On muon decay in left-right-symmetric electroweak models

P. Herczeg
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
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We investigate the implications of a recent measurement of the positron-momentum-spectrum end point in polarized muon decay for general $SU(2)_L \times SU(2)_R \times U(1)$ electroweak models.

I. INTRODUCTION

The main decay mode of the muon is an important source of information on the structure of the leptonic interactions. The dominant interaction responsible for the decay is known to have a $V-A$ structure.[1] In the minimal standard model of the electroweak interactions[2] the decay is entirely due to such an interaction. Although the minimal standard model is consistent at present with all data, for many theoretical reasons it cannot be viewed as a complete theory. This situation led to the formulation of various extensions of the model. In many theoretical schemes that go beyond the minimal standard model, the main decay mode of the muon receives contributions from interactions whose structure is different from $V-A$. This inspired new efforts to improve the existing accuracy of muon-decay experiments.[3]

One of the recent experimental results comes from a precise measurement of the positron momentum spectrum end point in polarized μ^+ decay.[4,5] This result was interpreted[4–6] in terms of the parameters of some special versions of $SU(2)_L \times SU(2)_R \times U(1)$ electroweak models.[7] In this paper we analyze the implications of the experimental results of Refs. 4 and 5 for more general realizations of $SU(2)_L \times SU(2)_R \times U(1)$ models, including the most general one.[8] For each scenario we compare the resulting constraints on the pertinent parameters with the constraints provided on them by other data.

In the next section we describe the experimental results of Refs. 4 and 5. Section III is a brief review of the relevant aspects of $SU(2)_L \times SU(2)_R \times U(1)$ models. In Sec. IV we study the constraints imposed on the parameters of various versions of $SU(2)_L \times SU(2)_R \times U(1)$ models by the experimental results of Refs. 4 and 5. In Sec. V we summarize our conclusions.

II. THE EXPERIMENTAL RESULT ON THE POSITRON-MOMENTUM-SPECTRUM END POINT

The energy-angle distribution of positrons from polarized μ^+ decays at rest is of the form[9,10]

$$d\Gamma(x,\theta)=\frac{d^3p}{(2\pi)^4}\frac{m_\mu E_0}{12}A[N(x)-P(x)P_\mu\cos\theta + \text{radiative corrections}] , \quad (1)$$

where p and E are the positron momentum and energy, E_0 is the maximum positron energy, $x=E/E_0$, and θ is the angle between the positron momentum and the spin direction of μ^+. $(-P_\mu)$ is the degree of longitudinal polarization of the μ^+ at the instant of μ^+ decay. The constant A is related to the muon lifetime. $N(x)$, $P(x)$, and A depend on the parameters of the underlying theory.

The experiments of Refs. 4 and 5 determined the quantity

$$w' \equiv -P_\mu \lim_{x\to 1}\left[\frac{P(x)+\text{radiative corrections}}{N(x)+\text{radiative corrections}}\right] \quad (2)$$

in the ratio

$$R'(1,\theta)\equiv \lim_{x\to 1} R'(x,\theta)=1+w'\cos\theta \quad (3)$$

of the full positron spectrum and the part of the spectrum independent of P_μ near the end point. w' was measured using two different techniques. In the first experiment[4] the positron spectrum was measured near the end point and for momenta in the direction opposite to the direction of the μ^+ spin, with the muon spin held by a longitudinal magnetic field. The second experiment[5] measured the positron-spectrum asymmetry above $x=0.88$ using a muon-spin-rotation technique.

The combined result of the two experiments is[11]

$$w>0.997\,53 \text{ (90\% confidence level)} \quad (4)$$

or equivalently

$$R\equiv R(1,\pi)<0.002\,47 \text{ (90\% confidence level)} . \quad (5)$$

In Eqs. (4) and (5) w and R are the quantities w' and R' with the radiative corrections and the effects of the electron mass subtracted. The result (5) is consistent with the prediction $R=0$ of the minimal standard model.

III. $SU(2)_L \times SU(2)_R \times U(1)$ ELECTROWEAK MODELS

The contrast between the $V-A$ structure of the charged-current weak interactions and the vector nature of the electromagnetic and strong interactions is a puzzling aspect of the fundamental interactions. An intriguing possibility is that the observed $V-A$ structure of the charged-current weak interactions is only approximate and that in reality both $V-A$ and $V+A$ currents participate. A model involving both $V-A$ and $V+A$ currents was suggested before the advent of gauge theories by Lipmanov.[12] In his model the $V-A$ and the $V+A$ currents are coupled to distinct vector-boson fields. Parity violation appears as a consequence of a difference in the masses of the two vector bosons.

The simplest viable gauge theory that leads to a structure analogous to the Lipmanov model requires $SU(2)_L\times SU(2)_R\times U(1)$ as the gauge group. $SU(2)_L\times SU(2)_R\times U(1)$ models of the weak and electromagnetic interactions[13] emerged first in the framework of a class of grand unified theories.[14]

In $SU(2)_L\times SU(2)_R\times U(1)$ electroweak models the fermions are assigned to representations of the group in a left-right-symmetric manner: the left- [right-] handed fermions are doublets of $SU(2)_L$ [$SU(2)_R$] and singlets of $SU(2)_R$ [$SU(2)_L$]:[15]

quarks,

$$\begin{bmatrix} u' \\ d' \end{bmatrix}_L, \begin{bmatrix} c' \\ s' \end{bmatrix}_L, \ldots, (T_L T_R Y)=(\tfrac{1}{2}0\tfrac{1}{3}) ,$$
$$\begin{bmatrix} u' \\ d' \end{bmatrix}_R, \begin{bmatrix} c' \\ s' \end{bmatrix}_R, \ldots, (T_L T_R Y)=(0\tfrac{1}{2}\tfrac{1}{3}) ; \quad (6)$$

leptons,

$$\begin{bmatrix} \nu_e' \\ e' \end{bmatrix}_L, \begin{bmatrix} \nu_\mu' \\ \mu' \end{bmatrix}_L, \ldots, (T_L T_R Y)=(\tfrac{1}{2}0-1) ,$$
$$\begin{bmatrix} \nu_e' \\ e' \end{bmatrix}_R, \begin{bmatrix} \nu_\mu' \\ \mu' \end{bmatrix}_R, \ldots, (T_L T_R Y)=(0\tfrac{1}{2}-1) , \quad (7)$$

where $\mathbf{T}_L$, $\mathbf{T}_R$, and Y are the generators of $SU(2)_L\times SU(2)_R\times U(1)$. The corresponding coupling constants are g_L, g_R, and g'. $SU(2)_L$ and $SU(2)_R$ generate left-handed $(V-A)$ and right-handed $(V+A)$ interactions, respectively. The model contains four charged gauge bosons [$W_1^\pm, W_2^\pm$; see Eq. (11)], the photon, and two massive neutral gauge bosons. Dirac fermion masses are generated by nonzero vacuum expectation values of Higgs fields (one or more) of the type $\phi(\tfrac{1}{2}\tfrac{1}{2}0)$. Additional Higgs fields must be introduced to break the gauge symmetry down to $U_{EM}(1)$. A possible choice is to add the triplet fields $\Delta_L(102)$ and $\Delta_R(012)$, which can also generate Majorana mass terms for the neutrinos.[16]

In Eqs. (6) and (7) the primed fields are the gauge-group eigenstates. They are linear combinations of the mass eigenstates, $u,d,\ldots,e,\mu,\ldots,\nu_1,\nu_2,\ldots$. In terms of the mass eigenstates the couplings of the charged gauge-boson fields W_L and W_R to the fermions can be written as

$$L=\frac{g_L}{2\sqrt{2}}W_L(\bar{P}\Gamma_L U_L N+\bar{N}^{(0)}\Gamma_L U^\dagger E) + \frac{g_R}{2\sqrt{2}}W_R(\bar{P}\Gamma_R U_R N+\bar{N}^{(0)}\Gamma_R V^\dagger E)+\text{H.c.} , \quad (8)$$

where $\Gamma_L=\gamma_\lambda(1-\gamma_5)$, $\Gamma_R=\gamma_\lambda(1+\gamma_5)$ (the Dirac indices have been suppressed), and

$$P=\begin{bmatrix} u \\ c \\ \vdots \end{bmatrix}, \quad N=\begin{bmatrix} d \\ s \\ \vdots \end{bmatrix} , \quad (9)$$

$$E=\begin{bmatrix} e \\ \mu \\ \vdots \end{bmatrix}, \quad N^{(0)}=\begin{bmatrix} \nu_1 \\ \nu_2 \\ \vdots \end{bmatrix} . \quad (10)$$

The fields W_L and W_R are linear combinations of the mass eigenstates W_1 and W_2:

$$W_L=\cos\zeta W_1+\sin\zeta W_2 ,$$
$$W_R=e^{i\omega}(-\sin\zeta W_1+\cos\zeta W_2) , \quad (11)$$

where ζ is a mixing angle and ω is a *CP*-violating phase. The matrices U_L and U_R are $n\times n$ unitary matrices (n is the number of generations). U_L and U_R contain (together) $n(n-1)$ mixing angles and n^2-n+1 *CP*-violating phases. For three generators their general form is[17]

$$U_L=\begin{bmatrix} c_1^L & -s_1^L c_3^L & -s_1^L s_3^L \\ s_1^L c_2^L & c_1^L c_2^L c_3^L - s_2^L s_3^L e^{i\delta_L} & c_1^L c_2^L s_3^L + c_3^L s_2^L e^{i\delta_L} \\ s_1^L s_2^L & c_1^L c_3^L s_2^L + c_2^L s_3^L e^{i\delta_L} & c_1^L s_2^L s_3^L - c_2^L c_3^L e^{i\delta_L} \end{bmatrix} , \quad (12)$$

$$U_R=\begin{bmatrix} e^{i\alpha}c_1^R & -e^{i\beta}s_1^R c_3^R & e^{-i\rho}s_1^R s_3^R \\ e^{-i\gamma}s_1^R c_2^R & e^{i(\beta-\alpha-\gamma)}(c_1^R c_2^R c_3^R - s_2^R s_3^R e^{i\delta_R}) & e^{-i(\alpha+\gamma+\rho)}(c_1^R c_2^R s_3^R + c_3^R s_2^R e^{i\delta_R}) \\ e^{-i\eta}s_1^R s_2^R & e^{i(\beta-\alpha-\eta)}(c_1^R c_3^R s_2^R + c_2^R s_3^R e^{i\delta_R}) & e^{-i(\alpha+\eta+\rho)}(c_1^R s_2^R s_3^R - c_2^R c_3^R e^{i\delta_R}) \end{bmatrix} , \quad (13)$$

where $s_k^{L,R}\equiv\sin\theta_k^{L,R}$ and $c_k^{L,R}\equiv\cos\theta_k^{L,R}$. The matrices (12) and (13) contain six mixing angles and seven *CP*-violating phases.

If the neutrinos are Dirac fermions, U and V are $n\times n$ unitary matrices that can be parametrized in the same way as the matrices U_L and U_R. Together they contain $n(n-1)$ mixing angles and n^2-n+1 *CP*-violating phases. In general, both Dirac and Majorana mass terms are present in the Lagrangian. The mass eigenstates are then $2n$ Majorana neutrinos,[18] so that U and V are $n\times 2n$ matrices. The $2n\times 2n$ matrix[19]

$$\begin{bmatrix} U \\ V^* \end{bmatrix} \quad (14)$$

is unitary. The matrix (14) contains $n(2n-1)$ mixing angles and $2n^2$ *CP*-violating phases.[20] In what follows, the explicit forms of the matrices U, V will not be needed.

The effective Hamiltonian for muon decay resulting from Eq. (8) (Ref. 21) is given by

$$\begin{aligned} H^{(\mu)} = {} & c_{LL}\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e^{(L)}\bar{\nu}_\mu^{(L)}\gamma^\lambda(1-\gamma_5)\mu \\ & + c_{RR}\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e^{(R)}\bar{\nu}_\mu^{(R)}\gamma^\lambda(1+\gamma_5)\mu \\ & + c_{LR}\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e^{(L)}\bar{\nu}_\mu^{(R)}\gamma^\lambda(1+\gamma_5)\mu \\ & + c_{RL}\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e^{(R)}\bar{\nu}_\mu^{(L)}\gamma^\lambda(1-\gamma_5)\mu + \text{H.c.} , \end{aligned} \tag{15}$$

where

$$\begin{aligned} c_{LL} &= \frac{g_L{}^2}{8m_1{}^2}\cos^2\zeta + \frac{g_L{}^2}{8m_2{}^2}\sin^2\zeta , \\ c_{RR} &= \frac{g_R{}^2}{8m_1{}^2}\sin^2\zeta + \frac{g_R{}^2}{8m_2{}^2}\cos^2\zeta , \\ c_{LR} &= \left[-\frac{g_L g_R}{8m_1{}^2} + \frac{g_L g_R}{8m_2{}^2} \right] \sin\zeta\cos\zeta e^{i\omega} , \\ c_{RL} &= \left[-\frac{g_L g_R}{8m_1{}^2} + \frac{g_L g_R}{8m_2{}^2} \right] \sin\zeta\cos\zeta e^{-i\omega} = c_{LR}^* \end{aligned} \tag{16}$$

(m_1 and m_2 are the masses of W_1 and W_2), and

$$\nu_l^{(L)} = \sum_j U_{lj}\nu_j \quad (l=e,\mu) , \tag{17}$$

$$\nu_l^{(R)} = \sum_j V_{lj}\nu_j \quad (l=e,\mu) . \tag{18}$$

Note that $|c_{RL}| = |c_{LR}|$.

For the ensuing discussion we shall also need the effective Hamiltonian for $\Delta S = 0$ semileptonic processes. From Eq. (8) (Ref. 21) one obtains

$$\begin{aligned} H^{(l)}_{\Delta S=0} = {} & a_{LL}\bar{l}\gamma_\lambda(1-\gamma_5)\nu_l^{(L)}\bar{u}\gamma^\lambda(1-\gamma_5)d \\ & + a_{RR}\bar{l}\gamma_\lambda(1+\gamma_5)\nu_l^{(R)}\bar{u}\gamma^\lambda(1+\gamma_5)d \\ & + a_{LR}\bar{l}\gamma_\lambda(1-\gamma_5)\nu_l^{(L)}\bar{u}\gamma^\lambda(1+\gamma_5)d \\ & + a_{RL}\bar{l}\gamma_\lambda(1+\gamma_5)\nu_l^{(R)}\bar{u}\gamma^\lambda(1-\gamma_5)d + \text{H.c.} , \end{aligned} \tag{19}$$

($l=e,\mu$), where

$$\begin{aligned} a_{LL} &= c_{LL}\cos\theta_1^L , \\ a_{RR} &= c_{RR}e^{i\alpha}\cos\theta_1^R , \\ a_{LR} &= c_{LR}e^{i\alpha}\cos\theta_1^R , \\ a_{RL} &= c_{RL}\cos\theta_1^L . \end{aligned} \tag{20}$$

In Eq. (20) α is a *CP*-violating phase from U_R [see Eq. (13)].

IV. IMPLICATIONS OF THE EXPERIMENTAL BOUND ON R

In a general $SU(2)_L \times SU(2)_R \times U(1)$ model the right-handed mixing angles, the *CP*-violating phases, and the leptonic mixing matrices U and V are arbitrary. Also, the neutrino mass eigenstates can be Dirac or Majorana fermions. Before considering the general case, we shall discuss the implications of the bound (5) in several special cases of $SU(2)_L \times SU(2)_R \times U(1)$ models, characterized by specific assumptions about the unknown quantities and the nature of the neutrino states. In all the cases discussed below we make the restrictive assumption that the masses of the neutrinos that can be produced in μ decay are sufficiently small that their effect on the spectrum can be neglected.[22]

A. Dirac neutrinos, no mixing in the leptonic sector

In this case the states (17) and (18) are given by

$$\begin{aligned} \nu_e^{(L)} &= U_{e1}\nu_1, \quad \nu_e^{(R)} = V_{e1}\nu_1 , \\ \nu_\mu^{(L)} &= U_{\mu 2}\nu_2, \quad \nu_\mu^{(R)} = V_{\mu 2}\nu_2 . \end{aligned} \tag{21}$$

Unitarity of U and V implies that $|U_{e1}| = |U_{\mu 2}| = |V_{e1}| = |V_{\mu 2}| = 1$. There is only one decay channel: $\mu^+ \to e^+ + \nu_1 + \bar{\nu}_2$. The functions $N(x)$ and $P(x)$ in Eq. (1) have the familiar form[9]

$$\begin{aligned} N(x) = {} & 6(1-x) + 4\rho\left[\tfrac{4}{3}x - 1 - \frac{1}{3}\frac{m_e{}^2}{E_0{}^2 x}\right] \\ & + 6\eta\frac{m_e}{E_0}\left[\frac{1-x}{x}\right] , \end{aligned} \tag{22}$$

$$P(x) = -\frac{p}{E}\xi\left[2(1-x) + 4\delta\left[\tfrac{4}{3}x - 1 - \frac{1}{3}\frac{m_e{}^2}{m_\mu E_0}\right]\right] ,$$

with[23]

$$\rho = \frac{3}{4}\frac{1+|\kappa_{RR}|^2}{1+|\kappa_{RR}|^2+|\kappa_{LR}|^2+|\kappa_{RL}|^2} , \tag{23}$$

$$\eta = 0 , \tag{24}$$

$$\xi = \frac{1-|\kappa_{RR}|^2+3(|\kappa_{LR}|^2-|\kappa_{RL}|^2)}{1+|\kappa_{RR}|^2+|\kappa_{LR}|^2+|\kappa_{RL}|^2} , \tag{25}$$

and

$$\delta = \frac{3}{4}\frac{1-|\kappa_{RR}|^2}{1-|\kappa_{RR}|^2+3(|\kappa_{LR}|^2-|\kappa_{RL}|^2)} , \tag{26}$$

where we have denoted

$$\kappa_{ik} = c_{ik}/c_{LL} \quad (ik = RR, LR, RL) . \tag{27}$$

The constant A is given by

$$A = 16|c_{LL}|^2(1+|\kappa_{RR}|^2+|\kappa_{LR}|^2+|\kappa_{RL}|^2) . \tag{28}$$

Equations (23), (25), (26), and (28) could have been simplified using the relation $|\kappa_{RL}| = |\kappa_{LR}|$, but we shall keep them general for future reference.

The muon polarization is given by[24]

$$P_\mu = \frac{|1-\eta_{LR}|^2 - |\eta_{RR}-\eta_{RL}|^2}{|1-\eta_{LR}|^2 + |\eta_{RR}-\eta_{RL}|^2} , \tag{29}$$

where

$$\eta_{ik} = a_{ik}/a_{LL} \quad (ik = RR, LR, RL) . \tag{30}$$

For the quantity R [defined in Eq. (5)] one obtains

$$R = 1 - \frac{\delta\xi}{\rho} P_\mu \ . \tag{31}$$

Equations (23), (25), and (26) yield

$$R = 1 - \frac{1-|\kappa_{RR}|^2}{1+|\kappa_{RR}|^2} P_\mu \ . \tag{32}$$

In the following we shall expand the quantities $P_\mu, \rho, \xi, \ldots$ in terms of the parameters η_{ik} and κ_{ik}, neglecting terms higher than second order.[25] In addition, we shall assume that one can neglect m_1^2/m_2^2 relative to one, and $\tan^2\zeta$ relative to m_1^2/m_2^2, and also that $\tan\zeta \simeq \zeta$ (Ref. 26). Introducing the notation

$$t \equiv \frac{g_R^2 m_1^2}{g_L^2 m_2^2} \ , \tag{33}$$

$$t_\theta \equiv \frac{g_R^2 m_1^2 \cos\theta_1^R}{g_L^2 m_2^2 \cos\theta_1^L} \ , \tag{34}$$

and

$$\zeta_g \equiv \frac{g_R}{g_L} \zeta \ , \tag{35}$$

we have

$$\kappa_{RR} = \frac{g_R^2}{g_L^2} \frac{(m_1^2/m_2^2) + \tan^2\zeta}{1+(m_1^2/m_2^2)\tan^2\zeta} \simeq t \ , \tag{36}$$

$$\kappa_{LR} = -\frac{g_R}{g_L} \frac{(1-m_1^2/m_2^2)\tan\zeta}{1+(m_1^2/m_2^2)\tan^2\zeta} e^{i\omega} \simeq -\zeta_g e^{i\omega} \ , \tag{37}$$

$$\kappa_{RL} = -\frac{g_R}{g_L} \frac{(1-m_1^2/m_2^2)\tan\zeta}{1+(m_1^2/m_2^2)\tan^2\zeta} e^{-i\omega} \simeq -\zeta_g e^{-i\omega} \ , \tag{38}$$

$$\eta_{RR} \simeq t_\theta e^{i\alpha} \ , \tag{39}$$

$$\eta_{LR} \simeq -\zeta_g \frac{\cos\theta_1^R}{\cos\theta_1^L} e^{i(\alpha+\omega)} \ , \tag{40}$$

and

$$\eta_{RL} \simeq -\zeta_g e^{-i\omega} \ . \tag{41}$$

The spectrum parameters and P_μ are given by

$$\rho \simeq \tfrac{3}{4}(1-|\kappa_{LR}|^2 - |\kappa_{RL}|^2) \simeq \tfrac{3}{4}(1-2\zeta_g^2) \ , \tag{42}$$

$$\begin{aligned}\xi &\simeq 1 - 2|\kappa_{RR}|^2 + 2|\kappa_{LR}|^2 - 4|\kappa_{RL}|^2 \\ &\simeq 1 - 2(t^2 + \zeta_g^2) \ ,\end{aligned} \tag{43}$$

$$\delta[\simeq \tfrac{3}{4}(1-3|\kappa_{LR}|^2 + 3|\kappa_{RL}|^2)] = \tfrac{3}{4} \ , \tag{44}$$

$$\delta\xi/\rho \simeq 1 - 2|\kappa_{RR}|^2 \simeq 1 - 2t^2 \ , \tag{45}$$

and

$$\begin{aligned}P_\mu &\simeq 1 - 2|\eta_{RR}|^2 - 2|\eta_{RL}|^2 + 4\,\mathrm{Re}\,\eta_{RR}\eta_{RL}^* \\ &\simeq 1 - 2t_\theta^2 - 2\zeta_g^2 - 4t_\theta\zeta_g\cos(\alpha+\omega) \ .\end{aligned} \tag{46}$$

For the quantity R we find

$$R \simeq 2t^2 + 2t_\theta^2 + 2\zeta_g^2 + 4t_\theta\zeta_g\cos(\alpha+\omega) \ . \tag{47}$$

Let us consider some special cases.

Manifest left-right symmetry. This term is used to describe $SU(2)_L \times SU(2)_R \times U(1)$ models, where $g_R = g_L$, $\omega = 0$, and $U_R = U_L$ (Refs. 27 and 15). The last relation implies that $t_\theta = t$ and $\alpha = 0$. Thus Eq. (47) simplifies to

$$R = 4\left[\frac{m_1^2}{m_2^2}\right]^2 + 2\zeta^2 + 4\frac{m_1^2}{m_2^2}\zeta \ . \tag{48}$$

This expression was used in Refs. 4 and 5 to interpret the experimental bound on R. The experimental result (5) implies[28]

$$\frac{m_1^2}{m_2^2} < 0.035 \quad \text{for any } \zeta \tag{49}$$

(with $m_1 \simeq 83$ GeV, this means that $m_2 > 443$ GeV; for $\zeta = 0$, one would have $m_1^2/m_2^2 < 0.026$), and

$$|\zeta| < 0.050 \quad \text{for any } m_1^2/m_2^2 \tag{50}$$

(for $m_2 \to \infty$ one would have $\zeta < 0.035$).

The limit (49) is the most stringent constraint on m_1^2/m_2^2 from leptonic and semileptonic processes.[29] A tighter bound comes from the nonleptonic sector. Requiring that the contribution from right-handed currents to the K_L-K_s mass difference ΔM_k would not exceed the experimental value of ΔM_k leads to the limit[30]

$$\frac{m_1^2}{m_2^2} \lesssim 3\times 10^{-3} \ . \tag{51}$$

The best limit on $|\zeta|$ from leptonic and semileptonic processes is provided by the ρ parameter in μ decay.[29] The experimental value[31] $\rho = 0.7517 \pm 0.0026$ and Eq. (42) imply

$$|\zeta| < 0.033 \ (90\% \text{ confidence level}) \ . \tag{52}$$

A stronger bound

$$|\zeta| \lesssim 4\times 10^{-3} \tag{53}$$

follows from an analysis of nonleptonic K decays.[32]

Thus, for manifestly left-right-symmetric models the constraints on m_1^2/m_2^2 and ζ derived from nonleptonic transitions are stronger at present than those from leptonic and semileptonic processes. It should be noted however that they are less reliable, in view of the uncertainties involved in calculations of nonleptonic amplitudes.

Pseudomanifest left-right symmetry. In this case the left- and right-handed quark mixing angles are still equal, but CP violation is present.[15] R is now

$$R = 4\left[\frac{m_1^2}{m_2^2}\right]^2 + 2\zeta^2 + 4\frac{m_1^2}{m_2^2}\zeta\cos(\alpha+\omega) \ , \tag{54}$$

which implies the same bounds on $|\zeta|$ and m_1^2/m_2^2 as (48) (Ref. 33). The limit (52) from the ρ parameter is, of course, unaffected.

The constraints from the nonleptonic transitions described above are also unchanged. The bound (51) becomes[17,34]

$$\frac{m_1^2}{m_2^2}|\cos(\alpha-\beta)| \lesssim 3\times 10^{-3} \tag{55}$$

[β is defined in Eq. (13)]. A new constraint is provided by the CP-violating parameter ϵ in $K_L \to 2\pi$ decays. From the requirement $|\epsilon| \leq |\epsilon_{\text{expt}}|$ one obtains

$$\frac{m_1{}^2}{m_2{}^2} |\sin(\alpha-\beta)| \lesssim 1.5\times 10^{-5} . \qquad (56)$$

Equations (55) and (56) imply again the bound (51) (Refs. 17 and 34). In the presence of CP violation the bound (53) becomes

$$|\zeta \cos(\alpha+\omega)| \lesssim 4\times 10^{-3} . \qquad (57)$$

A new constraint[17,35]

$$|\zeta \sin(\alpha+\omega)| \lesssim 2\times 10^{-3} \qquad (58)$$

follows[36] from searches for a time-reversal-odd correlation $\sim\langle \mathbf{J}\rangle\cdot\mathbf{p}_e\times\mathbf{p}_\nu$ in nuclear β decay.[37] Equations (57) and (58) yield approximately the bound (53).

Nonmanifest left-right symmetry. Here $\theta_1^R \neq \theta_1^L$, $g_R \neq g_L$, and CP violation is in general present.[15] The quantity R is then given by Eq. (47).

For the quantity t [Eq. (33)], which replaces $m_1{}^2/m_2{}^2$ in the muon-decay-spectrum parameters (but not in P_μ), Eq. (47) implies

$$t < 0.035 \text{ for any } t_\theta,\ \zeta_g, \text{ and } \cos(\alpha+\omega) . \qquad (59)$$

Next we observe that $\cos\theta_1^L \simeq$ "$\cos\theta_C$" $\simeq 1$ ($\theta_C \equiv$ Cabibbo angle), so that

$$|t_\theta| \lesssim t , \qquad (60)$$

and therefore

$$4t_\theta{}^2 + 2\zeta_g{}^2 + 4t_\theta\zeta_g\cos(\alpha+\omega) \lesssim R . \qquad (61)$$

Hence[38]

$$|t_\theta| \lesssim 0.035 \text{ for any } t,\ \zeta_g, \text{ and } \cos(\alpha+\omega) \qquad (62)$$

and

$$|\zeta_g| \lesssim 0.050 \text{ for any } t, t_\theta, \text{ and } \cos(\alpha+\omega) . \qquad (63)$$

The nonleptonic transitions in this case do not place limits on t, t_θ, or ζ_g. For example, the constraints from the contribution to ΔM_k and ϵ of box diagrams involving two intermediate c quarks imply

$$\frac{g_R{}^2 m_1{}^2}{g_L{}^2 m_2{}^2} \left| \left[\frac{c_1^R c_2^R c_3^R - s_2^R s_3^R e^{i\delta_R}}{c_1^L c_2^L c_3^L - s_2^L s_3^L e^{i\delta_L}} \right] \left[\frac{s_1^R c_2^R}{s_1^L c_2^L} \right] \right| \lesssim 3\times 10^{-3} . \qquad (64)$$

The bounds (57) and (58) become

$$|\zeta_g \cos(\alpha+\omega)\cos\theta_1^R/\cos\theta_1^L| \lesssim 4\times 10^{-3} \qquad (65)$$

and

$$|\zeta_g \sin(\alpha+\omega)\cos\theta_1^R/\cos\theta_1^L| \lesssim 2\times 10^{-3} , \qquad (66)$$

respectively. Hence Eq. (53) is replaced by

$$|\zeta_g \cos\theta_1^R/\cos\theta_1^L| \lesssim 4.5\times 10^{-3} . \qquad (67)$$

Thus Eqs. (59) and (62) are the most stringent bounds available on t and t_θ. The best limit on $|\zeta_g|$ is

$$|\zeta_g| < 0.033 , \qquad (68)$$

implied by the ρ parameter.

B. Majorana neutrinos; no mixing in the leptonic sector

The states (17) and (18) are

$$\begin{aligned} \nu_e^{(L)} &= U_{e1}\nu_1, \quad \nu_e^{(R)} = V_{e(n+1)}\nu_{n+1} , \\ \nu_\mu^{(L)} &= U_{\mu 2}\nu_2, \quad \nu_\mu^{(R)} = V_{\mu(n+2)}\nu_{n+2} \end{aligned} \qquad (69)$$

(n is the number of generations), where

$$|U_{e1}| = |V_{e(n+1)}| = |U_{\mu 2}| = |V_{\mu(n+2)}| = 1 .$$

If both ν_{n+1} and ν_{n+2} can be produced in the decay, there are four possible final states, each governed by a different part of the Hamiltonian (15). The observed spectrum is indistinguishable from the spectrum of Sec. IV A, as long as the effects of the neutrino masses on the spectrum can be neglected. If both ν_{n+1} and ν_{n+2} are heavy, the muon-decay Hamiltonian contains only the $V-A$ part (involving ν_1 and ν_2). Note that $R=0$ also when only the state ν_{n+2} is heavy.

For models with manifest or pseudomanifest left-right symmetry the constraints (51) and (53) apply. For such models one has in this case also the bound

$$|\zeta| < 5\times 10^{-3} \qquad (70)$$

from data on semileptonic decays, provided that the right-handed neutrinos are heavy and if further quark generations are absent or couple to the u quark only weakly.[39]

C. Dirac neutrinos; mixing in the leptonic sector

For general matrices U and V the spectrum is a sum of the spectra of $\mu^+ \to e^+ + \nu_i + \bar{\nu}_j$ decays over the pairs $(\nu_i, \bar{\nu}_j)$ produced in the decay. As we are assuming that the produced neutrinos are light (see introduction to this section), the sum over the pairs can be replaced by the sum $\sum_i' \sum_j'$, where the primes indicate that the sums extend only over the mass eigenstates produced in the decay. By the same assumption the set of neutrino mass eigenstates participating in $\pi \to \mu\nu$ decays is the same as the one in muon decay.

The observed spectrum is given by Eqs. (1) and (22) with parameters $\rho, \xi, \ldots$ that can be obtained from Eqs. (23)–(26) by the substitutions

$$\begin{aligned} &|c_{LL}|^2 \to |c_{LL}|^2 u_e u_\mu, \quad |\kappa_{RR}|^2 \to |\kappa_{RR}|^2 \tilde{v}_e \tilde{v}_\mu , \\ &|\kappa_{LR}|^2 \to |\kappa_{LR}|^2 \tilde{v}_\mu, \quad |\kappa_{RL}|^2 \to |\kappa_{RL}|^2 \tilde{v}_e , \end{aligned}$$

where

$$u_l = {\sum_i}' |U_{li}|^2 \quad (l=e,\mu) , \qquad (71)$$

$$v_l = {\sum_i}' |V_{li}|^2 \quad (l=e,\mu) , \qquad (72)$$

and

$$\tilde{v}_l = v_l/u_l \quad (l=e,\mu) . \qquad (73)$$

Similarly, P_μ is obtained from Eq. (29) by the substitution[24]

$$|\eta_{RR}-\eta_{RL}|^2 \to |\eta_{RR}-\eta_{RL}|^2 \tilde{v}_\mu \ . \tag{74}$$

Note that if all the mass eigenstates can be produced in the decay we have $u_l = v_l = 1$ and therefore the mixing has no observable effect on the spectrum.[6]

The approximate expressions for the spectrum parameters and P_μ are now[40]

$$\rho \simeq \tfrac{3}{4}(1-\zeta_g{}^2\tilde{v}_\mu - \zeta_g{}^2\tilde{v}_e) \ , \tag{75}$$

$$\eta = 0 \ , \tag{76}$$

$$\xi \simeq 1 - 2t^2\tilde{v}_e\tilde{v}_\mu + 2\zeta_g{}^2\tilde{v}_\mu - 4\zeta_g{}^2\tilde{v}_e \ , \tag{77}$$

$$\delta \simeq \tfrac{3}{4}[1-3\zeta_g{}^2(\tilde{v}_\mu - \tilde{v}_e)] \ , \tag{78}$$

$$\delta\xi/\rho \simeq 1 - 2t^2\tilde{v}_e\tilde{v}_\mu \ , \tag{79}$$

and

$$P_\mu \simeq 1 - 2t_\theta{}^2\tilde{v}_\mu - 2\zeta_g{}^2\tilde{v}_\mu - 4t_\theta\zeta_g\tilde{v}_\mu\cos(\alpha+\omega) \ . \tag{80}$$

The quantity R is given by[41]

$$R \simeq 2t^2\tilde{v}_e\tilde{v}_\mu + 2t_\theta{}^2\tilde{v}_\mu + 2\zeta_g{}^2\tilde{v}_\mu + 4t_\theta\zeta_g\tilde{v}_\mu\cos(\alpha+\omega) \ . \tag{81}$$

The experimental result (5), implies

$$t\sqrt{\tilde{v}_e\tilde{v}_\mu} < 0.035 \tag{82}$$

for any $t_\theta\sqrt{\tilde{v}_\mu}$, $\zeta_g\sqrt{\tilde{v}_\mu}$, and $\cos(\alpha+\omega)$, and

$$[t_\theta{}^2\tilde{v}_\mu + \zeta_g{}^2\tilde{v}_\mu + 2t_\theta\zeta_g\tilde{v}_\mu\cos(\alpha+\omega)]^{1/2} < 0.035 \tag{83}$$

for any $t\sqrt{\tilde{v}_e\tilde{v}_\mu}$. For a given $c \equiv \cos(\alpha+\omega)$, Eq. (83) yields $|t_\theta\sqrt{\tilde{v}_\mu}| < 0.035\ (1-c^2)^{-1/2}$ and $|\zeta_g\sqrt{\tilde{v}_\mu}| < 0.035(1-c^2)^{-1/2}$. Hence for $c=\pm 1$ (which are not ruled out), Eq. (83) sets no uncorrelated constraint on $t_\theta\sqrt{\tilde{v}_\mu}$ or $\zeta_g\sqrt{\tilde{v}_\mu}$. For $c=\pm 1$, Eq. (83) implies

$$|t_\theta \pm \zeta_g|\sqrt{\tilde{v}_\mu} < 0.035 \ . \tag{84}$$

The best limit on $|\zeta_g\sqrt{\tilde{v}_\mu}|$ is provided by the ρ parameter. The experimental value and Eq. (75) imply

$$|\zeta_g\sqrt{\tilde{v}_\mu}| < 0.047 \ \text{(90 confidence level)} \ . \tag{85}$$

Combining (84) and (85) yields

$$|t_\theta\sqrt{\tilde{v}_\mu}| < 0.082 \ , \tag{86}$$

which is the most stringent available bound on the quantity $|t_\theta\sqrt{\tilde{v}_\mu}|$.

For models with manifest or pseudomanifest left-right symmetry the bounds implied by the limit (5) and the experimental value of ρ are the same as in the general case [Eqs. (82)–(86)], except for $\zeta_g \to \zeta$ and $t, t_\theta \to m_1{}^2/m_2{}^2$. The limits (51) and (53) [or (55)–(58)] imply

$$\frac{m_1{}^2}{m_2{}^2}\sqrt{\tilde{v}_e\tilde{v}_\mu} \lesssim 3\times 10^{-3} \ , \tag{87}$$

$$|\zeta\sqrt{\tilde{v}_\mu}| \lesssim 4\times 10^{-3} \ , \tag{88}$$

and

$$\frac{m_1{}^2}{m_2{}^2}\sqrt{\tilde{v}_\mu} \lesssim 3\times 10^{-3} \ , \tag{89}$$

since $\tilde{v}_l \lesssim 1$ $(l=e,\mu)$ (Ref. 42).

D. Majorana neutrinos; mixing in the leptonic sector

The most general Lagrangian contains both Dirac and Majorana mass terms for the neutrinos. The complete set of mass eigenstates consists then of $2n$ Majorana neutrinos (n is the number of generations).[18] For Majorana neutrinos additional terms (not proportional to neutrino masses) appear in the muon-decay spectrum. However, with the effects of neutrino masses on the spectrum neglected, these terms do not survive in the limit[43] $x \to 1$. Hence, extending the definition of u_l and v_l [Eqs. (71) and (72)] to include the general case, the quantity R is given by Eq. (81) regardless of the nature and number of the neutrino mass eigenstates. With these definitions of u_l and v_l, Eq. (81) (given our approximations) is the most general expression for R in $SU(2)_L \times SU(2)_R \times U(1)$ electroweak models. The corresponding bounds on the parameters are given in Eqs. (82) and (84) (Ref. 44). Assuming that the additional terms in the spectrum do not affect appreciably the experimental value of ρ (which is probably the case, as the ρ parameter describes the high-energy part of the spectrum), the bound (85) and consequently also the bound (86) remain valid.

V. CONCLUSIONS

The purpose of this paper was to study the constraints on the parameters of various realizations of $SU(2)_L \times SU(2)_R \times U(1)$ electroweak models, implied by recent measurements of the end point of the positron momentum spectrum in polarized muon decay. For all cases considered we have assumed that the neutrinos that are produced in the decay are sufficiently light that the effects of their masses on the spectrum can be neglected.

The various versions of $SU(2)_L \times SU(2)_R \times U(1)$ models discussed can be divided into two classes.

(a) Models where the quantities $\tilde{v}_e$ and $\tilde{v}_\mu$ [defined in Eqs. (71)–(73)] are equal to one. Examples are models (with or without leptonic mixing) where all the neutrinos can be produced in muon decay, and also models where the right-handed leptonic mixing matrix is equal to the left-handed one [such as $SU(2)_L \times SU(2)_R \times U(1)$ models with Dirac neutrinos and a discrete left-right symmetry].

(b) Models with arbitrary $\tilde{v}_e$ and $\tilde{v}_\mu$.

In models of class (a) the quantity R depends in the most general case (models with nonmanifest left-right symmetry) on four parameters: t, t_θ, ζ_g, and $t_\theta\zeta_g\cos(\alpha+\omega)$ [Eq. (47)]. The spectrum parameters $\rho, \xi, \ldots$ are described by two of these (t, ζ_g); the remaining two (and ζ_g) are involved in the muon polarization P_μ. The experimental result for R [Eq. (5)] provides for this class of models the best available limit on t and t_θ [Eqs. (59) and (62)]. It implies also a stringent limit on ζ_g, not much weaker than the best present limit (which comes from the experimental value of the ρ parameter). For models constrained further to have manifest or pseudomanifest left-right symmetry, bounds on t

$=t_\theta=m_1{}^2/m_2{}^2$ and ζ derived from nonleptonic transitions and the β-decay limit (58) are more stringent at present than any of the constraints from leptonic or semileptonic processes.

In models of class (b) R depends again on four parameters, which are now $t\sqrt{\tilde{v}_e\tilde{v}_\mu}$, $t_\theta\sqrt{\tilde{v}_\mu}$, $\zeta_g\sqrt{\tilde{v}_\mu}$, and $t_\theta\zeta_g\tilde{v}_\mu\cos(\alpha+\omega)$. The experimental bound (5) yields the best available limit on $t\sqrt{\tilde{v}_e\tilde{v}_\mu}$ [Eq. (82)]. There are no uncorrelated constraints from R on $t_\theta\sqrt{\tilde{v}_\mu}$ or $\zeta_g\sqrt{\tilde{v}_\mu}$ but combined with the limit on $\zeta_g\sqrt{\tilde{v}_\mu}$ provided by the ρ parameter the constraint from R implies the most stringent available bound on $t_\theta\sqrt{\tilde{v}_\mu}$. The muon-decay spectrum depends also on the parameter $\zeta_g\sqrt{\tilde{v}_e}$ not involved in R. The best available limit on $\zeta_g\sqrt{\tilde{v}_e}$, the same as for $\zeta_g\sqrt{\tilde{v}_\mu}$ [Eq. (85)], comes from the ρ parameter. For models with manifest or pseudomanifest left-right symmetry the constraints derived from nonleptonic transitions and the β-decay limit (58) are again the most stringent. We note that the parameters [for models of class (b)] contained in R are not constrained by nuclear β decay.[45] For Majorana neutrinos further constraints on the parameters of $SU(2)_L\times SU(2)_R\times U(1)$ models come from searches for neutrinoless nuclear double-β decay.[46] However, unlike the parameters contained in R, these depend on the matrix elements U_{ej} and V_{ej}, while independent of $U_{\mu j}$ and $V_{\mu j}$.

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