# Regularization schemes for stochastic quantization

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We show that some regularization schemes in conventional quantum field theory can be extended to the Parisi-Wu stochastic-quantization scheme. In particular, we discuss the Pauli-Villars and the dimensional-regularization methods.

## I. INTRODUCTION

The stochastic quantization scheme of Parisi and Wu[1,2] has attracted the attention of many physicists since it does not require gauge fixing for constrained systems. In their original formulation Parisi and Wu start with a Langevin equation in fictitious time $t$,

$$\frac{\partial\phi}{\partial t}=-\frac{\delta S}{\delta\phi}+\eta(x,t)\ , \tag{1.1}$$

where $S$ is the (Euclidean) action and $\eta(x,t)$ is a Gaussian white-noise source having the properties

$$\langle\eta(x,t)\rangle=0\ , \tag{1.2}$$

$$\langle\eta(x,t)\eta(x',t')\rangle=2\delta(x-x')\delta(t-t')\ . \tag{1.3}$$

Using (1.1)—(1.3) the correlation functions of $\phi$ can be computed. Parisi and Wu show that in the steady-state limit the equal-time correlation functions go over to the Euclidean field theory Green's functions. However, this equivalence is only formal because of ultraviolet divergences. If the Parisi-Wu scheme is to be an acceptable theory in its own right, the analysis of divergences and the renormalization program must be completed for the stochastic perturbation theory for the correlation functions. A careful treatment of the ultraviolet divergences requires regularization schemes for the stochastic perturbation theory. It is desirable to have regularization schemes which allow easy computations and a minimal subtraction procedure. For gauge theories an additional requirement of preserving the underlying gauge symmetry must be met.

In the original Parisi-Wu formulation of gauge theories based on the Langevin equation

$$\frac{\partial A_\mu^a(x,t)}{\partial t}=-\frac{\delta S}{\delta A_\mu^a(x,t)}+\eta_\mu^a(x,t)\ , \tag{1.4}$$

the "free" two-point correlation function of the gauge fields, with initial conditions set at $t=0$, reads

$$\langle A_\mu^a(k,t)A_\nu^b(k',t')\rangle=\delta^{ab}\delta(k+k')\left[\frac{1}{k^2}\left[\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right][\exp(-k^2|t-t'|)-\exp(-k^2|t+t'|)]+2\min(t,t')\frac{k_\mu k_\nu}{k^2}\right]\ . \tag{1.5}$$

It must be noted that the last term in (1.5) diverges in the field theory limit $t=t'\to\infty$ and even for finite $t,t'$ it has a bad ultraviolet behavior. Both these difficulties are bypassed by a suitable Zwanziger stochastic gauge-fixing term.[3]

Several schemes of regularization have been proposed in the literature[4-10] for stochastic quantization. In some of these schemes the properties of the random source are modified and in some others the Langevin equation is modified. It has been recently demonstrated[10] by explicit computations that the stochastic regularization scheme in which (1.3) is modified to have a form

$$\langle\eta(x,t)\eta(x',t')\rangle=2\delta(x-x')\alpha(t-t') \tag{1.6}$$

is inconsistent with Zwanziger gauge fixing. A recently proposed regularization scheme[8] circumvents the problem of inconsistency with Zwanziger gauge fixing. This scheme has the added advantage of being a nonperturbative continuum regularization scheme which is a welcome feature.

In this paper we discuss some other regularizations for the stochastic-quantization method (SQM). These are a straightforward extensions of regularization schemes known to preserve gauge invariance in the conventional field-theory formalism.

The perturbative expansion of the correlation functions can be obtained in a systematic and efficient manner using the operator formalism of Namiki and Yamanaka.[11] The operator formalism offers a very simple and at present the only available route to a discussion of the renormalization and the derivation of Ward-Takahashi identities in the stochastic formulation of field theories.[9] In the operator formalism the Parisi-Wu stochastic quantization of $N$-dimensional field theory becomes equivalent to a field theory in $N+1$ dimensions, the extra dimension being the fictitious time. The perturbation expansion for the $(N+1)$-dimensional field theory runs parallel to that of conventional field theories.

In the following by regularization of stochastic perturbation theory we shall mean the regularization of the corresponding five-dimensional theory suggested by the operator formalism. A short summary of the main features of the stochastic perturbation theory are given in Sec. II utilizing the operator formalism. In Sec. III power counting for the degree of divergence for $\partial^n\phi^m$-type interactions is presented. We show that the degree of divergence of a stochastic diagram is bounded by the degree of divergence of a corresponding field-theory diagram. This enables us to adopt the Pauli-Villars regularization scheme for SQM. This regularization scheme can be implemented nonperturbatively as it corresponds to modifying the action of the $(N+1)$-dimensional field theory. We briefly discuss a higher-derivative-type regularization procedure for gauge theories. In Sec. IV we give the dimensional-regularization method for the stochastic perturbation theory, closely following the dimensional-regularization procedure for conventional field theories.

## II. PERTURBATION THEORY IN THE PARISI-WU SCHEME

In this section we briefly summarize the main features of the stochastic perturbation theory. For this purpose we shall be using the operator formalism of Namiki and Yamanaka. For details of this formalism we refer the reader to the original paper[11] and our review.[2]

The equal-time correlation functions

$$\langle \phi(x_1,t_1)\cdots\phi(x_n,t_n)\rangle_\eta$$

obtained from the Langevin equation

$$\frac{\partial\phi}{\partial t}=-K(\phi)+\eta \tag{2.1}$$

can also be computed by means of probability distribution $P[\phi,t]$ obeying the Fokker-Planck equation

$$\frac{\partial P}{\partial t}=FP\ , \tag{2.2}$$

where $F$ is the Fokker-Planck operator given by

$$F=\int dx\frac{\delta}{\delta\phi}\left[K(\phi)+\frac{\delta}{\delta\phi}\right]. \tag{2.3}$$

The operator formalism associates an operator $\hat{\phi}(x,t)$ to each $\phi(x,t)$ and a corresponding $\hat{\pi}(x,t)$ such that

$$[\hat{\phi}(x,t),\hat{\pi}(x',t)]=\delta(x-x')\ . \tag{2.4}$$

These operators obey Heisenberg equations of motion governed by a Hamiltonian obtained from $F$ by replacing $\delta/\delta\phi$ by $-\hat{\pi}$: i.e.,

$$\hat{H}=\int dx\,[-\hat{\pi}K(\phi)+\hat{\pi}^2(x,t)]\ . \tag{2.5}$$

Namiki and Yamanaka have shown that the steady-state averages $\langle\phi(x_1,t_1)\cdots\phi(x_n,t_n)\rangle_\eta$ computed from the Langevin equation (2.1) are identical with the expectation value $\langle 0\,|\,T(\hat{\phi}(x_1,t_1)\cdots\hat{\phi}(x_n,t_n))\,|\,0\rangle$ of the Heisenberg operators in a suitably defined vacuum state. The perturbation expansion then becomes identical with the one obtained from the functional representation for the generating functional

$$Z[J]=\int\mathscr{D}\phi\,\mathscr{D}\pi\exp\left[\int d^4x\,dt\,(-\mathscr{L}+iJ\phi)\right], \tag{2.6}$$

where

$$\begin{aligned}\mathscr{L}&=\pi\frac{\partial\phi}{\partial t}-H\\&=\pi\frac{\partial\phi}{\partial t}-\pi^2+\pi K(\phi)\ .\end{aligned} \tag{2.7}$$

As an example, for the $\phi^4$ theory ascribed by the action

$$S=\int d^4x\left[\tfrac{1}{2}\partial_\mu\phi\partial_\mu\phi+\frac{M^2}{2}\phi^2+\frac{g}{4!}\phi^4\right] \tag{2.8}$$

we have

$$K=\frac{\delta S}{\delta\phi}=(-\Box+M^2)\phi+\frac{g}{3!}\phi^3 \tag{2.9}$$

and the five-dimensional Lagrangian density assumes the form

$$\mathscr{L}=\pi\frac{\partial\phi}{\partial t}-\pi^2+\pi(-\Box+M^2)\phi+\frac{g}{3!}\pi\phi^3\ . \tag{2.10}$$

For perturbative calculations $\mathscr{L}$ can be split into a sum of "free" and interaction parts:

$$\mathscr{L}=\mathscr{L}_0+\mathscr{L}_{\rm int}\ , \tag{2.11}$$

with

$$\mathscr{L}_0=\pi\frac{\partial\phi}{\partial t}+\pi(-\Box+M^2)\phi\ , \tag{2.12}$$

$$\mathscr{L}_{\rm int}=\frac{g}{3!}\pi\phi^3\ . \tag{2.13}$$

A word about notation: the symbols $\mathscr{L},\mathscr{L}_0,\mathscr{L}_{\rm int},\mathscr{S}$, etc., will refer to the five-dimensional field theory whereas the symbols $L,L_0,L_{\rm int},S$ will refer to the corresponding quantities for the underlying four-dimensional field theory.

We now summarize some of the important features of the perturbation theory in SQM for a self-interacting scalar field $\phi$ with $L_{\rm int}$ containing the $m$-fold product of $\phi$ fields and $n$ derivatives ($L_{\rm int}\sim\partial^n\phi^m$). For this case $\mathscr{L}_0$ is the same as given in (2.12) and $\mathscr{L}_{\rm int}$ will be of the form $\pi\partial^n\phi^{m-1}$.

(1) The free propagators obtained from $\mathscr{L}_0$ are

$$\langle 0\,|\,T(\phi(k,t)\pi(k',t'))\,|\,0\rangle=\delta(k+k')G(k,t-t')\ , \tag{2.14}$$

$$\langle 0\,|\,T(\phi(k,t)\phi(k',t'))\,|\,0\rangle=\delta(k+k')D(k,t'-t')\ , \tag{2.15}$$

$$\langle 0\,|\,T(\pi(k,t)\pi(k',t'))\,|\,0\rangle=0\ , \tag{2.16}$$

where

$$G(k,t-t')=\theta(t-t')\exp[-(k^2+M^2)(t-t')]\ , \tag{2.17}$$

$$D(k,t-t')=(k^2+M^2)^{-1}\exp[-|t-t'|(k^2+m^2)] \tag{2.18}$$

$$=\int_0^\infty d\beta\,\theta(\beta-|t-t'|)\exp[-\beta(k^2+M^2)] \tag{2.19}$$

$$=\int_1^\infty d\beta\,|t-t'|\exp[-\beta|t-t'|(k^2+M^2)]\ . \tag{2.20}$$

(2) The Green's functions

$$\langle 0|T(\phi(x_1,t_1)\cdots\pi(x_1',t_1')\cdots)|0\rangle$$

of the five-dimensional theory have the usual Feynman-diagram expansion with vertices determined by $\mathscr{L}_{\rm int}$ and the propagators given by (2.14)–(2.20). There are two types of lines arising from the pairings of a $\phi$ and a $\pi$ field and from the pairing of two $\phi$ fields. These will be called the directed and crossed lines and will also be referred to as the $G$ and the $D$ propagators, respectively, and will be diagrammatically represented as shown in Figs. 1(a) and 1(b).

(3) The vertices of the Feynman diagrams are determined by the $\mathscr{L}_{\rm int}$ part of the five-dimensional action. To write the contribution of a given diagram each line is assigned a four-momentum in accordance with momentum conservation at each vertex. Each external vertex is assigned a time $t_i$ and a time $\tau_i$ is associated with each internal vertex. The contribution of a diagram is obtained by writing suitable propagators for the lines, appropriate factors for the vertices as dictated by $\mathscr{L}_{\rm int}$ and combinatorial factors as is done in conventional field theory. Apart from the integrations over all loop momenta we must also integrate over times assigned to the internal vertices.

(4) The directed lines or the $G$ propagators alone cannot form a closed loop. The self-closing loops of a single $G$ line which starts and ends at the same vertex are excluded. All diagrams containing a loop of two or more $G$ lines alone must vanish due the presence of the $\theta$ functions for the $G$ propagators.

(5) The self-closing loops of a single $D$ line must be excluded if and only if the SQM is to correspond to a normal-ordered interaction Lagrangian $L_{\rm int}$ for the underlying four-dimensional field theory.

(6) Given any internal vertex there is one and only one directed line attached to it such that it points towards the vertex. As a result the internal directed lines will form a closed loop and the diagram will vanish [remark (4)] unless at least one $\pi$ field coming from one of the internal vertices is paired with an external $\phi$ field. In other words we must have at least one external directed line pointing away from the external vertex.

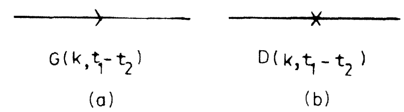


FIG. 1. The internal lines of the $(N+1)$-dimensional field theory.

(7) The contribution of an arbitrary Feynman diagram has the following structure:

$$\int\left[\prod_{\rm vertices} d\tau_i\right]\int\left[\prod_{\rm loops} d^4k_i\right]I(k_i,q_i,\tau_i,t_i)\ , \tag{2.21}$$

where $k_i$ are independent internal (loop) momenta and $q_i$ are the momenta of the external lines. The integrand has the form

$$I=\prod_{\rm all\ lines}\Delta^j(Q_j,\tau_{j_1}-\tau_{j_2})\prod_{\rm vertices}\mathscr{P}_a\ . \tag{2.22}$$

Here $\mathscr{P}_a$ is a polynomial in momenta of the lines meeting at the vertex labeled $a$. The precise form of $\mathscr{P}_a$ is determined by the way derivatives appear in $\mathscr{L}_{\rm int}$. For the $j$th line $Q_j$ is the momentum carried by the line, $\tau_{j_1}$ and $\tau_{j_2}$ are the fictitious times associated with the vertices joined by the line $j$. Also

$$\Delta^l=\begin{cases}G(Q_l,\tau_{l_1}-\tau_{l_2}) & \text{if } l\text{th line is directed line ,}\\ D(Q_l,\tau_{l_1}-\tau_{l_2}) & \text{otherwise .}\end{cases}$$

For an internal line $Q_l$ is a linear combination of $k$'s and $q$'s whereas for an external line $Q_l$ is simply $q_l$.

Writing the $D$ propagators in the form (2.19) or (2.20) we associate a parameter $\beta_j$ with each crossed line. For each directed line also we associate a parameter $\beta_j$ which is simply the time difference $\tau_{j_1}-\tau_{j_2}$. Then the contribution of any diagram can be cast in the form

$$\int\left[\prod_{D\ \rm lines\ only} d\beta_m\right]\int\left[\prod_{\rm vertices} d\tau_i\right]\int\left[\prod_{\rm loops} d^4k_i\right]\left[\prod_{\rm vertices}\mathscr{P}_a\right]\prod_{G\ \rm lines}\Delta^l(Q_l,\beta_l)\prod_{D\ \rm lines}\exp[-\beta_j|\tau_{j_1}-\tau_{j_2}|(Q_j{}^2+M^2)]\ . \tag{2.23}$$

(8) Let us for the moment consider the stochastic diagrams for the Green's functions involving the $\phi$ fields alone for the case when $\mathscr{L}_{\rm int}$ has the form $\pi\partial L_{\rm int}/\partial\phi$. For the graphs contributing to Green's functions of $\phi$ alone there is a one-to-one correspondence between the $G$ lines and the internal vertices. The parameters $\beta_j$ for the $G$ lines can be used as integration variables in place of $\{\tau_i\}$ and we have

$$\prod_{\rm internal\ vertices} d\tau_i=\prod_{G\ \rm lines} d\beta_l\ .$$

The contribution of an arbitrary diagram can now be written in the form

$$\int \left[ \prod_{\text{all lines}} d\beta_l \right] \int \left[ \prod_{\text{loops}} d^4k_i \right] \left[ \prod_{\text{vertices}} \mathscr{P}_a \right] \prod_{\text{all lines}} \exp[-\beta_l(Q_l{}^2+M^2)] \prod_{G\text{ lines}} \theta(\beta_l) \prod_{D\text{ lines}} \theta(\beta_m - |\tau_{m_1}-\tau_{m_2}|) \;. \quad (2.24)$$

Here an explicit expression for $\Delta_l$ has been written. The times $\tau_m$ appearing in the last set of $\theta$ functions must be expressed in terms of the parameters $\beta_l$. The $\theta$ function in (2.24) determines the region of integration for the parameters $\{\beta_l\}$. The different stochastic diagrams of a fixed topology correspond to different regions of integration.

The sets of diagrams with a fixed topology are in a one-to-one correspondence with the field-theory diagrams. Recalling that the propagator $(k^2+M^2)^{-1}$ can be written in the Schwinger parametric form

$$(k^2+M^2)^{-1} = \int_0^\beta d\beta \exp[-\beta(k^2+M^2)] \quad (2.25)$$

the integrand of (2.24) looks exactly like the field-theory diagram contribution except for the region of integration for the parameters $\beta_l$. It was first shown by Gonzales-Arroyo[12] that when all external times are set equal and the steady-state limit taken, the stochastic diagrams of a fixed topology on addition produce the range $(0,\infty)$ for each parameter $\beta_l$. Thus this sum becomes identical with the field-theory contribution written in the Schwinger parametric form. In Sec. IV we shall define dimensional regularization for stochastic diagrams in a way so as to retain this property for the regularized Green's functions also.

(9) It is easy to see that there exists a maximal loop passing through all the vertices of a one-particle-irreducible diagram. This loop integral will diverge only if all the times $\tau_i$, associated with all the internal vertices, are equal. If integrals over the loop momenta are carried out, the divergence of a diagram manifests itself as a nonintegrable power in difference of times $\tau$'s associated with the internal vertices. This fact has been used to suggest a new regularization scheme in Ref. 9.

## III. POWER COUNTING

For the sake of generality let us assume that we are dealing with a $(N+1)$-dimensional theory with $\mathscr{L}_{\text{int}}$ having a $p$ number of $\pi$ fields, a $q$ number of $\phi$ fields, and an $n$ number of derivatives. Also we assume that the $G$ and $D$ propagators have the form

$$G(k,t-t') = \theta(t-t')\exp[-f(k)(t-t')] \;, \quad (3.1)$$

$$D(k,t-t') = \frac{1}{f(k)}\exp[-|t-t'|f(k)] \quad (3.2)$$

$$= |t-t'| \int_1^\infty d\beta \exp[-\beta|t-t'|f(k)] \;, \quad (3.3)$$

where $f(k) \sim k^{2r}$ for large $k$. Writing a typical stochastic diagram contribution as

$$\int \left[ \prod_{\text{loops}} d^N k_i \right] J(q_i,k_i,t_i) \;, \quad (3.4)$$

where

$$J = \int \left[ \prod_{\text{vertices}} d\tau_i \right] \prod_{\text{internal } G \text{ lines}} \theta(\tau_{j_1}-\tau_{j_2})\exp[-(\tau_{j_1}-\tau_{j_2})f(Q_j)]$$
$$\times \prod_{\text{internal } D \text{ lines}} \int d\beta_l G(\beta_l-1)\,|\tau_{l_1}-\tau_{l_2}|\exp[-\beta_l|\tau_{l_1}-\tau_{l_2}|f(Q_l)]$$
$$\times \prod_{\text{external lines}} \Delta^l(q_l,\tau_{l_1}-\tau_{l_2}) \;. \quad (3.5)$$

For a stochastic diagram we define the following quantities: $V$=member of vertices, $L$=number of loops, $E_\pi$=number of external $\pi$ fields, $E_D$=number of external crossed lines, $E_G$=number of external $G$ lines pointing away from the external vertices, $E=E_D+E_G+E_\pi$=total number of external lines, $I_D$=number of internal crossed lines, $I_G$=number of internal directed lines, $I=I_G+I_D$=total number of internal lines, $N$=dimension of each loop integration.

To compute the behavior of $J$ as the loop momenta $k_i \to \infty$, we replace $k_i$ by $\lambda k_i$ and in (3.5) retain only leading terms for $\lambda \to \infty$. The large-$\lambda$ behavior of (3.5) is easily obtained by replacing integration variables $(\tau_i-\tau_j)$ by scaled time differences $\lambda^{2r}(\tau_i-\tau_j)$. We then have

$$J(q,\lambda k_i,t_i) \to \lambda^M J(q,k_i,t_i) \;, \quad (3.6)$$

where

$$M = nV - 2I_D r - 2(V-1)r \;. \quad (3.7)$$

The loop integration diverges if

$$NL + M \geq 0 \;. \quad (3.8)$$

The degree of divergence is now defined to be

$$\omega = NL + M \;. \quad (3.9)$$

Therefore

$$\omega = NL + nV - 2(I_D + V - 1)r \;. \quad (3.10)$$

The number of loop integrations is given by

$$L = I - V + 1 \;. \quad (3.11)$$

Counting the number of $\pi$ and $\phi$ fields needed for the lines of a diagram in two different ways we get

$$pV + E_\pi = I_G + E_G + E_\pi \quad (3.12)$$

and

$$qV + E_D + E_G = 2(I_D + E_D) + (I_G + E_G) + E_\pi \;. \quad (3.13)$$

Solving for $I_G$ and $I_D$ and expressing the degree of divergence in terms of $V$, $E_G$, $E_D$, and $E_\pi$ we get

$$\omega = N + 2r + (N/2 - r)[(q-p)V - E_D - E_\pi + E_G] - NE_G + (Np + n - N - 2r)V . \quad (3.14)$$

Now we show that for a $L_{\text{int}} \sim \partial^n \phi^m$-type interaction the degree of divergence of stochastic diagrams is bounded by the degree of divergence of corresponding field-theory diagram. This would imply that if underlying quantum field theory (QFT) has been regularized by means of regulator fields and kinetic-energy-like terms with higher-order derivatives, the corresponding stochastic theory is also regularized.

For $L_{\text{int}} \sim \partial^n \phi^m$ the $\mathscr{L}_{\text{int}}$ for the stochastic theory has the form $\pi \partial^n \phi^{m-1}$. Thus we set $p=1$, $q=m-1$ in (3.14) and we obtain for $N=4$

$$\omega = 4 + 2r + (2-r)[(m-2)V - E] - 2rE_G + (n-2r)V . \quad (3.15)$$

Now recall remark (6) of Sec. II that $E_G \geq 1$. Hence

$$\omega \leq E(r-2) + 4 + V[n + (2-r)m - 4] . \quad (3.16)$$

The right-hand side is just the degree of divergence of a diagram with $V$ vertices and $E$ external lines for interaction of type $\partial^n \phi^m$ in conventional QFT with propagator behaving as $k^{-2r}$.

The result that the degree of divergence of a stochastic diagram is bounded by the degree of divergence of the corresponding field-theory diagram[13] suggests new regularization schemes for SQM. Given a field theory in four dimensions we first regularize it by means of higher-order derivatives in bilinear terms and by introducing regulator fields. The regularized stochastic theory is then obtained by introducing a $\pi$ field for each regulator field also and constructing the five-dimensional action as described in Sec. II. The resulting five-dimensional theory is automatically regularized.

The regularization scheme discussed by Lee and Zinn Justin[14] can therefore be extended to the Parisi-Wu formalism of gauge theories.

## IV. DIMENSIONAL REGULARIZATION

We now discuss an extension of the dimensional-regularization scheme to Parisi-Wu stochastic quantization. It has already been noted that the contribution of ordinary QFT diagram written in the Schwinger parametric representation resembles the contribution of a stochastic diagram. Therefore to dimensionally regularize the stochastic diagrams we follow the steps for the dimensional regularization in QFT. We first calculate integrations over loop momenta in (2.24) as $N$-dimensional integration. The resulting expression is then analytically continued to complex values of $N$. The divergences will appear as poles in $N$. This regularization scheme has the advantage of being very simple and offers a minimal subtraction scheme to renormalize the stochastic theory.

It must be remarked that the scheme suggested here does not correspond to continuing all the five dimensions to complex values; it leaves the fictitious time untouched. An important consequence of this feature is that the correlation functions $\langle 0 | T(\phi(x_1,t_1) \cdots \phi(x_n,t_n)) | 0 \rangle$ regularized in this way will become identical with the dimensionally regularized amplitudes of conventional QFT at equal times $t_1 = \cdots = t_n$. This is because of the observation, in remark (8) of Sec. II, that the regions of integrations for the parameters for different diagrams of fixed topology give the range $0-\infty$ for each parameter.

The dimensional regularization for the Parisi-Wu formalism has been independently used in Ref. 15 for some explicit calculations in Yang-Mills theories.

## V. CONCLUSION

In this paper we have shown how conventional regularization schemes can be utilized to obtain new regularization methods for the stochastic formalism. We expect that these will give us regularizations which will be consistent with gauge invariance and stochastic gauge fixing. This point merits further detailed investigation.

In the end we note that the regularization schemes discussed in this paper do not modify the relation

$$G(k,t-t') = \theta(t-t') \frac{\partial}{\partial t} D(k,t-t')$$

between the $G$ and the $D$ propagators. This relation was crucial for finding a supersymmetry for the stochastic theory which was in turn used to prove equivalence of the Parisi-Wu and the conventional formulations of field theory.[16]


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