# Covariant soliton dynamics: Role of the gluon condensate in hadron spectroscopy

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(Received 9 June 1986)

We provide a covariant description of states of the charmonium and $b$-quarkonium systems. A very good fit to the spectrum is obtained in both cases in a model whose degrees of freedom are quarks and order parameters of QCD condensates. The current-quark masses used are similar to those suggested in QCD sum-rule studies.

We have recently studied covariant (nontopological) soliton solutions of a simple Lagrangian[1,2]

$$\mathscr{L}(x)=\tfrac{1}{2}\partial_\mu\chi(x)\partial^\mu\chi(x)-\frac{m_\chi{}^2}{2}\chi^2(x)+\bar{q}(x)[i\gamma^\mu\partial_\mu-m_q-g_\chi\chi(x)]q(x)\ . \tag{1}$$

Here $q(x)$ is the quark field, $m_q$ is the sum of a current-quark mass ($m_q^{\rm cur}$) and a dynamical mass ($m_q^{\rm dyn}$), $g_\chi$ is a coupling constant, and $\chi(x)$ measures the deviation of a QCD order parameter $\phi$ from its vacuum value, $\phi_0$ [$\phi(x)=\phi_0+\chi(x)$]. We were able to provide *covariant* descriptions of the $\rho$ and $\omega$ mesons, and states of the charmonium and $b$-quarkonium systems.[2] However, with our choice of parameters, we found that the states of $J/\psi$ and $\Upsilon$ were too widely spaced. In this work we overcome this difficulty and suggest that a useful effective Lagrangian is

$$\mathscr{L}'(x)=\tfrac{1}{2}\partial_\mu\chi(x)\partial^\mu\chi(x)-\frac{m_\chi{}^2}{2}\chi^2(x)+\bar{q}(x)[i\gamma^\mu\partial_\mu-m_q'-g_\chi^{\rm eff}\chi(x)]q(x)\ , \tag{2}$$

where $g_\chi^{\rm eff}=\beta g_\chi$, $m^{\rm dyn}=g_\chi\phi_0$, with

$$\beta=\left[\frac{m_q{}^{\rm dyn}}{m_q'}+\frac{m_q{}^{\rm cur}m_q^{q\bar q}}{m_q{}^{\rm dyn}m_q'}\right]\ , \tag{3}$$

$$m_q^{\rm dyn}=[(m_q^{q\bar q})^2+(m_q^{\rm gl})^2]^{1/2}\ , \tag{4}$$

and

$$m_q'=[(m_q^{q\bar q}+m_q^{\rm cur})^2+(m_q^{\rm gl})^2]^{1/2}\ . \tag{5}$$

Note that if $m_q^{\rm cur}=0$, $\beta=1$, and $m_q'\to m_q^{\rm dyn}$. In these equations $m_q^{q\bar q}$ is the dynamical quark mass which arises from the coupling of the quark to the chiral condensate[3] and $m_q^{\rm gl}$ is a dynamical mass which arises from coupling to the gluon condensate.[4,5] At a later point, we will present some results obtained for the charmonium and $b$-quarkonium systems using the field equations which follow from Eq. (2):

$$(\partial_\mu\partial^\mu+m_\chi{}^2)\chi(x)=-g_\chi^{\rm eff}\bar{q}(x)q(x)\ , \tag{6}$$

$$[i\gamma^\mu\partial_\mu-m_q'-g_\chi^{\rm eff}\chi(x)]q(x)=0\ ; \tag{7}$$

however, we will first attempt to motivate the model described in Eqs. (2)–(7).

Recently we have developed a phenomenology for the study of QCD at low-momentum transfer.[4,5] We suggested the following effective Lagrangian:

$$\begin{aligned}\mathscr{L}_{\rm eff}(x)=&-\tfrac{1}{4}\mathscr{G}^a_{\mu\nu}(x)\mathscr{G}_a^{\mu\nu}(x)+\frac{m_{\rm gl}{}^2}{2}\frac{\phi^2(x)}{\phi_0^2}\mathscr{A}^a_\mu(x)\mathscr{A}_a^\mu(x)+\bar{q}(x)\{i\not{D}-m_q{}^{\rm cur}-g_\pi[\sigma(x)+i\boldsymbol{\pi}(x)\cdot\boldsymbol{\tau}\gamma_5]\}q(x)\\&+\tfrac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x)+a\phi^2(x)-b\phi^4(x)+\tfrac{1}{2}\partial_\mu\sigma(x)\partial^\mu\sigma(x)+\tfrac{1}{2}\partial_\mu\boldsymbol{\pi}(x)\cdot\partial^\mu\boldsymbol{\pi}(x)\\&-\lambda\left[\sigma^2(x)+\boldsymbol{\pi}^2(x)-f_\pi{}^2\frac{\phi^2(x)}{\phi_0{}^2}\right]^2\ .\end{aligned} \tag{8}$$

This Lagrangian was obtained by separating the vector potential and field tensor into a condensate field and a fluctuation field:[4,5]

$$A_a^\mu(x)=\mathbb{A}_a^{\mu}(x)+\mathscr{A}_a^\mu(x)\ , \tag{9}$$

$$G^a_{\mu\nu}(x)=\mathbb{G}^a_{\mu\nu}(x)+\mathscr{G}^a_{\mu\nu}(x)\ . \tag{10}$$

A scale was set into the theory by writing

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$$\left\langle \text{vac} \left| \frac{g^2(\mu^2)}{4\pi^2} G^a_{\mu\nu}(0) G^{\mu\nu}_a(0) \right| \text{vac} \right\rangle \simeq \frac{3}{32\pi^2} [\langle \text{vac} \,|\, g^2(\mu^2) \, \mathrm{A}^{\mu}_a(0) \, \mathrm{A}^a_{\mu}(0) \,|\, \text{vac} \rangle]^2 \tag{11}$$

$$\equiv \frac{3}{32\pi^2} g^4(\mu^2) \phi_0^4 \,. \tag{12}$$

[Here $g(\mu^2)$ is the QCD coupling constant renormalized at the mass scale $\mu$.] We then used the value obtained for the left-hand side of Eq. (11) in QCD sum-rule studies[6] to provide a value for $g^2\phi_0^2$. [In the presence of quarks $\phi_0$ becomes spatially dependent, $\phi_0 \rightarrow \phi(x) = \phi_0 + \chi(x)$.] In Eq. (8) $m_{\mathrm{gl}}$ is a dynamical gluon mass[7] ($m_{\mathrm{gl}}{}^2 = \frac{3}{8} g^2 \phi_0^2$), $\sigma(x)$ and $\pi(x)$ are order parameters of the chiral condensate, and $D_\mu$ is the usual covariant derivative. Note that if $m_q^{\mathrm{cur}} = 0$, there is only a single mass scale in our model, since we have *assumed* that the chiral and gluon order parameters are proportional:

$$\sigma(x) = f_\pi \phi(x)/\phi_0 \,. \tag{13}$$

(In vacuum $\langle \sigma \rangle = f_\pi$.) We define $m_q^{q\bar{q}} = g_\pi f_\pi$ and set $m_q^{q\bar{q}} = 400$ MeV, which is generally consistent with values which have been suggested for this quantity.[3] In order to understand the origin of $m_q^{\mathrm{gl}}$, we consider the quark field equation:

$$\left[ i\gamma^\mu \left[ \partial_\mu - igA^a_\mu \frac{\lambda^a}{2} \right] - m_q^{\mathrm{cur}} - g_\pi [\sigma(x) + i\pi(x) \cdot \tau\gamma_5] \right] q(x) = 0 \,. \tag{14}$$

Since the gluon field has no vacuum expectation value, we consider a "second-order" version of this equation and keep only those terms that can have vacuum expectation values. We find[4,5]

$$[\mathbf{p}_{\mathrm{op}}{}^2 - \tfrac{1}{6} \langle g^2 \, \mathrm{A}_a \cdot \mathrm{A}_a \rangle + (m_q^{\mathrm{cur}} + m_q^{q\bar{q}})^2] q(x) = \frac{-\partial^2}{\partial t^2} q(x) \tag{15}$$

in a temporal gauge. We define

$$(m_q^{\mathrm{gl}})^2 = -\tfrac{1}{6} \langle g^2 \, \mathrm{A}_a \cdot \mathrm{A}_a \rangle \tag{16}$$

$$= \tfrac{1}{6} g^2 \phi_0^2 \,. \tag{17}$$

and define $m_q'$ as in Eq. (5). To provide a model for the coupling of the quark to the condensate order parameters we introduce

$$m_q'(x) = \{ [m_q^{q\bar{q}}(x) + m_q^{\mathrm{cur}}]^2 + [m_q^{\mathrm{gl}}(x)]^2 \}^{1/2} \,, \tag{18}$$

with

$$m_q^{q\bar{q}}(x) \equiv m_q^{q\bar{q}} \left[ \frac{\phi_0 + \chi(x)}{\phi_0} \right] , \tag{19}$$

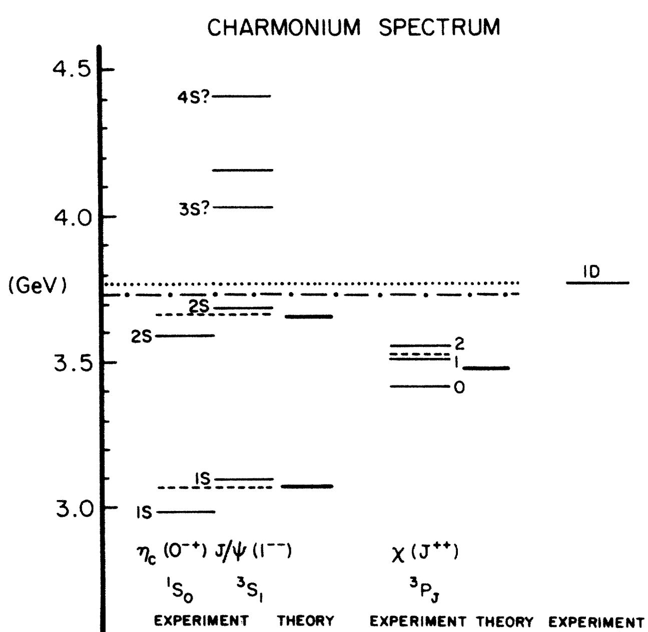


FIG. 1. Light lines denote various states of the charmonium system whose energies have been determined experimentally. The dashed lines are the spin-weighted averages, while the heavy lines are the theoretical predictions with $m_c^{\mathrm{cur}} = 1433$ MeV. The continuum of our model starts at $2m_q' = 3768$ MeV (dotted line), which is quite close to the $D,\bar{D}$ threshold (dot-dashed line).

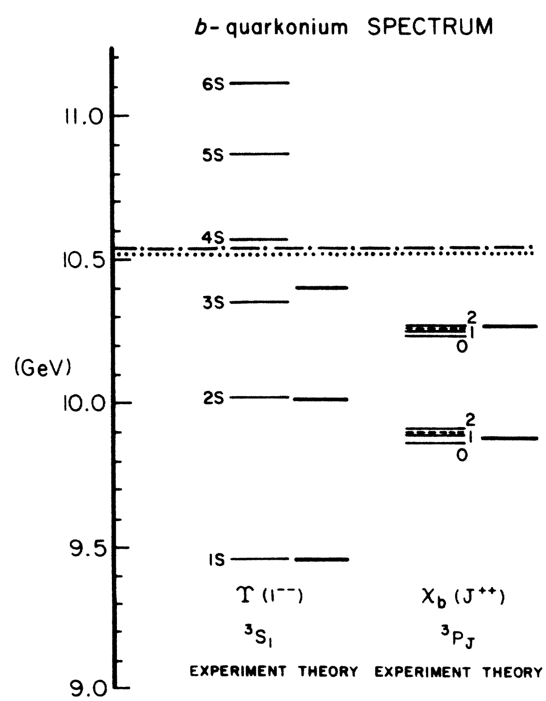


FIG. 2. Light lines denote various states of the $b$-quarkonium system. The heavy lines are theoretical predictions with $m_b^{\mathrm{cur}} = 4840$ MeV. (See caption of Fig. 1.) The continuum of our model starts at $2m_q' = 10.516$ GeV (dotted line), which is quite close to the $B,\bar{B}$ threshold (dot-dashed line).

$$m_q^{\rm gl}(x) \equiv m_q^{\rm gl}\left[\frac{\phi_0+\chi(x)}{\phi_0}\right] . \tag{20}$$

Now

$$m_q'(x) \simeq m_q'\left[1+\beta g_\chi \frac{\chi(x)}{m_q'} + \tfrac{1}{2}(1-\beta^2) g_\chi{}^2 \frac{\chi^2(x)}{m_q'^2} + \cdots\right] \tag{21}$$

with $\beta$ as defined in Eq. (3). Keeping the first two terms of the expansion in Eq. (21) is expected to be a good approximation and that leads us back to the Lagrangian, $\mathcal{L}'(x)$, of Eq. (2).

We have chosen $g_\chi = 7$ in our study[8] of the nucleon[1] and the $\rho$ and $\omega$ mesons.[2] The analysis of Ref. 5 fixes $m_\chi{}^2$ in terms of $g^2\phi_0{}^2$. We found $m_\chi = 459$ MeV. Further, from $(m_q^{\rm gl})^2 = \frac{1}{6} g^2 \phi_0{}^2$, we obtain $m_q^{\rm gl} = 434$ MeV. Therefore, using Eq. (4), we have $m_q^{\rm dyn} = 590$ MeV. Thus, only a single parameter is to be determined: the flavor-dependent, current-quark mass $m_q^{\rm cur}$. For the charmonium system the value of $m_c^{\rm cur}$ is fixed by fitting to the weighted average of the ground state ($0^-$; 2980 MeV) and first excited state ($1^-$; 3100 MeV). For the $b$-quarkonium system, $m_b^{\rm cur}$ is obtained by fitting the lowest $J^\pi = 1^-$ state. Our predictions for the other states are given in Figs. 1 and 2. A more detailed fit requires a model for the spin-dependent interactions, presumably arising from some short-range interaction ("one-gluon exchange").[2]

In summary, we note that we have provided a fully covariant description of a number of the low-lying states of the charmonium and $b$-quarkonium using a model which has been used previously to provide a good fit to the properties of the nucleon[1] and the $\rho$ and $\omega$ mesons.[2] The new feature in this work is a generalized model for quark-condensate coupling for the case $m_q^{\rm cur} \neq 0$. The details of our theory of $p$-wave solitons will be presented elsewhere as will a comprehensive study of the second-order quark field equation:

$$\left\{ \mathbf{p}_{\rm op}{}^2 + m_{\rm gl}^2 \left[\frac{\phi_0+\chi(x)}{\phi_0}\right]^2 + \left[ m_q^{\rm cur} + m_q^{q\bar q}\left[\frac{\phi_0+\chi(x)}{\phi_0}\right]\right]^2 \right\} q(x) = -\frac{\partial^2}{\partial t^2} q(x) . \tag{22}$$

This work was supported in part by the Faculty Research Award Program of the City University of New York and the National Science Foundation.

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[3]Many works deal with the chiral condensate order parameters. See, for example, M. C. Birse and M. K. Banerjee, Phys. Lett. **136B**, 284 (1984); Phys. Rev. D **31**, 118 (1985); W. Broniowski and M. K. Banerjee, Phys. Lett. **158B**, 335 (1985); H. D. Politzer, Nucl. Phys. **B117**, 397 (1976); V. Elias and M. D. Scadron, Phys. Rev. Lett. **53**, 1129 (1984); V. Elias, M. Scadron, and R. Tarrach, Phys. Lett. **162B**, 176 (1985); M. D. Scadron, Ann. Phys. (N.Y.) **148**, 257 (1983).

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[6]M. A. Shifman, Annu. Rev. Nucl. Part. Sci. 33, 199 (1983), and references therein.

[7]We find $m_{\rm gl} = 649$ MeV. The dynamical gluon mass obtained in our model is similar to that found in recent lattice gauge calculation [J. Mandula (private communication)]; C. Bernard, Phys. Lett. **108B**, 436 (1982). See also, J. M. Cornwall and A. Soni, *ibid.* **120B**, 431 (1983); J. M. Cornwall, Nucl. Phys. **B157**, 392 (1979); Phys. Rev. D **26**, 1453 (1982).

[8]The small parameter in this expansion is $g_\chi \chi(x)/m_q'$. If $g_\chi = 7$, the *largest* value for $\chi(x)$ is $-84$ MeV, since

$$m_q^{\rm dyn}(x) = m_q^{\rm dyn}[\phi_0+\chi(x)]/\phi_0 = m_q^{\rm dyn} + g_\chi \chi(x) \geq 0 .$$

The value of $\chi(x)$ averaged over hadron wave functions will be smaller than the maximum value. For charmonium, $m_q' = 1884$ MeV and for the $b$ quarkonium, $m_q' = 5258$ MeV.