Flux tube or bremsstrahlung?

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The classical problem of radiation emitted by an accelerated charge is considered. We use the analogy to suggest that the space-time development of the bremsstrahlung which accompanies the color exchange in hadron-hadron inelastic scattering is very similar to that of the color flux-tube model. The bremsstrahlung yields a considerable contribution to the effective string tension. It is argued that one should distinguish between the string tension measured in static and dynamic processes.

I. INTRODUCTION

Two basic features of quantum chromodynamics are asymptotic freedom at short distances and color confinement at larger distances. Describing hadronic processes at high energies, one has to make an artificial cut between the first stage of evolution governed by perturbation theory, and the second, nonperturbative stage with soft momentum transfer. However, this cut varies very much in different models describing the evolution of hadronic jets. As an example, in the string fragmentation models[1] only the few initial partons are produced perturbatively, their further evolution is described phenomenologically as fragmentation of the color string. The underlying idea is that the additional quarks are produced by the chromoelectric field in the flux tube stretched between the original quarks.[2]

An important parameter of the color flux-tube (or string) models is the string tension. It can be measured by low-energy phenomena as the value of the Regge slopes or the form of the heavy-quark potential, yielding a value of $\kappa \approx 1$ GeV/fm. At the same time in the high-energy hadron-nucleus interactions the energy and A dependence of different processes is sensitive to the value of κ (Ref. 3). This is because the string tension causes the incoming partons to decelerate after the first color exchange inside the nucleus, and the secondary reactions of these partons take place at lower momenta. The string tension extracted in this way from different high-energy h-A reactions is somewhat larger: $\kappa \approx 3$ GeV/fm.

In contrast with the string fragmentation models, in the Webber-Marchesini model[4] additional partons are produced by gluon bremsstrahlung. Nonperturbative effects are involved only when the virtualities of color partons decrease to $Q^2 \sim 1$ GeV^2. The success of this model indicates that gluon bremsstrahlung plays an important role in producing the hadronic final state.

The analogy between the multiparticle production and electromagnetic bremsstrahlung has been noticed a long time ago.[5] The radiation emitted by an accelerated charge has a specific space-time evolution as can be observed in the Landau-Pomeranchuk effect.[6] When a charge undergoes multiple scattering in a medium the resulting radiation is not the sum of intensities from individual scatterings. If the scattering centers are closer than the "formation length" of the radiation, $l_f \approx 1/\omega(1-\cos\theta)$, then the contribution from intermediate scatterings cancel out, the radiation is determined only by the initial and final velocities of the charge. Following Feinberg we may say that the electron "shakes off" its Coulomb field in the first scattering, and until this field is regenerated, subsequent scatterings of the "half dressed" electron occur without radiation. As discussed extensively by Nikolaev,[7] the notion of "hadronic formation length" is important in understanding the interactions of high-energy particles with nuclei. The formation length in classical electrodynamics is also discussed in Ref. 8.

In this paper we would like to point out—by using a simple and physically transparent formalism—that the analogy between bremsstrahlung and multiparticle production is even closer than usually thought.

We will show that the space-time evolution of radiation is similar to that of the color flux-tube model. The momentum loss during the bremsstrahlung can be described by an effective string tension. Curiously, using α_s instead of α_{em}, and the characteristic hadron size in the corresponding expressions one gets an effective string tension of several GeV/fm. The mass density of the hadronic plateau and the formation time of hadrons are also in rough agreement with the observed values.

II. BREMSSTRAHLUNG IN ELECTRODYNAMICS

The bremsstrahlung is a purely classical phenomenon—it is a consequence of local gauge invariance and Lorentz invariance. The charged particle is surrounded by a stationary gauge field given by the Gauss law. When the charge is accelerated, its field cannot follow it instantaneously—their relative configuration is distorted. This nonstationary configuration radiates out its energy excess until the new stationary field configuration is reached.

The current of a charged particle moving on the world line $x^\mu = x^\mu(\tau)$ is

$$j^\mu(x') = e \int d\tau \frac{dx^\mu}{d\tau} \delta(x' - x(\tau)) . \quad (1)$$

It has a Fourier transform:

$$\tilde{j}^\mu(k) = e \int d\tau \frac{dx^\mu}{d\tau} e^{ikx(\tau)} . \quad (2)$$

The energy density carried by the radiation is[9]

$$dE=\frac{d^3k}{2(2\pi)^3}[-\tilde{j}^{\mu}(k)\tilde{j}_{\mu}(k)],\quad k^0=|\mathbf{k}|\ . \tag{3}$$

For simplicity we shall consider one-dimensional accelerations along the $x\equiv x^1$ axis. Instead of k^{μ} it is useful to introduce the rapidity of the photon as a new variable

$$k^{\mu}=(k_T\cosh y,k_T\sinh y,\mathbf{k}_T)\ . \tag{4}$$

The rapidity y is connected with the angle θ between the momentum vector $\mathbf{k}$ and the x axis:

$$\tan\theta=\frac{1}{\sinh y}\ .$$

First we consider some simple examples.

(a) Instantaneous acceleration:

$$x^{\mu}=\begin{cases}v_1{}^{\mu}\tau & \text{for } \tau<0\ ,\\ v_2{}^{\mu}\tau & \text{for } \tau>0\ .\end{cases}$$

In this case we obtain

$$\tilde{j}^{\mu}(k)=-ie\left[\frac{v_1{}^{\mu}}{v_1k}-\frac{v_2{}^{\mu}}{v_2k}\right]$$

(coinciding, of course, with the soft-photon emission amplitude in QED). Introducing the rapidity variable of the moving charge

$$v^{\mu}=(\cosh\eta,\sinh\eta,0,0)\ ,$$

one obtains

$$\epsilon(y,k_T)\equiv\frac{dE}{d^2k_T dy\cosh y}$$
$$=\frac{k_T}{2(2\pi)^3}\left[\frac{e^2}{k_T{}^2}[\tanh(\eta_2-y)-\tanh(\eta_1-y)]^2\right]\ . \tag{5}$$

$\epsilon(y,k_T)$ has the meaning of the rest mass density of the portion of radiation with given y and k_T. For fixed k_T it has a plateau stretched between the initial and final rapidities of the accelerated charge, and exponentially vanishes outside (see Fig. 1). (This plateau is nothing else than the well-known radiation cone in θ.)

The total "rest mass" of the radiation at a given y,

$$\epsilon(y)\equiv\int\epsilon(y,k_T)d^2k_T\ , \tag{6}$$

is linearly divergent for the case of instantaneous acceleration. This is due to the fact that the acceleration is infinite at $\tau=0$.

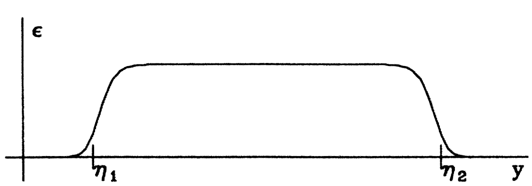

FIG. 1. The density of radiation ϵ at fixed k_T vs the photon's rapidity y.

(b) Finite proper acceleration:

$$x^{\mu}(\tau)=\frac{1}{a_0}(\sinh a_0\tau,\cosh a_0\tau,0,0),\quad -\infty<\tau<+\infty\ .$$

This corresponds to a motion with proper acceleration a_0, in an infinite rapidity interval, $\eta_1=-\infty$, $\eta_2=+\infty$. The exponent in (2) is

$$kx(\tau)=\frac{k_T}{a_0}\sinh(a_0\tau-y)\ .$$

It is convenient to boost the system of reference to K_y moving with rapidity y of the portion of radiation ("photon") considered. In K_y the given photon has no longitudinal momentum.

The current (2) has a simple form in this system:

$$\tilde{j}^0(k)=e\int d\tau\cosh(a_0\tau-y)\exp\left[i\frac{k_T}{a_0}\sinh(a_0\tau-y)\right]$$
$$=0\ ,$$
$$\tilde{j}^1(k)=e\int d\tau\sinh(a_0\tau-y)\exp\left[i\frac{k_T}{a_0}\sinh(a_0\tau-y)\right]$$
$$=i\frac{2e}{k_T}uK(u)\ ,$$

where $u=k_T/a_0$ and $K(u)$ is the modified Bessel function. It has an asymptotic behavior

$$[uK(u)]^2\approx\begin{cases}1 & \text{for } u\ll 1\ ,\\ u\pi e^{-2u}/2 & \text{for } u\gg 1\ .\end{cases}$$

That is, for $k_T\ll a_0$ the distribution $\epsilon(y,k_T)$ is given by (5), while it is cut off above $k_T\approx a_0$. The rest-mass distribution (6) is finite in this case:

$$\epsilon(y)=\frac{3e^2}{64}|a_0|\ . \tag{7}$$

[This can be easily calculated by Eq. (10) derived below.]

On concrete examples one can also show that the cutoff in k_T at a given y is in fact determined by the proper acceleration $a_0(\eta)$ of the charge when its rapidity approximately equals the photon's rapidity: $\eta\approx y$.

We turn now to the generic case where all these features can be easily understood.

(c) General case of one-dimensional acceleration: We have

$$kx(\tau)=k_T[x^0(\tau)\cosh y-x^1(\tau)\sinh y]\ .$$

Passing again to the system of reference K_y, the new coordinates are

$$t_y=x^0(\tau)\cosh y-x^1(\tau)\sinh y\ ,$$
$$x_y=-x^0(\tau)\sinh y+x^1(\tau)\cosh y\ .$$

Now we have $\tilde{j}^0=0$ and

$$\tilde{j}^1=e\int dt_y\frac{dx_y}{dt_y}e^{i\omega_y t_y}\ , \tag{8}$$

with $\omega_y \equiv k_T$ being the photon's frequency in K_y. By partial integration we obtain a more useful expression:

$$\tilde{j}^1 = \frac{ie}{\omega_y} \int dt_y a_y(t_y) e^{i\omega_y t_y} \equiv \frac{ie}{\omega_y} \tilde{a}_y(\omega_y) , \tag{9}$$

where

$$a_y(t_y) \equiv \frac{d^2 x_y}{dt_y{}^2}$$

is the acceleration measured in K_y.

From (2) and (3) we obtain

$$\epsilon(y) = \frac{e^2}{8\pi} \int_{-\infty}^{+\infty} dt_y [a_y(t_y)]^2 . \tag{10}$$

To see the meaning of expressions (9) and (10) let us consider a charge accelerated from $v_1 = \tanh\eta_1$ to $v_2 = \tanh\eta_2$. In K_y it has a velocity

$$v_y(t) = \tanh[\eta(t) - y] \tag{11}$$

with an acceleration

$$a_y = a_0 (1 - v_y{}^2)^{3/2} , \tag{12}$$

a_0 being the proper acceleration at given t. Consider values of photon rapidity: $\eta_1 < y < \eta_2$. The velocity v_y and the acceleration a_y vs $t \equiv t_y$ are shown in Fig. 2. Obviously, for finite a_0 the acceleration $a_y(t)$ is concentrated at those values of t where $v_y{}^2$ is essentially different from one:

$$| \eta(t) - y | \sim 1 ; \tag{13}$$

i.e., the radiation at given y is sensitive only to a limited part of the particle's trajectory, where its rapidity is near y. The width of this region is given by the proper acceleration

$$\Delta t_y \approx \frac{1}{a_0} . \tag{14}$$

For small transverse momenta,

$$\omega_y \equiv k_T \ll \frac{1}{\Delta t_y} \approx a_0 \tag{15}$$

one obtains, from (9),

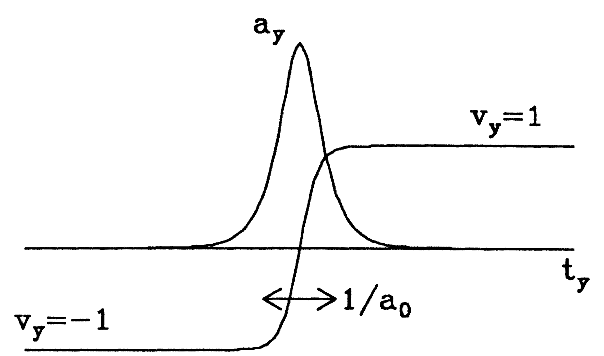

FIG. 2. Time dependence of velocity and acceleration of a charge as seen in the system of reference K_y moving with the photon's rapidity.

$$\begin{aligned}\tilde{j}^1 &\approx \frac{ie}{k_T} \int a_y(t_y) dt_y \\ &= \frac{ie}{k_T}(v_{2y} - v_{1y}) \\ &= \frac{ie}{k_T}[\tanh(\eta_2 - y) - \tanh(\eta_1 - y)]\end{aligned} \tag{16}$$

[cf. (5)]. It is also clear from Eq. (9) that the cutoff in k_T is given by the proper acceleration of that part of the trajectory when $\eta \approx y$:

$$k_T \le \frac{1}{\Delta t_y} \approx a_0 .$$

These results are interpreted in the following way. The charge moving with a constant velocity is surrounded by its static Coulomb field. When the charge is accelerated, the field cannot follow it instantaneously. The charge loses a part of its Coulomb field corresponding to $k_T < a_0$, i.e., outside some region with characteristic radius $r \sim 1/a_0$. The time scale of the acceleration is given by $\Delta t_y \sim 1/a_0$. One can say that the fast Fourier components of the field $\omega_y \equiv k_T > a_0$ can follow the movement of the charge, while the slow components $k_T < a_0$ are separated from it and keep moving with the given rapidity $y \approx \eta$ along the x axis.

It is instructive to rewrite the "parton" picture of a fast-moving charge, the Weizsäcker-Williams approximation from conventional variables to rapidity y and transverse momentum k_T. The number of equivalent photons is given by the standard expression

$$n(k) = \frac{e^2}{4\pi^3} \frac{k_T{}^2}{\left[k_T{}^2 + \omega^2 \dfrac{M^2}{E^2} \right]^2} \frac{d^3 k}{\omega} . \tag{17}$$

Here M, E is the mass and the energy of the charged particle, correspondingly. It is assumed that $\cosh\eta_1 \equiv E/M \gg 1$. Since $\omega = k_T \cosh y$, we have

$$n(y, k_T) = \frac{e^2}{2(2\pi)^3} \frac{1}{k_T{}^2} [1 + \tanh(\eta_1 - y)]^2 \tag{18}$$

with a relative error of order $e^{-2\eta_1}$.

Let us consider a motion where $\eta_1 \gg \eta_2 \gg 1$. Equations (18) and (5) agree completely for $y > \eta_2$. Parton photons of the initial state with $y > \eta_2$ do not belong to the final-state partons—they are emitted; those belonging to the final state ($y < \eta_2$) keep moving with the charge.

Equation (5) describes instantaneous acceleration. For finite proper acceleration the above statement is not completely true. Only those partons are emitted from the interval $\eta_2 < y < \eta_1$ which oscillate slowly in the proper frame: $k_T < a_0$, as said before.

We have seen that the field inside $r \approx 1/a_0$ is undisturbed, it moves with the charge. Outside this region the field is distorted with respect to the static Coulomb field—that is why this portion is emitted. (It is instructive to see this also from the Lienard-Wiechert potentials.)

Now we turn to the important quantity called the regeneration time of the Coulomb field or the formation time of the emitted radiation. Consider the following problem. A charge is initially at rest, then it is accelerated to some velocity v. During some time interval Δt the charge moves with this velocity and then it is stopped again. It was found that the radiation depends in a characteristic way from Δt. The solution to this classical problem can be simply obtained by using Eq. (9).

Let the charge be accelerated by a large proper acceleration a_0 from the original velocity $v_1=\tanh\eta_1$ at $t=0$ to the velocity $v_2=\tanh\eta_2$, and after a time interval Δt_1 slowed down to the initial velocity v_1. (Δt_1 is measured in K_{η_1}.) The acceleration $a_y(t)$ viewed from K_y is shown in Fig. 3. The time interval between the peaks of acceleration and deceleration measured in K_y is

$$\Delta t_y=\Delta t_1\frac{\cosh(\eta_2-y)}{\cosh(\eta_2-\eta_1)}\approx\Delta t_1 e^{\eta_1-y} \tag{19}$$

for $\eta_1<y<\eta_2$. For $k_T\ll a_0$ in Eq. (9) one can neglect the change of phase during the time intervals $1/a_0$ of accelerations and we obtain

$$|\tilde{j}^1|\approx\left|\frac{e}{k_T}[\tanh(\eta_2-y)-\tanh(\eta_1-y)](1-e^{ik_T\Delta t_y})\right| . \tag{20}$$

The last factor cuts off the emission of photons with $\cosh(y-\eta_1)>k_T\Delta t_1$. This can be interpreted in the following way. The portion of radiation emitted at $y\approx\eta$ with a given frequency $\omega_y=k_T$ has a characteristic "formation time" $\tau\sim 1/\omega_y$. Measured in the system of reference K_{η_1} this becomes

$$t_f\approx\frac{1}{k_T}\cosh(y-\eta_1) . \tag{21}$$

Naturally, only those photons are emitted in the process which are "formed" by time between the acceleration and deceleration, i.e., $t_f\leq\Delta t_1$. Of course, the formation time decreases with increasing k_T and at $k_T\sim a_0$ it is comparable with the time needed for acceleration—it becomes meaningless.

Since we want to apply this picture to the color exchange in hadron-hadron collisions, we notice that the cutoff $k_T\approx a_0$ was a consequence of our pointlike charges. Let us instead consider a smeared charge distribution. It suffices to smear only in transversal coordinate—to consider a charged disc with radius R perpendicular to the x axis. (The exchanged transverse momentum in usual hadronic collisions is limited by the inverse hadronic size; hence, the resulting color charges are smeared by this size in the transverse direction. We do not smear the charge in the longitudinal direction to preserve mathematical simplicity. The coupling of the electromagnetic field to a charged disc is invariant with respect to the Lorentz boosts perpendicular to the plane of the disc. This is not true for a three-dimensional rigid body.) In this case, Eq. (9) is only slightly modified—the current for the pointlike case gets multiplied by the two-dimensional Fourier transform of the charge distribution $\rho(x_T)$:

$$\tilde{j}^1(k)=\frac{i}{\omega_y}\int dt_y a_y(t_y)e^{i\omega_y t_y}\int d^2x_T\rho(x_T)e^{ik_Tx_T} . \tag{22}$$

Here one has another natural cutoff: for large accelerations $a_0\gg 1/R$ only those photons are emitted which have small transverse momenta $k_T\leq 1/R$. This is clear in the partonic picture: photons with $k_T>1/R$ are not present in the smeared charge distribution.

Let us now consider the time evolution of the system after an instantaneous ($a_0\gg 1/R$) acceleration from rapidity η_1 to η_2. The density of radiation in this case is given by (5) multiplied by a charge-distribution factor

$$\epsilon(y,k_T)=\frac{1}{2(2\pi)^3}[\tanh(\eta_2-y)-\tanh(\eta_1-y)]^2\frac{1}{k_T}|\tilde{\rho}(k_T)|^2 . \tag{23}$$

For further reference we also note that the height of the plateau (integrated over d^2k_T) for a Gaussian charge distribution in x_T is

$$\bar{\epsilon}=e^2(2\pi)^{-3/2}\langle k_T^2\rangle^{1/2} . \tag{24}$$

Because of the sudden acceleration the system becomes excited: the charge moves with rapidity η_2 while the field around it is not its static Coulomb field—instead it is the original Coulomb field of a charge moving with rapidity η_1.

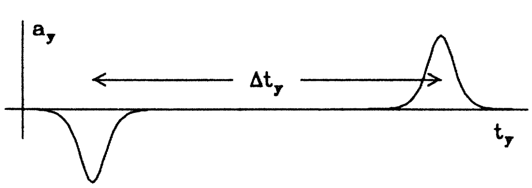

FIG. 3. Time dependence of acceleration in K_y of a charge which undergoes deceleration, flies some time with a constant velocity and then is accelerated again.

This disturbed charge-field system starts to radiate its energy excess. The amount of energy emitted during some time interval t is

$$\Delta E\approx\int d^2k_T\int_{\eta_1}^{y(k_T,t)}dy'\epsilon(y',k_T)\cosh(y'-\eta_1) , \tag{25}$$

where $y(k_T,t)$ is given by Eq. (21), and we assume that $\eta_2-\eta_1\gg 1$. The function $\epsilon(y',k_T)$ is practically independent of y' in the interval of integration [assuming $y(k_T,t)<\eta_2$]; its k_T dependence is given simply by (23). Hence we have

$$\begin{aligned}\Delta E&\approx\int d^2k_T\epsilon(k_T)\sinh[y(k_T,t)-\eta_1]\\&\approx t\int d^2k_T\epsilon(k_T)k_T=t\frac{e^2}{2\pi^2}\langle k_T^2\rangle .\end{aligned} \tag{26}$$

(In the last step we assumed a Gaussian charge distribution.)

The energy loss ΔE turned out to be proportional to the time interval t—just as in the case of a constant retarding force. The corresponding "dynamical string tension" is

$$\kappa_{\rm dyn}=\frac{e^2}{2\pi^2}\langle k_T{}^2\rangle \ . \qquad (27)$$

(Of course, it is senseless to speak about string in electrodynamics. We mean that when applied to color exchange in hadronic collisions the bremsstrahlung would result in a constant force which can be confused with the string tension.) The radiation does not continue forever—it stops when the fastest photons with $y\approx\eta_2$ are emitted after a time $t\approx\cosh(\eta_2-\eta_1)/k_T$.

The total energy radiated by such charge distribution when suddenly stopped from initial rapidity η_1 can be easily calculated from Eq. (23):

$$E_{\rm rad}=\int d^2k_T\int_{-\infty}^{\infty}dy\,\epsilon(y,k_T)\cosh y$$
$$=(\cosh\eta_1-1)\int\frac{d^2k_T}{(2\pi)^2}\,|\tilde\rho(k_T)|^2\frac{1}{4k_T} \ . \qquad (28)$$

The factor multiplying $(\cosh\eta_1-1)$ can be identified as the electrostatic energy of the charged disc at rest. (In our convention the Coulomb force is $e^2/4\pi r$.)

III. APPLICATION TO HADRONIC PROCESSES

Hadrons and their strong interactions are described by quantum chromodynamics. A peripheral interaction of hadrons is due to a soft-gluon (color) exchange followed by some mechanism of multiparticle production. In the color flux-tube (or string) model quark-antiquark pairs are created in the chromoelectric field of the tube. The energy required for multiparticle production is obtained from the incident hadron by the string tension pulling it backward. The deceleration of the incident hadron can be observed in collisions of hadrons with nuclei. However, color exchange means a sudden acceleration of color charge, and this leads necessarily to gluon bremsstrahlung by the same reasons as in electrodynamics.

In usual hadronic collisions the transverse momentum exchange is small. Hence the color charges created suddenly as a result of color exchange between two colliding hadrons are smeared in the transverse direction. It is interesting to notice that the space-time evolution of radiation emitted by a charged disc under similar motion resembles very much what we see in hadronic collisions. (A pair created with rapidities η_1, η_2 radiates the same way as a single charge accelerated suddenly from η_1 to η_2.)

Although not quite legitimate, one can play with numbers and substitute in the previous formulas $\alpha_{\rm em}=e^2/4\pi\rightarrow\alpha_s\approx1$, and $k_T\approx0.9$ GeV as reasonable values for the strong coupling constant and average transverse momentum of the primary gluons, respectively.

Using Eq. (24) we obtain for the height of the hadronic plateau (energy per unit rapidity)

$$\bar\epsilon=\alpha_s\left[\frac{2k_T{}^2}{\pi}\right]^{1/2}\approx0.7\text{ GeV} \ ,$$

in rough agreement with data (2—3 pions per unit rapidity with transverse mass ~0.3 GeV).

For the retarding force due to color bremsstrahlung ("dynamic string tension") Eq. (27) gives $\kappa_{\rm dyn}\approx2.5$ GeV/fm. The hadronic formation time deduced[7] from high-energy hadron-nucleus scattering experiments is also compatible with Eq. (21).

Although the agreement should not be taken too seriously, it shows nevertheless that color bremsstrahlung is important to understand the dynamics of multiparticle production. In particular, one should distinguish between the static string tension and the retarding force acting on the incident hadron after the color exchange. The first one is measured in static circumstances, e.g., from the slope of the Regge trajectories, charmonium spectrum, from lattice QCD. The latter could be observed in high-energy hadronic processes, e.g., in hadron-nucleus collisions.

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