Large-N two-dimensional QCD and chiral symmetry

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Chiral-symmetry breaking of two-dimensional QCD is studied in the $N\to\infty$ limit. The gap equation for the meson condensation function is solved in the axial gauge. The quark condensate $\langle\bar{\psi}\psi\rangle$ is calculated. The effects of finite temperature and baryon density are discussed.

I. INTRODUCTION

The study of dynamical chiral-symmetry breaking is important for understanding quantum chromodynamics (QCD). Although Monte Carlo lattice-gauge studies have made impressive progress in this direction, the nonperturbative nature of chiral-symmetry breaking makes it difficult to study through the QCD non-Abelian Lagrangian. Much effort is still needed to gain insight into the nonperturbative phenomena of QCD.

In this paper, we study the chiral-symmetry breaking in the 't Hooft model[1] of two-dimensional QCD in the $N\to\infty$ limit and in the axial gauge. Compared with four-dimensional QCD, two-dimensional QCD is apparently much simpler. The large-N expansion method, pioneered by 't Hooft[1] in this model, simplifies the calculation significantly and is more tractable than in four dimensions. We hope that through this model we can extract information about its vacuum structure and chiral-symmetry breaking, and shed some light on understanding the realistic four-dimensional QCD.

Despite many similarities between QCD_2 and realistic QCD_4, we note that there are some important differences due to special kinematics in two dimensions. For example, in four dimensions interactions of a quark pair lead to a flux tube at large distances and an approximate spherical bag at small distances; this leads to a confining potential at large distances and a Coulomb-type potential at small distances. In two dimensions the tube becomes trivially a flux line and this leads to a linear potential. Also, the infrared divergences in two dimensions are much more severe than those in four dimensions so that one cannot use the asymptotic perturbative expansion at all. Another essential difference lies in the fact that in four dimensions the theory is renormalizable, while in two dimensions it is super-renormalizable. Nevertheless QCD_2 is still a highly nontrivial theoretical model, as can be easily seen from its low-energy Lagrangian which was derived using bosonization.[2] Thus it provides us a theoretical model with rich content.

The paper is organized as follows. In Sec. II we discuss some subtleties in taking the chiral limit. In Sec. III we derive the gap equation for the meson condensation function in the axial gauge by the methods of both the Bogoliubov-Valatin transformation and Feynman diagrams. We then solve it numerically, using the solution to calculate the quark condensate $\langle\bar{\psi}\psi\rangle$. In Sec. IV we discuss the implications of results of Sec. III for finite temperature and baryon density.

II. QCD_2 AND SUBTLETIES IN TAKING THE CHIRAL LIMIT

The Lagrangian density for QCD_2 is defined as

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\sum_{\alpha}\bar{\psi}_\alpha(i\not{D}-m_\alpha)\psi_\alpha\ , \tag{2.1}$$

where

$$\begin{aligned}&F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-ig[A_\mu,A_\nu]\ ,\\ &D_\mu=\partial_\mu-igA_\mu\ ,\\ &A_\mu=T^aA_\mu^a\ .\end{aligned} \tag{2.2}$$

The index α runs over all colors and flavors of quarks. The T^a are the generators of the SU(N) gauge group. For clarity we restrict ourselves to one quark flavor; it is straightforward to generalize to multiquark flavors.

In the following, we sketch some important features of the theory in the limit as N (the number of colors) goes to infinity and in the chiral limit, in which m (the mass of quarks) goes to zero. For more details the reader is referred to the work of Zhitnitsky.[3]

In the $N\to\infty$ limit, the leading Feynman diagrams are planar diagrams. All other diagrams are in higher order of $1/N$. In this limit any Green's function is given by summing over only planar diagrams. Since we are in two dimensions, the gauge coupling constant g has the dimensions of mass. Thus we have a dimensionless combination g^2N/m^2; g^2N has to be finite in the $N\to\infty$ limit in order to have a physical, sensible, and nontrivial theory. The meson spectrum in this limit has been obtained by 't Hooft in the light-cone gauge by solving the $q\bar{q}$ bound-state Bethe-Salpeter equation. It assumes the following approximate linear form for large n:

$$\mu^2(n)\sim\pi g^2Nn+C(m),\quad n=0,1,2,\dots\ . \tag{2.3}$$

One important feature is that

$$m\to 0\Longrightarrow C(m)\to 0,\quad \mu^2(0)\to 0\ . \tag{2.4}$$

We remark that the above solution is obtained for the following relations among the parameters of the theory:[4,5]

$$N\to\infty,\quad g^2N=\text{const},\quad m\gg g\sim 1/N^{1/2}\ . \tag{2.5}$$

From above we see that the chiral limit ($m \to 0$) can only be reached after taking the limit $N \to \infty$. The following two ways of taking limits are essentially different:[6] (1) $N \to \infty$ first, then $m \to 0$; (2) $m \to 0$ first, then $N \to \infty$.

In the first case we always have $g \ll m$, and are thus in the weak-coupling regime, while in the second case we always have $g \gg m$, and are thus in the strong-coupling regime. The explanation is that these two cases correspond to different phases. Since $g(\sim 1/N^{1/2})$ tends to zero in the limit of $N \to \infty$, the phase transition region ($m \sim g$) and the chiral limit ($m \to 0$) overlap. Thus corresponding to case (1) or case (2), we approach the point $m=0$, $N=\infty$ of the phase diagram from different directions.

The meson spectrum (2.3) obtained by 't Hooft corresponds to the weak-coupling regime in which chiral symmetry is broken. Since we are in two dimensions, the symmetry breaking, similar to the SU(N) Thirring model,[7] is of the Berezinski-Kosterlitz-Thouless type;[8] for finite N and large x, the chiral-symmetry-violating bilinear combination behaves like

$$\langle \bar{\psi}(x)(1+\gamma_5)\psi(x)\bar{\psi}(0)(1-\gamma_5)\psi(0)\rangle \sim |x|^{-1/N} . \tag{2.6}$$

It is only in the $N \to \infty$ limit that true long-range order, and hence the Goldstone boson, occurs, and chiral symmetry is spontaneously broken. This case does not contradict the Coleman theorem.[9,10]

The picture in the strong-coupling regime is quite different from that of the weak-coupling regime. In the strong-coupling regime, chiral symmetry is not broken and massless composite fermions appear in the mass spectrum of bound states instead.[11]

In the following, we focus on the weak-coupling regime only.

III. THE GAP EQUATION

To study chiral-symmetry (CS) breaking, we follow closely the methods of Ref. 12 to write down the gap equation for the meson condensation function in the axial gauge. In a previous paper,[13] we solved the Bethe-Salpeter equation for the meson bound states in the axial gauge and zero-momentum frame, and obtained a mass spectrum which agrees with the solutions obtained in the light-cone gauge by 't Hooft. We start here by deriving the gap equation, first by using the Bogoliubov-Valatin transformation, from which the meson condensation function is defined, then by using the Feynman-diagram method, from which we solve the gap equation numerically.

In the axial gauge, where $A_1=0$, the Hamiltonian density derived from (2.1) becomes

$$\mathcal{H} = -\bar{\psi} i \partial_1 \gamma^1 \psi - \frac{2\pi f}{N} j^b_{0a} \frac{1}{\partial_1{}^2} j^a_{0b} , \tag{3.1}$$

where

$$j^b_{0a} = \psi^\dagger_a \psi^b - \frac{\delta^b_a}{N} \psi^\dagger_a \psi^a, \quad f \equiv \frac{g^2 N}{4\pi} ,$$

and a,b are the color indices. Using the Bogoliubov-Valatin transformation we transform the operators c,d, which annihilate state $|0\rangle$, into a new basis set C,D which annihilate the trial state $|\Phi\rangle$ containing a $q\bar{q}$ condensate:

$$C^a(p) = \frac{1}{[1+\Phi(p)^2]^{1/2}} [c^a(p) + \Phi(p) d^a(-p)] , \tag{3.2a}$$

$$D^a(p) = \frac{1}{[1+\Phi(p)^2]^{1/2}} [d^a(p) - \Phi(p) c^a(-p)] . \tag{3.2b}$$

The quark field can be reexpressed in terms of this new basis [$p^\mu = (p^0, p)$ is the momentum]:

$$\psi^a(x) = \int \frac{dp}{2\pi} [C^a(p) M_1(p) \exp(ipx) + D^{a\dagger}(p) M_2(p) \exp(-ipx)] \tag{3.3}$$

with

$$\langle \Phi | \Phi \rangle = 1 \tag{3.4}$$

$$M_1(p) = \frac{1}{[1+\Phi(p)^2]^{1/2}} [1+\gamma_0 \Phi(p)] U(p) , \tag{3.5a}$$

$$M_2(p) = \frac{1}{[1+\Phi(p)^2]^{1/2}} [1-\gamma_0 \Phi(p)] V(p) , \tag{3.5b}$$

where $U(p), V(p)$ are the helicity spinors. $|\Phi\rangle$ is related to $|0\rangle$ by

$$|\Phi\rangle = \frac{1}{\prod_p [1-\Phi(p)^2]^{1/2}} \times \prod_{p,a} [1 - \Phi(p) c^{a\dagger}(p) \tau d^{a\dagger}(-p)] |0\rangle , \tag{3.6}$$

where τ is the volume of an elemental cell in momentum space. The quark propagator is given by

$$\langle \Phi | \tfrac{1}{2}[\psi(0,x), \bar{\psi}(0,y)] | \Phi \rangle = \int \frac{dp}{2\pi} \exp[ip(x-y)] \left[\frac{\Phi(p)}{1+\Phi(p)^2} - \frac{1}{2} \frac{1-\Phi(p)^2}{1+\Phi(p)^2} \gamma^1 \hat{p} \right] , \tag{3.7}$$

where $\hat{p} = p/|p|$. With this we can calculate $\langle \Phi | H | \Phi \rangle$. The gap equation is obtained by minimizing $\langle \Phi | H | \Phi \rangle$ with respect to Φ:

$$\frac{\delta}{\delta \Phi} \langle \Phi | H | \Phi \rangle = 0 . \tag{3.8}$$

It takes the form

$$p\Phi(p) = \frac{f}{2} \int \frac{dk}{(p-k)^2} \frac{[1-\Phi(p)^2]\Phi(k) - [1-\Phi(k)^2]\Phi(p)}{[1+\Phi(k)^2]} . \tag{3.9}$$

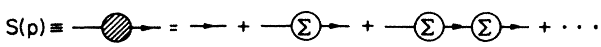

FIG. 1. Diagrammatic representation of the quark propagator $S(p)$.

Using the Feynman-diagram method in the axial gauge, we derive the gap equation by repeating much of the derivation from the work of Bars and Green.[14] The quark propagator and self-energy are given by (see Figs. 1 and 2)

$$S(p^\mu)=\frac{i}{\not p-m-\Sigma(p)+i\epsilon}\ , \tag{3.10}$$

$$\Sigma(p)=\frac{ig^2}{4\pi^2}T^aT^a\int\frac{dk^0dk}{(p-k)^2}\gamma^0\frac{1}{\not k-m-\Sigma(k)+i\epsilon}\gamma^0\ . \tag{3.11}$$

Let us define

$$\Sigma(p)=A(p)+\gamma^1B(p)\ ; \tag{3.12}$$

the Dirac matrices are $\gamma^0=\sigma_1$, $\gamma^1=i\sigma_2$, $\gamma^5=\gamma^0\gamma^1=\sigma_3$. Although it has been shown in the axial gauge, by Bars and Green,[14] that any physical quantity is Lorentz covariant in the color-singlet sector, the quark sector is not color singlet and therefore is not Lorentz covariant; thus we anticipate that, in particular, $\omega(p)$ may not always be positive after regularization. We cannot write it in the covariant form of

$$\omega(p)=\{[m+A(p)]^2+[p+B(p)]^2\}^{1/2}$$

as we usually do, but we can always parametrize it in terms of

$$A(p)=\omega(p)\cos[\theta(p)]-m\ , \tag{3.13a}$$

$$B(p)=\omega(p)\sin[\theta(p)]-p\ , \tag{3.13b}$$

and write

$$\Sigma(p)=f\int\frac{dk\,dk^0}{(p-k)^2}\frac{\omega(k)\cos[\theta(k)]-\gamma^1\omega(k)\sin[\theta(k)]}{[k^0-\omega(k)+i\epsilon][k^0+\omega(k)-i\epsilon]}\ . \tag{3.14}$$

After performing the k^0 integration with the caution that $\omega(k)$ may be negative for some range of k, we obtain

$$\omega(p)\cos[\theta(p)]=m+\frac{f}{2}\int\frac{dk}{(p-k)^2}\cos[\theta(k)]\ , \tag{3.15a}$$

$$\omega(p)\sin[\theta(p)]=p+\frac{f}{2}\int\frac{dk}{(p-k)^2}\sin[\theta(k)]\ . \tag{3.15b}$$

By adding and subtracting Eqs. (3.15a) and (3.15b), we can write them in the form

$$p\cos[\theta(p)]-m\sin[\theta(p)]=\frac{f}{2}\int\frac{dk}{(p-k)^2}\sin[\theta(p)-\theta(k)]\ , \tag{3.16a}$$

$$\omega(p)=m\cos[\theta(p)]+p\sin[\theta(p)]+\frac{f}{2}\int\frac{dk}{(p-k)^2}\cos[\theta(p)-\theta(k)]\ . \tag{3.16b}$$

Notice that from parity consideration, $A(p)=A(-p)$, $B(p)=-B(-p)$. So $\omega(p)=\omega(-p)$, $\theta(p)=-\theta(-p)$. By comparing the quark propagator derived through the two different methods of Bogoliubov-Valatin transformation and Feynman diagrams, we obtain the following relations in the limit $m\to 0$:

$$\frac{A(p)}{\omega(p)}=\frac{2\Phi(p)}{1+\Phi(p)^2}=\cos\theta(p)\ , \tag{3.17a}$$

$$\frac{p+B(p)}{\omega(p)}=\frac{1-\Phi(p)^2}{1+\Phi(p)^2}=\sin\theta(p)\ \text{ for } p\ge 0\ . \tag{3.17b}$$

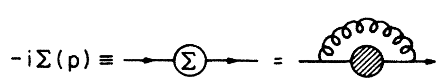

FIG. 2. Diagrammatic representation of the self-energy $\Sigma(p)$.

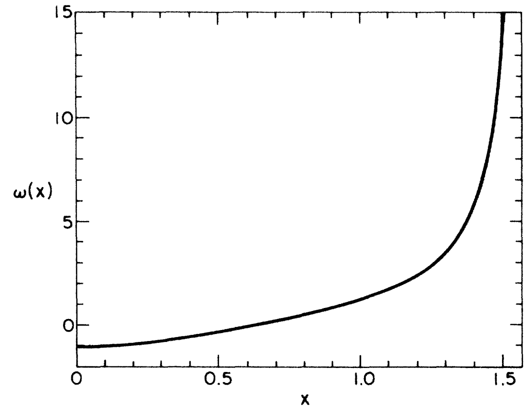

FIG. 3. Solution of $\omega(p)$ in the chiral limit $m\to 0$. There has been a variable change $p=\tan(x)$. All quantities are in proper units of $(2f)^{1/2}$. Note that $\omega(p)$ is an even function of p, so it is plotted for positive p only.

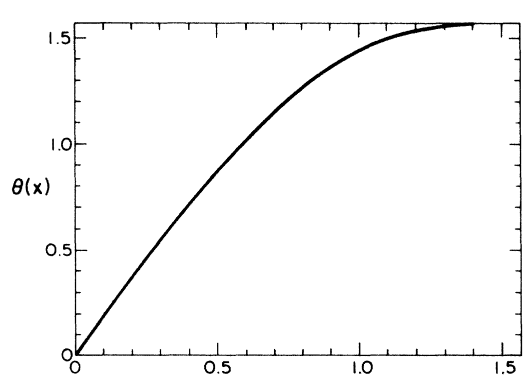

FIG. 4. Solution for $\theta(p)$ in the chiral limit, $m\to 0$. The x comes from the variable change $p=\tan(x)$. All quantities are in proper units of $(2f)^{1/2}$. Note that θ is an odd function of p.

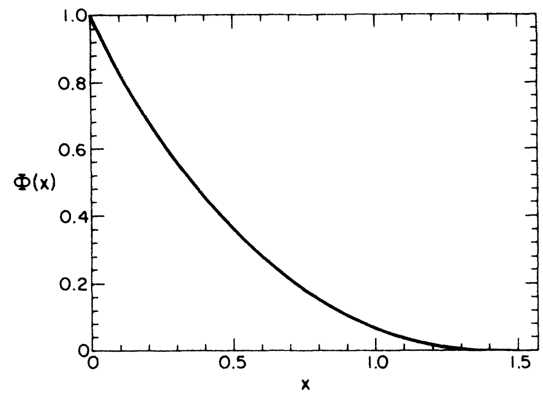

FIG. 5. The meson condensation wave function Φ in the chiral limit $m \to 0$. Note that Φ is an even function of p. All quantities are in proper units of $(2f)^{1/2}$.

We can use the parity property $\Phi(p)=\Phi(-p)$ to construct $\Phi(p)$ for $p<0$. The $\theta(p)$ is just the Bogoliubov-Valatin transformation angle. The gap equation (3.9) can now be obtained using the relation derived from Eqs. (3.17a) and (3.17b):

$$p\Phi(p)=\tfrac{1}{2}[1-\Phi(p)^2]A(p)-B(p)\Phi(p) \tag{3.18}$$

and using Eqs. (3.13a), (3.13b), (3.15a), (3.15b), (3.17a), and (3.17b) to express $A(p), B(p)$ in terms of the $\Phi(p)$. It is the same as that of Eq. (3.9).

In order to solve $\Phi(p)$, we start with Eqs. (3.16a) and (3.16b) and let $m \to 0$. Since the integral over the gluon propagator is infrared singular for $p=k$, we regularize it by the following scheme:[13,14]

$$\int \frac{dk}{(p-k)^2}F(k) \to \int \frac{dk}{(p-k)^2}\left[F(k)-F(p)-\frac{dF(p)}{dp}(k-p)\right] \tag{3.19}$$

for any smooth function $F(k)$ which decreases rapidly enough at infinity. In this way, the integrands in the integral equations become smooth. By making the variable change $p=\tan(x)$ and approximating the integrals in Eqs. (3.16a) and (3.16b) by sums over a set of discrete grid points $x_i \in [-\pi/2, \pi/2]$, $i=1,2,3,\ldots$, we can reduce them to two nonlinear algebraic equations. We solve these by using Newton's method. The solutions of $\omega(x)$, $\theta(x)$, and $\Phi(x)$ are plotted in Figs. 3, 4, and 5, respectively.

One further comment is in order here. In the chiral limit, $m \to 0$, there is another solution to Eq. (3.16a): namely,

$$\theta(p)=\tfrac{1}{2}\pi\Theta(p) , \tag{3.20}$$

where $\Theta(p)$ is the step function

$$\Theta(p)=\begin{cases}-1, & p>0,\\ -1, & p<0 .\end{cases} \tag{3.21}$$

This is the solution for which $\Phi(p)$ vanishes [Eqs. (3.17a) and (3.17b)]. The vanishing of the meson condensation signals the point where the 't Hooft approximation becomes invalid. Indeed by the 't Hooft-anomaly-condition argument,[15] either chiral symmetry is broken, in which case we have a nonzero $q\bar{q}$ condensate, or chiral symmetry is not broken, in which case the $q\bar{q}$ condensate vanishes and massless composite fermions appear. (This latter has the features of the strong-coupling regime.) Since throughout this paper we are interested only in chiral-symmetry breaking and in the weak-coupling regime, we disregard this solution.

In the chiral limit, $m \to 0$, the quark condensate $\langle \bar{\psi}\psi \rangle$ is given as

$$\begin{aligned}\langle \bar{\psi}\psi \rangle &= \frac{d}{dm}[-\mathrm{Tr}\ln iS^{-1}(m)]_{m=0}\\ &= -N\int_{-\infty}^{+\infty}\frac{dp\,A(p)}{2\pi\omega(p)}\\ &= -2N\int_0^{+\infty}\frac{dp}{2\pi}\cos\theta(p) ,\end{aligned} \tag{3.22}$$

where the N is the color factor. Substituting our solution of $\theta(p)$ into (3.22) we get

$$\langle \bar{\psi}\psi \rangle = -0.29N \tag{3.23}$$

in units of $(2f)^{1/2}$, which agrees with the result of Zhitnitsky[3] who obtained it by using the operator expansion.

IV. AT FINITE TEMPERATURE AND BARYON DENSITY

At finite temperature and baryon density, the expectation value of an operator (or a product of operators) Q is given as

$$\langle Q \rangle = \frac{\mathrm{Tr}\exp[-\beta(\hat{H}-\mu\hat{N})]T\{Q\}}{\mathrm{Tr}\exp[-\beta(\hat{H}-\mu\hat{N})]} , \tag{4.1}$$

where $\hat{H}$ is the Hamiltonian of the system, $\hat{N}$ is the baryon-number operator (not to be confused with the color factor), $T\{Q\}$ denotes the generalized time-ordering operator, and β is the inverse of temperature. Since we work in the color-singlet sector, the chemical potential μ here is the flavor chemical potential. We work only in the imaginary time formalism. In this case the Feynman rules and diagrams of perturbation expansion are the same as that of zero temperature, except different boundary conditions require that the field operators must be periodic (antiperiodic) for bosons (fermions).[16] Consequently the following changes are needed:

$$\begin{aligned}&k_0 \to i\omega_n+\mu ,\\ &\int \frac{dk^0 dk}{(2\pi)^2} \to \frac{i}{\beta}\sum_n \int \frac{dk}{2\pi} ,\\ &(2\pi)^2\delta^2(k_1+k_2+\cdots) \to \frac{\beta}{i}(2\pi)\delta(\mathbf{k}_1+\mathbf{k}_2+\cdots)\\ &\qquad\times\delta_{\omega_{n_1}+\omega_{n_2}+\cdots} ,\end{aligned} \tag{4.2}$$

where $\omega_n = 2\pi n/\beta$ for bosons and $(2n+1)\pi/\beta$ for fermions.

For clarity and to avoid arithmetic complication, we start with the $\mu=0$ case and later make some straightforward generalizations to finite μ. Notice that in our derivation of the gap equation and $\langle\bar{\psi}\psi\rangle$, the finite-temperature effects will only come in through the quark propagator

$$S(k)=\frac{i}{\not{k}-m+\Sigma(k)+i\epsilon}$$

when we change the k_0 integral into a summation over integer n. We know that

$$\frac{i}{\beta}\sum_n \frac{1}{i\omega_n \pm \omega(k) \mp i\epsilon} = \oint_c \frac{dk^0}{2\pi i}\frac{1}{k^0 \pm \omega(k)\mp i\epsilon} \times \tanh(\tfrac{1}{2}\beta k^0) , \quad (4.3)$$

where the contour c is a closed path around the imaginary axis. By taking into account that after the regularization $\omega(k)$ may be negative, we perform the contour integral and obtain

$$\frac{i}{\beta}\sum_n \frac{1}{k^0 \pm \omega(k)\mp i\epsilon} = \mp\tanh[\tfrac{1}{2}\beta\,|\,\omega(k)\,|\,] . \quad (4.4)$$

Thus, at finite temperature, Eqs. (3.7a), (3.7b), and (3.13) become (in the limit $m\to 0$)

$$p\cos[\theta(p)] = \frac{f}{2}\int \frac{dk}{(p-k)^2}\sin[\theta(p)-\theta(k)] \times\tanh[\tfrac{1}{2}\beta\,|\,\omega(k)\,|\,] , \quad (4.5a)$$

$$\omega(p) = p\sin[\theta(p)] + \frac{f}{2}\int \frac{dk}{(p-k)^2}\cos[\theta(p)-\theta(k)] \times\tanh[\tfrac{1}{2}\beta\,|\,\omega(k)\,|\,] , \quad (4.5b)$$

$$\begin{aligned}\langle\bar{\psi}\psi\rangle &= \frac{d}{dm}[-\mathrm{Tr}\ln iS^{-1}(m)]_{m=0}\\ &= -N\int_{-\infty}^{+\infty}\frac{dpA(p)}{2\pi\omega(p)}\tanh[\tfrac{1}{2}\beta\,|\,\omega(k)\,|\,]\\ &= -2N\int_0^{+\infty}\frac{dp}{2\pi}\cos\theta(p)\tanh[\tfrac{1}{2}\beta\,|\,\omega(k)\,|\,] . \quad (4.6)\end{aligned}$$

We know that Eqs. (3.16a) and (3.16b) have a meson solution for any $f>0$, and since $0<\tanh[\frac{1}{2}\beta\,|\,\omega(k)\,|\,]\le 1$ for any nonzero β, we conclude that chiral symmetry is broken at any finite temperature and restored only at $T\to\infty$.

At $\mu\neq 0$, we need to replace $i\omega_n$ to $(i\omega_n+\mu)$ in previous derivations and to write

$$\Sigma(p) = A(p) + \gamma_1 B(p) + \gamma_0 C(p) . \quad (4.7)$$

We use the formulas[17]

$$\frac{i}{\beta}\sum_n \frac{1}{i\omega_n+\mu\pm\omega(k)\mp i\epsilon} = \frac{1}{2\pi}\sum_{n=0}^{\infty}\left[\frac{1}{n+\delta}-\frac{1}{n+\bar{\delta}}\right] , \quad (4.8)$$

where

$$\delta = \tfrac{1}{2} - i\beta\frac{\mu\pm\omega(k)}{2\pi} \quad (4.9)$$

and

$$\sum_{n=0}^{\infty}\left[\frac{1}{(n+\delta)^s}-\frac{1}{(n+\bar{\delta})^s}\right] = \zeta(s,\delta)-\zeta(s,\bar{\delta}) . \quad (4.10)$$

Here $\zeta(s,\delta)$ is the generalized Riemann ζ function which has the following properties:

$$\lim_{s\to 1}\left[\zeta(s,\delta)-\frac{1}{s-1}\right] = -\psi(\delta) , \quad (4.11)$$

$$\mathrm{Im}\psi(\tfrac{1}{2}+iy) = \tfrac{1}{2}\pi\tanh(\pi y) . \quad (4.12)$$

After some algebra we get

$$\begin{aligned}S(p) &= \sum_n \frac{i}{\not{p}+\Sigma(k)+\gamma^0\mu}\\ &= \tfrac{1}{2}(\{\cos[\theta(p)]-\gamma^1\sin[\theta(p)]\}I(p)-\gamma^0K(p)) , \quad (4.13)\end{aligned}$$

where $I(p)=1-n(p)-\bar{n}(p)$, $K(P)=n(p)-\bar{n}(p)$, and

$$n(p) = \frac{1}{\exp[\beta(\,|\,\omega(p)\,|\,-\bar{\mu}(p))]} , \quad (4.14a)$$

$$\bar{n}(p) = \frac{1}{\exp[\beta(\,|\,\omega(p)\,|\,-\bar{\mu}(p))]} , \quad (4.14b)$$

$$\bar{\mu}(p) = \mu - C(p) . \quad (4.15)$$

Thus from Eqs. (3.11) and (3.22), we derive the following equations similar to Eqs. (4.5a), (4.5b), and (4.6):

$$p\cos[\theta(p)] = \frac{f}{2}\int\frac{dk}{(p-k)^2}\sin[\theta(p)-\theta(k)]I(k) , \quad (4.16a)$$

$$\omega(p) = p\sin[\theta(p)] + \frac{f}{2}\int\frac{dk}{(p-k)^2}\cos[\theta(p)-\theta(k)]I(k) , \quad (4.16b)$$

$$C(p) = -\frac{f}{2}\int\frac{dk}{(p-k)^2}K(k) , \quad (4.17)$$

and

$$\begin{aligned}\langle\bar{q}q\rangle &= -(\mathrm{Tr}S)_{m=0}\\ &= -N\int_{-\infty}^{+\infty}\frac{dpA(p)}{2\pi\omega(p)}I(k)\\ &= -2N\int_0^{+\infty}\frac{dp}{2\pi}\cos\theta(p)I(k) . \quad (4.18)\end{aligned}$$

We have a situation similar to the $\mu=0$ case which has been discussed before. Since

$$\begin{aligned}I(p) &= 1-n(p)-\bar{n}(p)\\ &= \{\exp[2\beta\,|\,\omega(p)\,|\,]-1\}n(p)\bar{n}(p) , \quad (4.19)\end{aligned}$$

it follows that $0<I(p)\le 1$ for any nonzero β and finite μ. We see again that chiral symmetry is broken at any finite

temperature and baryon density, and restored only in the limit $T \to \infty$ or $\mu \to \infty$.

McLerran and Sen[18] have studied the confinement-deconfinement phase transition of this model. They reached the conclusion that the confinement-deconfinement phase transition also does not occur at any finite temperature and baryon density. There is a close relation between these two cases. From our discussions on the effects of finite temperature and baryon density on chiral-symmetry breaking, we see that the gluon propagator, which gives rise to a linear confinement potential at zero temperature, is modified by multiplying a thermal factor which becomes zero only in the limit of $T \to \infty$ or $\mu \to \infty$. If we view the gauge coupling constant multiplying by the thermal factor as an effective gauge coupling constant, the resulting confinement potential is strong enough to bind a quark-antiquark pair, and therefore produce a quark condensate at any finite temperature and baryon density. It is only in the limit of $T \to \infty$ or $\mu \to \infty$ that the effective gauge coupling constant becomes zero and the quarks become free of interactions, and consequently the quark condensate vanishes.

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