Spectroscopy of Higgs scalars and exotic mesons up to 1 TeV in a composite model

T. Kobayashi*
Sektion Physik der Universität München and Max-Planck-Institut für Physik und Astrophysik, Werner-Heisenberg-Institut für Physik, D-8000 München 40, Federal Republic of Germany

S. Tokitake
Institute of Physics, University of Tsukuba, Ibaraki 305, Japan
(Received 19 May 1986)

Masses of Higgs mesons and exotic mesons are investigated in a composite model for leptons, quarks, and Higgs mesons based on the left-right-symmetric model. It is predicted that three Higgs scalars and four exotic mesons may appear in the energies lower than 1 TeV. They will be detected as monojet, dijet, single-lepton, dilepton, and lepton + jet events. We also suggest that a rich new-particle spectrum including the right-handed weak boson will appear in the energy region lower than 5 TeV.

The most crucial test of models is the observation of new particles predicted by the model. The prediction of the masses of the new particles is, however, not easy in the models beyond the standard model,[1] especially, in composite models. The difficulty arises from the ambiguity of the Higgs mechanism, that is, the ambiguity of the number of Higgs doublets, coupling constants, vacuum expectation values, and so on. In the composite model based on the left-right-symmetric model proposed in Ref. 2 we can write down the explicit form of the Higgs couplings[2,3] and predict the masses of the Higgs mesons.[3] The characteristics of the model are the existence of exotic mesons represented with the triplet (or antitriplet) representation of $SU(3)_C$. In this paper we shall study the masses of the exotic mesons as well as those of the Higgs mesons in the model proposed in Refs. 2—4, and show that some mesons have masses less than 1 TeV.

The model is written in terms of preons t^l, t^q, and S^0 with the following representations of the left-right-symmetric gauge group: $G \equiv SU(3)_H \times SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where $SU(3)_H$ and $SU(3)_C$, respectively, stand for hypercolor and color interactions:[4]

	$SU(3)_H$	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	N_{B-L}	J^P
$t^l_{L(R)}$	3	1	2(1)	1(2)	-1	$\frac{1}{2}^+$
$t^q_{L(R)}$	3	3	2(1)	1(2)	$\frac{1}{3}$	$\frac{1}{2}^+$
S^0	3	1	1	1	0	0^+

(1)

where N_{B-L} = (baryon number) − (lepton number) and $J^P = (\text{spin})^{\text{parity}}$. The preons t^l and t^q are, respectively, described by the charge doublets $(t^{l(0)}, t^{l(-1)})$ and $(t^{q(2/3)}, t^{q(-1/3)})$, where Q in $t^{a(Q)}$ denotes the electric charge of t^a. The S^0 boson is introduced in order to generate the generation of fermions. (For details, see Ref. 2.) It is noted that the assignment of $SU(3)_C$ in (1) referred from Ref. 4 is different from those given in Refs. 2 and 3. When we take account of the anomaly-matching condition,[5] the above choice is better because the model is anomaly-free and has no constraints on the number of the generations.[4] Higgs mesons generating the spontaneous parity violation[6] are described in terms of the bound states as $\Delta_L^* \equiv t_L^l t_L^l \widetilde{S}^0$, $\Delta_R^* \equiv t_R^l t_R^l \widetilde{S}^0$, $\phi^l \equiv t_L^l \overline{t_R^l}$, and $\phi^q \equiv t_L^q \overline{t_R^q}$, and also exotic scalar mesons written as $E_1 \equiv t_L^l \overline{t_R^q}$ and $E_2 \equiv t_L^q \overline{t_R^l}$ must exist in the model. They are represented by the following representation of the $SU(3)_C \times SU(2)_L \times SU(2)_R$ group:

	Δ_L	Δ_R	ϕ^l	ϕ^q	E_1	E_2
$SU(3)_C$	1	1	1	1	$\bar{3}$	3
$SU(2)_L$	3	1	2	2	2	2
$SU(2)_R$	1	3	2	2	2	2

(2)

where it is noted that ϕ^l couples only to leptons, while ϕ^q couples only to quarks.[2,3,7] The notation $\widetilde{S}^0$ is written in terms of the linear combination of the triplet representations of $SU(3)_H$ as

$$\widetilde{S}^0 = \{S^0 + (S^{0\dagger}S^{0\dagger})_3 + [(S^0S^0)_{\bar{3}}(S^0S^0)_{\bar{3}}]_3 + \cdots\}/\sqrt{N+1}\,,$$

where N stands for the number of fermion generations. (For details, see Ref. 2.) In this paper we shall not touch on scalar preons[4] with Yukawa couplings to t^l and t^q which allows us to avoid the Vafa-Witten theorem for the vectorlike composite model,[8] because they do not change anything derived in the following discussions. The effective couplings among these composite mesons are described in terms of preon-line diagrams[2,3] as

$$
\begin{aligned}
V_{\rm eff}=&(\alpha_0+\alpha_\epsilon)[\mathrm{Tr}(\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R)]^2+\alpha_1\mathrm{Tr}(\Delta_L^\dagger\Delta_L\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R\Delta_R^\dagger\Delta_R)\\
&+\beta_1\mathrm{Tr}[\Delta_L^\dagger\Delta_L(\phi^l\phi^{l\dagger}+E_1E_1^\dagger)+\Delta_R^\dagger\Delta_R(\phi^{l\dagger}\phi^l+E_2^\dagger E_2)]\\
&+\beta_2\mathrm{Tr}(\Delta_L^\dagger\phi^l\Delta_R\phi^{l\dagger}+\Delta_R^\dagger\phi^{l\dagger}\Delta_L\phi^l)+\beta_\epsilon\mathrm{Tr}(\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R)\mathrm{Tr}\left[\sum_{a=l,q}\phi^{a\dagger}\phi^a+\sum_{i=1,2}E_i^\dagger E_i\right]\\
&+\gamma\,\mathrm{Tr}\left[\sum_{a=l,q}\phi^{a\dagger}\phi^a\phi^{a\dagger}\phi^a+\sum_{i=1,2}E_i^\dagger E_iE_i^\dagger E_i+2(\phi^{l\dagger}E_1E_1^\dagger\phi^l+\phi^lE_2^\dagger E_2\phi^{l\dagger}+\phi^qE_1^\dagger E_1\phi^{q\dagger}+\phi^{q\dagger}E_2E_2^\dagger\phi^q)\right.\\
&\qquad\left.+2(\phi^lE_2^\dagger\phi^qE_1^\dagger+\phi^{l\dagger}E_1\phi^{q\dagger}E_2)\right]+\gamma_\epsilon\left[\mathrm{Tr}\left[\sum_{a=l,q}\phi^{a\dagger}\phi^a+\sum_{i=1,2}E_i^\dagger E_i\right]\right]^2\\
&-\mu^2\mathrm{Tr}(\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R)+\mathrm{Tr}\left[\sum_{a=l,q}m_{\phi a}{}^2\phi^{a\dagger}\phi^a+\sum_{i=1,2}m_1{}^2E_i^\dagger E_i\right]+V_\rho(\phi)\,,
\end{aligned}
\tag{3}
$$

where $V_\rho(\phi)$ does not contribute to the Higgs potentials but to the mass terms of the imaginary parts of the neutral Higgs mesons, and is written as

$$V_\rho(\phi)=\rho\sum_{i=1}^{4}\sum_{j=1}^{4}(\mathrm{Im\,Tr}\Phi_i^\dagger\Phi_j)^2\,. \tag{4}$$

In (3) and (4) the definitions $\Delta_A\equiv\tau\cdot\Delta_A/\sqrt{2}$, $\Phi_1\equiv(\phi^l+\tilde\phi^l)/\sqrt{2}$, $\Phi_2\equiv(\phi^l-\tilde\phi^l)/\sqrt{2}$, $\Phi_3\equiv(\phi^q+\tilde\phi^q)/\sqrt{2}$, $\Phi_4\equiv(\phi^q-\tilde\phi^q)/\sqrt{2}$, and $\tilde\phi^a\equiv\tau_2\phi^{a*}\tau_2$ are used, $\mathrm{Tr}A$ and $\mathrm{Im}X$, respectively, stand for the trace of the matrix A and the imaginary part of X and the meson matrices are represented as

$$
\begin{aligned}
\Delta_A&=\begin{bmatrix}\Delta^+/\sqrt{2} & \Delta^{++}\\ \Delta^0 & -\Delta^+/\sqrt{2}\end{bmatrix}_A\quad(A=L,R)\,,\\
\phi^a&=\begin{bmatrix}\phi_1^{a(0)} & \phi^{a(+)}\\ \phi^{a(-)} & \phi_2^{a(0)}\end{bmatrix}\quad(a=l,q)\,,\\
E_1&=\begin{bmatrix}E_{11}^{-2/3} & E_1{}^{1/3}\\ E_1{}^{-5/3} & E_{12}{}^{-2/3}\end{bmatrix}\,,\\
E_2&=\begin{bmatrix}E_{21}{}^{2/3} & E_2{}^{5/3}\\ E_2{}^{-1/3} & E_{22}{}^{2/3}\end{bmatrix}\,,
\end{aligned}
$$

where Q in E_i^Q denotes the electric charge of the meson. Since the terms with α_ϵ, β_ϵ, γ_ϵ, and ρ are introduced as the corrections,[2,3,7] we should take account of the relations $|\alpha_\epsilon|<|\alpha_0|$, $|\beta_\epsilon|<|\beta_1|$, $|\gamma_\epsilon|\sim|\rho|<|\gamma|$. In (3) it must be noted that only Δ mesons can have a negative squared mass, $-\mu^2$, via the S^0-boson condensation,[2,3] whereas the ϕ^a and E_i mesons still have the positive squared masses $m_{\phi^a}{}^2$ and $m_i{}^2$, in the model. Following the discussion on the S^0-boson condensation, we always take account of the relation $\mu^2\gg m_{\phi^a}{}^2\simeq m_i{}^2$ hereafter. The vacuum expectation values appear as[2]

$$
\begin{aligned}
V_L&\equiv\langle\Delta_L^{0*}=t_L^{l(0)}t_L^{l(0)}\tilde S^0\rangle\,,\\
V_R&\equiv\langle\Delta_R^{0*}=t_R^{l(0)}t_R^{l(0)}\tilde S^0\rangle\,,\\
a_l&\equiv\langle\phi_1^{l(0)}=t_L^{l(0)}\overline{t_R^{l(0)}}\rangle\,,\\
b_l&\equiv\langle\phi_2^{l(0)}=t_L^{l(-)}\overline{t_R^{l(-)}}\rangle\,,\\
a_q&\equiv\langle\phi_1^{q(0)}=t_L^{q(2/3)}\overline{t_R^{q(2/3)}}\rangle\,,\\
b_q&\equiv\langle\phi_2^{q(0)}=t_L^{q(-1/3)}\overline{t_R^{q(-1/3)}}\rangle\,,
\end{aligned}
\tag{5}
$$

and the Higgs potential is given by

TABLE I. Squared masses of Higgs and exotic mesons where $V_R{}^2\gg\kappa^2$, $|\beta_\epsilon|\ll\beta_1$, and $|\gamma_\epsilon|\ll\gamma$ are taken into account and $\Delta m^2=m_E{}^2-m_{\phi^q}{}^2$.

Mesons	m^2	Mesons	m^2
Δ_L^{++}, Δ_R^{++}	$2\lvert\alpha_1\rvert V_R{}^2$	$\mathrm{Im}\Delta_L^0$	$2\lvert\alpha_1\rvert V_R{}^2$
Δ_L^+	$2\lvert\alpha_1\rvert V_R{}^2$	$\mathrm{Im}\phi_1{}^{l(0)}$	$\beta_1V_R{}^2$
Mixing states of	$\beta_1V_R{}^2$	Mixing states of	$36\rho\kappa^2$
$\Delta_R{}^+$ and $\phi^{l(-)*}$	0	$\mathrm{Im}\phi_2^{l(0)}$, $\mathrm{Im}\phi_1^{q(0)}$,	$12\rho\kappa^2$
		and $\mathrm{Im}\phi_2^{q(0)}$	0
$\phi^{l(+)}$	0	$\mathrm{Im}\Delta_R^0$	0
Mixing states of	$4\gamma\kappa^2$		
$\phi^{q(+)}$ and $\phi^{q(-)*}$	0	$E_{21}{}^{2/3}$, $E_2{}^{-1/3}$	$\beta_1V_R{}^2$
$\mathrm{Re}\Delta_L^0$	$2\lvert\alpha_1\rvert V_R{}^2$	$E_u\equiv E_{11}{}^{-2/3*}$	Δm^2
$\mathrm{Re}\Delta_R^0$	$4\bar\alpha V_R{}^2$	$E_d\equiv E_1{}^{-1/3*}$	Δm^2
$\mathrm{Re}\phi_1^{l(0)}$	$\beta_1V_R{}^2$	Mixing states of	$4\gamma\kappa^2$
$\mathrm{Re}\phi_1^{l(0)}$	$4\gamma\kappa^2$	$E_1{}^{-5/3}$ and $E_2{}^{5/3*}$	Δm^2
$\mathrm{Re}\phi_1^{q(0)}$	$4\gamma\kappa^2$	Mixing states of	$4\gamma\kappa^2$
$\mathrm{Re}\phi_2^{q(0)}$	$4\gamma\kappa^2$	$E_{12}{}^{-2/3}$ and $E_{22}{}^{2/3*}$	Δm^2

$$V_H=\bar{\alpha}V^4+\alpha_1(V_L{}^4+V_R{}^4)+\bar{\beta}V^2a_l{}^2+\beta_\epsilon V^2v^2$$
$$+2\beta_2V_LV_Ra_l{}^2+\gamma(a_l{}^2+b_l{}^2+a_q{}^2+b_q{}^2)$$
$$+\gamma_\epsilon(a_l{}^2+v^2)^2-\mu^2V^2+m_{\phi^l}{}^2(a_l{}^2+b_l{}^2)$$
$$+m_{\phi^q}{}^2(a_q{}^2+b_q{}^2)\ , \tag{6}$$

where $V^2=V_L{}^2+V_R$, $v^2=b_l{}^2+a_q{}^2+b_q{}^2$, $\bar{\alpha}=\alpha_0+\alpha_\epsilon$, and $\bar{\beta}=\beta_1+\beta_\epsilon$. As was discussed in Refs. 3 and 9, the Higgs potential has the absolute minimum at the point represented with the relations, $V_R{}^2\gg b_l{}^2\simeq a_q{}^2=b_q{}^2$ and $V_L=a_l=0$, provided that the constraints $\bar{\alpha}+\alpha_1>0$, $\gamma+\gamma_\epsilon>0$, $\alpha_1<0$, $\beta_1>0$, $\beta_1>|\beta_2|$ or $-2\alpha_1\gamma>(\beta_1-|\beta_2|)^2$, and $\beta_\epsilon<0$ are satisfied. From now on we shall confine our interest in this solution, where the nonzero vacuum expectation values are given as

$$V_R{}^2=[2(\gamma+3\gamma_\epsilon)\mu^2-|\beta_\epsilon|(m_{\phi^l}{}^2+2m_{\phi^q}{}^2)]D^{-1}\ ,$$
$$b_l{}^2=\{\,|\beta_\epsilon|\gamma\mu^2=[2\tilde{\alpha}(\gamma+2\gamma_\epsilon)-\beta_\epsilon{}^2]m_{\phi^l}{}^2$$
$$+(4\tilde{\alpha}\gamma_\epsilon-\beta_\epsilon{}^2)m_{\phi^q}{}^2\}(\gamma D)^{-1}\ , \tag{7}$$
$$a_q{}^2=b_q{}^2=\{\,|\beta_\epsilon|\gamma\mu^2+(2\tilde{\alpha}\gamma_\epsilon-\beta_\epsilon{}^2/2)m_{\phi^l}{}^2$$
$$-[2\tilde{\alpha}(\gamma+\gamma_\epsilon)-\beta_\epsilon{}^2/2]m_{\phi^q}{}^2\}(\gamma D)^{-1}\ ,$$

where $D\equiv4\tilde{\alpha}(\gamma+3\gamma_\epsilon)-3\beta_\epsilon{}^2$ and $\tilde{\alpha}\equiv\alpha_0+\alpha_1+\alpha_\epsilon$. The three equations may be written as

$$V_R{}^2\simeq\mu^2/2\tilde{\alpha}\ ,$$
$$\kappa^2\equiv b_l{}^2\simeq a_q{}^2=b_q{}^2\simeq(\,|\beta_\epsilon|\mu^2-2\tilde{\alpha}m_\phi{}^2)/4\tilde{\alpha}\gamma\ . \tag{8}$$

We can estimate $\kappa^2=(100\ \text{GeV})^2$ from the relation $3\kappa^2\simeq v^2\simeq 2g^{-2}m_{W_L}{}^2$, where m_{W_L} and g are, respectively, the mass of left-handed weak boson and the coupling constant of SU(2) gauge interaction. From the equations it is estimated that

$$V_R{}^2\gtrsim m_\phi{}^2/|\beta_\epsilon|\ ,\quad m_\phi{}^2\sim2\gamma\kappa^2\ . \tag{9}$$

Since m_ϕ is of the order of the characteristic energy scale (Λ_H) of $SU(3)_H$ which may be estimated to be larger than 1 TeV, we shall study mesons with masses of the order of $|\beta_\epsilon|\kappa^2$, $|\gamma_\epsilon|\kappa^2$, and $\rho\kappa^2$, and less than those order.

Squared masses derived from (3) are listed in Table I, where the relations $V_R{}^2\gg\kappa^2$, $|\beta_\epsilon|\ll\beta_1$, and $|\gamma_\epsilon|\ll\gamma$ are taken into account. Since we do not find any reason to derive $m_1\neq m_2$ in the model, the definition $\Delta m^2\equiv m_E{}^2-m_{\phi^q}{}^2$ with $m_E\equiv m_1=m_2$ is also used. We see that two mixing states among the three imaginary parts of $\phi_2^{l(0)}$, $\phi_1^{q(0)}$, and $\phi_2^{q(0)}$ have the masses given by $6\sqrt{\rho}\kappa$ and $2\sqrt{3\rho}\kappa$, which are estimated to be $600\sqrt{\rho}$ GeV and $350\sqrt{\rho}$ GeV from $\kappa\simeq100$ GeV. For the reasonable

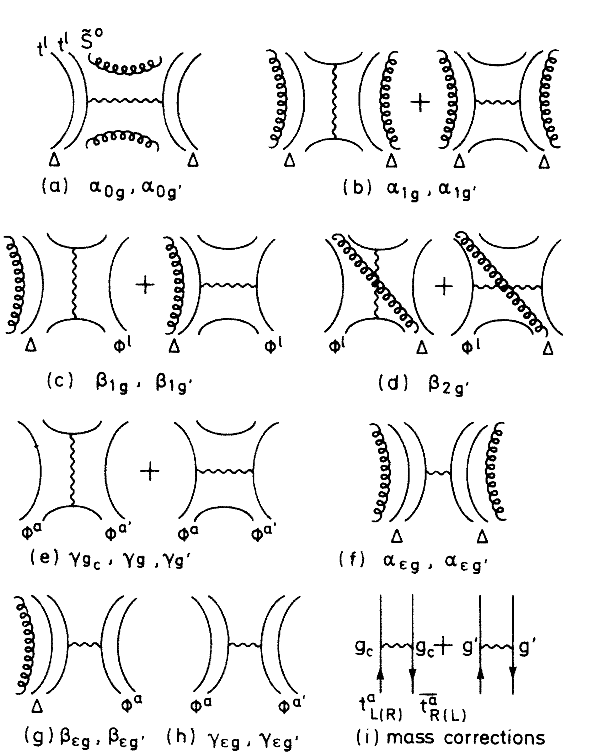

FIG. 1. Diagrams corrected in terms of the exchange of one gauge boson of $SU(3)_C$, $SU(2)_{L,R}$, or U(1), where $\Delta\equiv\Delta_L$ or Δ_R, $\phi^a=\phi^l$ or ϕ^q, and wavy lines stand for gauge bosons of $SU(3)_C$, $SU(2)_{L,R}$, and U(1).

choice of $\rho\lesssim1$ we obtain $6\sqrt{\rho}\kappa\lesssim600$ GeV and $2\sqrt{3\rho}\kappa\lesssim350$ GeV.

Squared masses of the four exotic states, $E_\mu\equiv E_{11}{}^{-2/3*}$, $E_d\equiv E_1{}^{1/3*}$, one of the mixing states of $E_1{}^{-5/3}$ and $E_2{}^{5/3*}$ and that of $E_1{}^{-2/3}$ and $E_{22}{}^{2/3*}$, are given by Δm^2. The mass difference between the exotic and ϕ^q mesons arise only from the interactions of $SU(3)_C\times SU(2)_R\times SU(2)_L\times U(1)_{B-L}$. Taking account of such mass differences, we also have to take account of the corrections of the effective potentials in terms of the same interactions at the same time. We may estimate such corrections by the one-gauge-boson-exchange diagrams shown in Fig. 1, where g_C, g, and g', respectively, stand for the gauge couplings of $SU(3)_C$, SU(2), and U(1), and the coupling constants for the correction diagrams are represented with α_{0g}, $\alpha_{0g'}$ and so on.

The contributions of the correction diagrams to the Higgs potential are given by

$$V_H^C=4\alpha_{0g'}V^4+2(2\alpha_{0g}+\alpha_{1g}+\alpha_{1g'}+2\alpha_{\epsilon g}+2\alpha_{\epsilon g'})(V_L{}^4+V_R{}^4)+\beta_{1g}V^2(a_l{}^2+2b_l{}^2)$$
$$+4\beta_{2g'}V_LV_Ra_l{}^2+2\beta_{\epsilon g}V^2(a_l{}^2-b_l{}^2+a_q{}^2-b_q{}^2)+2\gamma_g(a_l{}^4+b_l{}^4+4a_l{}^2b_l{}^2+a_q{}^4+b_q{}^4+4a_q{}^2b_q{}^2)$$
$$+2\gamma_{g'}(a_l{}^4+b_l{}^4)+2(\tfrac{4}{3}\gamma_{g_C}+\tfrac{1}{9}\gamma_{g'})(a_q{}^4+b_q{}^4)+2\gamma_{\epsilon g}(a_l{}^2-b_l{}^2+a_q{}^2-b_q{}^2)^2\ . \tag{10}$$

For the minimum with $V_L=a_l=0$ we can derive

$$m_{\phi^q}{}^2-m_{\phi^l}{}^2\simeq 2\beta_{1g}V_R{}^2+\gamma(2b_l{}^2-a_q{}^2-b_q{}^2)\;,$$
$$b_q{}^2-a_q{}^2\simeq 2(\beta_{\epsilon g}V_R{}^2-2\gamma_{\epsilon g}b_l{}^2)/\gamma\;, \tag{11}$$

where the relations $\gamma \gg |\gamma_\epsilon|$, $|\gamma_{g_C}|$, $|\gamma_g|$, and $|\gamma_{g'}|$ are taken into account.

The important effect of the corrections appears in the masses of the charged Higgs mesons, Δ_R^+, $\phi^{l(+)}$, $\phi^{l(-)*}$, $\phi^{q(+)}$, and $\phi^{q(-)*}$. After the introduction of the corrections the five mesons are mixed and rearranged into the new states having the masses $m^2\simeq\beta_1 V_R{}^2$, $4\gamma\kappa^2$, $6\gamma_{\epsilon g}(b_q{}^2-a_q{}^2)$ and the two massless bosons. The two massless bosons are absorbed into the left- and right-handed weak bosons as well as two massless bosons of the imaginary parts of the neutral Higgs mesons. Since two heavy mass states with $\beta_1 V_R{}^2$ and $4\gamma\kappa^2$ are not interesting at the moment, let us estimate a rather light mass given by, $m^2\simeq 6\gamma_{\epsilon g}(b_q{}^2-a_q{}^2)$. From (11) it is estimated as

$$6\gamma_{\epsilon g}(b_q{}^2-a_q{}^2)\simeq 12\gamma_{\epsilon g}(\beta_{\epsilon g}V_R{}^2-2\gamma_{\epsilon g}\kappa^2)/\gamma$$
$$\sim 24g^4\gamma_\epsilon(C-\gamma_\epsilon/\gamma)\kappa^2\;, \tag{12}$$

where $\gamma_{\epsilon g}\sim g^2\gamma_\epsilon$, $\beta_{\epsilon g}\sim g^2\beta_\epsilon$, and $\beta_\epsilon V_R{}^2=2\gamma C\kappa^2$ are used. From the relation given in (9) the choice $C<10$ may be reasonable. Taking account also of $\gamma_\epsilon \lesssim 1$ and $\kappa\simeq 100$ GeV, we obtain

$$m\simeq[6\gamma_{\epsilon g}(b_q{}^2-a_q{}^2)]^{1/2}<500\text{ GeV}\;. \tag{13}$$

In the critical estimation for $C\simeq 1$ we have $m\lesssim 150$ GeV.

Main corrections in the exotic meson masses appear in the masses of $E_2{}^{5/3}$ and $E_{22}{}^{2/3}$. The correction $2\beta_{1g}V_R{}^2$ is added to their squared masses. From the first relation of (11) its order is estimated to be that of the mass difference between ϕ^l and ϕ^q, i.e., $2\beta_{1g}V_R{}^2\sim m_{\phi^q}{}^2-m_{\phi^l}{}^2$. The mass difference can be described in terms of one-gauge-boson-exchange contributions of the $SU(3)_C$ and U(1) interactions; that is, the attractive force induced by one-U(1)-gauge-boson exchange acts on both states, while that induced from $SU(3)_C$ acts only on the ϕ^q state. Then it is estimated as

$$m_{\phi^q}-m_{\phi^l}\simeq\frac{1}{4\pi}\left(-\tfrac{4}{3}g_C{}^2+\tfrac{8}{9}g'^2\right)\left\langle\frac{1}{r}\right\rangle\;. \tag{14}$$

where $\langle 1/r\rangle$ stands for the expectation value of the inverse of the relative distance between t^a and $\bar{t}^a$ which may be taken as $\langle 1/r\rangle\sim\Lambda_H\sim m_\phi$. We derive

$$\beta_{1g}V_R{}^2\sim m_\phi(m_{\phi^q}-m_{\phi^l})\simeq\frac{1}{6\pi}\left(-2g_C{}^2+\tfrac{4}{3}g'^2\right)m_\phi{}^2\;. \tag{15}$$

Then we may estimate

$$|2\beta_{1g}V_R{}^2|\ll m_\phi{}^2\sim 2\gamma\kappa^2\;. \tag{16}$$

In the same consideration the mass difference $\Delta m^2=m_E{}^2-m_\phi{}^2$ is written as

$$\Delta m^2\simeq\frac{1}{2\pi}\left(\tfrac{4}{3}g_C{}^2+\tfrac{4}{9}g'^2\right)m_\phi{}^2\ll m_\phi{}^2\sim 2\gamma\kappa^2\;. \tag{17}$$

Now we may say that the mixing states of $E_1{}^{-5/3}$ and $E_2{}^{5/3*}$ and those of $E_1{}^{2/3}$ and $E_{22}{}^{2/3*}$ are not much affected by the corrections. Considering that $\gamma_g\kappa^2\sim\gamma_{g'}\kappa^2<\beta_{1g}V_R{}^2\sim\Delta m^2$, we also see that the corrections to the E_u and E_d masses are small. Then we still find four exotic states having the masses of the order of the mass difference Δm^2 after the introduction of the corrections. Provided that the next sublevel appears at an energy less than 10 TeV, these four exotic states may be observed in the energies less than 1 TeV. We may say that the observation of these exotic states like the bound states of leptons and quarks is the most crucial test of the model.

Here let us summarize our results for the light new mesons:

Mesons	m^2	Main decay modes
Two imaginary parts	$\lesssim 600$ GeV	$q_u+\bar{q}_u$, $q_d+\bar{q}_d$ (dijet)
of the neutral Higgs	$\lesssim 350$ GeV	and l^-+l^+ (dilepton)
One charged	<500 GeV	$q_u+\bar{q}_d$, $q_d+\bar{q}_u$ (dijet)
Higgs boson	($\lesssim 150$ GeV)	$l^-+\bar{\nu}$, $\nu+l^+$ (single lepton)
$E_u=E_{11}{}^{-2/3*}$	$\lesssim 1$ TeV	$q_u+\bar{\nu}$ (monojet)
$E_d=E_1{}^{1/3*}$	$\lesssim 1$ TeV	$q_d+\bar{\nu}$ (monojet)
Mixing states of $(E_1{}^{-5/3},\ E_2{}^{5/3*})$	$\lesssim 1$ TeV	$l^-+\bar{q}_u$ (lepton + jet)
and $(E_{12}{}^{-2/3},\ E_{22}{}^{2/3*})$	$\lesssim 1$ TeV	$l^-+\bar{q}_d$ (lepton + jet)

where $l=(e,\mu,\tau,\ldots)$, $q_u=(u,c,t,\ldots)$, and $q_d=(d,s,b,\ldots)$, and q_u and q_d can be jets. We may say that these seven new particles will be detected as dijet, dilepton, single-lepton, monojet, or lepton + jet events. They can also be sources of new generations if the new generations exist in sufficiently low energies.

Here we estimate the order of m_ϕ. If there is no accidental cancellation in the equation $|\beta_\epsilon|\mu^2-2\tilde{\alpha}m_\phi{}^2$ of (8), we can put $m_\phi{}^2\simeq 2\gamma c\kappa^2$ with $c<10$. Requiring that the correction of the γ coupling in terms of the one-loop diagram estimated as $\sim\gamma^2/8\pi^2\sim 10^{-2}\gamma^2$ does not exceed the original coupling γ, we should take $\gamma\lesssim 10^2$. These choices of c and γ give us

$$m_\phi < 50\kappa \simeq 5 \text{ TeV} . \tag{18}$$

For the choice $|\beta_\epsilon| > 10^{-1}$ we also derive the upper bound for the right-handed weak boson as

$$m_{W_R} \simeq g^2 m_\phi / \sqrt{|\beta_\epsilon|} \lesssim m_\phi < 5 \text{ TeV} . \tag{19}$$

These equations mean that a very rich new-particle spectrum may be expected below 5 TeV. If this is not the case, the model must give a meaning to the accidental cancellation.

Finally we would like to comment on quark masses. As was shown in (7) and (11), we have $a_q{}^2 \simeq b_q{}^2 \gg (b_q{}^2 - a_q{}^2)$ in this model. This relation, however, does not mean $m_{q_u} = m_{q_d}$, because there exist loop corrections which distinguish between the coupling constant of q_u with $\phi_1^{q(0)}$ and that of q_d with $\phi_2^{q(0)}$ (Ref. 10). Such corrections for the Higgs couplings are absorbed into the diagrams with α_ϵ, β_ϵ, and γ_ϵ.

One of us (T.K.) would like to express his gratitude to Professor H. Fritzsch and Professor W. Zimmermann for their kind hospitality in Munich. The other (S.T.) would like to thank Fūju-kai for financial support. One of us (T.K.) was supported in part by Deutsche Forschungsgemeinschaft Contract No. Fr. 412/6-2.

*On leave from the Institute of Physics, University of Tsukuba, Ibaraki 305, Japan.

[1]S. L. Glashow, Nucl. Phys. **22**, 57 (1961); S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (Nobel Symposium No. 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[2]T. Kobayashi, Phys. Rev. D **31**, 2340 (1985); **32**, 1222 (1985).

[3]T. Kobayashi and S. Tokitake, Phys. Rev. D **33**, 2017 (1986).

[4]T. Kobayashi (unpublished).

[5]G. 't Hooft, in *Quarks and Leptons,* proceedings of the Cargese Summer Institute, Cargese, 1979, edited by J.-L. Basdevant *et al.* (NATO Advanced Study Institute Series, Series B: Physics, Vol. 61) (Plenum, New York, 1980).

[6]R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980); Phys. Rev. D **23**, 165 (1981).

[7]T. Kobayashi, Lett. Nuovo Cimento **44**, 417 (1985).

[8]C. Vafa and E. Witten, Nucl. Phys. **B234**, 173 (1984).

[9]T. Kobayashi, Phys. Lett. **167B**, 79 (1986).

[10]D. Chang, R. N. Mohapatra, P. B. Pal, and J. C. Pati, Phys. Rev. Lett. **55**, 2756 (1985); see also Ref. 3.