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Semiclassical projection of hedgehog models with quarks

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A simple semiclassical method is presented for calculating physical observables in states with good angular momentum and isospin for models whose mean-field solutions are hedgehogs. The method is applicable for theories which have both quark and meson degrees of freedom. The basic approach is to find slowly rotating solutions to the time-dependent mean-field equations. A nontrivial set of differential equations must be solved to find the quark configuration for these rotating hedgehogs. The parameters which specify the rotating solutions are treated as the collective degrees of freedom. They are requantized by imposing a set of commutation relations which ensures the correct algebra for the SU(2)$\times$SU(2) group of angular momentum and isospin. Collective wave functions can then be found and with these wave functions all matrix elements can be calculated. The method is applied to a simple version of the chiral quark-meson model. A number of physical quantities such as magnetic moments, charge distributions, g_A, $g_{\pi NN}$, N-Δ mass splitting, properties of the N-Δ transition, etc., are calculated.

I. INTRODUCTION

During the past several years there has been intense interest in various chiral models of the structure of baryons. These models range from the Skyrmion,[1-4] in which quark degrees of freedom are completely eliminated in favor of effective meson degrees of freedom, to models such as the chiral (or hybrid) bag[5-10] and the chiral quark-meson model,[11-14] which include both quarks and mesons. These models all share an important feature that the mean-field soliton solutions are hedgehogs—they have a specified correlation between angular momentum and isospin. These solitons are not eigenstates of either J^2 or I^2 and represent a superposition of N and Δ states. In order to study the properties of baryons with good quantum numbers, it is necessary to project the hedgehog.

For fixed-particle-number quantum mechanics there is a well-established method, due to Peierls and Yoccoz,[15,16] for projecting from a mean-field state. In principle, this method can be extended in a straightforward fashion to field-theoretic problems with hedgehog symmetry. In practice, this leads to a formidable quantum calculation. At present, such calculations have only been undertaken with the use of approximate methods such as the use of a coherent-state formalism with a plane-wave basis.[17,18]

An alternative projection method, based on semiclassical physics, was developed by Adkins, Nappi, and Witten[4] (ANW), and applied to the Skyrme model. This method involves the use of collective coordinates which parametrize rotations. These collective rotations are allowed to become time dependent and are eventually quantized to yield states with the appropriate quantum numbers. The method is consistent as long as the motion of the collective variables is adiabatic compared to the motion of the intrinsic variables. This semiclassical projection scheme has the virtue of simplicity and is exact in the large-N_c limit.

It is appealing to use the semiclassical projection method for models with quarks. Some authors have attempted to use the ANW approach with quarks by collectively rotating the hedgehog quark spinor together with the hedgehog meson fields.[10,19] In these calculations, it was implicitly assumed that the quark spinor, as viewed from the rotating frame, is identical to the hedgehog spinor of the static mean-field solution. Unfortunately, this is not consistent and yields the spurious results that quarks make no contribution to the angular momentum (and, hence, no contribution to the moment of inertia) (Ref. 10) and that the isoscalar magnetic moment is zero (Ref. 19). The problem is that the quark equations are first order in time while the meson equations are second order. Thus, as we shall see, the time-dependent collective rotations perturb the mesons only to the second order in the rotation frequency and, in the adiabatic approximation, these effects can be dropped. In contrast, the quark spinors are perturbed to first order. To treat the quarks consistently, it is necessary to calculate the changes in the spinors due to the time-dependent collective rotation. Such a treatment is analogous to cranking calculations which have long been used in the study of deformed nuclei. A first step towards a semiclassical projection technique suitable for quark models was a cranking calculation of the N-Δ mass splitting.[20] In this paper we will generalize this technique and show how it can be used to calculate the other static properties of N's and Δ's, such as the isovector charge radius, the magnetic moments, g_A, and the pion-nucleon coupling constant, etc.

In order to facilitate the inclusion of quarks, we use a formalism somewhat different from ANW. The basic difference is the class of time-dependent rotations considered. ANW allow arbitrary time dependence, while we consider only those rotations which solve (to first order in the frequency) the time-dependent mean-field (classical) equations. Both methods, when suitably quantized, yield identical results for the Skyrme model. However, the treatment of quarks is greatly simplified with our approach. We will attempt to cast our results in a form which parallels ANW as closely as possible.

As our work was nearing completion, we became aware of the work of Ohta and Seki.[21] They treat the same problem in a manner very similar to ours. The fundamental difference in the two approaches is the treatment of quarks. We explicitly solve differential equations for the first-order changes in the quark spinors due to the collective time-dependent rotations. In contrast, Ohta and Seki estimate these changes by invoking what they refer to as the "closure approximation." Unfortunately, this approximation is *ad hoc*. Its validity can be assessed by comparing with the explicit solutions of the differential equations. For the model which we have studied the approximation appears to be rather poor. Kahana and Jackson have also done cranking calculations for the chiral bag model.[22]

For concreteness, we will study a particular simple model, the chiral quark-meson model of Ref. 11. The model is based on the linear σ model[23] with quarks. In Sec. II we will review this model and show how mean-field hedgehog solutions emerge. Next (Sec. III), we consider rotating solutions to the model (cranking). In Sec. IV we show how to quantize the collective variables. Various static properties of the N and Δ are calculated in Sec. V, as are some simple properties of the N-Δ transition.

While the discussion of this paper applies to a particular model, the extension to other models, such as chiral bags, is straightforward. One important generalization is to hedgehog models, which have vector-meson degrees of freedom. Like the quarks, the vector-meson fields have changes which are linear in the rotation frequency. We treat the problem of cranking[24] and semiclassical projection for these models elsewhere.[25]

II. THE CHIRAL-QUARK MODEL AND MEAN-FIELD THEORY

The chiral quark-meson (CQM) model was proposed by Birse and Banerjee[11] and independently by Kahana, Ripka, and Soni.[12] The basic approach is to couple quarks to mesons in a chirally invariant manner without invoking a bag. The simplest realization of this idea has a Lagrangian of the form of a Gell-Mann—Levy σ model[23] with quarks in the place of nucleons. The CQM model has been studied using both the linear[11] and nonlinear[12] variants of the σ model. If the σ mass is large ($\sim$1000 MeV) it makes little difference phenomenologically whether the linear or nonlinear variant is used. Following Ref. 11 we will consider a Lagrangian based on the linear σ model:

$$\mathscr{L}=\bar{\psi}[i\partial\!\!\!/+g(\hat{\sigma}+i\gamma_5\tau\cdot\hat{\boldsymbol{\phi}})]\psi+\tfrac{1}{2}(\partial_\mu\hat{\sigma})^2+\tfrac{1}{2}(\partial_\mu\hat{\boldsymbol{\phi}})^2-U(\hat{\sigma},\hat{\boldsymbol{\phi}}) , \tag{2.1a}$$

where the mesonic potential functional is given by

$$U(\sigma,\boldsymbol{\phi})=\tfrac{1}{4}\Lambda^2(\sigma^2+\boldsymbol{\phi}^2-\nu^2)^2+C\sigma . \tag{2.1b}$$

In Eqs. (2.1), $\hat{\sigma}$ and $\hat{\boldsymbol{\phi}}$ are the quantum-mechanical field operators for σ mesons and pions; ψ is the quark field operator. A summation over color indices is implicit. The constants Λ, ν, and C are related to the masses for the pion and σ meson and to the pion decay constant, $F_\pi=93$ MeV:

$$\begin{aligned}\Lambda^2&=(m_\sigma{}^2-m_\pi{}^2)/2F_\pi{}^2,\\ \nu^2&=F_\pi{}^2-m_\pi{}^2/\Lambda^2 ,\\ C&=F_\pi m_\pi{}^2 .\end{aligned} \tag{2.2}$$

Spontaneous chiral-symmetry breaking gives a nonzero vacuum expectation for $\hat{\sigma}$,

$$\langle\hat{\sigma}\rangle_{\text{vac}}=-F_\pi , \tag{2.3}$$

and the explicit chiral-symmetry breaking term in Eq. (2.1b) gives the pion its mass. The quark-pion coupling constant g is a free parameter of the model. In the numerical calculations which are presented in this paper, we will use the preferred parameters of Ref. 11, $m_\sigma=1200$ MeV, $m_\pi=139.6$ MeV, and $gF_\pi=500$ MeV.

There is some ambiguity in the choice of m_σ. In boson-exchange models of N-N forces one uses a σ "meson" with a mass of about 500 MeV. This σ should not, in our view, be considered as the meson which is the chiral partner of the pion. The boson exchange σ is, after all, not really a meson—there is no experimental evidence for a 500-MeV resonance in π-π scattering. Rather, this σ should probably be viewed as simulating many types of higher-order effects in multipion channels (box diagrams, effects of Δ's, etc.). Moreover, an attractive scalar potential with a range comparable to $(500\text{ MeV})^{-1}$ may well be found between two solitons (which represent nucleons) for models of the sort considered here even if m_σ itself is large. The point is that the σ field for a static soliton will, in such a case, have a spatial extent similar to that of the source of the σ field which falls off at large distances with a characteristic distance of $(2m_\pi)^{-1}$. Various effects (the finite mass of σ, the quark contributions to the mass, the effects of renormalizing the N-Δ part of the potential, and both quantum and nonstatic effects) can increase the effective mass of the σ part of the potential to something larger than $2m_\pi$. Thus, an effective mass of around 500 MeV is not unreasonable.

We note that π-π resonances with the quantum numbers of the σ have been observed in the 1-GeV region: The $S(975)$ and the $\epsilon(1300)$. Unfortunately, one cannot deduce from the observation of a resonance its properties under chiral transformations. Thus, one does not know if one of these resonances is, in fact, the chiral partner of the pion. It seems reasonable to suppose that perhaps one of these resonance is, in fact, the pion's chiral partner and we will take the somewhat arbitrary value of 1200 MeV for the σ mass. The other resonance might correspond to a chiral singlet—the glueball. Alternatively, the mass eigenstates for the two resonances may represent linear superpositions of the σ and the glueball. While 1200 MeV was chosen arbitrarily, the results are, as mentioned above, rather insensitive to variations in the σ mass.

We will study the model using mean-field theory. There are many different ways to introduce mean-field theory, each carrying its own interpretation. The treatment which is simplest for our purposes is based on a particular class of quantum states. Any state in this class will be labeled $|\sigma,\boldsymbol{\phi},q\rangle$, where $\sigma,\boldsymbol{\phi}$ are c-number fields (which describe the motion of pions and σ mesons) and q is a Dirac spinor (which describes the quark motion). The detailed form of these quantum states is of little importance. The state must, however, have the property that

$$\begin{aligned}
&\langle\sigma,\boldsymbol{\phi},q\,|\,\hat{\sigma}(\mathbf{x})\,|\,\sigma,\boldsymbol{\phi},q\rangle=\sigma(\mathbf{x}),\\
&\langle\sigma,\boldsymbol{\phi},q\,|\,\boldsymbol{\phi}(\mathbf{x})\,|\,\sigma,\boldsymbol{\phi},q\rangle=\sigma(\mathbf{x}),\\
&\langle\sigma,\boldsymbol{\phi},q\,|\,\bar{\psi}_c(\mathbf{x}')M\psi_{c'}(\mathbf{x})\,|\,\sigma,\boldsymbol{\phi},q\rangle=\delta_{cc'}\bar{q}(\mathbf{x}')Mq(\mathbf{x})\ ,
\end{aligned}\tag{2.4}$$

where ψ_c is the quark operator for color c and M is an arbitrary matrix in Dirac spinor space and isospin. It is apparent from the color structure of the quark equations that the mean-field states are *locally* color singlet. The form of the mean-field state also indicates that only the effects of the N_c valence quarks (where N_c is the number of colors) are included; a single spinor for the valence quarks is the only quark information in the state.

The dynamical assumption of mean-field theory is that quantum fluctuations are not important in the evaluation of expectation values of products of field operators in a mean-field state. Thus, one assumes

$$\begin{aligned}
&\langle\sigma,\boldsymbol{\phi},q\,|\,\hat{\sigma}\bar{\psi}_cM\psi_{c'}\,|\,\sigma,\boldsymbol{\phi},q\rangle=\sigma\delta_{cc'}\bar{q}Mq\ ,\\
&\langle\sigma,\boldsymbol{\phi},q\,|\,\boldsymbol{\phi}\bar{\psi}_cM\psi_{c'}\,|\,\sigma,\boldsymbol{\phi},q\rangle=\boldsymbol{\phi}\delta_{cc'}\bar{q}Mq\ ,\\
&\langle\sigma,\boldsymbol{\phi},q\,|\,\hat{\sigma}^j\hat{\phi}_a{}^k\hat{\phi}_b{}^l\hat{\phi}_c{}^m\,|\,\sigma,\boldsymbol{\phi},q\rangle=\sigma^j\phi_a{}^k\phi_b{}^l\phi_c{}^m\ ,
\end{aligned}\tag{2.5}$$

where, in the third equation, the subscripts a, b, and c indicate isospin; the superscripts are powers.

The c-number fields σ, $\boldsymbol{\phi}$, and q used to describe the mean-field states are, in general, time dependent. Equations of motion for σ and $\boldsymbol{\phi}$ can be obtained from the quantum Euler-Lagrange equations by taking expectation values in the mean-field states:

$$\langle\sigma,\boldsymbol{\phi},q\,|\,[\partial_\mu(\delta\mathcal{L}/\delta\partial_\mu\hat{\sigma})-\delta\mathcal{L}/\delta\hat{\sigma}]\,|\,\sigma,\boldsymbol{\phi},q\rangle=0\ .\tag{2.6}$$

$$\langle\sigma,\boldsymbol{\phi},q\,|\,[\partial_\mu(\delta\mathcal{L}/\delta\partial_\mu\hat{\boldsymbol{\phi}})-\delta\mathcal{L}/\delta\hat{\boldsymbol{\phi}}]\,|\,\sigma,\hat{\boldsymbol{\phi}},q\rangle=0\ .$$

By invoking the dynamical assumption of Eq. (2.5), we obtain

$$\begin{aligned}
&\Box\sigma-N_cgq^\dagger\beta q+\delta U(\sigma,\boldsymbol{\phi})/\delta\sigma=0\ ,\\
&\Box\boldsymbol{\phi}-N_cgq^\dagger\beta\gamma_5\boldsymbol{\tau}q+\delta U(\sigma,\boldsymbol{\phi})/\delta\boldsymbol{\phi}=0\ .
\end{aligned}\tag{2.7}$$

One must take slightly more care to obtain the equations of motion for the quarks as the Euler-Lagrange equation has only a single quark field operator, while the equations giving the expectation value of mean-field states involves quark bilinears. This problem can be circumvented by multiplying the Euler-Lagrange equation for ψ by $\psi^\dagger M_A$ on the left, where M_A is an arbitrary 4×4 matrix:

$$\langle\sigma,\boldsymbol{\phi},q\,|\,\psi^\dagger M_A[\partial_\mu(\delta\mathcal{L}/\delta\partial_\mu\psi^\dagger)-\delta\mathcal{L}/\delta\psi^\dagger]\,|\,\sigma,\boldsymbol{\phi},q\rangle=0\ .\tag{2.6$'$}$$

Using the relations in Eqs. (2.5), we obtain

$$q^\dagger M_A[-i\boldsymbol{\alpha}\cdot\nabla-g\beta(\sigma+i\boldsymbol{\tau}\cdot\boldsymbol{\phi}\gamma_5)]q=iq^\dagger M_A\partial_tq\ .$$

Since M_A is an arbitrary matrix, the preceding equation can only be satisfied, in general, if the equation of motion for q,

$$[-i\boldsymbol{\alpha}\cdot\nabla-g\beta(\sigma+i\boldsymbol{\tau}\cdot\boldsymbol{\phi}\gamma_5)]q=i\partial_tq\ ,\tag{2.7$'$}$$

is satisfied.

The overall effect of taking an expectation value and ignoring quantum fluctuations as in Eq. (2.5) is simply to replace $\hat{\sigma}$, $\hat{\boldsymbol{\phi}}$, and ψ with σ, $\boldsymbol{\phi}$, and $\sqrt{N_c}q$ in the equations of motion (2.6). It is clear that the mean-field approximation is essentially classical in nature. Eventually, we will requantize the collective degrees of freedom of the system. Of course, even with this essentially classical interpretation of the mean-field equations of motion, it is necessary to recognize that q is a Dirac spinor and thus contains some quantal information (in the sense of first quantization). In later sections we will often use mean-field theory to study various operators. We will adopt the language that an operator evaluated in mean-field theory is simply the expectation of that operator in a mean-field state as obtained by ignoring fluctuations as in Eq. (2.5).

Let us now turn to the question of finding localized stationary solutions (solitons) for the equations of motion (2.7). In a stationary solution the meson fields are static and the quark spinor has the trivial time dependence $q(t)=e^{-i\epsilon t}q$. The eigenvalue, ϵ, is a Lagrange multiplier which enforces the normalization of the quark spinor:

$$\int d^3r\,q^\dagger q=1\ .\tag{2.8}$$

The equations for a stationary solution are therefore given by

$$\begin{aligned}
&-\nabla^2\sigma-N_cgq^\dagger\beta q+\delta U(\sigma,\boldsymbol{\phi})/\delta\sigma=0\ ,\\
&-\nabla^2\boldsymbol{\phi}-N_cigq^\dagger\beta\gamma_5\boldsymbol{\tau}q+\delta U(\sigma,\boldsymbol{\phi})/\delta\boldsymbol{\phi}=0\ ,\\
&[-i\boldsymbol{\alpha}\cdot\nabla-g\beta(\sigma+i\boldsymbol{\tau}\cdot\boldsymbol{\phi}\gamma_5)]q=\epsilon q\ .
\end{aligned}\tag{2.9}$$

Solutions to Eqs. (2.9) have been found using the hedgehog form of Chodos and Thorn:[5]

$$\begin{aligned}
&\sigma_h(\mathbf{r})=\sigma_h(r)\ ,\\
&\boldsymbol{\phi}_h(\mathbf{r})=\phi_h(r)\hat{\mathbf{r}}\ ,\\
&q_h=\frac{1}{\sqrt{4\pi}}\begin{bmatrix}G(r)\chi_h\\ i\hat{\boldsymbol{\sigma}}\cdot\mathbf{r}F(r)\chi_h\end{bmatrix}\ ,
\end{aligned}\tag{2.10a}$$

where the subscript h indicates the hedgehog solution and χ_h is the hedgehog Pauli spinor,

$$\chi_h=\frac{1}{\sqrt{2}}(u\downarrow-d\uparrow)\ .\tag{2.10b}$$

The substitution of this form into Eqs. (2.9) automatically satisfies the angular parts of the equations. One obtains four coupled ordinary differential equations for the radial functions σ_h, ϕ_h, G, and F. These can be solved numerically. One iterates to find the eigenvalue, ϵ_h, which gives a solution that satisfies the normalization condition (2.8). The solutions, based on this model with the parameters as given earlier, are plotted in Fig. 1.

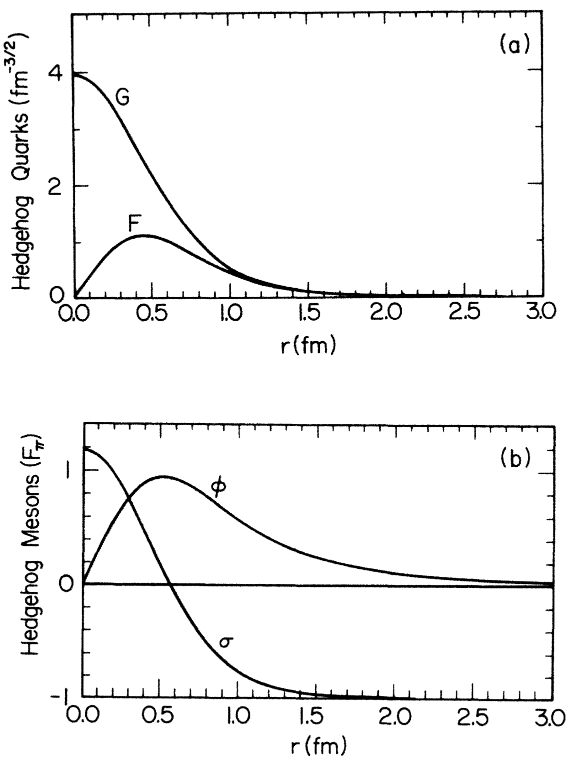

FIG. 1. The hedgehog mean-field solution. Quark upper and lower components are shown in (a), and meson fields in units of F_π in (b).

III. CRANKING THE HEDGEHOG

The hedgehog form (2.10) is not invariant under either rotations or isorotations. However, because of correlations, it is invariant under a combined rotation in space and isospace. The generators of such a grand rotation are the grandspin operators **K**, which are defined by $\mathbf{K}=\mathbf{J}+\mathbf{I}$. Because the hedgehog breaks the **J**-**I** symmetry, there are an infinite number of solutions to the stationary Euler-Lagrange equations which are degenerate to the hedgehog solution. These solutions can be obtained by rotating the hedgehog in either space or isospace. The grand-rotational symmetry implies that one can reach all of these degenerate solutions by considering either rotations or isorotations. We choose to rotate in isospace. A convenient parametrization for an arbitrary isorotation is

$$\begin{aligned} &q \to Aq \;, \\ &\boldsymbol{\phi}\cdot\boldsymbol{\tau} \to A\,\boldsymbol{\phi}\cdot\boldsymbol{\tau} A^\dagger \;, \\ &\sigma \to \sigma \;, \end{aligned} \tag{3.1}$$

where A is a space-independent SU(2) matrix.

We now turn our attention to time-dependent rotations and cranking. The basic idea is to find stationary solutions to the Euler-Lagrange equations in a rotating (or isorotating) frame.[16,20,26] Thus we write

$$\begin{aligned} &q(t)=A(t)q' \;, \\ &\boldsymbol{\phi}(t)=\tfrac{1}{2}\mathrm{Tr}[\boldsymbol{\tau}A(t)\boldsymbol{\phi}\cdot\boldsymbol{\tau}A^\dagger(t)] \;, \\ &\sigma(t)=\sigma' \;, \end{aligned} \tag{3.2}$$

where q', $\boldsymbol{\phi}'$, and σ' represent the spinor, pion, and sigma fields in the rotating frame and $A(t)$ specifies the time-dependent rotation. We seek stationary solutions for q', $\boldsymbol{\phi}'$, and σ'. For an arbitrary $A(t)$, there are generally no stationary solutions for the primed fields. If, however, $A(t)$ corresponds to rotational motion in which the expectation value (i.e., the classical value) of the angular momentum and isospin are time independent, then stationary solutions exist. Of course, by Noether's theorem, it is precisely those rotations which lead to static classical **I** and **J** that can be time-dependent solutions to the mean-field equations of motion. We will consider the form

$$A(t)=B\exp(it\boldsymbol{\lambda}\cdot\boldsymbol{\tau}/2) \;, \tag{3.3}$$

where B is a time-independent SU(2) matrix and $\boldsymbol{\lambda}$ is a time-independent vector. As will be shown in Sec. IV, this form is sufficiently general to allow arbitrary classical values of **J** and **I** consistent with the physical constraint from grand-rotational symmetry that $|\mathbf{I}|=|\mathbf{J}|$. While solving the cranking problem, B and $\boldsymbol{\lambda}$ will be treated as parameters. They are the collective degrees of freedom for the rotational problem. Eventually, we will requantize the system, and B and λ will acquire nontrivial commutation relations.

We start by studying the motion of the primed fields. Equations of motion for q', ϕ', and σ' can be found by substituting the equations describing collective rotations, (3.2) and (3.3), into the mean-field equations of motion for q, $\boldsymbol{\phi}$, and σ [Eqs. (2.7)]:

$$\begin{aligned} &[-i\boldsymbol{\alpha}\cdot\nabla-g\beta(\sigma'-i\,\boldsymbol{\tau}\cdot\boldsymbol{\phi}'\gamma_5)+\boldsymbol{\lambda}\cdot\boldsymbol{\tau}/2-i\partial_t]q'=0 \;, \\ &-\nabla^2\phi_j-N_c ig q'^\dagger\beta\gamma_5\tau_j q'+\frac{\partial U(\sigma',\boldsymbol{\phi}')}{\partial\phi'_j} \\ &\qquad\qquad +\partial_t{}^2\phi'_j+(\boldsymbol{\lambda}\times\partial_t\boldsymbol{\phi}')_j=0 \;, \\ &-\nabla^2\sigma'-N_c g q'^\dagger\beta q'+\frac{\partial U(\sigma,\boldsymbol{\phi})}{\partial\sigma}+\partial_t{}^2\sigma'=0 \;. \end{aligned} \tag{3.4}$$

We seek stationary solutions of Eqs. (3.4), which implies that $\partial_t{}^2\sigma'=\partial_t{}^2\phi'=\partial_t\phi'=0$ and that $i\partial_t q'=\epsilon' q'$. The equations for stationary σ', $\boldsymbol{\phi}'$, and q' are formally identical to the equations for stationary σ, $\boldsymbol{\phi}$, and q [Eq. (2.9)], except for the $\boldsymbol{\lambda}\cdot\boldsymbol{\tau}/2$ term in the equation for q'. Therefore, it is convenient to expand Eqs. (3.4) about the stationary hedgehog solutions of Eq. (2.10) and to treat $\boldsymbol{\lambda}\cdot\boldsymbol{\tau}/2$ as a perturbation. We write

$$\begin{aligned} &q'=q_h+\delta q, \quad \epsilon'=\epsilon_h+\delta\epsilon \;, \\ &\sigma'=\sigma_h+\delta\sigma, \quad \boldsymbol{\phi}'=\boldsymbol{\phi}_h+\delta\boldsymbol{\phi} \;, \end{aligned} \tag{3.5}$$

where the subscript h indicates the hedgehog solution (2.10), and solve perturbatively for δq, $\delta\sigma$, $\delta\boldsymbol{\phi}$, and $\delta\epsilon$ at first order in λ. It has been shown[20] on the basis of grand-reversal symmetry (time reversal combined with an isorotation through π about the 2 axis) that, to first order,

$$\delta\sigma = \delta\phi = 0 \ . \tag{3.6}$$

Grand-reversal symmetry and the proof that $\delta\sigma$ and $\delta\phi$ vanish to first order are reviewed in Appendix A.

An expression for δq and $\delta\epsilon$ can be obtained from the quark equation of motion (3.4a) with the use of stationarity, the adiabatic approximation, and Eq. (3.6):

$$h'\delta q + (\boldsymbol{\lambda}\cdot\boldsymbol{\tau}/2)q_h - \epsilon_h \delta q - \delta\epsilon q_h = 0 \ , \tag{3.7}$$

where h' is the effective Dirac Hamiltonian in the rotating frame,

$$h' = -i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} - g\beta(\sigma_h + i\boldsymbol{\tau}\cdot\boldsymbol{\phi}_h\gamma_5) \ . \tag{3.8}$$

Multiplying Eq. (3.7) on the left by $q_h^\dagger$ gives $\delta\epsilon = q_h^\dagger(\boldsymbol{\lambda}\cdot\boldsymbol{\tau}/2)q_h = 0$. Finally, we are left with the cranking equation for δq:

$$(h' - \epsilon_h)\delta q = (-\boldsymbol{\lambda}\cdot\boldsymbol{\tau}/2)q_h \ . \tag{3.9}$$

Simple tensorial properties of grand spin are of great help in solving Eq. (3.9). The hedgehog spinor q_h has even parity and transforms under grand rotations as a $K=0$ tensor. Therefore $(\boldsymbol{\lambda}\cdot\boldsymbol{\tau}/2)q_h$ is an even-parity $K=1$ tensor with **K** parallel to $\boldsymbol{\lambda}$. Since the differential operator $(h' - \epsilon_h)$ is invariant under grand rotations, δq must be a $K=1$ even-parity spinor with **K** aligned with $\boldsymbol{\lambda}$. The most general spinor with these transformation properties can be expressed in terms of four radial functions in the following manner:

$$\delta q = \frac{1}{\sqrt{4\pi}} \begin{bmatrix} A(r)\boldsymbol{\lambda}\cdot\boldsymbol{\sigma}\chi_h + B(r)(\boldsymbol{\lambda}\cdot\boldsymbol{\sigma}/3 - \boldsymbol{\lambda}\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}\cdot\boldsymbol{\sigma})\chi_h \\ iC(r)\boldsymbol{\lambda}\cdot\hat{\mathbf{r}}\chi_h - D(r)(\boldsymbol{\lambda}\times\hat{\mathbf{r}})\cdot\boldsymbol{\sigma}\chi_h \end{bmatrix} , \tag{3.10}$$

where χ is the hedgehog spinor of Eq. (2.10b). The form (3.10) reduces the cranking equation (3.9) to four ordinary differential equations (ODE's) for the radial functions:

$$\begin{aligned} \partial_r(-A + 2B/3) &= -2B/r - g\phi_h(-A + 2B/3) \\ &\quad - (g\sigma_h - \epsilon_h)C + F/2 \ , \\ \partial_r(A + B/3) &= -B/r - g\phi_h(-A - B/3) \\ &\quad - (g\sigma_h - \epsilon_h)D - F/2 \ , \\ \partial_r(C - 2D)/3 &= -2(C - 2D)/3r + g\phi_h(C + 2D)/3 \\ &\quad - (g\sigma_h + \epsilon_h)A + G/2 \ , \\ \partial_r(-C - D) &= -(C + D)/r - g\phi_h(C - D) + (g\sigma_h + \epsilon_h)B \ , \end{aligned} \tag{3.11}$$

where σ and ϕ are the radial sigma and pion functions from the hedgehog solution (2.10). These inhomogeneous ODE's are driven by the hedgehog spinor functions, G and F. Equations (3.11) can be solved numerically with boundary conditions such that the functions are everywhere finite. The results are plotted in Fig. 2(a).

For comparison we plot the same four functions obtained using the "closure approximation" of Ref. 21 in Fig. 2(b). We note that at least for the model considered here the closure approximation does a poor job of reproducing the exact solutions.

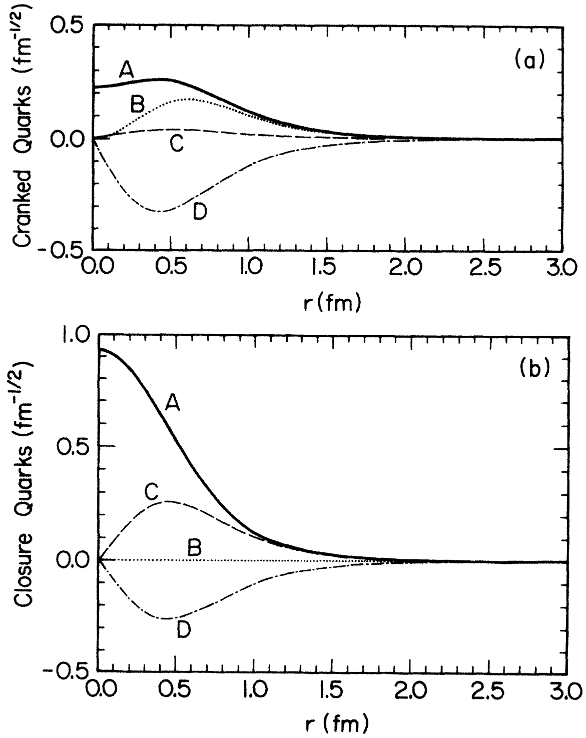

FIG. 2. Solutions to the cranking equations in terms of the functions in Eq. (3.10). Exact solution is shown in (a), and results from the "closure approximation" in (b).

IV. QUANTIZING THE COLLECTIVE VARIABLES

Having found rotating solutions to the time-dependent mean-field equations, we turn to the question of quantizing the collective variables. This is done by considering the angular momentum and isospin, both of which are conserved by the system. The expressions for **I** and **J** following from Eq. (2.1) are

$$\begin{aligned} I_a &= \int d^3r[\psi^\dagger(\tau_a/2)\psi + \epsilon_{abc}\hat{\phi}_b\partial_t\hat{\phi}_c] \\ J_k &= \int d^3r\{\psi^\dagger[\sigma_k/2 - i(\mathbf{r}\times\boldsymbol{\partial})_k]\psi + (\partial_t\hat{\phi}_a)(\mathbf{r}\times\boldsymbol{\partial})_k\hat{\phi}_a\} \ . \end{aligned} \tag{4.1}$$

Using the mean-field approximation described in Sec. II and the rotating solution for the quark and meson fields given in Eqs. (3.2), (3.3), (3.5), (3.6), (3.10), and (3.11), we find mean-field expressions for the isospin and angular momentum which depend only on the collective variables B and λ:

$$\begin{aligned} I_a &= \int d^3r[N_c\delta q^\dagger\tau_b q_h - (\lambda_a - \boldsymbol{\lambda}\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}_a)\phi_h{}^2\lambda_b] \\ &\quad \times \tfrac{1}{2}\mathrm{tr}(\tau_a B\tau_b B^\dagger) \\ &= -\mathscr{I}\tfrac{1}{2}\mathrm{tr}(\tau_a B\tau_b B^\dagger)\lambda_b \ , \\ J_k &= \int d^3r\{N_c\delta q^\dagger[\sigma_k - 2i(\mathbf{r}\times\boldsymbol{\partial})_k]q_h \\ &\quad + (\lambda_a - \boldsymbol{\lambda}\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}_a)\phi_h{}^2\lambda_k\} \\ &= \mathscr{I}\lambda_k \ , \end{aligned} \tag{4.2}$$

where $\mathscr{I}$, the moment of inertia, can be expressed in terms of the radial functions which specify ϕ_h, δq, and q_h,

$$\mathscr{I}=\int dr\, r^2[N_c(AG+CF/3-2DF/3)+(8\pi/3)\phi_h{}^2] \ . \tag{4.3}$$

We have evaluated the integral in (4.3) for the model being studied here and have obtained the result $\mathscr{I}=1.17$ fm. The meson contribution accounts for 62% of the total value. This represents the fraction of both the angular momentum and isospin in a rotating soliton which is carried by the pion.

As mentioned previously, these mean-field expressions for **I** and **J** are time independent and give $|\mathbf{I}|=|\mathbf{J}|$. By choosing an appropriate matrix B, the angle between **I** and **J** can be arbitrary. Thus we see that the rotation in Eq. (3.3) is sufficiently general. We also note that the quark contributions to **I** and **J** (and therefore to $\mathscr{I}$) depend on δq and thus cannot be found without a cranking calculation.

The mean-field expressions for **I** and **J** are essentially classical in nature. We requantize the system by insisting that the commutation relations for the angular momentum and the isospin,

$$\begin{aligned}&[J_k,J_l]=i\epsilon_{klm}J_m \ ,\\ &[J_k,I_a]=0 \ ,\\ &[I_a,I_b]=i\epsilon_{abc}I_c \ ,\end{aligned} \tag{4.4}$$

are satisfied by the collective expressions given in Eq. (3.13). The adiabatic assumption implies that the intrinsic degrees of freedom are decoupled from the collective ones and thus commute. Since $\mathscr{I}$ depends only on the intrinsic degrees of freedom,

$$[\mathscr{I},\lambda_k]=[\mathscr{I},\lambda_b\tfrac{1}{2}\mathrm{tr}(\tau_a B\tau_b B^\dagger)]=0 \ . \tag{4.5}$$

The commutation relations (4.4) will be satisfied if the three parameters needed to specify B commute with each other and if

$$\begin{aligned}&[\lambda_a,\lambda_b]=i\epsilon_{abc}(\lambda_c/\mathscr{I}) \ ,\\ &[\lambda_a,B]=B\tau_a/(2\mathscr{I}) \ .\end{aligned} \tag{4.6}$$

One can use the commutation relations of (4.4) to quantize the system and find collective functions. The matrix B must be parametrized in terms of collective variables. There are many equivalent ways to do this. We will follow Adkins, Nappi, and Witten (ANW) and write B as

$$B=b_0+i\boldsymbol{\tau}\cdot\mathbf{b} \ , \tag{4.7}$$

with the constraint that

$$b_0{}^2+\mathbf{b}\cdot\mathbf{b}=1 \ . \tag{4.8}$$

With this parametrization, we find that Eqs. (4.4) are satisfied if $\boldsymbol{\lambda}$ is given by

$$\lambda_k=[i/(2\mathscr{I})]\left[b_k\frac{\partial}{\partial b_0}-b_0\frac{\partial}{\partial b_k}-\epsilon_{klm}b_l\frac{\partial}{\partial b_m}\right] . \tag{4.9}$$

Using the collective expressions for **I** and **J**, (4.2), and the differential operator for $\boldsymbol{\lambda}$, (4.9), one can find collective wave functions which are simultaneous eigenstates of J^2, I^2, J_z, and I_3. Because the collective motion satisfied $|\mathbf{I}|=|\mathbf{J}|$ at the mean-field level, all of the collective eigenstates obtained by quantizing this motion will have the same eigenvalue for I^2 and J^2. One finds

$$\begin{aligned}&|p\uparrow\rangle=(1/\pi)(b_1+ib_2), \quad |p\downarrow\rangle=-(i/\pi)(b_0-ib_3) \ ,\\ &|n\uparrow\rangle=(i/\pi)(b_0+ib_3), \quad |n\downarrow\rangle=-(1/\pi)(b_1-ib_2) \ ,\\ &|\Delta^{++}s=\tfrac{3}{2}\rangle=(\sqrt{2}/\pi)(b_1+b_2)^3\end{aligned} \tag{4.10}$$

(the other Δ states can be obtained from $|\Delta^{++}s=\frac{3}{2}\rangle$ by use of the lowering operators $I_-=I_1-iI_2$ and $J_-=J_x-J_y$). It is not surprising that the collective wave functions are identical to those of ANW. Indeed, the mean-field expressions for **I** and **J** written in terms of $\boldsymbol{\lambda}$, B, and $\mathscr{I}$ [Eqs. (4.2)] are identical for the Skyrme model and for models with quarks. The treatment of the collective motion differs only in the expression for the moment of inertia.

We note, in passing, that the parametrization of the matrix B given in Eq. (4.7) is not the only convenient form. A useful alternative is to express B in terms of the three Euler angles, α, β, and γ, used to specify a rotation in space:

$$R=\exp(-i\alpha J_z)\exp(-i\beta J_y)\exp(-i\gamma J_z) \ .$$

With this parametrization, B is given by

$$B=\exp(i\gamma\tau_3/2)\exp(i\beta\tau_2/2)\exp(i\alpha\tau_3/2) \ . \tag{4.11}$$

The collective wave functions parametrized this way are nothing but the conventional Wigner D matrices with appropriate normalization constants:

$$|J,m_j,m_i\rangle=\frac{(2J+1)^{1/2}}{2\pi}D^J_{m_j,-m_i}(\alpha,\beta,\gamma) \ , \tag{4.12}$$

where $J(J+1)$, mj, and mi are the eigenvalues of J^2, J_z, and I_3, respectively.

V. PROPERTIES OF N's AND Δ's

In this section we will use the collective wave functions of Sec. IV to calculate the various static properties of nucleons and deltas. The basic method is straightforward. Suppose one wishes to study the matrix element of some operator θ between arbitrary states of N or Δ. We start by using the mean-field approximations of Sec. II and the rotating solution of Sec. III to obtain an expression for θ in terms of the collective variables, $\boldsymbol{\lambda}$ and B. This is completely analogous to obtaining the collective expressions for **I** and **J** in Eq. (4.2). We will denote the collective expression for θ as $\theta_{\rm coll}$. Since the intrinsic variables are assumed to commute with the collective variables, $\theta_{\rm coll}$ as a collective operator depends only on $\boldsymbol{\lambda}$ and B. Of course, the time dependence of the rotation in Eq. (3.3) implies that, in general, $\theta_{\rm coll}$ will be time dependent and we will view it as a collective Heisenberg operator, $\theta_{\rm coll}(t)$. Matrix elements of the Schrödinger operator θ in a nucleon state are obtained by evaluating $\theta_{\rm coll}(t=0)$ in the appropriate collective wave function from Eq. (4.10). The validity of this procedure depends upon the validity of both the mean-field treatment of Sec. II and of the adiabatic approximation.

We will start by calculating the isoscalar electromagnetic properties of the nucleon. The electromagnetic current may be written as

$$j^\mu = \tfrac{1}{2} B^\mu + V_3^\mu \ , \qquad (5.1)$$

where B^μ is the baryon current and V_a^μ is the vector current for the ath component of isospin. The isoscalar properties depend only on the baryon current:

$$B^\mu = (1/N_c)\bar{\psi}\gamma^\mu\psi \ . \qquad (5.2)$$

The mean-field expressions for the time and space components of B^μ in terms of the collective variables and the functions describing q_h and δq are

$$\begin{aligned} B^0_{\rm coll} &= q_h^\dagger q_h = (4\pi)^{-1}(G^2+F^2) + O(\lambda^2) \ , \\ B^k_{\rm coll} &= 2\delta q^\dagger \gamma^0 \gamma^k q_h + O(\lambda^2) \qquad (5.3) \\ &= (4\pi r)^{-1}(2AF + 2BF/3 - 2DG)(\boldsymbol{\lambda}\times\mathbf{r})_k + O(\lambda^2) \ . \end{aligned}$$

In the adiabatic limit we may drop the terms of order λ^2.

The expression for the space components of the baryon current indicates one of the differences between the semiclassical treatment of a model with quarks, such as the chiral quark-meson model, and the Skyrme model. One expects on naive geometrical grounds that $B^k_{\rm coll} = b(r)(\boldsymbol{\lambda}\times\mathbf{r})_k$, where $b(r)$ is a radial function. We find that for models with quarks, $b(r)$ is given in terms of δq. Since δq is determined from the cranking calculation, we see that $b(r)$ depends in detail on the dynamics of the model. In sharp contrast, the Skyrme model gives $B^k_{\rm coll}$ purely as a rigid rotation of the static mean-field baryon density[11] and therefore is independent of the dynamics. For our model, we have compared $b(r)$ as correctly calculated using cranking with what it would have been if a rigid rotation of the baryon density had been naively applied. We plot

$$\begin{aligned} b_{\rm crank} &= (4\pi r)^{-1}(2AF + 2BF/3 - 2DG) \ , \\ b_{\rm rigid} &= (4\pi)^{-1}(G^2+F^2) \end{aligned} \qquad (5.4)$$

in Fig. 3 and note that the cranked result is somewhat reduced from the rigid result.

The isoscalar mean-square radius $\langle r^2\rangle_{I=0}$ is defined as $\langle r^2\rangle_{\rm prot} + \langle r^2\rangle_{\rm neut}$. It can be found by evaluating $\int d^3r\, r^2 B^0_{\rm coll}$ in either a proton or neutron state. This gives

$$\langle r^2\rangle_{I=0} = \int dr\, r^4(G^2+F^2) \ . \qquad (5.5)$$

Numerical evaluation of this model yields a result of (0.70 fm)^2 which compares well with the experimental value of (0.72 fm)^2. The isoscalar magnetic moment operator is defined as

$$\mu_{I=0} = \int d^3r\, \mathbf{r}\times\mathbf{j}_{I=0} = \tfrac{1}{2}\int d^3r\, \mathbf{r}\times\mathbf{B} \ , \qquad (5.6)$$

where $\mathbf{B}$ is the baryon space current. To find the isoscalar magnetic moment, we evaluate the third component of $\boldsymbol{\mu}$

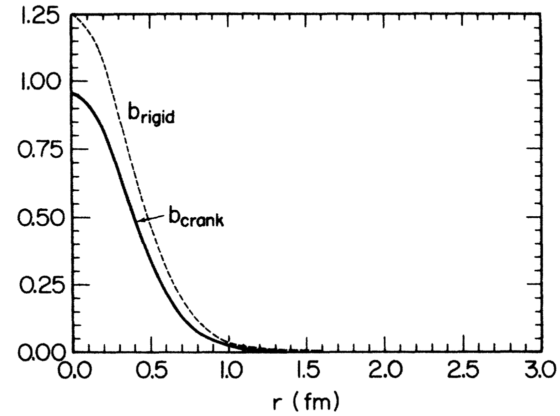

FIG. 3. The radial function $b(r)$ which characterizes the isovector space current density: $B^k_{\rm coll} = b(r)(\boldsymbol{\lambda}\times\mathbf{r})_k$. Both the exact result of the cranking calculation and the result of a naive rigid rotation are plotted.

in a proton spin-up state. Using our mean-field approach we have

$$\mu_{I=0} = \left\langle p\uparrow \left| \tfrac{1}{2}\int d^3r(\mathbf{r}\times\mathbf{B}_{\rm coll})_z \right| p\uparrow \right\rangle \ .$$

Upon substituting the form (5.3) for $\mathbf{B}_{\rm coll}$, we find

$$\mu_{I=0} = \langle p\uparrow | \lambda_3 | p\uparrow\rangle \tfrac{2}{3}\int dr\, r^3(AF + BF/3 - DG) \ .$$

Finally, we note that $\lambda_3 = J_3/\mathscr{I}$ and that by construction the collective $|p\uparrow\rangle$ in Eq. (4.10) is an eigenstate J_3 with eigenvalue $\frac{1}{2}$. The expression for $\mu_{I=0}$ becomes

$$\mu_{I=0} = \frac{1}{3\mathscr{I}}\int dr\, r^3(AF + BF/3 - DG) \ . \qquad (5.7)$$

Numerically, this gives $0.381\mu_N$ (where μ_N is the nuclear magneton), which is less than one-half of the observed value of $0.88\mu_N$.

The square of the isoscalar magnetic radius is defined in analogy to Eq. (5.6) as

$$\langle r^2\rangle^m_{I=0} = \left\langle p\uparrow \left| \tfrac{1}{2}\int d^3r\, (\mathbf{r}\times\mathbf{B})_z\, r^2 \right| p\uparrow \right\rangle \Big/ \mu_{I=0} \ .$$

The semiclassical evaluation yields

$$\langle r^2\rangle^m_{I=0} = \frac{1}{3\mathscr{I}\mu_{I=0}}\int dr\, r^5(AF + BF/3 - DG) \ . \qquad (5.8)$$

Numerically we find $\langle r^2\rangle^m_{I=0} = (0.91\ {\rm fm})^2$; experimentally, it is $(0.81\ {\rm fm})^2$.

Next we consider the isovector electromagnetic properties. These depend solely on the third isospin component of the vector current V_3^μ. The quantum operator for V_3^μ is given by

$$V_3^\mu = \tfrac{1}{2}\,\bar{\psi}\gamma^\mu\tau_3\psi + \epsilon_{3ab}\hat{\phi}_a\partial^\mu\hat{\phi}_b \ . \qquad (5.9)$$

The mean-field expressions for the time and space components for V_3^μ are

$$\begin{aligned} V^0_{3\,\mathrm{coll}} &= N_c \delta q^\dagger B^\dagger \tau_3 B q_h + \epsilon_{3ab}\phi_{ha}\partial_0\phi_{hb} + O(\lambda^2) \\ &= \{(N_c/4\pi)[AG\lambda_d + BG(-\boldsymbol{\lambda}\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}_d + \lambda_d/3) + CF(\boldsymbol{\lambda}\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}_d) - DF(\lambda_d - \boldsymbol{\lambda}\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}_d)] + \phi_h{}^2(-\lambda_d + \boldsymbol{\lambda}\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}_d)\}C_{3d} + O(\lambda^2) , \\ V^k_{3\,\mathrm{coll}} &= (N_c/2) q^\dagger B^\dagger \gamma^0\gamma^k \tau_3 B q - \epsilon_{3ab}\phi_{ha}\partial_k\phi_b + O(\lambda^2) = [(N_c/4\pi)\epsilon_{kmd} GF\hat{\mathbf{r}}_m + \epsilon_{kmd}\hat{\mathbf{r}}\phi_h{}^2/r]C_{3d} + O(\lambda^2) , \end{aligned} \tag{5.10}$$

where C_{ab} is defined as

$$C_{ab} = \tfrac{1}{2}\mathrm{Tr}(\tau_a B \tau_b B^\dagger) . \tag{5.11}$$

The expression for I_3, (4.2), can be obtained from Eqs. (5.10) and (5.11) by integration over space and the evaluation of the trace. It is useful to have the expectation value of C_{3d} in proton on neutron collective wave functions. Using the collective wave functions, one finds

$$\begin{aligned} \langle p\uparrow | C_{3d} | p\uparrow\rangle &= \langle n\downarrow | C_{3d} | n\downarrow\rangle = -\tfrac{1}{3}\delta_{3d} , \\ \langle n\uparrow | C_{3d} | n\uparrow\rangle &= \langle p\downarrow | C_{3d} | p\downarrow\rangle = +\tfrac{1}{3}\delta_{3d} . \end{aligned} \tag{5.12}$$

The electric charge density ρ is given by the space component of the electromagnetic current. Since we have collective expressions for both the isovector and isoscalar contributions to the current, we can determine the charge densities. We have plotted in Fig. 4(a) the isoscalar and isovector radial charge densities,

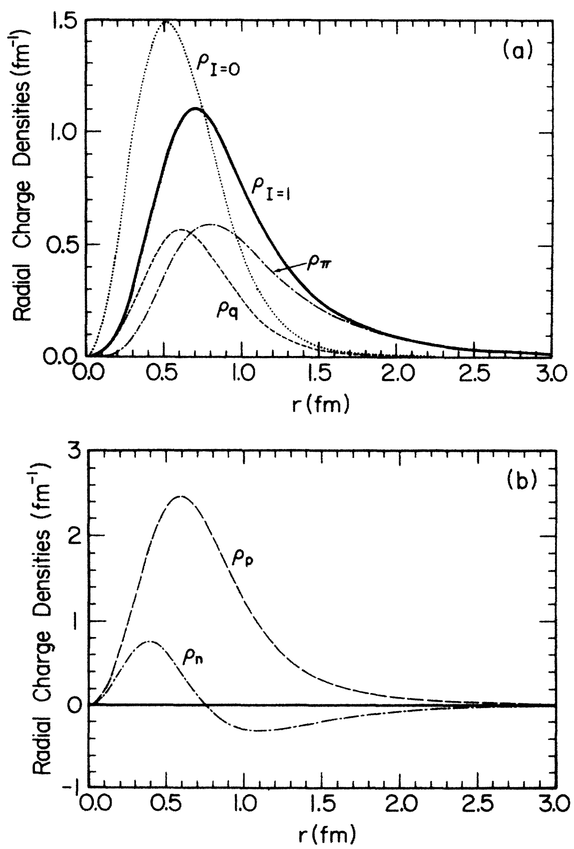

FIG. 4. Radial charge densities. The isoscalar and isovector radial charge densities and separately the pion and quark contribution to the isovector distribution are shown in (a), and the proton and neutron radial charge densities in (b).

$$\left\langle p\uparrow \left| \int d\Omega\, r^2 j^0 \right| p\uparrow \right\rangle \pm \left\langle n\uparrow \left| \int d\Omega\, r^2 j^0 \right| n\uparrow \right\rangle ,$$

where $d\Omega$ indicates an integral over angles. We show separately the quark and meson contributions to the isovector density. In Fig. 4(b) we plot the radial charge densities for the proton and the neutron.

The isovector charge radius operator is given by

$$\langle r^2\rangle_{r=1} = (2/I_3)\left\langle p\uparrow \left| \int d^3r\, r^2 V^0_3 \right| p\uparrow \right\rangle .$$

From this one obtains the semiclassical result

$$\langle r^2\rangle_{I=1} = \frac{1}{\mathscr{I}} \int dr\, r^4 [N_c(AG + CF/3 - 2DF/3) + (8\pi/3)\phi_h{}^2] . \tag{5.13}$$

Numerically, we get $\langle r^2\rangle_{I=-1} = (1.11\ \mathrm{fm})^2$. Experimentally, $\langle r^2\rangle_{I=1} = \langle r^2\rangle_{\mathrm{prot}} - \langle r^2\rangle_{\mathrm{neut}}$ is somewhat smaller: $(0.88\ \mathrm{fm})^2$. The pion contribution to $\langle r^2\rangle$ accounts for 81%. It is not surprising that $\langle r^2\rangle_{I=1}$ is dominated by the pions. The factor $r^2 V_3{}^0$ in the integrand is very sensitive to the long-range part of the fields. Since the pion is the longest-range field in the problem, one expects it to make a large contribution.

The isovector magnetic moment operator is given by

$$\boldsymbol{\mu}_{I=1} = \int d^3r\, \mathbf{r}\times \mathbf{j}_{I=1} = \int d^3r\, \mathbf{r}\times\mathbf{V}_3 . \tag{5.14}$$

We can determine $\mu_{I=1}$ by evaluating the third component of $\boldsymbol{\mu}_{I=1}$ in a proton spin-up state, using the mean-field approximations:

$$\mu_{I=1} = \left\langle p\uparrow \left| \int d^3r\, \mathbf{r}\times\mathbf{V}_{3\,\mathrm{coll}} \right| p\uparrow \right\rangle .$$

Using the expression for $\mathbf{V}_{3\,\mathrm{coll}}$, (5.10), the fact that

$$\lambda_3 | p\uparrow\rangle = J_3/\mathscr{I}\, | p\uparrow\rangle = \tfrac{1}{2}\, | p\uparrow\rangle ,$$

and the expectation value for C_{3d} in $| p\uparrow\rangle$, (5.12), we obtain

$$\mu_{I=1} = \int dr\, r^2 (2N_c r GF/9 + 8\pi\phi_h{}^2/9) . \tag{5.15}$$

We notice that the meson contribution to $\mu_{I=1}$ is exactly one-third of the meson contribution to $\mathscr{I}$. This feature is also found in the Skyrme model.[4] However, in the Skyrme model the mesons account for the entire contributions for both $\mathscr{I}$ and $\mu_{I=1}$. We also notice that the quark contributions to $\mathbf{V}_{3\,\mathrm{coll}}$, and hence to $\mu_{I=1}$, depend only on the stationary hedgehog solution and not on the changes due to cranking. Thus the calculation of the isovector magnetic moment in Ref. 19 is correct despite the fact that cranking effects were ignored. Upon evaluating (5.15), we find $\mu_{I=1} = 4.00\mu_N$, which should be compared with an experimental value of $4.70\mu_N$. In our calculation, the pion contribution accounts for 56% of the total. The square of the isovector magnetic radius is given by

$$\langle r^2\rangle^m_{I=1}=\frac{1}{\mu_{I=1}}\int dr\, r^4(2N_c rGF/9+8\pi\phi_h{}^2/9) . \quad (5.16)$$

The quark and pion contributions to $\langle r^2\rangle^m_{I=1}$ are 0.321 and 0.925 fm^2, respectively, for a total of (1.12 fm)^2; the experimental value is (0.80 fm)^2. Again, we are not surprised to find that the mesons dominate the contributions to the magnetic square radius as the integrand is highly sensitive to the long-range parts of the fields.

Now let us turn our attention to the problem of calculating g_A. For a system with finite pion mass, the expression for g_A is

$$\tfrac{1}{2}g_A=\left\langle p\uparrow\left|\int d^3x\, A_3^z\right|p\uparrow\right\rangle , \quad (5.17)$$

where the axial-vector current is given by

$$A_a^\mu=\tfrac{1}{2}\bar\psi\gamma^\mu\gamma_5\tau_a\psi+(\hat\phi_a\partial^\mu\hat\sigma-\hat\sigma\partial^\mu\hat\phi_a) .$$

The collective expression for g_A is obtained straightforwardly using our mean-field methods:

$$g_A=\langle p\uparrow|C_{33}|p\uparrow\rangle\int dr\, r^2\left[-N_c(G^2-F^2/3)+\frac{8\pi}{3}(\phi_h\partial_r\sigma_h-\sigma_h\partial_r\phi_h-2\sigma_h\phi_h/r)\right] , \quad (5.18)$$

where $\langle p\uparrow|C_{33}|p\uparrow\rangle$ is $-\frac{1}{3}$ from Eq. (5.12). Numerically, we find that the quark and meson contributions to g_A are 0.667 and 0.748, respectively, for a total of 1.42. This can be compared with the experimental value of 1.26.

As noted by ANW, there is a subtlety in the chiral limit. For a system with m_π set to zero at the level of the Lagrangian, the three-dimensional integral used to obtain Eq. (5.18) is not absolutely convergent. However, because the pion pole goes to $q=0$, expression (5.18) no longer gives g_A. Instead, g_A is given by $\frac{3}{2}$ of the value in Eq. (5.18). A simple explanation of this factor of $\frac{3}{2}$ is given in Appendix B.

The calculation for $g_{\pi NN}$ for models with quarks exactly parallels the ANW treatment of the Skyrme model, as $g_{\pi NN}$ can be determined solely from the pion tail. Suppose one has an old-fashioned description of hadronic physics based on the interactions of nucleons and pions. Far from the nucleon, the pion field is given by

$$\langle N|\hat\phi_a(\mathbf{x})|N'\rangle=-\frac{g_{\pi NN}}{8\pi M_N}\langle N|\sigma_i\tau_a|N'\rangle\hat x_i(m_\pi+1/r) \times\exp(-m_\pi r)/r . \quad (5.19)$$

The soliton solution has a pion tail which at large distances decays like a p-wave Yukawa form:

$$\phi_a(\mathbf{x})=DC_{ai}\hat x_i(m_\pi+1/r)\exp(-m_\pi r)/r , \quad (5.20)$$

where C_{ai} is defined in (5.11) and the constant D can be read directly from the numerical solution. Using the wave functions in Eq. (4.10), one finds that $\langle N|C_{ai}|N'\rangle$ equals $-\langle N|\tau_a\sigma_i|N'\rangle/3$. By comparing Eqs. (5.20) and (5.19) and evaluating C_{ai} in the collective wave functions, one finds

$$g_{\pi NN}=8\pi DM_N/3 . \quad (5.21)$$

Using our solution to find D and a nucleon mass of 939 MeV, we obtain $g_{\pi NN}=14.9$; the experimental result is 13.5.

Of course, pion-nucleon scattering depends on q^2 and the coupling between N and π should be given by a form factor $g_{\pi NN}(q^2)$. The pion-nucleon coupling constant calculated in Eq. (5.21) corresponds to $g_{\pi NN}(q^2=-m_\pi^2)$. One can easily find the full form factor using the semiclassical methods of this paper by considering Fourier-Bessel transforms of the collective expression for the pion source. We do not do so as we do not believe the model to be very useful at large energies. We wish to observe, however, that any model that satisfies the PCAC (partial conservation of axial-vector current) relation, $\partial_\mu A_a^\mu=m_\pi^2F_\pi\hat\phi_a$, also satisfies the Goldberger-Treiman relation,[28] which gives $g_{\pi NN}(q^2=0)$ in terms of F_π, $M<$ and g_A:

$$g_{\pi NN}(q^2=0)=Mg_A/F_\pi . \quad (5.22)$$

The model considered here, of course, satisfies the PCAC relation at the quantum level. The mean-field expressions for A_a^μ also satisfy a classical version of the PCAC relation $\partial_\mu A^\mu_{a\,\mathrm{coll}}=m^2F_\pi\phi_a$. Thus, the Goldberger-Treiman relation is satisfied for the g_A and $g_{\pi NN}(q^2=0)$ as calculated semiclassically. Goldberger and Treiman give $g_{\pi NN}(q^2=0)=14.3$, which is 4% smaller than $g_{\pi NN}(q^2=-m_\pi{}^2)$.

Having calculated a number of properties for the nucleon, we now turn our attention to the properties of the Δ and N-Δ transition. Most of the properties we consider are related to the nucleon properties by model-independent relations.[4] These model-independent relations are analogous to the well-known relations of transition operators and moments in the conventional Bohr-Mottelson[29] treatment of deformed even-even nuclei and follow directly from the tensorial structure of the various operators considered and the fact the collective wave functions are Wigner D matrices. For some operators these relations are trivial. For example, the isoscalar g factor for Δ is identical to the nucleon value. The isovector magnetic properties are somewhat more interesting. The isovector magnetic transition matrix element is given by

$$\langle p\uparrow|\mu^3_{I=1}|\Delta^{++}s=1\rangle=-\sqrt{2}\langle p\uparrow|\mu^3_{I=1}|p\uparrow\rangle , \quad (5.23a)$$

while the Δ isovector magnetic moment is given by

$$\langle \Delta^{++} s=\tfrac{3}{2}\,|\,\mu^3_{I=1}\,|\,\Delta^{++} s=\tfrac{3}{2}\rangle=\tfrac{24}{5}\langle p\uparrow|\,\mu^3_{I=1}\,|\,p\uparrow\rangle\ . \tag{5.23b}$$

The π-N-Δ coupling constant is given by

$$g_{\pi N\Delta}=\tfrac{3}{2}g_{\pi NN}\ . \tag{5.24}$$

There is one interesting property of the N-Δ transition which cannot be related to nucleonic properties: namely, the electric quadrupole transition matrix element. Of course, the electric quadrupole moment for a nucleon vanishes as the quadrupole operator is a rank-two tensor under rotations, while the nucleon is a rank one-half. This is analogous to the quadrupole properties of deformed nuclei—the ground state is $J=0$ and thus has a vanishing quadrupole moment, but the deformed nature of the state guarantees a collective $E2$ transition from the ground state to the first $J=2$ state. For our system we can calculate the quadrupole transition by studying the quadrupole operator, which is given, in Cartesian coordinates, by

$$Q_{ij}=r^2(\hat{x}_i\hat{x}_j-\tfrac{1}{3})\rho\ , \tag{5.25}$$

where ρ is the electric charge density. Since the isoscalar part of the charge density is spherically symmetric only the isovector part will contribute. Using $\pi=V_3{}^0$ and the form in Eq. (5.10), we can obtain a collective form for Q_{ij}. Evaluating this in the collective wave functions of Eq. (4.10) yields

$$\langle p\uparrow|\,Q_{33}\,|\,\Delta^+ s=\tfrac{1}{2}\rangle =(\sqrt{2}/45\mathscr{I})\int dr\, r^4[N_c(BG-CF-DF)+4\pi\phi^2]\ . \tag{5.26a}$$

The other components of Q can be related to Q_{33}:

$$\langle p\uparrow|\,Q_{11}\,|\,\Delta^+ s=\tfrac{1}{2}\rangle=\langle p\uparrow|\,Q_{22}\,|\,\Delta^+ s=\tfrac{1}{2}\rangle =-\tfrac{1}{2}\langle p\uparrow|\,Q_{33}\,|\,\Delta^+ s=\tfrac{1}{2}\rangle\ ,$$
$$\langle p\uparrow|\,Q_{ij}\,|\,\Delta^+ s=\tfrac{1}{2}\rangle=0\quad \text{for } i\neq j\ . \tag{5.26b}$$

Numerically, we find the quark and meson contributions are 6.9×10^{-3} and 4.70×10^{-2} fm^2 for a total of 5.39×10^{-2} fm^2. Experimentally, it is known that the quadrupole transition matrix element is small. A recent theoretical attempt to extract the $E2$ resonant transition from the experimental data[30] gives an electric quadrupole partial decay width which corresponds to a quadrupole transition matrix element, which agrees, in turn, with our calculation in order of magnitude. This extraction from the data gives a negative relative sign of the electric quadrupole and magnetic dipole transition matrix elements, which agrees with the semiclassical result. Moreover, we note that this transition matrix element vanishes for spherical models of N's and Δ's, while in an intrinsically deformed description such as a hedgehog it is automatically nonzero. It is interesting to observe that the meson contribution of $\langle p\uparrow|\,Q_{33}\,|\,\Delta^+=\tfrac{1}{2}\rangle$ is equal to $\sqrt{2}/30$ times the meson contribution of $\langle r^2\rangle_{I=1}$, precisely the relation found in the Skyrme model.[31] Since the pion is light, the meson contribution dominates both $\langle p\uparrow|\,Q_{33}\,|\,\Delta^+ s=\tfrac{1}{2}\rangle$ and $\langle r^2\rangle_{I=1}$. Thus, even if one consider models with quarks there is an approximate relation:

$$\langle p\uparrow|\,Q_{33}\,|\,\Delta^+ s=\tfrac{1}{2}\rangle\simeq(\sqrt{2}/30)\langle r^2\rangle_{I=-1}\ . \tag{5.27}$$

The relation (5.27) is in some sense model independent—it depends only on the pions dominating both $\langle r^2\rangle_{I=1}$ and one quadrupole transition matrix element. We note that as one approaches the chiral limit both the quadrupole transition and the isovector square radius diverge and relation (5.27) approaches an equality. For the model considered here, the right-hand side of (5.27) is 7% larger than the left-hand side.

Finally, let us consider the energy of N's and Δ's. The Hamiltonian corresponding to the Lagrangian in Eq. (2.1) is

$$H=\int d^3x\{\psi^\dagger[-i\boldsymbol{\alpha}\cdot\nabla-g\beta(\hat{\sigma}+i\boldsymbol{\tau}\cdot\hat{\boldsymbol{\phi}}\gamma_5)]\psi +\tfrac{1}{2}(\nabla\hat{\boldsymbol{\phi}})^2+\tfrac{1}{2}(\nabla\hat{\sigma})^2 +\tfrac{1}{2}\hat{\Pi}_\sigma{}^2+\tfrac{1}{2}\hat{\Pi}_\phi{}^2+U(\hat{\sigma},\hat{\boldsymbol{\phi}})\}\ , \tag{5.28}$$

where $\hat{\Pi}_\sigma$ and $\hat{\Pi}_\phi$ are the momenta conjugate to $\hat{\sigma}$ and $\hat{\boldsymbol{\phi}}$. We obtain a collective expression for H in the usual fashion and obtain

$$H_{\text{coll}}=E_h+\mathscr{I}\,\boldsymbol{\lambda}\cdot\boldsymbol{\lambda}/2\ , \tag{5.29a}$$

where E_h is the mean-field expression for the energy in the static hedgehog solution:

$$E_h=\int d^3x\{N_c q_h^\dagger[-i\boldsymbol{\alpha}\cdot\nabla-g\beta(\hat{\sigma}+i\boldsymbol{\tau}\cdot\hat{\boldsymbol{\phi}}\gamma_5)]q_h +\tfrac{1}{2}(\nabla\boldsymbol{\phi})^2+\tfrac{1}{2}(\nabla\hat{\sigma}_h)^2+U(\sigma_h,\boldsymbol{\phi}_h)\}\ . \tag{5.29b}$$

If one were to follow the semiclassical projection method as outlined thus far, one obtains the following expression for the masses of the N's and Δ's:

$$M_N=E_h+J_N(J_N+1)/2=E_h+3/8\mathscr{I}\ ,$$
$$M_\Delta=E_h+J_\Delta(J_\Delta+1)/2=E_h+15/8\mathscr{I}\ . \tag{5.30a}$$

In Eqs. (5.30a) there appears to be a clean separation of the energy into a static contribution from the hedgehog and a contribution from the collective rotation. In fact, this is somewhat misleading. Some of the rotational kinetic energy is contained in E_h. Recall that the mean-field treatment of quarks is not completely classical. There is some quantal information contained in q_h. In particular, we note that the quark contribution to the mean-field expectation value of J^2 does not vanish in the static hedgehog solution, even though the expectation value of $\mathbf{J}$ is zero. In contrast, the meson contribution to J^2 is zero with our mean-field approximations. To avoid double counting of the quark rotational kinetic energy, one should subtract from the masses in Eq. (5.30a) the rotational energy of quarks in the ground state. We can approximate this energy by $\langle J_q{}^2\rangle_h/2\mathscr{I}$, where $\langle J_q{}^2\rangle_h$ is the expectation of the quark contribution to J^2 in the

TABLE I. Summary of physical quantities for the chiral quark-meson model of Ref. 11 as calculated using the semiclassical projection method. For comparison, the experimental results are also listed.

Quantity	Semiclassical calculation	Experiment
$\langle r^2\rangle_{I=0}$	$(0.70\ \mathrm{fm})^2$	$(0.72\ \mathrm{fm})^2$
$\mu_{I=0}\ (\mu_N)$	0.381	0.88
$\langle r^2\rangle^m_{I=0}$	$(0.91\ \mathrm{fm})^2$	$(0.81\ \mathrm{fm})^2$
$\langle r^2\rangle_{I=1}$	$(1.11\ \mathrm{fm})^2$	$(0.88\ \mathrm{fm})^2$
$\mu_{I=1}\ (\mu_N)$	4.00	4.70
$\langle r^2\rangle^m_{I=1}$	$(1.12\ \mathrm{fm})^2$	$(0.80\ \mathrm{fm})^2$
g_A	1.42	1.26
$g_{\pi NN}$	14.9	13.5
$\langle p\uparrow \mid \mu^3_{I=1} \mid \Delta^+ s=\frac{1}{2}\rangle\ (\mu_N)$	-2.8	3.3
$\langle p\uparrow \mid Q_{33} \mid \Delta^+ s=\frac{1}{2}\rangle\ (\mathrm{fm}^2)$	$+5.4$	
$(M_N+M_\Delta)/2$ (MeV)	1119	1086
M_N-M_Δ (MeV)	253	293

hedgehog state. This subtraction of $\langle J_q{}^2\rangle_h/2\mathscr{I}$ is completely analogous to subtractions of $\langle J^2\rangle/2\mathscr{I}$ from Hartree-Fock energies of deformed nuclei to obtain bandhead energies. For the Skyrme model there are no quarks and $\langle J_q{}^2\rangle_h$ is obviously zero. For models with three valence quarks, such as the ones being discussed here, $\langle J_q{}^2\rangle$ equals $\frac{9}{4}$. Thus, the masses for N and Δ become

$$M_N=E_N+J_N(J_N+1)/2\mathscr{I}-\langle J_q^2\rangle_h/2\mathscr{I}=E_h-3/4\mathscr{I}\ , \tag{5.30b}$$

$$M_\Delta=E_h+J_\Delta(J_\Delta+1)/2\mathscr{I}-\langle J_q{}^2\rangle_h/2\mathscr{I}=E_h+3/4\mathscr{I}\ .$$

We see that arithmetic mean of M_N and M_Δ is given by E_h, while the N-Δ mass splitting is given by $3/2\mathscr{I}$. Numerically, we find an average N-Δ mass of 1119 MeV and a splitting of 252 MeV. This should be compared with an observed average mass of 1086 MeV and a splitting of 293 MeV. The calculated and experimental properties of N and Δ are summarized in Table I.

All of the calculations in this paper were based on the semiclassical motion of three valence quarks interacting with mesons. We have, without justification, ignored the effects of sea quarks. It is straightforward, in principle, to extend the approach developed here to include the effects of sea quarks. The numerical work for such a computation will be substantially greater than for the calculations reported here. We note that the effects of sea quarks become particularly important when applied to chiral bag models. In the recent interpretation of chiral bags sea-quark effects are of critical importance.[7-12]

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APPENDIX A: GRANT REVERSAL

In this appendix the grand-reversal symmetry will be reviewed and it will be shown how this symmetry leads to a $\delta\delta=\delta\phi=0$ to first order. The grand-reversal operator consists of time reversal and a simultaneous isorotation about the 2 axis through π. Formally, the grand-reversal operator, $\hat{R}$, operating on a state specified by the quantum numbers L,J,I,M_j,m_i has the following effect:

$$\hat{R}\mid L,J,I,m_j,j_i\rangle=(-1)^c\mid L,J,I,-m_j,m_i\rangle\ , \tag{A1}$$

where, in the Condon-Shortely phase convention, $c=L,\ +J+I-m_j-m_i$. The effect of grand reversal on the various field operators can easily be deduced:

$$\begin{aligned}&\psi(\mathbf{x},t)\rightarrow\sigma_2\tau_2\psi(\mathbf{x},-t)\mathscr{K}\ ,\\ &\hat{\sigma}(\mathbf{x},t)\rightarrow\hat{\sigma}(\mathbf{x},-t)\ ,\\ &\hat{\boldsymbol{\phi}}(\mathbf{x},t)\rightarrow\hat{\boldsymbol{\phi}}(\mathbf{x},-t)\ .\end{aligned} \tag{A2}$$

From Eq. (A2) one finds the effect of grand reversal on our mean fields:

$$\begin{aligned}&q(\mathbf{x},t)\rightarrow\sigma_2\tau_2 q^*(\mathbf{x},-t)\ ,\\ &\sigma(\mathbf{x},t)\rightarrow\sigma(\mathbf{x},-t)\ .\\ &\boldsymbol{\phi}(\mathbf{x},t)\rightarrow\boldsymbol{\phi}(\mathbf{x},-t)\ .\end{aligned} \tag{A3}$$

In Eq. (A3) we see that the meson mean fields are even under grand-reversal transformations.

From the properties of Eq. (A3) we see easily that the static hedgehog solution is even under grand reversal. Let us now consider the set of equations for the mean fields in the rotating frame (3.4). All terms in the these equations are even under grand reversal except the $\boldsymbol{\lambda}\cdot\boldsymbol{\tau}/2$ term in the quark equation, which is odd. In our perturbative calculation for $\delta\sigma$ and $\delta\boldsymbol{\phi}$, it is precisely the $\boldsymbol{\lambda}\cdot\boldsymbol{\tau}/2$ term which we are treating as a perturbation. Thus, to first order, $\delta\sigma$ and $\delta\boldsymbol{\phi}$ must be odd under grand reversal. However, from Eq. (A3) we see that $\sigma=\sigma_h+\delta\sigma$ and $\boldsymbol{\phi}=\boldsymbol{\phi}_h+\delta\boldsymbol{\phi}$ are even; since σ_h and $\boldsymbol{\phi}_h$ are even, we see that $\delta\sigma$ or $\delta\boldsymbol{\phi}$ must also be even. For any nonzero $\delta\sigma$ or $\delta\boldsymbol{\phi}$, there is a contradiction; thus $\delta\sigma$ and $\delta\boldsymbol{\phi}$ must vanish.

A more tedious way to see that $\delta\sigma$ and $\delta\boldsymbol{\phi}$ vanish is to simply expand Eqs. (3.4) without regard for grand reversal. After some long but straightforward algebra one finds that the first-order shifts are given by $\delta\sigma=\delta\boldsymbol{\phi}=0$ and δq is given by Eq. (3.9).

APPENDIX B: g_A AND THE CHIRAL LIMIT

In this appendix we will explain the origin of the factor of $\frac{3}{2}$ which must be included in Eq. (5.18) to give g_A correctly for systems with $m_\pi=0$ in the Lagrangian. We will calculate the integral which gives g_A as a function of m_π and show that there is a discontinuity at $m_\pi=0$. The limit of this integral in the limit of m_π going to 0^+ will be shown to be $\frac{3}{2}$ times the integrals with $m_\pi=0$.

The semiclassical expression for g_A is

$$\tfrac{1}{2}g_A=\int d^3x\,\langle p\uparrow\mid(A_{3\,\mathrm{coll}})^z\mid p\uparrow\rangle\ . \tag{B1}$$

One can divide the integral into two regions: $r<R$ and

$r>R$, where R is an arbitrary large radius chosen so that for $r>R$ all of the fields have essentially reached their asymptotic forms. The discontinuity comes from the $r>R$ region. Clearly, in this asymptotic region, $A_{3\,\mathrm{coll}}$ is completely dominated by the pions which are the lightest mass fields in the problem; σ will be at its vacuum expectation value of $-F_\pi$ and the quarks will not contribute. Thus, in the asymptotic region

$$\langle p\uparrow|(A_{3\,\mathrm{coll}})^2|p\uparrow\rangle=F_\pi\langle p\uparrow|\partial^2\phi_{3\,\mathrm{coll}}|p\uparrow\rangle\ . \tag{B2}$$

The asymptotic form of the pion field in proton state for $m_\pi\neq 0$ can be obtained from Eqs. (5.20) and (5.12):

$$\langle p\uparrow|\phi_{3\,\mathrm{coll}}|p\uparrow\rangle=-(D/3)\hat{x}_3(m_\pi+1/r)\exp(-m_\pi r)/r\ . \tag{B3}$$

From Eqs. (B2) and (B3) we can evaluate the integral in the asymptotic region:

$$\begin{aligned}\int_{r>R} d^3x\,\langle p\uparrow|(A_{3\,\mathrm{coll}})^z|p\uparrow\rangle &=\frac{4\pi D F_\pi m_\pi{}^2}{9}\int_R^\infty r^2dr\,\exp(-m_\pi r)/r\\ &=-\frac{4\pi D F_\pi}{9}[\exp(-m_\pi r)+Rm_\pi\exp(-m_\pi r)]_R^\infty\ .\end{aligned} \tag{B4}$$

This gives a value of $4\pi DF_\pi(1+m_\pi R)\exp(-m_\pi/R)/9$. Taking the $m_\pi\rightarrow 0$ limit, one finds

$$\lim m_\pi\rightarrow 0\int_{r>R} d^3x\,\langle p\uparrow|(A_{3\,\mathrm{coll}})^2|p\uparrow\rangle=4\pi DF_\pi/9\ . \tag{B5a}$$

On the other hand, if one evaluates (B4) with m_π set to zero at the outset, one finds

$$\int_{r>R} d^3x\,\langle p\uparrow|(A_{3\,\mathrm{coll}})^z|p\uparrow\rangle=0\ . \tag{B5b}$$

The difference between Eqs. (B5a) and (B5b) is precisely the discontinuity we seek. It is fairly clear what is happening. A finite amount of the contributions to the integral in (B1) come from distance scales of $1/m_\pi$. If one sets m_π to zero in the Lagrangian, this contribution is lost, yielding the discontinuity.

For the chiral limit one can use an argument of ANW to express the full space integral of $(A_{3\,\mathrm{coll}})^z$ in terms of the pionic tail. The divergence of the axial-vector current vanishes in the chiral limit and one can use the divergence theorem

$$\begin{aligned}\int d^3x(A_{3\,\mathrm{coll}})^2 &=\int d^3x\,\partial_i[x_3(A_{3\,\mathrm{coll}})^i]\\ &=\int_S x_3\mathbf{A}_{3\,\mathrm{coll}}\cdot d\mathbf{S}\ .\end{aligned} \tag{B6}$$

Using the asymptotic form for $\mathbf{A}_{3\,\mathrm{coll}}$, one obtains $8\pi DF_\pi/9$. Note that the discontinuity in Eq. (B5) is $4\pi DF_\pi/9$. Thus if one wishes g_A to be a continuous function of m_π, one must multiply the result of Eq. (B6) [or equivalently, Eq. (5.18)] by $\frac{3}{2}$.

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