pi R}\ ,$$

where m_D is the nonperturbative Debye mass occurring in the denominator of the ϵ propagator. Although V_1 does have the expected exponential decay, it is not perturbatively calculable either.

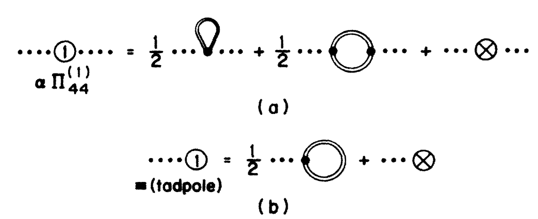

FIG. 2. One-loop perturbation-theory diagrams in SU(2) diagonal gauge: (a) self-energy of ϵ and (b) tadpole graphs. The dotted lines represent ES propagators, the double lines off-diagonal MS gluons, and the $\otimes$'s are counterterms.

V. CONCLUSION

Our previous attempts at computing the non-Abelian Debye screening mass had suggested that it be defined in terms of a gauge-invariant electrostatic correlation function dominated by single-gluon exchange. We showed in this paper that the only viable candidate is the Polyakov loop eigenvalue correlation, which yields the color-singlet potential energy of a quark-antiquark pair at finite temperature. In order to compute the singlet potential perturbatively, we used diagonal (unitarity) gauge, in which A_4 is endowed with a gauge-symmetry-breaking vacuum expectation value v, to be determined self-consistently, order by order. Through the Higgs mechanism, v is converted into masses for the off-diagonal magnetostatic gluons, thereby providing some infrared softening.

We found $v=0$ at the tree level, since there is no classical symmetry breaking, and also at one loop, consistent with absence of magnetic screening up to that order. One would expect a nonvanishing VEV to first show up in a two-loop computation, which is beyond the scope of perturbative diagonal gauge and in any case would be meaningless in view of the infrared problem. Nevertheless, we were able to assess the effects a nonzero v would have on perturbative results. We found that drastic changes would be induced.

The properly defined Debye screening mass would characterize only the diagonal electrostatic gluons of the theory and the coefficients in its g^2 expansion,

$$m^2=(Ag^2+Bg^4+\cdots)T^2 ,$$

would be incalculable in perturbation theory; an interesting side result is the absence of any $O(g^3)$ term. The decoupling of the massive ES field in the deep infrared would not be as straightforward as before, but would instead freeze it to its background value v. The leading magnetostatic screening mass, related to the VEV of A_4, would remain incalculable as before. Most importantly, the perturbative expansion of the thermodynamic potential would be affected beyond the g^2 term; we have (for references see Ref. 3)

$$\Omega=-\frac{(N^2-1)\pi^2T^4}{45}+\frac{(N^2-1)Ng^2T^4}{144}+O(g^3) ,$$

where the $O(g^3)$ term, coming as it does from resumming the $O(g^2)$ Debye mass, would become perturbatively incalculable.

Much of the early excitement about the quark-gluon plasma was due to its supposed perturbative nature, arising from the smallness of the QCD coupling constant at high temperature and/or density. The discovery of the infrared problem, however, soon dispelled this enthusiasm, by showing that perturbation theory had only limited applicability. Symmetry breakdown only worsens the situation, by making most perturbative results invalid at any temperature.

Despite its negative implications for perturbative calculations at finite temperature, the occurrence of symmetry breaking in hot QCD is an attractive idea, since it provides elegant explanations for the early breakdown of perturbation theory in the ES sector and the way in which the infrared divergences of hot QCD cure themselves. We find the resulting scenario, as outlined in Refs. 8–10, very plausible: The condensation of the Higgs field stabilizes color-magnetic monopoles, which form a plasma and screen magnetic charge, thereby providing the MS gluons with $O(g^2T)$ masses. All infrared divergences are then eliminated, though of course the infrared problem still remains. The monopole plasma and magnetic screening are compatible with the three-dimensional confinement expected of the static modes and in fact the magnetic screening mass and the string tension of spacelike Wilson loops are probably related.[11]

While we have not presented any concrete or direct evidence for symmetry breaking in this paper, there seems to be every indication of it from the works mentioned above and from the PLC calculation. The question can only be decided by a fully nonperturbative computation. We feel that at this point a Monte Carlo simulation with improved statistics is needed to conclusively settle this important issue.

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APPENDIX: SU(N) PROJECTION OPERATORS AND INTERQUARK POTENTIALS

We shall develop expressions for SU(N) interquark potentials by applying the appropriate projection operators to static quark correlation functions. Specifically, we shall derive the adjoint (adj) and singlet (1) potentials for a quark-antiquark pair; for completeness, we shall also include the symmetric (sym) and antisymmetric (ant) potentials between two quarks. We shall consider the zero-temperature Minkowski theory here; the generalization to the finite-temperature Euclidean theory is given in Sec. II.

The operator describing the propagation of a static ("infinitely heavy") quark at position $\mathbf{x}$ between the real times $-T/2$ and $T/2$ is given by[13]

$$W(\mathbf{x})\equiv P\exp\left[ig\int_{-T/2}^{T/2}dt\,A_0(\mathbf{x},t)\right] , \quad T\to\infty ;$$

for antiquarks, the corresponding operator is

$$\overline{W}(\mathbf{x})\equiv P\exp\left[ig\int_{-T/2}^{T/2}dt\,\overline{A}_0(\mathbf{x},t)\right]=\widetilde{W}^{\dagger}(\mathbf{x}) ,$$

where the tilde denotes transpose, and the generators of the complex-conjugate represenation are taken to be $\overline{T}^a=-\widetilde{T}^a$. Note that these operators describe static "quarks" of arbitrary spin in the fundamental representation of color SU(N); the expressions we shall obtain therefore yield *spin-averaged* potentials.

qq potentials

Two SU(N) quarks can interact via the symmetric or the antisymmetric channels, according to the decomposition

$$N\otimes N=\tfrac{1}{2}N(N+1)\oplus\tfrac{1}{2}N(N-1) .$$

Therefore the quark-quark correlation function

$$G_{qq}(R)\equiv\langle W(\mathbf{R})W(\mathbf{0})\rangle\sim\langle qq\,|\,e^{iTH}\,|\,qq\rangle$$

is of the form

$$G_{qq}(R)=e^{iTV_{\rm sym}(R)}P_{\rm sym}+e^{iTV_{\rm ant}(R)}P_{\rm ant} .$$

Here $P_{\rm sym},P_{\rm ant}$ are projection operators of the form

$$P_{NN}=AI\otimes I+BT^a\otimes T^a\equiv(A\delta^i{}_j\delta^k{}_l+BT^{ai}{}_jT^{ak}{}_l)$$

and possessing the properties

$$(P_{\rm sym,ant})^2=P_{\rm sym,ant} , \quad P_{\rm sym}P_{\rm ant}=0 ,$$
$$P_{\rm sym}+P_{\rm ant}=I\otimes I .$$

Using the multiplication law for SU(N) generators,

$$T^aT^b=\frac{\delta^{ab}I}{2N}+\tfrac{1}{2}(d^{abc}+if^{abc})T^c ,$$

we readily obtain

$$P_{\text{sym}}=\frac{N+1}{2N}I\otimes I+T^a\otimes T^a ,$$

$$P_{\text{ant}}=\frac{N-1}{2N}I\otimes I-T^a\otimes T^a .$$

We have identified the symmetric and antisymmetric solutions by the coefficients of the $I\otimes I$ terms; we also verify, using the Fierz identities

$$\delta^i{}_j\delta^k{}_l\,|_{\text{exch}}\equiv\delta^i{}_l\delta^k{}_j=\frac{1}{N}\delta^i{}_j\delta^k{}_l+2T^{ai}{}_jT^{ak}{}_l ,$$

$$T^{ai}{}_jT^{ak}{}_l\,|_{\text{exch}}\equiv T^{ai}{}_lT^{ak}{}_j$$

$$=\frac{N^2-1}{2N}\delta^i{}_j\delta^k{}_l-\frac{1}{N}T^{ai}{}_jT^{ak}{}_l ,$$

that

$$P_{\text{sym}}\,|_{\text{exch}}=+P_{\text{sym}} ,\quad P_{\text{ant}}\,|_{\text{exch}}=-P_{\text{ant}} .$$

Defining

$$\text{Tr}P\equiv P^i{}_iP^k{}_k ,$$

we also note that

$$\text{Tr}P_{\text{sym}}=\frac{N(N+1)}{2} ,\quad \text{Tr}P_{\text{ant}}=\frac{N(N-1)}{2} .$$

We now calculate

$$\exp(iTV_{\text{sym}})=\text{Tr}(P_{\text{sym}}G_{qq})/\text{Tr}P_{\text{sym}}$$

$$=\frac{N}{N+1}\langle\tilde{\text{Tr}}W(\mathbf{R})\tilde{\text{Tr}}W(\mathbf{0})\rangle$$

$$+\frac{1}{N+1}\langle\tilde{\text{Tr}}W(\mathbf{R})W(\mathbf{0})\rangle ,$$

$$\exp(iTV_{\text{ant}})=\text{Tr}(P_{\text{ant}}G_{qq})/\text{Tr}P_{\text{ant}}$$

$$=\frac{N}{N-1}\langle\tilde{\text{Tr}}W(\mathbf{R})\tilde{\text{Tr}}W(\mathbf{0})\rangle$$

$$-\frac{1}{N-1}\langle\tilde{\text{Tr}}W(\mathbf{R})W(\mathbf{0})\rangle ,$$

where we have used the SU(N) identity

$$2N\,\tilde{\text{Tr}}(T^aX)\tilde{\text{Tr}}(T^aY)\equiv\tilde{\text{Tr}}(XY)-(\tilde{\text{Tr}}X)(\tilde{\text{Tr}}Y) ,$$

for any $N\times N$ matrices X,Y.

$\bar{q}q$ potentials

An SU(N) quark-antiquark pair can interact via the singlet or the adjoint channels, according to the decomposition

$$\bar{N}\otimes N=1\oplus(N^2-1) .$$

Therefore the quark-antiquark correlation function

$$G_{\bar{q}q}(R)\equiv\langle\overline{W}(\mathbf{R})W(\mathbf{0})\rangle\sim\langle\bar{q}q\,|\,e^{iTH}\,|\,\bar{q}q\rangle$$

is of the form

$$G_{\bar{q}q}(R)=e^{iTV_1(R)}P_1+e^{iTV_{\text{adj}}(R)}P_{\text{adj}} .$$

Here P_1,P_{adj} are projection operators of the form

$$P_{\bar{N}N}=AI\otimes I+B\bar{T}^a\otimes T^a\equiv(A\delta_i{}^j\delta^k{}_l+B\bar{T}^a{}_i{}^jT^{ak}{}_l)$$

and possessing the properties

$$(P_{1,\text{adj}})^2=P_{1,\text{adj}} ,\quad P_1P_{\text{adj}}=0 ,\quad P_1+P_{\text{adj}}=I\otimes I .$$

Using the multiplication law for SU(N) generators given earlier, we readily obtain

$$P_1=\frac{1}{N^2}I\otimes I-\frac{2}{N}\bar{T}^a\otimes T^a ,$$

$$P_{\text{adj}}=\frac{N^2-1}{N^2}I\otimes I+\frac{2}{N}\bar{T}^a\otimes T ,$$

We have identified the singlet and adjoint solutions by the coefficients of the $I\otimes I$ terms. Unlike the qq case, they do not satisfy any simple exchange relations; however, we note that

$$\text{Tr}P_1=1 ,\quad \text{Tr}P_{\text{adj}}=N^2-1 .$$

We now calculate

$$\exp(iTV_1)=\text{Tr}(P_1G_{\bar{q}q})/\text{Tr}P_1=\langle\tilde{Tr}W^\dagger(\mathbf{R})W(\mathbf{0})\rangle ,$$

$$\exp(iTV_{\text{adj}})=\text{Tr}(P_{\text{adj}}G_{\bar{q}q})/\text{Tr}P_{\text{adj}}$$

$$=\frac{N^2}{N^2-1}\langle\tilde{\text{Tr}}W^\dagger(\mathbf{R})\tilde{\text{Tr}}W(\mathbf{0})\rangle$$

$$-\frac{1}{N^2-1}\langle\tilde{\text{Tr}}W^\dagger(\mathbf{R})W(\mathbf{0})\rangle ,$$

where we have again used the identity mentioned earlier.

Static Wilson loop

Except in $A_0=0$ gauge, which is singular, the gauge fields at $t=\pm\infty$ become pure gauge. Thus we can transform to a gauge where the fields vanish at $t=\pm\infty$. In that gauge, $\exp(iTV_1)$ is the same as the static Wilson loop.[13] But since the latter is gauge invariant, we can *define* a gauge-invariant singlet static potential via the equation

$$\exp[iTV_1(R)]=\langle\tilde{\text{Tr}}W^\dagger(\mathbf{R})W(\mathbf{0})\rangle\quad\text{(in special gauge)}$$

$$=\left\langle\tilde{\text{Tr}}P\exp\left[ig\oint_S dx^\mu A_\mu(x)\right]\right\rangle\quad\text{(in any gauge)} ,$$

where the rectangular contour S has spatial extent R and temporal extent $T\to\infty$. We thus recover the usual formula for the singlet quark-antiquark potential at zero temperature.

There seems to be no way of extracting gauge-invariant adjoint, symmetric, or antisymmetric potentials without compactifying the time direction.

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