Nongauge interactions of vector bosons and rare K decays

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We study the effect of nonstandard vector-boson couplings on the rare decay modes of K mesons. Our results show that as long as the Glashow-Iliopoulos-Maiani mechanism is operative, experimental results on rare K decays do not produce any constraints on nonstandard self-couplings of vector bosons.

The experimental discovery[1] of $W^{\pm}$ and Z bosons makes the standard $SU(2)\times U(1)$ gauge theory[2] of weak interactions the most plausible theory but certainly does not establish it totally. Hung and Sakurai[3] and independently Bjorken[4] had quite sometime back shown that the predictions of the standard theory in the lowest order could be reproduced in theories with global weak SU(2) symmetry and weak mixing of the neutral vector meson with the photon. Independently, the weak vector bosons have also been proposed as composites of preons where once again any direct relationship of the vector bosons with any gauge principle is absent.[5] Discriminating between the standard $SU(2)\times U(1)$ theory and others obviously involves going beyond the lowest-order tree approximation into some finer predictions involving loop diagrams. At the moment, there do not seem to be any definite experimental numbers available to test higher-order weak corrections, but soon they may become available. Pending that, it is useful to recognize from a theoretical point of view the processes and parameters that may serve as suitable testing grounds for higher-order effects.

One of the hallmarks of the gauge theory is the characteristic three- and four-point self-couplings of the vector bosons (and photon) among themselves with completely specified strength. A similar scheme of couplings is highly unlikely in any other theory. With a more general scheme of couplings, the weak correction to the $(g-2)$ of the muon[6,7] and the correction to the tree-level $W^{\pm}$-Z mass relationship[8] have recently been worked out. Suzuki[6] also has given bounds arising out of model-independent unitarity restrictions on the various coupling constants for the general self-interaction vertex of the vector bosons. In this Brief Report we address ourselves to a study of strangeness-changing neutral currents with a general rather than the standard $SU(2)\times U(1)$ self-coupling of the vector bosons.

In the standard theory, the Glashow-Iliopoulos-Maiani[9] (GIM) mechanism prevents the occurrence of any strangeness-changing neutral currents at the tree level. However, as Bjorken has emphasized, the GIM mechanism is far more general than the gauge theory and right now is the only known mechanism for suppressing strangeness-changing neutral currents. Without committing ourselves to any model, we assume that the GIM mechanism is operative. At the one-loop level, where once again at the order-of-magnitude level there is a potential conflict between theory and experiment for some of the K-decay processes, Gaillard and Lee[10] were able to show that the GIM-mechanism once again suppresses all processes well below the experimental upper bounds, except possibly in $K^{+}\rightarrow\pi^{+}e\bar{e}$. Several authors[11-17] have subsequently improved the calculation of Ref. 20 by including the effect of strong QCD corrections as well as the effect of heavy quarks. Typically these change the amplitude by a factor of 2 or 3. The calculation of Gaillard and Lee uses the full $SU(2)\times U(1)$ theory with its vector-boson self-couplings. We use here the more general vector-boson self-coupling as given by Suzuki in studying the same decay modes as Ref. 10, to see the kind of restrictions that are imposed on the coupling constants by the very strong experimental bounds on strangeness-changing neutral-current transitions.

The quark couplings to W, Z, and γ are taken to be the same as in the standard theory. For the $WW\gamma$ vertex, we use the effective Lagrangian[6,7]

$$
\begin{aligned}
L_{\text{eff}}^{WW\gamma} ={}& L^{WW\gamma}(\text{standard theory})+ie\kappa W_{\mu}^{\dagger}W_{\nu}(\partial^{\mu}A^{\nu}-\partial^{\nu}A^{\mu}) \\
&+\frac{ie\lambda}{M_W{}^2}\{[(\partial^{\lambda}W_{\beta}^{\dagger})(\partial_{\mu}W^{\beta})-(\partial_{\mu}W_{\beta}^{\dagger})(\partial^{\lambda}W^{\beta})]\partial_{\lambda}A^{\mu}-\partial^{\Lambda}W_{\beta}^{\dagger}[(\partial^{\beta}W_{\mu})(\partial_{\lambda}A^{\mu})-(\partial_{\lambda}W_{\mu})(\partial^{\beta}A^{\mu})-(\partial_{\lambda}W_{\mu})(\partial^{\beta}A^{\mu})] \\
&+[(\partial^{\alpha}W_{\mu}^{\dagger})(\partial^{\lambda}A_{\mu})-(\partial^{\lambda}W_{\mu}^{\dagger})(\partial^{\alpha}A^{\mu})]\partial_{\lambda}W_{\alpha}-(\partial^{\alpha}W_{\beta}^{\dagger})(\partial_{\mu}W_{\alpha})(\partial^{\beta}A^{\mu})+(\partial_{\mu}W_{\beta}^{\dagger})(\partial^{\beta}W_{\alpha})(\partial^{\alpha}A^{\mu})\}\ , \qquad (1)
\end{aligned}
$$

where k and λ measure the departure of the gyromagnetic ratio of the W away from its standard value according to

$$\mu=\frac{e(1+K+\lambda)}{2M_W} \quad \text{and} \quad Q=-e\frac{(K-\lambda)}{M_W{}^2}\ ; \qquad (2)$$

μ and Q are the magnetic and quadrupole moments, respectively. The effective WNZ interaction is taken as[6]

$$
\begin{aligned}
L_{\text{eff}}^{WNZ} = -if[&(W_{\nu}^{\dagger}\partial^{\mu}W^{\nu}-\partial^{\mu}W_{\nu}^{\dagger}W^{\nu})z_{\mu} \\
&+aW_{\mu}^{\dagger}W_{\nu}(\partial^{\mu}Z^{\nu}-\partial^{\nu}Z^{\mu}) \\
&-b(W_{\mu}^{\dagger}\partial^{\mu}W_{\nu}-\partial^{\mu}W_{\nu}^{\dagger}W_{\mu})Z^{\nu} \\
&+icZ_{\mu}Z^{\mu}W_{\nu}^{\dagger}W^{\nu}-idZ^{\mu}Z^{\nu}W_{\mu}^{\dagger}W_{\nu}]\ , \qquad (3)
\end{aligned}
$$

where the standard theory values of the coupling constants are $f = g\cos\theta_W$, $a = b = c = d = 1$. In calculating any higher-order graphs with Lagrangians (1) and (2), it should be borne in mind that one does not have any local gauge invariance for the weak interaction and hence one is committed to work with a unitary-gauge propagator for any internal W or Z line. Further, divergences occurring in any loop integration are to be cut off at a scale Λ, up to which (1) and (2) above remain useful effective Lagrangians. If (1) and (2) are thought of as low-energy remnants of a composite structure, Λ naturally is the composite scale, which is expected to be 1 TeV or higher.

The vertices responsible for electromagnetic decays of K mesons are the $sd\gamma$ and $sd\gamma\gamma$ ones. The various graphs contributing to them have been detailed in Ref. 10 and the one to which the anomalous term in (1) will make an extra contribution is shown in Fig. 1. The $sd\gamma$ vertex has the structure

$$\Gamma_\mu^{(\gamma)}\left[p-\frac{q}{2},p+\frac{q}{2}\right] = e(Q-1)\gamma_\mu La + i\sigma_{\mu\nu}q^\nu\frac{1}{2m}M + \gamma^\nu(q^2g_{\mu\nu}-q_\nu q_\mu)\tfrac{1}{6}\langle r^2\rangle L + O(q^3)\,. \tag{4}$$

In (4), Q is the charge of s and d, $L = \frac{1}{2}(1-\gamma_5)$, m is the mass of s and d (constituent) quarks assumed equal, and M and $\langle r^2\rangle$, respectively, are the transition moment and charge radius. The contribution of the K and λ terms to M, denoted, respectively, by M_K and M_λ via Fig. 1 works out to be

$$M_\kappa = -eQ\frac{G_F}{\sqrt{2}}\frac{12m^2}{\pi^2}\left[\frac{m_C{}^2-m_u{}^2}{M_W{}^2}\ln\frac{M_W{}^2}{m_{C^2}}\right] \times \cos\theta_C\sin\theta_C(\kappa-1)\,, \tag{5a}$$

$$M_\lambda = -e\frac{G_F}{\sqrt{2}}m^2\frac{\cos\theta_C\sin\theta_C}{2\pi^2}\left[\frac{m_C{}^2-m_u{}^2}{M_W{}^2}\right]\lambda\,. \tag{5b}$$

In Eq. (5) we have put the masses of the s and d quarks both equal to m. $M_\kappa + M_\lambda$ is comparable to the standard-model contribution to M, denoted by $M_{\rm SM}$:

$$M_{\rm SM} = -eQ\frac{G_F}{\sqrt{2}}\frac{m^2}{2\pi^2}\left[\frac{m_C{}^2-m_u{}^2}{M_{W^2}}\ln\frac{M_{W^2}}{m_{C^2}}\right]\cos\theta_C\sin\theta_C\,. \tag{6}$$

But, as has been pointed out by Gaillard and Lee,[10] for the decay $k\to\pi ee$, the dominant contribution comes not from the magnetic-moment term in (1) but the charge-radius one. For the charge radius the standard model contribution is dominated by the diagram in which the photon is hooked on to the fermion. The approximation $m_d = m_s$ therefore is in a term which is small anyway. The dominant contribution to charge radius is, up to a factor of $O(1)$,

$$\tfrac{1}{6}\langle r^2\rangle \cong -\frac{eQ}{6\pi^2}\frac{G_F}{\sqrt{2}}\cos\theta_C\sin\theta_C\,. \tag{7}$$

Diagrams in which the photon is hooked onto a W propagator are down by a factor $M_W{}^{-2}$ relative to (4). This continues to be true even with the κ and the λ terms in the coupling, Eq. (1). Thus, the induced $sd\gamma$ vertex turns out to be insensitive to the presence or otherwise of the anomalous moment coupling for values of photon momenta relevant to K decays.

For the $sd\gamma\gamma$ vertex, similar considerations once again make the κ term insignificant against the dominant contribution coming from the diagram in which the photons hook onto a fermion line.

The Feynman diagrams contributing to $K\to\mu^+\mu^-$ decay have been detailed in Ref. 10. The modified vertex now renders the graph shown in Fig. 2 quadratically divergent and hence will provide a constraint on the cou-

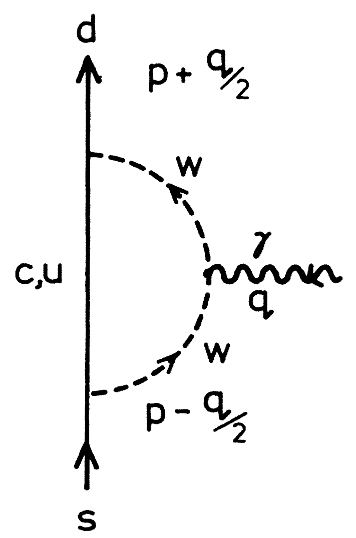

FIG. 1. Induced $sd\gamma$ vertex. The $WW\gamma$ vertex represented by a blob is given by Eq. (1).

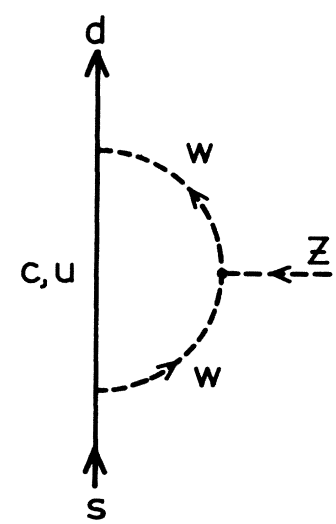

FIG. 2. Induced sdZ vertex considered in the text. The WWZ vertex represented by a blob is given by Eq. (2).

pling constants since it is known experimentally that the amplitude for the process $K \to \mu^+\mu^-$ is bounded approximately by a factor α^2 relative to the dominant mode of K^+ decay, $K^+ \to \mu\nu$. Straightforward calculation gives us for the ratio of S-matrix element for the process $K^+ \to \mu^+\mu^-$ via Fig. 2 and the vertex to the standard $K^+ \to \mu\nu$ amplitude, the expression

$$\frac{{}^{(iS)}K \to \mu^+\mu^-}{{}^{(iS)}K^+ \to \mu\nu} = \frac{\alpha}{2\pi} \frac{m_c{}^2 - m_u{}^2}{M_W{}^2 \sin 2\theta_W} \left[\frac{\Lambda^2}{16 M_Z{}^2} \right] \frac{f(1-b)}{g \cos\theta_W} . \tag{8}$$

Demanding that this ratio be less than α^2 gives us the numerical restriction

$$| f(b-1) | \lesssim 3.5 \left[\frac{M_W{}^2}{\Lambda^2} \right] . \tag{9}$$

However, on the basis of unitarity arguments Suzuki has obtained a more powerful restriction

$$| f(b-1) | \lesssim 1.5 \left[\frac{M_W{}^3}{\Lambda^3} \right] , \tag{10}$$

which effectively makes the former inequality redundant.

We conclude that once the GIM mechanism is incorporated and the unitarity bound of Suzuki taken into consideration, the rare K-meson decay experimental results do not place any restrictions on the various coupling parameters in the general vertex Eqs. (1) and (2).

Finally, in Glashow-Salam-Weinberg theories with the extended Higgs-boson sector one finds that the extra charged-Higgs-boson exchange at the one-loop level significantly enhances the flavor-changing neutral currents for *heavy* quarks.[18] For light quarks, however, this effect can be neglected.

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