Propagation properties and condensate formation of the confined Yang-Mills field

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The dynamical generation of a pole in the self-energy of a Yang-Mills field—an extension of the Schwinger mechanism—establishes a link between the tendency of the field to form nonperturbative vacuum condensates and its "noninterpolating" property in the confining phase—the fact that it has no particles associated with it. The nonvanishing residue of such a pole—a parameter b^4 of dimension (mass)^4—on the one hand provides for a nonvanishing value of $\langle 0 | (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 | 0 \rangle$, a contribution to the "gluon condensate." On the other hand, it implies a dominant nonperturbative form of the propagator that has no particle singularity on the real k^2 axis; instead, it describes a quantized field whose elementary excitations are short lived. The dispersion law for these excitations is given and shows that they grow more particlelike (are asymptotically free) at large momenta, thus providing a qualitative description of the short-lived excitation at the origin of a gluon jet. At large k^2, the nonperturbative propagator reproduces nonperturbative corrections derived from the operator-product expansion. Moreover, it is a solution to the Euclidean Dyson-Schwinger equation for the Yang-Mills field in the following sense: there exist nonperturbative three-vector vertices Γ_3 and auxiliary ghost-ghost-vector vertices G_3, satisfying all symmetry and invariance requirements, and in conjunction with which this propagator solves both the Euclidean Dyson-Schwinger equation through one-dressed-loop terms and the Γ_3 Slavnov-Taylor identity up to perturbative corrections of order g^2. The consistency conditions for this solution give $b^2 = \mu_0^2 \exp[-(4\pi)^2/11g^2]$ to this order, confirming the nonperturbative nature of the residue parameter, and providing a paradigm for the dynamical determination of condensates.

I. THE EXTENDED SCHWINGER MECHANISM

The spontaneous generation of mass for a gauge vector boson, whether by mechanisms visible already at the tree level or by pure quantum effects,[1] can be represented formally as the development of a pole at lightlike four-momentum in the polarization function—the general Schwinger mechanism.[2] Schematically,

$$\Pi^{(0)}(k^2)=0 \rightarrow \Pi(k^2) = -\frac{m^2(g,\mu_0)}{k^2} + O(g^2) , \quad (1.1)$$

where the polarization function Π is defined in the usual way through the transverse invariant

$$D_T(k^2) = \{-k^2[1+\Pi(k^2)]\}^{-1} \quad (1.2)$$

in the tensor decomposition

$$D^{\mu\nu}(k) = t^{\mu\nu}(k) D_T(k^2) + l^{\mu\nu}(k) D_L(k^2) , \quad (1.3)$$

$$t^{\mu\nu}(k) = g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} = g^{\mu\nu} - l^{\mu\nu}(k) \quad (1.4)$$

of the momentum-space gauge-field propagator

$$\delta_{ab} D^{\mu\nu}(k) = -i \int d^4x \langle 0 | T[A_a^\mu(x) A_b^\nu(0)] | 0 \rangle e^{ik\cdot x} . \quad (1.5)$$

In general, the residue m^2, representing the squared mass of the gauge boson grown massive, will depend on the renormalized gauge coupling g and the associated renormalization-mass scale μ_0. The essential, nonperturbative feature of the process indicated in (1.1) is that the function $1+\Pi(k^2)$, equal to unity in zeroth-order perturbation theory, develops a term with a lower *power* of the squared four-momentum, whereas perturbation theory, to any finite order, can at best produce logarithmic corrections, indicated by $O(g^2)$ in Eq. (1.1).

The present paper discusses what happens when this process of spontaneous generation of lower powers of k^2 proceeds one step further and leads to the formation of a second-order pole in $\Pi(k^2)$. For the object usually studied in quantum field theory, the (transverse) proper two-point vertex or inverse propagator $\Gamma_{2,T} = 1/D_T$, this implies the structure

$$\Gamma_{2,T}(k^2) = -k^2 + 2a^2(g,\mu_0) - \frac{b^4(g,\mu_0)}{k^2} + O(g^2) , \quad (1.6)$$

with a first-order pole at $k^2=0$, where the masses $a(g,\mu_0)$ and $b(g,\mu_0)$ [we write $2a^2$ instead of the m^2 of Eq. (1.1)] may depend nonanalytically on g, while $O(g^2)$ summarily denotes perturbative corrections. The term "perturbative," for the purposes of this paper, will denote contributions that can be represented as power series in g^2 and behave logarithmically for large k^2. We will refer to a dynamical effect leading to Eq. (1.6) as an *extended Schwinger mechanism.*

In what follows, arguments will be presented to indicate that a quantized Yang-Mills field with a proper two-point vertex as in Eq. (1.6), in a regime of parameters where

$$a^2(g,\mu_0) < b^2(g,\mu_0) , \quad (1.7)$$

is well suited in several ways for describing the physics of the chromodynamic gluon field. Before examining these arguments, however, one should reflect on a prejudice encountered frequently in the study of gauge-field propagators. It is often stated summarily that the propagator and other Green's functions for a Yang-Mills field (i) are gauge-noninvariant quantities and (ii) therefore have no physical meaning. This dogma is such an oversimplification as to be counterproductive. Even the first part of its statement is merely a half-truth—in the covariantly quantized theory with a standard gauge-fixing term $-[\partial_\mu A^\mu(x)]^2/2\xi_0$ and corresponding ghost coupling, on which the discussion in this paper will be based, the propagator, and in fact all Green's functions, enjoy the well-known Becchi-Rouet-Stora (BRS) invariance, a genuine nonclassical gauge invariance comprehensive enough to ensure (among other things) all those consequences of gauge invariance essential to the renormalization program. More significantly, while the radiative corrections generally do make a non-Abelian $D_T(k^2)$ dependent on the gauge-fixing parameters such as ξ_0, this dependence may well be absent for certain dominant nonperturbative parts of the transverse vector propagator, which may then have an eminently physical meaning. The prime examples, in theories such as the standard electroweak model, are the particle singularities: the facts that the W and Z propagators (i) possess a pole on the positive real k^2 axis and that (ii) this pole is at the physical $m_W{}^2$ or $m_Z{}^2$, are independent of gauge fixing, and indeed form the conceptual basis for all calculations of those masses. It is from these gauge-fixing-independent particle poles that we infer—by reversing the chain of arguments used in establishing a Lehmann-Källén representation—that the corresponding gauge fields are interpolating fields for certain asymptotically detectable particles. Utterly familiar as these truths are, it is clarifying to realize that an issue as "physical" as the particle interpretation of the electroweak theory rests on the theoretical side, *not* on the much-touted gauge-invariant Green's functions, but on gauge-fixing-independent features of an otherwise gauge-fixing-dependent function. Below we shall present arguments for, and indeed prove to a certain order, the conjecture that the nonperturbative terms in Eq. (1.6) may enjoy a similar independence of gauge fixing, and a similar status in determining the particle interpretation, for a Yang-Mills field whose conspicuous empirical feature is that it just has *no* asymptotically detectable particles associated with it.

In Sec. II we start our study of expression (1.6)—while deferring temporarily the question of how it may emerge dynamically—by recording those of its properties that make it peculiarly suitable for a description of confined-gluon propagation. These include, first, the fact that it implies the nonvanishing of certain nonperturbative vacuum expectation values, and second, that it leads, in the regime (1.7), to a form of the gluon propagation function that has no particle singularity on the real k^2 axis, and thus predicts the nonexistence of an asymptotically detectable gluon particle. Instead, its space-time structure describes a field whose elementary excitations are intrinsically of finite lifetime, a lifetime growing however with $|\mathbf{k}|$ so as to make these excitations look increasingly particlelike at large momenta. These propagation properties fit remarkably well the observed characteristics of the short-lived gluonic excitation at the origin of a gluon jet. The logical connection established by expression (1.6) between those two effects makes it possible, perhaps for the first time, to think in mathematical terms of the gluon *vacuum condensate as a confining agent,* and may influence our notion of gluon confinement.

The more intricate question of whether Eq. (1.6) fits into the specific dynamical framework of quantum chromodynamics is addressed in Sec. III—V. As a prelude, we may ask (Sec. III) whether the nonperturbative part of (1.6) is consistent with asymptotic, large-k^2, nonperturbative corrections to the free gluon propagator as derived previously from the operator-product expansion (OPE). This indeed turns out to be the case, and the comparison not only identifies the dimension-four condensate appearing in the OPE in terms of the residue parameter b^4 up to $O(g^2)$, but may also shed some light on the peculiar workings of the operator-product expansion in this sector. We then turn to the central question (Sec. IV) of whether the vertex (1.6) can be a solution to the dynamical equation determining the propagator, the Dyson-Schwinger equation, which for technical simplification we consider here up to its one-loop terms and for a pure gluon theory. Here we must first ask whether nonperturbative parts of the three-point vertices involved can be constructed which, together with (1.6), satisfy the relevant Slavnov-Taylor identities. Such a construction is possible, and not only fixes the nonperturbative parts of most of the invariant functions in these vertices, but also suggests a very specific dressing pattern which, as a heuristic principle, we may require to persist in the transverse vertex part not fully determined by the Slavnov-Taylor identities. In this way we are led to a linearly parametrized family of three-point vertices "compatible" with Eq. (1.6). Section IV then demonstrates that the Dyson-Schwinger equation *in the Euclidean domain,* when set up with these compatible vertices, indeed reproduces expression (1.6) up to "perturbative" corrections $O(g^2)$ when its inverse, D_T, is used in the (nonlinear) right-hand side, provided two *consistency conditions* are met which determine, to this order, the "condensate" parameters a^2 and b^2. These conditions give $a^2=0$ and the important relation (4.16), which agrees with renormalization-group requirements, and confirms the conjectured, nonanalytic g dependence. An essential point about this solution is that the *perturbative renormalization constants remain applicable,* as is expected for an asymptotically free theory.

While the general lines of an inclusion of the two-loop Dyson-Schwinger term will become quite clear from Sec. IV by way of analogy, the fact that the "compatible" three-point vertices get restricted but not yet fully determined in the process of this solution requires further consideration. We close in Sec. V with the conjecture that the solution may be regarded as the first stage of a scheme for determining nonperturbative parts of primitively divergent vertices—a scheme which, in spite of the infinite nature of the Green's-function hierarchy, may be a *closed* problem.

Nonperturbative solutions to the Dyson-Schwinger equation for the Yang-Mills field have been sought previously in what may be called the extreme opposite direction, where a second-order pole develops not in the polarization function $\Pi(k^2)$, but in the propagator $D^{\mu\nu}(k)$ itself.[3] Since the Dyson-Schwinger equation is strongly nonlinear, and its solution probably not unique, it is quite possible that both types of solutions exist. Here we only wish to state that the physical and mathematical properties of expression (1.6) discussed below would seem to make it at least a serious contender for the "physical" solution. We remark, however, that a strongly singular behavior of the one-gluon-exchange diagram in momentum transfer—the heuristic guideline for the search of Ref. 3—would emerge also in the present approach, but from the singular character of the vertices (Sec. IV) rather than from the gluon propagator, as suggested some time ago by Alabiso and Schierholz.[4]

Some limitations of the present paper should be emphasized from the outset. On the technical side, covariant gauge fixing and the $\overline{\text{MS}}$ renormalization scheme (where $\overline{\text{MS}}$ denotes the modified minimal subtraction scheme) are employed exclusively. On the physical side, nothing yet is said about the inclusion of fermions, nor about the behavior of the theory at nonzero temperature.

II. CONDENSATE FORMATION AND NONINTERPOLATING GLUON FIELD

Consider the partially renormalized vacuum expectation value

$$C_2=\left\langle 0\left|\frac{\alpha_s}{\pi}[\partial^\mu \bar{A}{}^{\nu}_{a}(x)-\partial^\nu \bar{A}{}^{\mu}_{a}(x)]^2\right|0\right\rangle \tag{2.1}$$

in $D=4-\epsilon$ Euclidean dimensions, where

$$\alpha_s=\mu_0^\epsilon\left[\frac{g^2}{4\pi}\right] \tag{2.2}$$

is the appropriate dimensionless coupling constant, and where the overbars denote the renormalized gauge field. This represents a contribution (not classically gauge invariant but BRS invariant) to the "gluon vacuum condensate" of Shifman, Vainshtein, and Zakharov,[5] whose partially renormalized version reads

$$C_{GG}=\left\langle 0\left|\frac{\alpha_s}{\pi}\bar{G}{}^{\mu\nu}_{a}(x)\bar{G}_{\mu\nu,a}(x)\right|0\right\rangle \tag{2.3}$$

in terms of the field-strength tensor

$$\bar{G}{}^{\mu\nu}_{a}(x)=\partial^\mu\bar{A}{}^{\nu}_{a}(x)-\partial^\nu\bar{A}{}^{\mu}_{a}(x)+\frac{Z_1}{Z_3}gf_{abc}\bar{A}{}^{\mu}_{b}(x)\bar{A}{}^{\nu}_{c}(x)\ . \tag{2.4}$$

This condensate, we recall, characterizes the leading nonperturbative vacuum properties in the context of operator-product expansions for current-correlation functions.[5,6] While (2.1) is just the simplest contribution to (2.3), with no direct physical significance by itself, it will be of interest here because it can be related directly to the nonperturbative part of the Euclidean gluon propagator. Taking the T product in (1.5) to be the naive one,[7] the relation

$$\tfrac{1}{2}T\{[\partial_\mu\bar{A}_{\nu,a}(x)-\partial_\nu\bar{A}_{\mu,a}(x)][\partial^\mu\bar{A}{}^{\nu}_{a}(y)-\partial^\nu\bar{A}{}^{\mu}_{a}(y)]\}$$
$$=\left[g_{\nu\mu}\left[\frac{\partial}{\partial x^\zeta}\frac{\partial}{\partial y_\zeta}\right]-\frac{\partial}{\partial x^\nu}\frac{\partial}{\partial y^\mu}\right]T[\bar{A}{}^{\mu}_{a}(x)\bar{A}{}^{\nu}_{a}(y)]+iZ_3{}^{-1}(N^2-1)(D-1)\delta^D(x-y)\mathbb{1} \tag{2.5}$$

is straightforward to establish, for an SU(N)-color group, by use of the canonical equal-time commutation relations. Here, as in (2.4), Z_3 denotes the gauge-field renormalization constant. Upon taking expectation values of both sides in the physical vacuum state, and using the inverse of (1.5) in D Euclidean dimensions, one finds

$$\langle 0|T\{[\partial_\mu\bar{A}_{\nu,a}(x)-\partial_\nu\bar{A}_{\mu,a}(x)][\partial^\mu\bar{A}{}^{\nu}_{a}(y)-\partial^\nu\bar{A}{}^{\mu}_{a}(y)]\}|0\rangle=2(N^2-1)(D-1)\int\frac{d^Dk_E}{(2\pi)^D}[k_E{}^2\bar{D}_T(-k_E{}^2)-Z_3{}^{-1}]e^{ik_E\cdot(x-y)}\ , \tag{2.6}$$

which involves the renormalized version $\bar{D}_T=Z_3{}^{-1}D_T$ of the transverse gluon propagation function (1.2). Here $k_E{}^2$ is the positive-definite, squared Euclidean four-momentum. By putting $y=x$ one obtains for the condensate (2.1)

$$C_2=\frac{g^2}{2\pi^2}(N^2-1)(D-1)\mu_0{}^{4-D}\times\int\frac{d^Dk_E}{(2\pi)^D}[k_E{}^2\bar{D}_T(-k_E{}^2)-Z_3{}^{-1}]\ . \tag{2.7}$$

The subtraction of $Z_3{}^{-1}$ in the integrand performs, for this particular matrix element, a function analogous to normal operator ordering with free fields. Indeed for the free field, $k_E{}^2D_T(-k_E{}^2)\to 1$ and $Z_3\to 1$, and the condensate vanishes.

We may now insert the particular transverse propagator corresponding to (1.6) and (1.7),

$$\bar{D}_T(-k_E{}^2)=\frac{k_E{}^2}{k_E{}^4+2a^2k_E{}^2+b^4}+O(g^2),\quad a^2<b^2\ , \tag{2.8}$$

and moreover stipulate that *in conjunction with (1.6) or (2.6), the perturbative renormalization constants remain* *valid*, that is in particular, $Z_3=1+O(g^2)$. This important point, to be verified in Sec. IV, is intuitively obvious: the large-k^2 behavior of (1.6) or (2.8) is still the free one, and so will be the large-momentum behavior of the higher vertices compatible with (1.6). So the divergent parts of loop diagrams, and thereby the renormalization constants, will be those of perturbation theory. (Put differently, the nonperturbative propagator modification, because it only affects the small-k^2 behavior, is compatible with asymptotic freedom.) The result then is, for $N=3$,

$$C_2=-\frac{12}{\pi^2}\left[\frac{g}{4\pi}\right]^2\left\{(b^4-4a^4)\left[-\ln\left[\frac{b^2}{\mu_0{}^2}\right]+N_\epsilon+\tfrac{1}{3}\right]\right.$$
$$\left.+a^2\left[4\frac{a^4}{b^2}-3b^2\right]\left[1-\left[\frac{a}{b}\right]^4\right]^{-1/2}\arccos\left[\frac{a}{b}\right]^2\right\}+O(g^4) \tag{2.9}$$

up to terms of order $\epsilon=4-D$, where an abbreviation[8] convenient for working in the $\overline{\rm MS}$ scheme,

$$N_\epsilon=\frac{2}{\epsilon}-\gamma_E+\ln 4\pi\ , \tag{2.10}$$

has been used for the divergent term. It is clear that (2.1), as a matrix element of a local composite operator, will exhibit a divergence not taken care of by field renormalization, and also that the extra renormalization required for (2.1) as a contribution to (2.3) cannot be discussed in the context of (2.9) alone, not only because (2.3) has non-Abelian terms involving the higher n-point functions of $\overline{A}(x)$ up to $n=4$, but also because the operator of (2.3), at least in the gauge-fixing scheme used here, will mix upon renormalization with classically gauge-noninvariant Lorentz scalars of dimensions four. Since this mixing problem is reasonably well studied,[9] it will not be of interest to us here. The essential point about (2.9) emerges when we anticipate from Sec. IV that the case of physical interest for the gluon field is $a^2=0$, and that the dynamical determination of b^2 implies $-(g/4\pi)^2\ln(b^2/\mu_0{}^2)=\frac{1}{11}+O(g^2)$. We then conclude that

$$C_2(a^2=0)=b^4\left[-\frac{12}{11\pi^2}+O(g^2)\right],\quad N=3\ , \tag{2.11}$$

where divergence and operator-mixing problems appear only in the terms $O(g^2)$. This result says that the residue parameter b^4 which controls the extended Schwinger mechanism induces—or, depending on taste, may be regarded as the effect of—a nonvanishing "condensate" (2.1). The leading-order coefficient connecting the two is, as a result of the nonperturbative g dependence of b^2, of order g^0 in spite of the explicit g^2 factor in the definition (2.1); it is also independent of either the gauge-fixing parameter ξ or the renormalization-mass scale μ_0. [In higher orders, the latter properties are not expected to persist for (2.1) alone, but only for the renormalized version of (2.3), which should be gauge-fixing independent and renormalization-group invariant.] It hardly needs emphasis that (2.11) is not an "approximation" to (2.3) in any quantitative sense; indeed to the leading order exhibited, it even has the wrong sign, since the "empirical" gluon condensate as extracted from QCD sum rules[6] is positive. Thus as a contribution to (2.3), (2.11) is overcompensated in nature by the non-Abelian and operator-mixing terms.

Next we may cast a glance at the simpler and more familiar condensate

$$C_m=\left\langle 0\left|\frac{\alpha_s}{\pi}\overline{A}{}^{\mu}_{a}(x)\overline{A}_{\mu,a}(x)\right|0\right\rangle\ , \tag{2.12}$$

which of course is essentially the trace of the tadpole term in Fig. 2 below and will therefore be termed the mass-type condensate. Its evaluation analogous to (2.7) gives

$$C_m=-\frac{g^2}{4\pi^2}(N^2-1)\mu_0{}^{4-D}\times\int\frac{d^Dk_E}{(2\pi)^D}[(D-1)\overline{D}_T(-k_E{}^2)+\overline{D}_L(-k_E{}^{-2})]\ . \tag{2.13}$$

It is elementary that the longitudinal contribution actually disappears due to BRS invariance—the simplest Slavnov-Taylor identity ensures that, to all orders,

$$\overline{D}_L(k^2)=\frac{-\xi}{k^2+i0},\quad \xi=Z_3{}^{-1}\xi_0\ , \tag{2.14}$$

so the longitudinal contribution to (2.13) vanishes by virtue of the well-known symmetric-integration theorem

$$\int d^Dk\frac{k^\mu k^\nu}{(k^2)^2}=\frac{g^{\mu\nu}}{D}\int d^Dk\frac{1}{k^2}=0\ . \tag{2.15}$$

It is intuitively necessary that only transverse gluons contribute to the generation of quantities such as (2.6) and (2.13). Since vacuum condensates are physical quantities leading to observable effects, one expects unphysical degrees of freedom, such as the longitudinal gluon fields or Faddeev-Popov ghosts, not to participate in their formation, except indirectly through higher perturbative corrections in $\overline{D}_T$.

On using again the propagator (2.8), the condensate (2.13) becomes

$$C_m = \frac{6}{\pi^2}\left[\frac{g}{4\pi}\right]^2 \left[2a^2\left[-\ln\frac{b^2}{\mu_0^2}+N_\epsilon+\tfrac{1}{3}\right] + b^2 f_m\left[\frac{a^2}{b^2}\right]\right] \quad (2.16a)$$

$$= a^2\left[\frac{12}{11\pi^2}+O(g^2)\right], \quad (2.16b)$$

where a divergence, and thus renormalization and operator-mixing problems,[10] appear only in $O(g^2)$. Here $f_m(\rho)$ is an elementary function vanishing at $\rho=0$. Once more, the parameters a^2, b^2 characterizing the extended Schwinger mechanism are seen to be connected to a condensate. This one is less important, since the most plausible solution for a^2 will turn out below to be $a^2=0$, in which case (2.16) shows that C_m vanishes.

In obtaining results such as (2.11) and (2.16), it is crucial that all Green's functions involved, and the equations relating them to condensates, be considered in the *Euclidean* domain. Had we started from the corresponding Minkowski-space relations and divided the integrations into an integral along a real k_0 axis and a $(D-1)$-dimensional Euclidean one, then a Wick rotation of the k_0 contour would have produced, in addition to the above real answers, complex contributions from integrand poles in the first and third quadrants of the k_0 plane [arising from those of Eq. (2.20) below] which would not make physical sense. To a much higher degree than in the perturbative theory—where switching back and forth between the two regimes by means of Wick rotations is an essentially trivial act—the prescription of Euclidean field theory must be taken seriously here: calculate all Green's functions and related quantities in the Euclidean domain, and only in the end continue to Minkowski space. This will become important again when we study the Dyson-Schwinger equation for $\Gamma_{2,T}$.

What are the propagation characteristics of the obviously condensation-prone field described by (1.6)? We have already written the Euclidean transverse propagator in (2.8); as continued to Minkowski space, it reads

$$D_T(k^2) = \frac{-k^2}{(k^2)^2 - 2a^2k^2 + b^4} + O(g^2), \quad a^2 < b^2 . \quad (2.17)$$

Here we discuss the nonperturbative part alone; it will become clear in Sec. IV that the "perturbative" terms denoted again by $O(g^2)$ will not alter the picture qualitatively.

The function in (2.17), which Fig. 1 displays for $a^2=0$, has no pole on the real k^2 axis. Thus the field it describes is not an interpolating field for any asymptotically detectable "gluon particle" of whatever mass. Instead one observes, by decomposing expression (2.17) as

$$D_T(k^2) = \frac{r_+}{k^2 - s_+} + \frac{r_-}{k^2 - s_-} + O(g^2) \quad (2.18)$$

with residues and pole positions

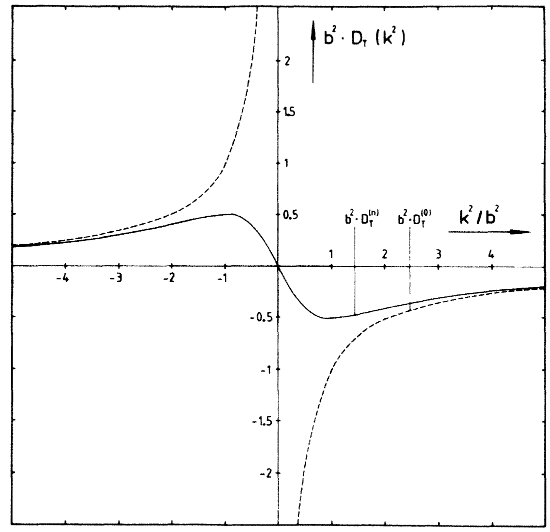

FIG. 1. Nonperturbative, transverse propagator function $D_T^{(n)}(k^2)$ corresponding to the extended Schwinger mechanism with $a^2=0$ (solid line) and bare transverse propagator $D_T^{(0)}(k^2) = -1/k^2$ (dashed line) in units of $1/b^2$ as functions of k^2/b^2.

$$r_\pm = \frac{1}{2}\left\{1 \mp i\frac{a^2}{b^2}\left[1-\left[\frac{a}{b}\right]^4\right]^{-1/2}\right\}, \quad (2.19)$$

$$s_\pm = a^2 \pm i b^2 \left[1-\left[\frac{a}{b}\right]^4\right]^{1/2}, \quad (2.20)$$

that the propagator exhibits two conjugate poles in the complex k^2 plane. Both features fit in remarkably well with what we know about physical gluons. Not only is there, on the theoretical side, no principle forbidding complex propagator poles in a non-Abelian, confining gauge theory, more importantly, the empirical case for them is fairly compelling. A photon can get converted into e^+e^- pairs, or more complicated channels, only when interacting with matter, when radiated into a perfect vacuum, it will travel on indefinitely as a stable particle. Field theory describes this by the gauge-independent pole at $k^2=0$ in the transverse photon propagator, which fixes the photon dispersion law as $k^2=0$. By contrast, what we call the gluon—the extremely short-lived excitation at the origin of a gluon jet—will rapidly convert itself into hadronization channels even when released into an absolute vacuum; it has an *intrinsically* finite lifetime. Regardless of how we describe the later stages of the hadronization process, the gluon's dispersion law itself must therefore contain a finite lifetime: that is, there must be a gauge-independent complex zero in the inverse gluon propagator. (This, of course, is true already for the effective propagators of fast-decaying hadronic resonances, but these are composites, while the gluon is an excitation of one of the elementary, fundamental fields.) The conjugate zero then follows from the nature of the propagator, which simultaneously describes (anti)gluon propagation backward in time.

To elaborate on these remarks, consider the coordinate-space expression

$$D_T^{\mu\nu}(x-y)=\int \frac{d^4k}{(2\pi)^4}\frac{-g^{\mu\nu}k^2+k^\mu k^\nu}{(k^2-s_+)(k^2-s_-)} \times e^{-ik\cdot(x-y)} . \tag{2.21}$$

Again we restrict ourselves to the case $a^2=0$, where

$$r_\pm=\tfrac{1}{2},\ \ s_\pm=\pm ib^2 , \tag{2.22}$$

the case anticipated to be the physically relevant one. This case is also distinguished as the only one where the residues in (2.18) are real and positive (thus the complex poles are not "ghost poles" in the old sense[11]) whereas the pole positions are purely imaginary. Then doing the k_0 integration by residues we have

$$D_T^{\mu\nu}(x-y)=\frac{1}{4b^2}(g^{\mu\nu}\Box-\partial^\mu\partial^\nu)_x\int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\left[\frac{1}{\omega(|\mathbf{k}|)}e^{-i\omega(|\mathbf{k}|)|x_0-y_0|}+\text{c.c.}\right] ; \tag{2.23}$$

that is, the elementary excitations of the field propagate as decaying plane waves, with a complex dispersion law given by

$$\omega(|\mathbf{k}|)=(|\mathbf{k}|^2-ib^2)^{1/2}=\epsilon(|\mathbf{k}|)-\frac{i}{2}\frac{1}{\tau(|\mathbf{k}|)} , \tag{2.24}$$

which consists of an energy-momentum relation

$$\epsilon(|\mathbf{k}|)=\{\tfrac{1}{2}[(|\mathbf{k}|^4+b^4)^{1/2}+|\mathbf{k}|^2]\}^{1/2} \tag{2.25}$$

and a momentum-dependent lifetime

$$\tau(|\mathbf{k}|)=\{2[(|\mathbf{k}|^4+b^4)^{1/2}-|\mathbf{k}|^2]\}^{-1/2} =\frac{\epsilon(|\mathbf{k}|)}{b^2} . \tag{2.26}$$

For momenta large compared to the "condensate mass" b,

$$\epsilon(|\mathbf{k}|)=|\mathbf{k}|\left[1+\frac{1}{8}\left[\frac{b}{|\mathbf{k}|}\right]^4+\cdots\right] , \tag{2.27a}$$

$$\tau(|\mathbf{k}|)=\frac{|\mathbf{k}|}{b^2}\left[1+\frac{1}{8}\left[\frac{b}{|\mathbf{k}|}\right]^4+\cdots\right] , \tag{2.27b}$$

so the lifetime grows linearly with momentum, while the energy-momentum relation rapidly approaches that of a free massless particle. These relations, which reflect "naive" asymptotic freedom, are in qualitative agreement with observation: when endowed with larger momenta, the gluonic excitations grow increasingly particlelike, and "jets get jettier." On the other hand, for momenta $|\mathbf{k}| \ll b$, both ϵ and τ approach finite lower limits,

$$\epsilon(|\mathbf{k}|)=\frac{b}{\sqrt{2}}\left[1+\frac{1}{2}\left[\frac{|\mathbf{k}|}{b}\right]^2+\cdots\right] , \tag{2.28a}$$

$$\tau(|\mathbf{k}|)=\frac{1}{b\sqrt{2}}\left[1+\frac{1}{2}\left[\frac{|\mathbf{k}|}{b}\right]^2+\cdots\right] , \tag{2.28b}$$

so that, in particular, $\epsilon\tau\to\frac{1}{2}$. Thus even at zero three-momentum there are short-lived elementary excitations of the field which follow a time-energy uncertainty relation. While further evaluation of (2.23) proceeds along standard lines leading to Bessel-function expressions not important to the present discussion, it might be mentioned that the space-time propagation function decays exponentially at large spacelike separations in roughly the same way as a scalar, massive, free propagator with mass $b/\sqrt{2}$, except that the decay is a damped oscillation, rather than monotonous. The essential difference from a particle propagator emerges at large *timelike* arguments, where a particle propagator is radiative, whereas (2.23) again decays exponentially according to

$$D_T^{\mu\nu}(x)\xrightarrow[x^2\to\infty]{}\frac{-i}{2b(2\pi)^{3/2}}\left[g^{\mu\nu}-\frac{x^\mu x^\nu}{x^2}\right]|t|^{-3/2} \times e^{-b|t|/\sqrt{2}}\cos\left[\frac{b}{\sqrt{2}}|t|-\frac{\pi}{8}\right] , \tag{2.29}$$

with $|t|=(x^2)^{1/2}$, which once more exhibits the lifetime $1/b\sqrt{2}$ of (2.28b).

The difference between (2.17) on one side, and the exact statement (2.14) about $D_L(k^2)$ on the other, is noteworthy. The longitudinal gluon, exactly like the longitudinal photon in QED, retains its zeroth-order propagator, and thus its particle pole, to all orders. Both can afford to do so precisely because they are unphysical degrees of freedom: since physical initial or final states containing them do not exist, they appear only in intermediate states, where the theory may with impunity propagate them in a formally particlelike manner. Field theory accounts for this by defining the S matrix as including projectors onto a "physical subspace" free of longitudinal gluons. This device *cannot* be applied to transverse gluons, which despite their color-nonsinglet nature must not be dismissed summarily as unphysical in the above, rigid sense. The reason is that we do see transverse-gluon jets, whereas longitudinal-gluon or Faddeev-Popov-ghost jets do not exist. If we make the plausible assumption that the already well-established tendency of jet gluons to become more particlelike at higher momenta may be extrapolated, and if relations such as (2.27) plus (3.8) may be used as guides for such an extrapolation in even the crudest qualitative sense, then eventually (though at gluon energies admittedly exorbitant, $\epsilon\approx10^{12}$ GeV) these excitations would travel over distances of millimeters during their intrinsic lifetime, and could in principle be intercepted by macroscopic detectors *before* hadronization. A theory that would simply seek to banish them to an unphysical subspace would then be manifestly insufficient.

By viewing together Eqs. (2.11) and (2.16) on the one hand, and Eqs. (2.18) and (2.24) on the other, the extended Schwinger mechanism may be seen to establish the following physical picture. Existence of a certain vacuum condensate $\propto b^4$ modifies the gluon self-energy nonperturbatively through a pole term with residue b^4. This term, in turn, pushes the particle pole of the free gluon propagator into the complex plane, to $k^2=\pm ib^2$, thereby providing an imaginary part to the dispersion law, or a finite lifetime $\propto 1/b$ for the elementary excitations of the field. By having to plow their way through a "vacuum" filled with condensates, the excitations get damped out in a way reminiscent of disturbances in a viscous medium. At momenta $\gg b$, particlelike behavior of the elementary excitations is recovered asymptotically, though never exactly.

By giving the connection between these effects a specific mathematical form, the extended Schwinger mechanism opens the possibility—perhaps for the first time—of thinking in quantitative terms about the *gluon vacuum condensate as a confining agent,* where confinement of gluons is understood here in the strict empirical sense that free gluons are not seen but short-lived gluonic excitations are. Of course, this possibility is contingent upon a demonstration that an extended Schwinger mechanism is not outside the specific dynamical scheme of quantum chromodynamics, and the next sections address themselves to this question.

III. CONDITIONS FROM THE OPERATOR-PRODUCT EXPANSION AND THE RENORMALIZATION GROUP

The first nontrivial test which expression (1.6) or (2.17) has to pass in quantum chromodynamics is that it must reproduce the asymptotic, large-k^2, "nonperturbative corrections" to the free gluon propagator that can be derived[8] from the Wilson operator-product expansion (OPE). Such corrections are obtained in the usual fashion by writing a Wilson expansion for the time-ordered field product in (1.5), taking vacuum expectation values, and treating the Wilson-coefficient functions in the small-x or large-k^2 regime by perturbation theory. The result takes the form $D=D^{\mathrm{P}}+D^{\mathrm{NP}}$, where the "perturbative" portion,

$$\bar{D}^{\mathrm{P}}_{\mu\nu}(k)=t_{\mu\nu}(k)\left[-\frac{1}{k^2+i0}+O(g^2)\right]+l_{\mu\nu}(k)\left[-\frac{\xi}{k^2+i0}\right], \tag{3.1}$$

arises from the terms associated with the identity operator in the Wilson expansion. The leading contribution at large k^2 to the "nonperturbative correction" D^{NP}, in the pure gluon theory considered here, comes from terms associated with an operator of mass dimension four—it is generally assumed that the dimension-two operators such as $A_\mu A_\nu$, while present in the general OPE (Ref. 10), do not contribute to the vacuum expectation value, i.e., that a condensate of type (2.12) is absent. For color-SU(N) and $D=4-\epsilon$ Minkowskian dimensions, this leading contribution is given by[8]

$$\bar{D}^{\mathrm{NP}(1)}_{\mu\nu}(k)=-i\frac{C'}{4(N^2-1)(D-1)}\left\{\frac{1}{D(D+2)}\left[(D+1)g_{\mu\nu}\left[\frac{\partial}{\partial k^\lambda}\frac{\partial}{\partial k_\lambda}\right]-2\frac{\partial}{\partial k^\mu}\frac{\partial}{\partial k^\nu}\right](2\pi)^D\delta^D(k)\right\}+O(g^2)\,, \tag{3.2}$$

where C' denotes the nonperturbative vacuum expectation value of the dimension-four operator. As indicated, this has its own perturbative corrections $O(g^2)$, which we do not consider here. The dimension-four operator is occasionally[8] written $G^{\mu\nu}G_{\mu\nu}$, so that C' becomes proportional to the full gluon condensate C_{GG} of Eq. (2.3), and this indeed would be the only choice if one were expanding a gauge-invariant operator product, as in the standard applications of the OPE to products of gauge-invariant fermion currents. In the present case, where one is expanding the gauge-*noninvariant* $T(A_\mu A_\nu)$, this choice however is not necessary and in fact implausible, and we shall see presently how to identify the C' in (3.2).

At first sight, (3.2) looks quite different from anything that might emerge from (2.17), and it is therefore interesting that the two in fact agree. Since (3.2) is distribution valued, we must consider its k-space integral with a test function $f(k)$ regular near $k=0$ and possessing thus a Taylor expansion

$$f(k)=f(0)+f'_\lambda(0)k^\lambda+\frac{1}{2!}f''_{\kappa\lambda}(0)k^\kappa k^\lambda+\cdots\,. \tag{3.3}$$

When translating these expressions to Euclidean space and evaluating the integral by partial integrations, (3.2) simply picks out the second-order coefficients, and we obtain

$$\mu_0{}^{4-D}\int\frac{d^Dk_E}{(2\pi)^D}\bar{D}^{\mathrm{NP}(1)}_{\mu\nu}(k_E)f(k_E)=-\frac{\mu_0{}^{4-D}C'}{4(N^2-1)(D-1)}\frac{1}{D(D+2)}[(D+1)\delta_{\mu\nu}\mathrm{tr}f''(0)-2f''_{\mu\nu}(0)]+O(g^2)\,. \tag{3.4}$$

We want to compare this with the nonperturbative part of (1.3) with (2.17), which is of the purely transverse form

$$\bar{D}^{\rm NP}_{\mu\nu}(k)=t_{\mu\nu}(k)\left[\bar{D}_T(k^2)-\left[-\frac{1}{k^2+i0}+O(g^2)\right]\right]. \tag{3.5a}$$

For the special case of the nonperturbative propagator (2.17) considered here, we have

$$\bar{D}_T(k^2)-\left[-\frac{1}{k^2+i0}+O(g^2)\right]=\frac{b^4-2a^2k^2}{k^2[(k^2)^2-2a^2k^2+b^4]}+O(g^2). \tag{3.5b}$$

When this is again translated to Euclidean space and subjected to symmetric D-dimensional integration with the test function (3.3), a contribution proportional to $f(0)$ can be avoided only by choosing $a^2=0$; the term then becomes proportional to a Feynman-parameter integral vanishing by symmetry:

$$\int_0^1 \frac{dy}{2y-1}=0\,.$$

Thus if the absence of a C_m-type condensate usually assumed in the OPE is correct, then as in (2.16) we are led to $a^2=0$ as the physically preferred value of the spontaneous mass parameter in (1.6). We continue, of course, to pick up contributions from the higher, even-order terms in the Taylor series (3.3), which in $D=4$ will be increasingly divergent, but these turn out to have mass factors of dimensions 6,8,10,..., and therefore must be compared with OPE terms of higher condensate dimension, which vanish more rapidly as $k^2\to\infty$. For the second-order term, we get the (Euclidean) result to be compared to (3.4):

$$\mu_0{}^{4-D}\int\frac{d^Dk_E}{(2\pi)^D}\bar{D}^{\rm NP}_{\mu\nu}(k_E)[\tfrac{1}{2}f''_{\kappa\lambda}(0)k_{E,\kappa}k_{E,\lambda}]=-\frac{1}{4(N^2-1)(D-1)}\frac{1}{D(D+2)}[(D+1)\delta_{\mu\nu}{\rm tr}f''(0)-2f''_{\mu\nu}(0)]\times\left[2(N^2-1)(D-1)\mu_0{}^{4-D}\int\frac{d^Dk_E}{(2\pi)^D}\{k_E{}^2\bar{D}_T(-k_E{}^2)-[1+O(g^2)]\}\right]. \tag{3.6}$$

This result, making use only of (3.5a) and not yet of the special form (3.5b), is quite general. The identical forms of the tensor structures on the right-hand sides of (3.4) and (3.6) confirm that (3.2) is indeed transverse.[8] Comparison of the other factors and a glance at (2.7) then shows that, generally,

$$\frac{g^2\mu_0{}^{4-D}}{4\pi^2}C'=C_2+O(g^2)\,. \tag{3.7}$$

The dimension-four condensate in the OPE expression (3.2) thus is essentially the C_2 of Eq. (2.1), up to "perturbative" corrections. Although, as we have mentioned, C_{GG} is occasionally written in place of the left-hand side (LHS) of (3.7), the simple and general relation (3.6) shows clearly that no conclusion stronger than (3.7) is justified at this level of the OPE. On using in (3.6) the special form (3.5b), with $a^2=0$, we are led back to (2.11).

We may thus state that the extended Schwinger mechanism is compatible with asymptotic predictions from the OPE in a distribution sense. As an aside, the comparison hints at the possibility that the workings of the operator-product expansion in the one-gluon channel considered here may be rather unusual from a physical point of view. By pulling apart the full nonperturbative propagator into a perturbative part (3.1) and a term such as (3.5), whose k^2 dependence is treated by asymptotic expansion, the OPE never quite gets rid of the "wrong" particle pole at $k^2=0$ in the perturbative propagator. Moreover, an integral such as $\int d^4k\, D^{\rm NP}(k)f(k)$, which for sufficiently well-behaved test functions $f(k)$ may even be convergent, gets decomposed rather artificially into a series of increasingly divergent terms. In contrast with the beautiful applications of the OPE to current correlations in color-singlet channels,[5] we may therefore expect it to be less useful in learning about the gluon itself.

Concerning the (g,μ_0) dependence of b^4, we recall long-known restrictions from renormalization-group arguments.[12] If b^2 is to be a measurable physical quantity, it should be of the gauge-fixing-independent and renormalization-group-invariant form:

$$b^2=\mu_0{}^2\exp\left[-2\int^g[\beta(g')]^{-1}dg'\right] \tag{3.8a}$$

$$=\mu_0{}^2\exp\left\{-\frac{1}{11(g/4\pi)^2}-\frac{102}{121}\ln\left[11\left[\frac{g}{4\pi}\right]^2\right]+2\lambda_b+O(g^2)\right\} \tag{3.8b}$$

$$=(\Lambda^{(2)}_{\overline{\rm MS}})^2e^{2\lambda_b}[1+O(g^2)]\,, \tag{3.8c}$$

for a pure gluon theory with $N=3$. The two g^2-singular terms in the exponent of (3.8b) have been written so as to give the standard two-loop definition, $(\Lambda^{(2)}_{\overline{\rm MS}})^2$, of the QCD Λ parameter in the $\overline{\rm MS}$ scheme; the dimensionless integration constant $2\lambda_b$ fixes b^2 in terms of Λ^2. The nonanalytic and renormalization-group-invariant dependence (3.8b) is a highly nontrivial requirement for the extended Schwinger mechanism to fit into the QCD framework. If the ansatz (1.6) would run counter to some principle of QCD dynamics, that dynamics presumably would react to (1.6) by forcing the b^4 parameter to have a noninvariant μ_0 dependence, and it will be interesting to see that this does not happen.

In this context, it should be pointed out that the extended Schwinger mechanism, if its crucial b^4 parameter would indeed show a nonperturbative coupling dependence such as (3.8b), could equivalently be regarded as a special realization of the mechanism of infrared-singularity generation discussed by West.[13] West's work demonstrates, on the basis of a general solution to the renormalization-group equation for a single-variable Green's function, that this type of nonperturbative coupling dependence can be associated with infrared-singular pole behavior in k^2. While his discussion has in mind the function $-k^2D_T(k^2)$, our Eq. (1.6) would correspond to the mechanism occurring in the function $\Gamma_{2,T}(k^2)$, which is equally possible.

IV. SOLUTION OF THE DYSON-SCHWINGER EQUATION

The Dyson-Schwinger (DS) equation for the gluon propagator in covariant gauges, displayed in Fig. 2, is not a closed equation for Γ_2 but in addition involves the proper three-point vertices Γ_3 (gluon-gluon-gluon) and $\tilde{\Gamma}_3$ (ghost-ghost-gluon), as well as the four-point gluon amplitude T_4. (Since we are considering a pure gluon theory, there is no fermion vertex.) Dynamical formation of nonperturbative parts, if any, in these higher vertices is not independent of the one postulated for Γ_2 in Eq. (1.6), since all vertices are related to each other via the Slavnov-Taylor (ST) identities[14,15] embodying BRS invariance.

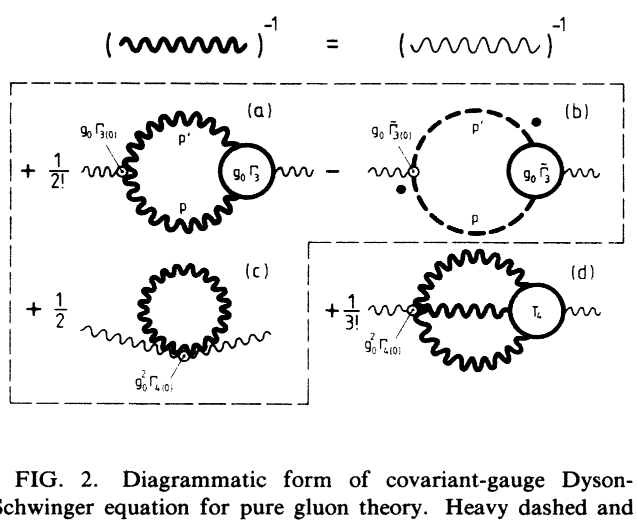

FIG. 2. Diagrammatic form of covariant-gauge Dyson-Schwinger equation for pure gluon theory. Heavy dashed and wavy lines represent, respectively, dressed ghost and gluon propagators. $\Gamma_{3(0)}$ and $\tilde{\Gamma}_{3(0)}$, bare three-point vertices, Γ_3 and $\tilde{\Gamma}_3$, dressed proper three-point vertices, $\Gamma_{4(0)}$, bare four-gluon vertex, T_4, four-gluon off-shell T matrix. The "truncated" equation considered in the text retains only the boxed, i.e., the one-dressed-loop diagrams for the gluon self-energy.

While the coupled nature of the Green's-function hierarchy would seem to lead to a consideration of the equations for all these higher vertices, the question of whether the extended Schwinger mechanism (1.6) can live in the specific dynamical environment of a gauge theory will be studied here only in a weaker and more restricted, but still nontrivial version. We shall inquire whether there exists a class of "acceptable" nonperturbative vertices Γ_3 and $\tilde{\Gamma}_3$ which satisfy all Lorentz-invariance and Bose-symmetry constraints, and which together with (1.6) solve the relevant ST identities up to perturbative corrections $O(g^2)$, such that the extended Schwinger mechanism can become self-consistent in the gluonic DS equation. The question of how vertices of this class can in turn be made self-consistent in the next-level equations must be deferred to a separate study, but it is remarkable that the detailed (and intricate) way in which (1.6) "solves" the DS equation will nevertheless lead (i) to a very definite conclusion about the coupling dependence of the nonperturbative mass parameter b and (ii) to interesting indications that the problem of nonperturbative parts on the higher levels of the Green's-function hierarchy may in fact be a *closed* problem. We have not mentioned the four-gluon amplitude T_4 because, in what follows, the above question will only be studied in the context of a "truncated" DS equation omitting the T_4 term. The reason is not that this term is unimportant, but that the kinematical complications presented by the three-point vertex (one-loop) terms are already substantial enough to render it imperative to try simpler things first. However, the strategy applied to these terms will be quite obviously extendable to the T_4 contribution.

In this "truncated" problem, only the two lowest ST identities for Γ_2 and Γ_3 are relevant. The former has been written in (2.14): $D_L(k^2)$ should not get dressed. The identity for Γ_3 is more intricate;[14,16] it connects Γ_3 to $\Gamma_{2,T}$, to the ghost propagator $\tilde{D}(k^2)$, and to an auxiliary amplitude $G_3^{\alpha\cdot\beta}$ defined graphically in Fig. 3 which also involves the ghost fields:

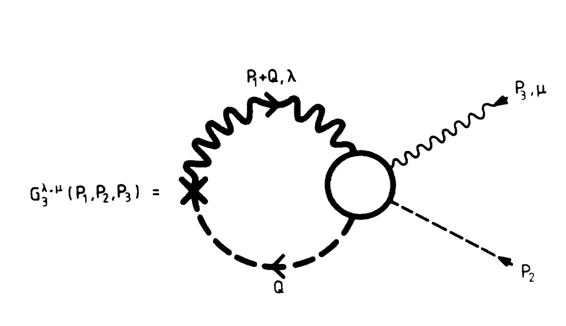

FIG. 3. Definition of auxiliary ghost-ghost-vector amplitude G_3. The circle denotes an amplitude which is connected, amputated, and one-particle irreducible in the gluon + ghost→gluon + ghost channel. Heavy dashed and wavy lines represent dressed ghost and gluon propagators, respectively; the cross on the left puts the "external" field points of the two propagators together in coordinate space, or calls for integration over the four-momentum Q in momentum space.

$$\Gamma_3^{\rho\sigma\nu}(P_1,P_2,P_3)P_{3,\nu}=P_3{}^2\widetilde{D}(P_3{}^2)\left[\frac{1}{D_T(P_1{}^2)}t^{\rho}{}_{\alpha}(P_1)G_3^{\alpha\sigma}(P_1,P_3,P_2)-\frac{1}{D_T(P_2{}^2)}t^{\sigma}{}_{\beta}(P_2)G_3^{\beta\rho}(P_2,P_3,P_1)\right] . \tag{4.1}$$

Here and below, the four-momenta are always understood to satisfy

$$P_1+P_2+P_3=0 , \tag{4.2}$$

and the notation of Eq. (1.4) is used. While (4.1) involves the transverse part of G_3 with respect to its first momentum argument, the longitudinal part will also enter the DS equation via the proper ghost-ghost-gluon vertex $\widetilde{\Gamma}_3$, which is given by

$$\widetilde{\Gamma}_3^{\mu}(P_1,P_2,P_3)=P_{1,\lambda}G_3^{\lambda\mu}(P_1,P_2,P_3) . \tag{4.3}$$

[All three-point vertices have a common color dependence[16] given by the SU(N) structure constants f_{abc}, which has been factored out in the above equations.]

Omitting all technical and heuristic detail, the essential steps and results leading to self-consistency of the extended Schwinger mechanism in the gluonic DS equation may now be outlined as follows.

(1) It is possible to construct "acceptable" nonperturbative vertices Γ_3 and G_3 (and $\widetilde{\Gamma}_3$ from the latter), which satisfy the symmetry and Lorentz-invariance requirements, and which are compatible with Eq. (1.6) in the sense that (i) the ST identity for Γ_2 is satisfied up to perturbative terms $O(g^4)$—that is, the longitudinal gluon self-energy vanishes through order g^2—and (ii) the ST identity (4.1) for Γ_3 is satisfied up to perturbative corrections $O(g^2)$.

The Lorentz-invariance and symmetry constraints are solved as usual through suitable tensor decompositions, which are straightforward to establish. A set of such formulas—similar to those of Ball and Chiu[16] but with some changes to suit our purpose—is given in the Appendix, which also lists the detailed form of the invariant functions determined by the construction. In this construction, the essential guidelines are (i) the ST identities themselves and (ii) a *heuristic principle* apparently first formulated by Eichten and Feinberg[17] in the context of the theory of dynamical symmetry breaking, which for our purpose may be phrased as follows:

> Vertices whose external legs *all* refer to unphysical degrees of freedom—longitudinal gluons or Faddeev-Popov ghosts—should remain "in the perturbative mode," i.e., should not develop nonperturbative parts proportional to b^4 or a^2. (4.4)

While obviously valid in the purely gluonic sector, this postulate is highly plausible in view of what has been said above about the connection between nonperturbative terms and measurable vacuum condensates. By incorporating it, our construction ensures in particular that up to perturbative corrections $O(g^2)$, the factor $P_3{}^2\widetilde{D}(P_3{}^2)$ in (4.1) may be replaced by unity.

(2) Although this construction determines only the "longitudinal" part (4.1) or (A22) of Γ_3 (with at least one longitudinal gluon leg) and thus leaves the nonperturbative parts of two among the six different invariant functions of Γ_3 undetermined except for symmetry properties, it establishes in the other invariant functions a very definite *pattern of nonperturbative dressing* visible, e.g., in Eqs. (A22) for the longitudinal Γ_3 or (A16) for $\widetilde{\Gamma}_3$: dimensionless invariant functions acquire, in addition to their zeroth-order forms, terms proportional to $n(P_i{}^2)$ with dimensionless kinematical factors, where $P_i{}^2$ is any of the three invariant variables of the vertex, and where the quantity

$$n(k^2)=-2\frac{a^2}{k^2}+\frac{b^4}{(k^2)^2} \tag{4.5}$$

represents, under the assumption of the extended Schwinger mechanism, the nonperturbative part of the gluon polarization function—that is,

$$\Gamma_{2,T}(k^2)=-k^2[1+n(k^2)+O(g^2)] =\frac{1}{D_T^{(n)}(k^2)}+O(g^2) . \tag{4.6}$$

The essential observation is that if we postulate a pole in the gluon's Γ_2, the ST identities enforce singularities in the next higher vertices Γ_3 and $\widetilde{\Gamma}_3$ as well. There is nothing wrong with this—since the nonperturbative parts among themselves solve the ST identities exactly, BRS invariance is maintained, and since they dominate only at small values of the $P_i{}^2$ and leave the perturbative large-$P_i{}^2$ behavior unchanged, renormalizability and asymptotic freedom are also maintained. In fact, such infrared singularities in the vertices conform to a long-standing theoretical prejudice about confining gauge theories, since exchange mechanisms controlled by such vertices will lead to spatially long-range forces. However, consistency questions do arise from the fact that the vertex singularities—at least in the vertices with one or more *unphysical* legs that we have constructed up to now—are of type $(P_i{}^2)^{-2}$, as shown by (4.5). If a simple pole in Γ_2 leads to a second-order pole in Γ_3, are we not facing the possibility of stronger and stronger singularities in the higher n-gluon vertices? This fortunately is not the case, since by the general prescriptions for renormalizable theories,[18] the vertices—more precisely, the irreducible kernels—for $n>4$ are to be built by *skeleton expansions* from the (dressed and renormalized) primary propagators and vertices with $n\leq 4$. Each of their external legs must end in a primary vertex, and therefore can have no stronger singularity in its momentum-squared variable than in those primary vertices. (Thus any singularities in Γ_3—and, if we go beyond our "truncated" scheme, in Γ_4 or T_4—will have to become consistent among themselves in the Γ_3 and Γ_4 equations.)

However, even in the context of this paper, which concentrates on the DS equation for Γ_2 alone, the consistency problem is inescapable: if $\tilde{\Gamma}_3$ and the partially longitudinal Γ_3 have second-order poles in the invariant masses of their transverse-gluon legs, will the contributions to $\Gamma_{2,T}$ controlled by these vertices not come out with $(k^2)^{-2}$ singularities, rather than the $(k^2)^{-1}$ of Eq. (1.6) postulated as input?

(3) The contributions in question are those from *unphysical two-particle intermediate states:* the dressed-ghosts loop of Fig. 2(b), and those parts of the dressed-gluons loop of Fig. 2(a) in which at least one of the two-loop gluons is longitudinal [and which by (2.14) therefore carry factors of ξ_0 or ξ_0^2]. These quantities are completely fixed to order g_0^2 by the preceding construction, and it is therefore a critical test of the survivability of an extended Schwinger mechanism in a Yang-Mills theory to see whether they assume an acceptable form. Remarkably, they do, without further hypotheses. Their combined contributions to the RHS of the transverse, unrenormalized DS equation for Γ_2 dimensionally regularized in $D=4-\epsilon$ Euclidean dimensions, are found to be

$$\frac{1}{D_T^{(n)}(-k_E^2)}\left[\frac{g_0}{4\pi}\right]^2\frac{N}{3}\left\{-(\tfrac{1}{4}-\tfrac{3}{2}\xi_0)\left[N_\epsilon-\ln\frac{k_E^2}{\mu_0^2}\right]-\tfrac{2}{3}-\tfrac{3}{2}\xi_0\left[1-f_\xi\left[\frac{a^2}{b^2},\frac{-k_E^2}{b^2}\right]\right]-\tfrac{3}{4}\xi_0^2\right\}+O(g_0^4)\,. \tag{4.7}$$

This is proportional to the inverse of the very propagator $D_T^{(n)}$ that was used as input to the equation, with a factor g_0^2 that identifies the whole contribution as belonging to the "perturbative" corrections $O(g^2)$ in (1.6). Note that those terms in the integrand arising from application of the ST identity (A22) that are not proportional to $1/D_T^{(n)}(-k_E^2)$ all cancel, that $(k_E^2)^{-2}$ singularities get softened to $(k_E^2)^{-1}$, and that the kinematical singularities present in the transverse kinematical tensors of (A22) are removed—the latter are in fact no more of a nuisance than the long-familiar ones in (1.3) and (1.4). Note also that the divergent (N_ϵ) and the associated logarithmic terms in (4.7) are exactly the same as for the corresponding contributions to the *perturbative* one-loop calculation. In fact the only nonperturbative feature in (4.7) is the appearance of the dimensionless function f_ξ, which is finite, i.e., free of divergent and logarithmic terms, and whose detailed form is unimportant here except for the property that in the "perturbative limit,"

$$(a^2,b^2)\to(0,0)\,, \tag{4.8}$$

it vanishes identically, as it should.

On the other hand, if (4.7) automatically emerges as proportional to $(1/D_T^{(n)})g_0^2$, it obviously will play no role in establishing the leading, nonperturbative part of (1.6), which is to be regarded formally as of zeroth order in g_0. This, too, is intuitively welcome—since the nonperturbative terms represent physical effects with observable consequences, one expects as in Sec. II that unphysical intermediate states cannot contribute to their generation.

(4) Self-consistency of the nonperturbative part of $\Gamma_{2,T}$ can then be expected to result, in the context of our "truncated" DS equation, only from the gluon loop of Fig. 2(a) with both loop gluons transverse. (This is an essential difference from the first of Ref. 3, where the nonperturbative effect is assumed to arise entirely from the vertex part controlled by the ST identity.) Again labeling nonperturbative parts by the suffix NP, we thus have, in D Euclidean dimensions,

$$[\Gamma_{2,T}(-k_E^2)]_{\rm NP}=k_E^2+\left[-g_0^2\mu_0^{4-D}\frac{N}{2}\frac{1}{D-1}\int\frac{d^Dq_E}{(2\pi)^D}\Gamma_{3(0)}^{\mu\kappa\lambda}(k_E,-p_E',-p_E)D_T^{(n)}(-p_E'^2)\right.$$
$$\left.\times D_T^{(n)}(-p_E^2)t_{\mu\nu}(k_E)t_{\kappa\rho}(p_E')t_{\lambda\sigma}(p_E)\,\Gamma_3^{\rho\sigma\nu}(p_E',p_E,-k_E)\right]_{\rm NP}\,. \tag{4.9}$$

Here $\Gamma_{3(0)}$ denotes the bare three-gluon vertex. The crucial question is whether the term in large parentheses on the RHS can become equal to $2a^2+(b^4/k_E^2)+O(g_0^2)$, with the added, important condition that the divergence present in the perturbative correction $O(g_0^2)$ should combine with that of (4.7) in such a way as to keep perturbative renormalization feasible.

The answer will partly depend on the totally transverse three-gluon vertex appearing in the last line of (4.9). This vertex, which has two different invariant functions (for technical detail, see again the Appendix), is not determined by the ST identities. Nevertheless, the ST identities, through the construction discussed before, establish a strong *heuristic guideline* for its choice, in the form of the nonperturbative-dressing pattern observed in step (2). (It would be highly implausible to assume that transverse-gluon legs in the partially longitudinal vertex and those in the totally transverse part dress themselves in completely different ways.) This pattern suggests that we construct the two unknown invariants as linear combinations of the building blocks

$$n(P_1^2),\quad n(P_2^2),\quad n(P_3^2)\,, \tag{4.10a}$$

$$n(P_2^2)n(P_3^2),\quad n(P_3^2)n(P_1^2),\quad n(P_1^2)n(P_2^2)\,, \tag{4.10b}$$

$$n(P_1^2)n(P_2^2)n(P_3^2)\,, \tag{4.10c}$$

with kinematic coefficients dimensionally appropriate for the invariant function considered, and with due regard for the symmetry requirements on that function.

A parametrization of this form, written out in detail in Eqs. (A27)—(A31) of the Appendix, will be used in the following. While not yet the most general parametrization of its kind, it represents a large class of transverse vertices, conforming to the above-described dressing pattern, for which nontrivial insights can already be gained into the restricted consistency question posed for this section. This parametrization ensures again removal of the kinematic singularities in the associated transverse tensors and, through the linear restrictions [(A30) and (A31)] placed on its parameters, the softening of k_E^{-4} to k_E^{-2} singularities in the gluon-loop contribution to the propagator. In all, it possesses a fourteen-parameter linear freedom.

Up to this point we have done no more than extending heuristically the dressing pattern established by the ST identities as far as possible. This naturally gives us terms $(2a^2/k_E^2)+(b^4/k_E^4)$ in (4.9) via (4.10), and since we have arranged for softening, it would now seem likely (though not yet guaranteed) that the desired self-consistency can be attained in (4.9). Appearances are deceptive, however, because a problem more difficult than establishing the singular momentum dependence is to obtain a sensible *coupling* dependence. The overall coupling factor g_0^2 in (4.9), even assuming that renormalization will have turned it into a (finite but μ_0-dependent) g^2 to this order, seems to spoil consistency hopelessly. We must avoid consistency conditions of type $b^4=g^2\times\text{const}\times b^4$; to nail down the intrinsically μ_0-dependent coupling to a specific number would be nonsensical. But the freedom in the vertex does not help at all—if we use it to choose some of the coefficients proportional to g^{-2} to cancel the g^2, then Γ_3 contains $g^{-2}b^4$ factors while Γ_2 contains b^4, and these two mass scales cannot be simultaneously renormalization-group invariant. At least one of the two vertices would have nonperturbative terms with an unphysical mass scale, and this would disqualify (1.6) as a physically meaningful mechanism.

The solution to this problem will emerge, not from the vertex, but from the nonperturbative denominators of the two $D_T^{(n)}$'s of (4.9), in which we have *no* freedom any more, given the postulated Eq. (1.6). It will also turn out to be tied intimately to the divergence of the loop integration, and will thus reveal one of the subtler aspects of the extended Schwinger mechanism. To see this, we cannot avoid looking at (4.9) in somewhat greater detail.

(5) For brevity we from now on *restrict ourselves to the case* $a^2=0$, and simply state that a self-consistent solution with $a^2=0$ is possible. We prefer such a solution because it is only for $a^2=0$ that we obtain the agreement with the OPE discussed in Sec. III. [One may do the following steps with $a^2\neq 0$, realizing that the tadpole of Fig. 2(c) then contributes, obtain additional a^2-dependent terms in (4.11) below, and convince oneself that they disappear for $a^2=0$.]

The two-transverse-gluons loop in the large parentheses of (4.9), with the family of transverse vertices defined in (A24)—(A28) subject to constraints (A30) and (A31), in this case assumes, after a long calculation using standard symmetric-integration machinery, the final form

$$\frac{1}{D_T^{(n)}(-k_E^2)}\left(\frac{g_0}{4\pi}\right)^2\frac{N}{3}\left[-\frac{25}{4}\left(N_\epsilon-\ln\frac{k_E^2}{\mu_0^2}\right)-\tfrac{89}{12}+T_0(\zeta',\eta')\left(\ln\frac{b^2}{\mu_0^2}-\ln\frac{k_E^2}{\mu_0^2}\right)+f_D\left(\zeta',\eta';-\frac{k_E^2}{b^2}\right)\right] \tag{4.11a}$$

$$+\left(\frac{b^4}{k_E^2}\right)\left(\frac{g_0}{4\pi}\right)^2\frac{N}{3}\left[W_n(\zeta',\eta')\left(N_\epsilon-\ln\frac{b^2}{\mu_0^2}\right)+W_0(\zeta',\eta')\left(N_\epsilon-\ln\frac{k_E^2}{\mu_0^2}\right)+f_n\left(\zeta',\eta';-\frac{k_E^2}{b^2}\right)\right]. \tag{4.11b}$$

Here (ζ',η') denotes the set of vertex parameters, T_0, W_0, W_n are linear combinations of these parameters [given in detail in (A32) and (A33)], and f_D, f_n are again "finite" functions, free of divergent or logarithmic terms, whose details are again unimportant except for the fact that they vanish as $b^2\to 0$ or $k_E^2\to\infty$.

The first new element in (4.11), as compared to the contribution (4.7), is the term (4.11b) proportional to b^4/k_E^2, which is precisely what we need. It implies that the vertex construction of step (1), heuristically extended to the totally transverse vertex as in step (4), can indeed supply the right kind of singularity needed for (1.6). The second, and crucial, new element is the term $-\ln(b^2/\mu_0^2)$ inside (4.11b); it arises, together with the divergent N_ϵ preceding it, from the divergent loop integration with two nonperturbative propagators (2.8) that contain the mass scale b^2. To see that this term resolves the dilemma of correct coupling dependence, we now insist that perturbative coupling-constant renormalization

$$g_0^2=Z_\alpha g^2,\quad Z_\alpha(g,\epsilon)=1-11\left(\frac{g}{4\pi}\right)^2 N_\epsilon+O(g^4) \tag{4.12}$$

(for $N=3$ in the $\overline{\text{MS}}$ scheme) remain applicable, since our propagator and vertices retain perturbative large-momentum behavior. We may then write

$$\left[\frac{g_0}{4\pi}\right]^2 W_n(\zeta',\eta')\left[N_\epsilon-\ln\frac{b^2}{\mu_0{}^2}+2\lambda_b\right]$$

$$=\left[\left[\frac{g}{4\pi}\right]^2 N_\epsilon+O(g^4)\right]$$

$$\times\left[X+W_n(\zeta',\eta')\left[\frac{g}{4\pi}\right]^2 N_\epsilon\right], \quad (4.13)$$

where

$$X=X(g,\mu_0)$$

$$=-W_n(\zeta',\eta')\left[\frac{g}{4\pi}\right]^2\left[\ln\left[\frac{b(g,\mu_0)}{\mu_0}\right]^2-2\lambda_b\right], \quad (4.14)$$

and where $2\lambda_b W_n$ is an as yet undetermined, finite constant that we have extracted from the f_n function of (4.11b). Perturbative renormalization will continue to work if (4.13) is arranged to be free of divergent N_ϵ's up to terms $O(g^4)$. This requires $X:W_n=1:11$ up to $O(g^4)$, a condition from which the freedom in the vertex entering through $W_n(\zeta',\eta')$ drops out completely, and which gives

$$\ln\left[\frac{b^2}{\mu_0{}^2}\right]=-\frac{1}{11(g/4\pi)^2}+2\lambda_b+O(g^2)\ . \quad (4.15)$$

This intricate way of getting rid of the $g_0{}^2$ factor is the physically correct one, as becomes apparent by exponentiating (4.15):

$$b^2=\mu_0{}^2\exp\left[-\frac{1}{11(g/4\pi)^2}[1+O(g^2)]\right]. \quad (4.16)$$

To the order we can hope to determine with our truncated, one-dressed-loop equation, this is just the gauge-fixing-independent and renormalization-group-invariant form (3.8). [The $\ln g^2$ term in the exponent of (3.8) could be obtained only with the T_4 contribution of Fig. 2(d).] Thus the b parameter of (1.6) indeed has a chance of living within Yang-Mills theory as a physical, invariant mass scale, and displays the conjectured nonanalytic coupling dependence typical of a nonperturbative phenomenon.

While the constant $2\lambda_b$ of (4.15) will be seen to be not yet determined on the level of the DS equation alone, and therefore has not been made explicit in (4.16), there is nevertheless an intriguing aspect to (4.16): in view of (2.11), it represents the beginning of a dynamical determination of a vacuum-condensate parameter from the Green's-function equations. The idea that *vacuum condensates may become calculable through their nonperturbative effects on propagators and vertices* is likely to be of general significance, and its usefulness may transcend the context of the extended Schwinger mechanism in which it has been used here.

(6) In order to remove the remaining divergence in (4.11b), and to make both the nonperturbative $b^4/k_E{}^2$ and the zeroth-order $k_E{}^2$ in (4.9) emerge with the correct coefficients of unity, we may impose three further consistency conditions, which amount to three additional linear constraints on the transverse vertex, and are listed in (A32) and (A33) of the Appendix. In the remaining terms, by (4.12), we may replace $g_0{}^2$ by g^2 to this order. However, the result

$$\frac{1}{D_T^{(n)}(-k_E{}^2)}\left[\frac{g}{4\pi}\right]^2\left[-\frac{25}{4}\left[N_\epsilon-\ln\frac{k_E{}^2}{\mu_0{}^2}\right]-\tfrac{89}{12}+f_D\left[\zeta',\eta';-\frac{k_E{}^2}{b^2}\right]\right]$$

$$+\frac{b^4}{k_E{}^2}\left\{1+\left[\frac{g}{4\pi}\right]^2\left[f_n\left[\zeta',\eta';-\frac{k_E{}^2}{b^2}\right]-22\lambda_b\right]\right\} \quad (4.17)$$

appears to produce different perturbative corrections $O(g^2)$ to the zeroth-order $(k_E{}^2)$ and the nonperturbative $(b^4/k_E{}^2)$ parts of $\Gamma_{2,T}$. This at first might seem to be just a clumsy feature, but a few lines of calculation will show that the $O(g^2)$ corrections to the *propagator*, $D_T=1/\Gamma_{2,T}$, would then possess *second*-order poles at $k^2=\pm ib^2$, and would thus destroy the appealing physical interpretation of the "quasiparticle" propagator poles at $k^2=\pm ib^2$ discussed in Sec. II.

Here another subtle property of (4.11) comes into play, which again is valid only for $a^2=0$: the function f_n is an even function of $s=-k_E{}^2/b^2$ with no singularities (in particular, no cuts) on the real s axis, and its value at $s=\pm i$,

$$f_p(\zeta',\eta')=f_n(\zeta',\eta';s=\pm i)\ ,$$

is real. Thus f_n-f_p vanishes both at $s=+i$ and at $s=-i$ [in fact, its behavior there is proportional to $(s\mp i)\ln(s\mp i)$, as we simply state here for brevity]. Contrary to appearances, the function $\tilde{f}_n$ defined by

$$f_n(\zeta',\eta';s)=f_p(\zeta',\eta')+(s^2+1)\tilde{f}_n(\zeta',\eta',s) \quad (4.18)$$

therefore has no poles (but only logarithmic branch points) at $s=\pm i$. Thus if we choose

$$2\lambda_b = \tfrac{1}{11} f_p(\xi', \eta') , \tag{4.19}$$

then because of

$$(b^4/k_E^2)(s^2+1) = 1/D_T^{(n)}(-k_E^2) ,$$

the $O(g^2)$ corrections in (4.17) assume an acceptable form. At this level, the scale constant $2\lambda_b$ is not yet fully determined, since the vertex parameters on which it depends will get fixed only on the next level of the dynamical hierarchy, the equation for Γ_3 (and that for Γ_4, if we go beyond the "truncated" DS equation and include the T_4 term). Nevertheless, (4.19) shows the logic that will lead to the calculation of the scale constant once the vertex is fixed. The open end of this calculation simply reflects the fact that the DS equation is not a closed problem; complete determination of the vertex is not *expected* to occur until the next level of the Green's-function hierarchy.

(7) Now adding (4.7), where g_0 and ξ_0 may be replaced by g and ξ to this order, to (4.17) as simplified by (4.19), we obtain the final result for the transverse DS equation,

$$\Gamma_{2,T}(-k_E^2) = \left[1 - \left[\frac{13-3\xi}{2} \right] \left[\frac{g}{4\pi} \right]^2 N_\epsilon + O(g^4) \right] \left[k_E^2 + \frac{b^4}{k_E^2} \right] \times \left[1 + \left[\frac{g}{4\pi} \right]^2 \left\{ \left[\frac{13-3\xi}{2} \right] \ln \frac{k_E^2}{\mu_0^2} - \frac{97}{12} - \tfrac{3}{2}\xi \left[1 - f_\xi \left[0; -\frac{k_E^2}{b^2} \right] \right] - \tfrac{3}{4}\xi^2 + \tilde{f}_D \left[\xi', \eta'; -\frac{k_E^2}{b^2} \right] \right\} + O(g^4) \right] , \tag{4.20}$$

which through $k_E^2 \to -k^2$ may be continued back to the Minkowskian domain. Here $\tilde{f}_D = f_D + \tilde{f}_n$. As anticipated, this may now be rendered finite by *perturbative* gauge-field renormalization:

$$\bar{\Gamma}_2 = Z_3 \Gamma_2 ,$$
$$Z_3(g,\epsilon) = 1 + \left[\frac{13-3\xi}{2} \right] \left[\frac{g}{4\pi} \right]^2 N_\epsilon + O(g^4) \tag{4.21}$$

(in the $\overline{\text{MS}}$ scheme). In the limit (4.8) the functions f_ξ and $\tilde{f}_D$ both vanish, and the perturbative one-loop result is recovered. The unphysical nature of the logarithmic cut in (4.20) is clearly seen from the dependence of its coefficient of gauge fixing ξ, which is the same as in the perturbative theory.

It is in this sense that the extended Schwinger mechanism (1.6) is a solution to the DS equation for the Yang-Mills field. When viewing (4.20), and in particular the result (4.16), together with the results of Sec. III, it seems reasonable to conclude that the mechanism stands a good chance of being sustained by the specific dynamics of QCD. It will be noted that it is Fig. 2(a), arising from the trilinear gluon self-interaction, that plays the crucial role in making the solution possible. In this sense, one is dealing with a genuine "non-Abelian" effect.

V. CONCLUDING REMARKS

As emphasized before, the consistency question has been considered here only in a restricted sense. The answer we found refers to a class of "compatible" three-gluon vertices which, in spite of its remaining eleven-parameter freedom, is of a rather special form strongly suggested (though not enforced) by the ST identities. In this sense it is a preliminary answer; the next logical step of studying the Γ_3 (and Γ_4) levels of the Green's-function hierarchy would go beyond the limits of this paper. One should realize, however, that already this restricted first step has revealed a nontrivial result that is largely independent of either the special class of vertices adopted, or of the freedom remaining within that class. This result is the emergence of factors $-\ln(b^2/\mu_0^2)$ from the self-energy-loop integration as a consequence of the nonperturbative-loop propagators, and the observation how a consistency condition designed to maintain perturbative renormalizability, Eq. (4.15), renders this term proportional to $1/g^2$ in leading order so as to cancel the overall g^2 of the loop. This mechanism of generating a nonperturbative contribution from an apparently perturbative diagram, and in accord with renormalization-group requirements, is remarkable because the crucial logarithmic factor is tied to the *divergence* of the loop integration and would not be present without it; in a sense, it represents a dynamical effect of the divergence structure of renormalizable Yang-Mills theory.

This has the further, interesting consequence that formation of nonperturbative terms by this mechanism is expected to be possible only in the small finite number of "primitively divergent" vertices (those with $n \leq 4$ in Yang-Mills theory) which have genuine divergences—the higher vertices, when built from dressed and renormalized building blocks through skeleton expansion, involve only convergent loop integrals. Thus in spite of the infinite nature of the Green's-function hierarchy, the consistency question for nonperturbative parts of the kind postulated in (1.6) beyond the level of the DS equation will be a "closed" problem that can be decided from the equations for the primitively divergent vertices alone. This question, as well as the inclusion of a fermion sector, is currently being studied.

Finally, one might think of a rather direct test of (1.6) in the QCD framework: the qualitative propagator behavior shown in Fig. 1, or rather its Euclidean form given in (2.8), could in principle be checked by lattice calculations with gauge fixing. It is one of the purposes of this paper to suggest such tests. The necessity of obtaining a reasonably accurate Fourier transform (1.5) at small k^2, which calls for large lattices, would undoubtedly strain the present technical capabilities of lattice calculations to their limits, but it is not inconceivable that at least the qualitative "bending over" of the D_T function—from its asymptotic form $\propto 1/k_E^2$ to the approximately linear infrared form $\propto k_E^2/b^4$—could be observed with tolerable error bars. Of course there remains a problem of principle with such tests which at the moment seems to have no ready solution—if $b^2 \neq 0$ is but one possible phase of the gluon field, there is no way of ensuring in advance that a *numerical* calculation will run into just this phase.

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APPENDIX: NONPERTURBATIVE THREE-POINT VERTICES

Here we record some details of the kinematics of the Γ_3 and G_3 vertices, and of their nonperturbative parts as resulting from the construction of Sec. IV, steps (1) and (4).

The auxiliary vertex G_3 can be written in terms of five dimensionless invariant functions $G_{31}, \ldots, G_{35}$ as

$$G_3^{\lambda\mu}(P_1,P_2,P_3)=g^{\lambda\mu}+t^{\lambda\mu}(P_3)G_{31}(P_1^2,P_2^2,P_3^2)+l^{\lambda\mu}(P_3)G_{32}(P_1^2,P_2^2,P_3^2)$$
$$+m^{\lambda\mu}(P_3,P_1)G_{33}(P_1^2,P_2^2,P_3^2)+l^{\lambda\mu}(P_1)G_{34}(P_1^2,P_2^2,P_3^2)$$
$$+\frac{P_1^\lambda P_3^\mu}{(P_1\cdot P_3)}G_{35}(P_1^2,P_2^2,P_3^2)\ . \qquad \text{(A1)}$$

Here we have introduced the kinematical tensor

$$m^{\mu\nu}(p,q)=g^{\mu\nu}-\frac{p^\mu q^\nu}{(p\cdot q)}=m^{\nu\mu}(q,p) \qquad \text{(A2)}$$

with transversality properties

$$q_\mu m^{\mu\nu}(p,q)=m^{\mu\nu}(p,q)p_\nu=0\ . \qquad \text{(A3)}$$

Since in zeroth order $G_3^{\lambda\mu}$ is given by $g^{\lambda\mu}$, nonperturbative terms (if any) in the invariant functions should be chosen so that in the "perturbative limit" (4.8), these functions satisfy the conditions

$$G_{3n}(P_1^2,P_2^2,P_3^2;a^2=b^2=0)=O(g^2)\ ,$$
$$n=1,\ldots,5\ . \qquad \text{(A4)}$$

In dealing with the full proper three-gluon vertex $f_{abc}\Gamma_3^{\rho\sigma\nu}(P_1,P_2,P_3)$, some kinematical definitions will be useful. We introduce the three permutation invariants

$$S_1(P_1^2,P_3^2)=\tfrac{1}{2}(P_1^2+P_2^2+P_3^2)\ , \qquad \text{(A5a)}$$

$$S_2(P_1^2,P_2^2,P_3^2)=-\tfrac{1}{4}[(P_1^2)^2+(P_2^2)^2+(P_3^2)^2-2(P_1^2P_2^2+P_2^2P_3^2+P_3^2P_1^2)]\ , \qquad \text{(A5b)}$$

$$S_3(P_1^2,P_2^2,P_3^2)=P_1^2P_2^2P_3^2\ . \qquad \text{(A5c)}$$

The second of these, by (4.2), may be written in the equivalent form

$$S_2(P_1^2,P_2^2,P_3^2)=P_1^2P_2^2-(P_1\cdot P_2)^2\ , \qquad \text{(A5d)}$$

or any cyclic permutation of the RHS. Also useful will be the four-vectors Q_1,Q_2,Q_3 defined by

$$Q_3^\mu=\frac{(P_2\cdot P_3)P_1^\mu-(P_1\cdot P_3)P_2^\mu}{[S_2(P_1^2,P_2^2,P_3^2)]^{1/2}} \qquad \text{(A6)}$$

and cyclic permutations. These vectors satisfy, in analogy to (4.2),

$$Q_1+Q_2+Q_3=0\ , \qquad \text{(A7)}$$

and are transverse to their respective P_i's in the sense that

$$Q_1\cdot P_1=Q_2\cdot P_2=Q_3\cdot P_3=0\ , \qquad \text{(A8a)}$$

whereas their scalar products among themselves are simply

$$Q_i\cdot Q_k=P_i\cdot P_k,\ \ i,k=1,2,3\ . \qquad \text{(A8b)}$$

Bose symmetry and the antisymmetry of the f_{abc} require the tensor function $\Gamma_3^{\rho\sigma\nu}(P_1,P_2,P_3)$ to be totally antisymmetric under permutations of the pairs (ρ,P_1), (σ,P_2), (ν,P_3). A complete tensor decomposition adapted to our purposes (which differs from the one of Ref. 16 in the choice of the third term) reads

$$\Gamma_3^{\rho\sigma\nu}(P_1,P_2,P_3)=\{[g^{\rho\sigma}(P_1-P_2)^\nu]F_0(P_1{}^2,P_2{}^2;P_3{}^2)+(g^{\rho\sigma}P_3^\nu)F_1(P_1{}^2,P_2{}^2;P_3{}^2)$$
$$+[m^{\rho\sigma}(P_2,P_1)P_3^\nu]F_3(P_1{}^2,P_2{}^2;P_3{}^2)+(\text{c.p.})\}+\frac{P_2^\rho P_3^\sigma P_1^\nu+P_3^\rho P_1^\sigma P_2^\nu}{[S_2(P_1{}^2,P_2{}^2,P_3{}^2)]^{1/2}}F_A(P_1{}^2,P_2{}^2,P_3{}^2)$$
$$+\left[[m^{\rho\sigma}(P_2,P_1)Q_3^\nu]\left[-2\frac{\sqrt{S_2}}{P_3{}^2}F_2(P_1{}^2,P_2{}^2;P_3{}^2)\right]+(\text{c.p.})\right]$$
$$+\left[\frac{P_2^\rho P_3^\sigma P_1^\nu-P_3^\rho P_1^\sigma P_2^\nu}{[S_2(P_1{}^2,P_2{}^2,P_3{}^2)]^{1/2}}-(g^{\rho\sigma}Q_3^\nu+g^{\sigma\nu}Q_1^\rho+g^{\nu\rho}Q_2^\sigma)\right]F_S(P_1{}^2,P_2{}^2,P_3{}^2)\,, \tag{A9}$$

where (c.p.) denotes cyclic permutations of $(\rho,P_1)\cdots(\nu,P_3)$. The six invariant functions F_0,F_1,F_2,F_3,F_A,F_S are dimensionless and have been chosen so that in zeroth order F_0 alone survives and reduces to unity. Thus these functions should satisfy

$$F_n(P_1{}^2,P_2{}^2,P_3{}^2;a^2=b^2=0)=\delta_{n0}+O(g^2)\,,\quad n=1,\ldots,3,A,S\,. \tag{A10}$$

Under interchange of the first two arguments $P_1{}^2$ and $P_2{}^2$, F_0 and F_2 are even while F_1 and F_3 are odd; F_S and F_A are totally symmetric and totally antisymmetric, respectively, in all three variables.

The construction mentioned in step (1) of Sec. IV now leads to the following forms of the invariant functions $G_{31}\cdots G_{35}$ in (A1):

$$G_{31}(P_1{}^2,P_2{}^2,P_3{}^2)=n(P_3{}^2)+O(g^2)\,, \tag{A11}$$

$$G_{33}(P_1{}^2,P_2{}^2,P_3{}^2)=\left[1-\frac{(P_2\cdot P_3)^2}{P_2{}^2P_3{}^2}\right]n(P_3{}^2)+O(g^2)\,, \tag{A12}$$

$$\begin{aligned}&G_{32}(P_1{}^2,P_2{}^2,P_3{}^2)=O(g^2)\,,\\&G_{34}(P_1{}^2,P_2{}^2,P_3{}^2)=O(g^2)\,,\end{aligned} \tag{A13}$$

$$G_{35}(P_1{}^2,P_2{}^2,P_3{}^2)=-\frac{(P_1\cdot P_3)^2}{P_1{}^2P_3{}^2}(G_{32}+G_{34})+O(g^2)=O(g^2)\,. \tag{A14}$$

Taken together, these imply the following nonperturbative form of G_3:

$$G_3^{\lambda\mu}(P_1,P_2,P_3)=g^{\lambda\mu}+\left[t^{\lambda\mu}(P_3)+\left[1-\frac{(P_2\cdot P_3)^2}{P_2{}^2P_3{}^2}\right]m^{\lambda\mu}(P_3,P_1)\right]n(P_3{}^2)+O(g^2)\,, \tag{A15}$$

from which, by (4.3), the ghost-ghost-gluon vertex follows as

$$\tilde{\Gamma}_3^{\mu}(P_1,P_2;P_3)=P_{1,\lambda}\left[t^{\lambda\mu}(P_3)\left[\frac{-1}{P_3{}^2D_T^{(n)}(P_3{}^2)}\right]+l^{\lambda\mu}(P_3)\right]+O(g^2)\,. \tag{A16}$$

The invariant functions F_0,F_1,F_3,F_A of (A9) can be determined from these through the ST identity (4.1) and read

$$F_0(P_1{}^2,P_2{}^2;P_3{}^2)=\frac{1}{2P_1{}^2D_T^{(n)}(P_1{}^2)}\left[\frac{1}{P_3{}^2D_T^{(n)}(P_3{}^2)}+\frac{P_2\cdot P_3}{P_2{}^2}n(P_2{}^2)\right]$$
$$+\frac{1}{2P_2{}^2D_T^{(n)}(P_2{}^2)}\left[\frac{1}{P_3{}^2D_T^{(n)}(P_3{}^2)}+\frac{P_1\cdot P_3}{P_1{}^2}n(P_1{}^2)\right]$$
$$+\tfrac{1}{2}P_3{}^2[\Phi_+(P_1{}^2,P_2{}^2)+\Phi_+(P_2{}^2,P_3{}^2)+\Phi_+(P_3{}^2,P_1{}^2)]+O(g^2)\,, \tag{A17}$$

$$F_1(P_1{}^2,P_2{}^2;P_3{}^2)=\frac{1}{2P_1{}^2D_T^{(n)}(P_1{}^2)}\left[1-\frac{P_1{}^2-P_2{}^2}{P_3{}^2}n(P_3{}^2)-\frac{P_2\cdot P_3}{P_2{}^2}n(P_2{}^2)\right]$$
$$-\frac{1}{2P_2{}^2D_T^{(n)}(P_2{}^2)}\left[1-\frac{P_2{}^2-P_1{}^2}{P_3{}^2}n(P_3{}^2)-\frac{P_1\cdot P_3}{P_1{}^2}n(P_1{}^2)\right]$$
$$+\tfrac{1}{2}(P_1{}^2-P_2{}^2)[\Phi_+(P_1^2,P_2{}^2)+\Phi_+(P_2{}^2,P_3{}^2)+\Phi_+(P_3{}^2,P_1{}^2)]+O(g^2)\,, \tag{A18}$$

$$F_3(P_1{}^2,P_2{}^2;P_3{}^2)=\frac{1}{S_3}\left[\frac{n(P_1{}^2)}{D_T^{(n)}(P_2{}^2)}\left[\frac{P_2{}^2}{P_3{}^2}S_2+(P_2\cdot P_3)(P_2\cdot P_1)\right]\right.$$
$$\left.-\frac{n(P_2{}^2)}{D_T^{(n)}(P_1{}^2)}\left[\frac{P_1{}^2}{P_3{}^2}S_2+(P_1\cdot P_3)(P_1\cdot P_2)\right]\right]+O(g^2)\ , \quad \text{(A19)}$$

$$F_A(P_1{}^2,P_2{}^2,P_3{}^2)=-[\Phi_-(P_1{}^2,P_2{}^2)+\Phi_-(P_2{}^2,P_3{}^2)+\Phi_-(P_3{}^2,P_1{}^2)]+O(g^2)\ . \quad \text{(A20)}$$

They contain the nonperturbative building blocks

$$\Phi_\pm(p^2,q^2)=\frac{1}{p^2q^2}\frac{1}{2}\left[\frac{n(q^2)}{D_T^{(n)}(p^2)}\pm\frac{n(p^2)}{D_T^{(n)}(q^2)}\right]\ . \quad \text{(A21)}$$

(The functions F_2 and F_S remain undetermined.) It is, however, simpler and more transparent to characterize this vertex by giving separately its partially longitudinal portion (with at least one longitudinal gluon leg) and its totally transverse part. The former is given by the ST identity (4.1) and in the present case therefore assumes the form

$$\Gamma_3^{\rho\sigma\nu}(P_1,P_2,P_3)P_{3,\nu}=\frac{1}{D_T^{(n)}(P_1{}^2)}[t^{\rho\sigma}(P_1)+s^{\rho\sigma}(P_1,P_2)n(P_2{}^2)]$$
$$-\frac{1}{D_T^{(n)}(P_2{}^2)}[t^{\sigma\rho}(P_2)+s^{\sigma\rho}(P_2,P_1)n(P_1{}^2)]+O(g^2)\ , \quad \text{(A22)}$$

featuring the kinematical tensor

$$s^{\rho\sigma}(P_1,P_2)=t_\alpha^\rho(P_1)t^{\alpha\sigma}(P_2)+\left[1-\frac{[(P_1+P_2)\cdot P_2]^2}{(P_1+P_2)^2P_2{}^2}\right]m^{\rho\sigma}(P_2,P_1)\ , \quad \text{(A23)}$$

which is transverse to both P_1^ρ and P_2^σ. These expressions reflect clearly the principle (4.4): nonperturbative dressing terms occur only in the momentum variables of transverse gluons.

The totally transverse vertex can be written in the form

$$t_{\kappa\rho}(P_1)t_{\lambda\sigma}(P_2)t_{\mu\nu}(P_3)\Gamma_3^{\rho\sigma\nu}(P_1,P_2,P_3)=\frac{t_{\kappa\rho}(P_1)t_{\lambda\sigma}(P_2)t_{\mu\nu}(P_3)}{[-P_1{}^2D_T^{(n)}(P_1{}^2)][-P_2{}^2D_T^{(n)}(P_2{}^2)][-P_3{}^2D_T^{(n)}(P_3{}^2)]}\Gamma_{3(0)}^{\rho\sigma\nu}(P_1,P_2,P_3)$$
$$-2\left[[t(P_1)\cdot t(P_2)]_{\kappa\lambda}\left[\frac{\sqrt{S_2}}{P_3{}^2}Q_{3,\mu}\right]F_2'(P_1{}^2,P_2{}^2;P_3{}^2)+\text{c.p.}\right]$$
$$-2\left[\frac{\sqrt{S_2}}{P_1{}^2}Q_{1,\kappa}\right]\left[\frac{\sqrt{S_2}}{P_2{}^2}Q_{2,\lambda}\right]\left[\frac{\sqrt{S_2}}{P_3{}^3}Q_{3,\mu}\right]\Phi_S(P_1{}^2,P_2{}^2,P_3{}^2)\ , \quad \text{(A24)}$$

where $\Gamma_{3(0)}$ is the bare three-gluon vertex [given by (A9) with $F_n=\delta_{n0}$]. Here the invariant functions F_2' (symmetric in its first two arguments) and Φ_s (symmetric in all three arguments) are linear combinations of F_2, F_s, and F_0 from (A9), and hence may be taken as the two invariants, undetermined by the ST identities, which according to Sec. IV should be parametrized in terms of the nonperturbative building blocks (4.10). In the present context it turns out more expedient to parametrize two equivalent functions Ψ_2' [dimension (mass)^2] and Ψ_S' [dimension (mass)^4] connected to the above by

$$F_2'(P_1{}^2,P_2{}^2;P_3{}^2)=\frac{1}{P_1{}^2P_2{}^2}\{(-P_1\cdot P_2)\Psi_2'(P_1{}^2,P_2{}^2;P_3{}^2)+\tfrac{1}{2}\Psi_S'(P_1{}^2,P_2{}^2,P_3{}^2)$$
$$+\tfrac{1}{6}S_2[2n(P_3{}^2)-n(P_1{}^2)-n(P_2{}^2)]\}\ , \quad \text{(A25)}$$

$$\Phi_S(P_1{}^2,P_2{}^2,P_3{}^2)=\frac{1}{S_3}(\Psi_S'(P_1{}^2P_2{}^2,P_3{}^2)-\{P_3{}^2\Psi_2'(P_1{}^2,P_2{}^2;P_3{}^2)+(\tfrac{1}{3}S_2+\tfrac{1}{2}P_3{}^2P_1\cdot P_2)[n(P_1{}^2)+n(P_2{}^2)]+\text{c.p.}\})\ ,$$
$$\text{(A26)}$$

since in terms of these invariants the integral of (4.9) assumes a simple form; it becomes

$$-g_0{}^2\mu_0{}^{4-D}\frac{N}{2}\frac{4}{D-1}\int\frac{d^Dq_E}{(2\pi)^D}\frac{S_2(-p_E'^2,-p_E{}^2,-k_E{}^2)}{S_3(-p_E'^2,-p_E{}^2,-k_E{}^2)}D_T^{(n)}(-p_E'^2)D_T^{(n)}(-p_E{}^2)$$

$$\times\left[3\left[\frac{D-2}{2}\right]\Psi_{S'}(-p_E'^2,-p_E{}^2,-k_E{}^2)-\{[(D-2)(-p_E'\cdot p_E)+S_1(p_E'^2,p_E{}^2,k_E{}^2)]\times\Psi_2'(-p_E'^2,-p_E{}^2;-k_E{}^2)+(\text{c.p.})\}\right],$$

where c.p. denotes cyclic permutations of p', p, and $-k$.

The class of "compatible" vertices we are considering in this paper is now given by the explicit parametrizations

$$\begin{aligned}\Psi_2'(p^2,q^2;r^2)=&U(\zeta_1',\zeta_2',\zeta_3';p,q)n(p^2)+U(\zeta_1',\zeta_2',\zeta_3';q,p)n(q^2)+U(\zeta_4',\zeta_5',0;p,q)n(r^2)\\&+U(\zeta_6',\zeta_7',\zeta_8';p,q)n(p^2)n(r^2)+U(\zeta_6',\zeta_7',\zeta_8';q,p)n(q^2)n(r^2)\\&+U(\zeta_9',\zeta_{10}',0;p,q)n(p^2)n(q^2)+U(\zeta_{11}',\zeta_{12}',0;p,q)n(p^2)n(q^2)n(r^2)\,,\end{aligned}\tag{A27}$$

$$\begin{aligned}\Psi_S'(p^2,q^2,r^2)=&V(\eta_1',\eta_2',\eta_3',\eta_4';r)n(p^2)n(q^2)+V(\eta_1',\eta_2',\eta_3',\eta_4';p)n(q^2)n(r^2)\\&+V(\eta_1',\eta_2',\eta_3',\eta_4';q)n(r^2)n(p^2)+V(\eta_5',\eta_6',0,0;r)n(p^2)n(p^2)n(r^2)\,.\end{aligned}\tag{A28}$$

Here (p,q,r) is any cyclic permutation of (P_1,P_2,P_3), and U,V denote the polynomials

$$U(\zeta_1',\zeta_2',\zeta_3';p,q)=\zeta_1'S_1+(\zeta_2'-\zeta_1')r^2+\zeta_3'\tfrac{1}{2}(p^2-q^2)\,,\tag{A29a}$$

$$V(\eta_1',\eta_2',\eta_3',\eta_4';r)=\eta_1'S_1{}^2+\eta_2'S_2+\eta_3'S_1r^2+\eta_4'(r^2)^2\,.\tag{A29b}$$

This form depends linearly on a total of 18 dimensionless coefficients $(\zeta_1'\cdots\zeta_{12}',\eta_1'\cdots\eta_6')$. It is very general and thus does not automatically ensure the softening of $(P_i{}^2)^{-2}$ into $(P_i{}^2)^{-1}$ singularities required for consistency of (1.6). The specific correlation needed between the various terms of (A27) and (A28) to produce this softening is expressed by the four linear constraints

$$R_1(\zeta')\equiv3(\zeta_6'+\zeta_7'-\tfrac{1}{2}\zeta_8'+\zeta_{10}'-\zeta_{11}'-2\zeta_{12}')=0\,,\tag{A30a}$$

$$R_2(\zeta')\equiv3(\zeta_2'-\tfrac{1}{2}\zeta_3'+\tfrac{1}{2}\zeta_4'-\zeta_6'-\zeta_7'+\tfrac{1}{2}\zeta_8'-\zeta_{10}'+\tfrac{1}{2}\zeta_{11}'+\zeta_{12}')=0\,,\tag{A30b}$$

$$R_1'(\zeta',\eta')\equiv3[\zeta_6'-\zeta_{11}'+\tfrac{3}{2}(\eta_1'+\eta_3'+\eta_4'-\eta_5')]=0\,,\tag{A31a}$$

$$R_2'(\zeta',\eta')\equiv3[\tfrac{1}{2}\zeta_4'-\zeta_6'+\tfrac{1}{2}\zeta_{11}'-\tfrac{3}{2}(\eta_1'+\eta_3'+\eta_4'-\tfrac{1}{2}\eta_5')]=0\,.\tag{A31b}$$

The additional consistency conditions necessary for turning expression (4.11) into (4.17) and thus for solving the DS equation are

$$T_0(\zeta',\eta')\equiv-\tfrac{1}{4}[10\zeta_6'+12\zeta_7'+4\zeta_8'+15(-\zeta_9'+\eta_1'+\eta_2')+18(\zeta_{10}'+\eta_3')]=0\,,\tag{A32a}$$

$$\begin{aligned}W_0(\zeta',\eta')\equiv-\tfrac{1}{4}[&-10\zeta_6'-12\zeta_7'-4\zeta_8'+15\zeta_9'-18\zeta_{10}'-5\zeta_{11}'+30\zeta_{12}'\\&-15(\eta_1'+\eta_2')-18\eta_3'+15(\eta_5'+\eta_6')]=0\,,\end{aligned}\tag{A32b}$$

and, finally

$$\begin{aligned}W_n(\zeta',\eta')\equiv-\tfrac{1}{4}[&10\zeta_1'+26\zeta_2'-11\zeta_3'+6\zeta_4'+18\zeta_5'+10\zeta_6'+12\zeta_7'+4\zeta_8'-15\zeta_9'+18\zeta_{10}'-16\zeta_{11}'-44\zeta_{12}'\\&+15(\eta_1'+\eta_2')+18\eta_3'-36\eta_5'-15\eta_6']=11\,.\end{aligned}\tag{A33}$$

These conditions, of course, are specific for the "truncated" DS equation considered here, and will change once the T_4 term is included.

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