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Weak decays of the H dibaryon

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We calculate decay rates and branching fractions for the postulated $S=-2$ H dibaryon, both for the expected $\Delta S=1$ channels ($n\Lambda, n\Sigma^0, p\Sigma^-$) and for the possible $\Delta S=2$ mode (nn). For $\Delta S=1$ decays we find the S waves are dominated by $\Delta I=\frac{3}{2}$ transitions due to the Pauli principle, which forces the six-quark final state to be in a SU(3) 27-plet. If observed, this would be the first breakdown of the $\Delta I=\frac{1}{2}$ rule. The lifetime is long, of order 10^{-8} sec, which should be considered in planning experiments. In the $\Delta S=2$ case, we add a consideration of dispersive effects to our previous calculation of the box diagram, reinforcing our conclusion that the H is too short lived to be the explanation of the Cygnus X-3 events.

I. INTRODUCTION

Dibaryon states, distinct from the deuteron, have been sought for many years. Although none have yet been detected, there is reasonable motivation to continue the search. In particular a theoretical analysis[1] suggests the most stable such state would be a neutral, strangeness -2 state of zero spin and isospin. This so-called H particle would be a composite of the six quarks *uuddss*.

The mass estimate for the H in Ref. 1 has it sufficiently bound to be stable under strong decay ($m_H < 2m_\Lambda$). Interestingly there has been a recent speculation[2] that a more deeply bound H, stable even under $\Delta S=1$ weak decay ($m_H < m_\Lambda + m_n$), might be associated with the observation of high-energy muons associated with a hadronic component in the emission spectrum of Cygnus X-3 (Ref. 3).

Regardless of whether this speculation is valid, the H is a very interesting particle in its own right, especially as it relates to our understanding of quark dynamics. It is hoped that its existence will be probed in a forthcoming experimental program.[4] For such an endeavor, it is important to have some sense of how the H would decay weakly (lifetimes and branching fractions). It is our purpose to provide this information here.

Throughout we shall take seriously the description presented in Ref. 1 of the H as a composite of six quarks which have *highly similar spatial wave functions.* Thus, although it has become traditional to refer to the H as a dibaryon, it is very unlike the deuteron where color correlations between the quarks produce a moleculelike bound state of two distinct baryons. Presumably if we were able to reduce the H binding energy to nearly zero, it would more and more resemble a deuteronlike composite of two Λ hyperons. The weak decay of this object would be dictated by Λ decay, accompanied by minor off-shell corrections. However, for the H particle considered in this paper the picture of weak decay we arrive at is rather different, involving in a central way the symmetry properties of six-quark ground-state configurations. It is important for the reader to appreciate this point; otherwise some of our results might appear unduly mystifying. Indeed, we find (see Sec. IV) for the $\Delta S=1$ decay of H into two baryons ($H \to B_1 B_2$) that the $\Delta I=\frac{3}{2}$ contributions are *dominant* in the S wave, in contrast with the usual $\Delta I=\frac{1}{2}$ rule. As we shall see, this result has a very simple origin. If observed, it would be a significant confirmation of both the scenario assumed here and our understanding of it. In addition we find a rather long H lifetime which may be important to the design of experiments which look for it.

In Sec. II we describe how to write both six-quark wave functions and also the weak Hamiltonian in the quark model. Section III contains a discussion of the P-matrix formalism which is used to relate bag-model multiquark configurations to baryonic scattering states. Sections IV and V are devoted to $\Delta S=1$ weak decays $H \to B_1 B_2$ and $H \to B_1 B_2 \pi$, respectively. We have previously given a calculation of the $\Delta S=2$ decay $H \to nn$ via the box diagram.[5] In Sec. VI we repeat this result for completeness and present a full discussion of the corresponding dispersive contribution. Our previous conclusion that the $\Delta S=2$ lifetime is too short for the H to be the explanation of the Cygnus X-3 events is reinforced. Section VII contains a summary of our results.

II. QUARK-MODEL WAVE FUNCTIONS AND OPERATORS

We shall compute weak decays of the H in the quark model. To do this we must first write down wave functions for the H initial state and for the baryon-baryon final states. Also we must give the quark structure of the $\Delta S=1,2$ nonleptonic weak Hamiltonians.

Dealing with the H wave function is a crucial aspect of the calculation. Obviously, to do computations involving six-quark states is a formidable task. However even learning about the quark content of the H is a problem of some subtlety. Fortunately there exists an approach which provides a powerful guide in how to proceed. Consider Table I, which we reproduce from Jaffe's paper.[1] It gives the spectroscopy of all possible color-singlet six-quark configurations of ground-state (i.e., S-wave) quarks. The list of

TABLE I. Spectroscopy of ground-state six-quark configurations.

SU(6) of color/spin	SU(3) of flavor	Spin
490	1	0
896	8	1,2
280	10	1
175	10*	1,3
189	27	0,2
35	35	1
1	28	0

allowed states is a remarkably restrictive one. Evidently the symmetry structure, especially as regards the Pauli principle, plays a significant role in constraining the multiquark configurations considered here. Observe that spin-zero states can occur only in the SU(3)-flavor representations **1**, **27**, and **28**. The singlet corresponds to the H particle and the 27-plet contains the possible baryon final states to which the H can couple via the weak Hamiltonian. The 28-plet has no relevance to our calculation because it does not occur in the Clebsch-Gordan series of two SU(3) octets.

The fact that the SU(3)-flavor representations each appear just once in Table I means that any six-quark configuration having the correct quantum numbers must be a valid wave function for the associated state. We illustrate this important statement with the following simple example, which also serves to introduce some useful notation. Suppose we wish to write down the 27-plet wave function having the quantum numbers of two neutrons. For a single neutron with spin component k, we have[6]

$$|n,k\rangle \frac{1}{\sqrt{18}}\epsilon^{\alpha\beta\gamma}\epsilon^{mn}d^{\dagger}_{\alpha m}u^{\dagger}_{\beta n}d^{\dagger}_{\gamma k}|0\rangle , \tag{1}$$

where greek symbols run from 1 to 3 and denote color, latin symbols run from 1 to 2 and denote spin, and $\epsilon^{\alpha\beta\gamma},\epsilon^{mn}$ are the usual antisymmetric tensors defined over the range of their indices. An unnormalized spin-zero six-quark state with the quantum numbers of two neutrons can then be written as $(duddud)$. The parentheses surrounding six quarks represent the construction of a colorless, spinless configuration:

$$\begin{aligned}(abcdef)=\epsilon^{\alpha\beta\gamma}\epsilon^{\eta\sigma\rho}\epsilon^{mn}\epsilon^{pq}\epsilon^{st}\\ \times a^{\dagger}_{\alpha m}b^{\dagger}_{\beta n}c^{\dagger}_{\gamma s}d^{\dagger}_{\eta p}e^{\dagger}_{\sigma q}f^{\dagger}_{\rho t} .\end{aligned} \tag{2}$$

Denoting a unit-normalized member of the flavor 27-plet carrying hypercharge Y, isospin I, and isospin-component I_3, as $|Y;I,I_3\rangle_{27}$, we can write for the "dineutron" state

$$|2;1,-1\rangle_{27}=\frac{1}{12\sqrt{10}}(duddud) . \tag{3}$$

Observe that the normalization constant of this six-quark confined configuration is not just the square of the neutron normalization factor. Such normalization terms must be computed independently for each six-quark state, unless of course inferred from some other state by a symmetry principle.

Since we have introduced the $(abcdef)$ notation for spinless and colorless six-quark composites, it is worthwhile to point out at this juncture certain of its properties. It is straightforward to show that

$$(abcdef)=(defabc) . \tag{4}$$

There is also permutation invariance, up to a phase, with respect to interchanging the first two, or fourth and fifth, members:

$$\begin{aligned}(abcdef)&=-(bacdef)\\ &=-(abcedf) .\end{aligned} \tag{5}$$

Finally, upon writing out the spin content of $(abcdef)$ one obtains eight distinct terms. For each of these there are an additional 36 color terms. The $(abcdef)$ notation thus characterizes a good deal of hidden complexity, which makes the six-quark states a real challenge to perform calculations with.

We have considered the $(duddud)$ configuration in part because it is the one reached by a $\Delta S=2$ weak transition from the H. Configurations reached by $\Delta S=1$ weak transitions from the H are the $|1;\frac{3}{2},-\frac{1}{2}\rangle_{27}$ and $|1;\frac{1}{2},-\frac{1}{2}\rangle_{27}$ member of the dibaryon 27-plet. Conventional ladder operations and orthogonality relations can be used to construct these from Eq. (3). We obtain

$$\begin{aligned}|1;\tfrac{3}{2},-\tfrac{1}{2}\rangle_{27}=\frac{1}{12\sqrt{15}}[(sududd)+(sduudd)\\ +(sddudu)]\end{aligned} \tag{6a}$$

and

$$\begin{aligned}|1;\tfrac{1}{2},-\tfrac{1}{2}\rangle_{27}=\frac{1}{60\sqrt{3}}[-2(sududd)+(sduudd)\\ +(sddudu)+3(udsudd)] .\end{aligned} \tag{6b}$$

These arise, respectively, from $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{1}{2}$ decays of the flavor singlet H.

There is a bit of a surprise associated with the $|1;\frac{1}{2},-\frac{1}{2}\rangle_{27}$ state of Eq. (6b). It has the form which anyone employing the usual algebraic ladder operations would obtain. Yet inspection reveals that among all possible states of Table I, it uniquely has the quantum numbers of a Λn composite. But the Λn composite has the structure $(udsudd)$. It must therefore *also* be true that (in unit-normalized form)

$$|1;\tfrac{1}{2},-\tfrac{1}{2}\rangle_{27}=\frac{1}{12\sqrt{3}}(udsudd) . \tag{7}$$

The reader might wonder how Eqs. (6b) and (7) can each be correct. Yet they are, and we can demonstrate so in the following manner. Two vectors associated with the same ray in Hilbert space must have the property that the absolute value of their inner product equals the product of their norms. We have verified that this is the case for the states in Eqs. (6b) and (7). They are indeed equivalent representations of $|1;\frac{1}{2},-\frac{1}{2}\rangle_{27}$.

We can now construct the H wave function. The following linear combination is found to be a singlet under the SU(3) of flavor

$$|H\rangle=\tfrac{1}{144}[(udsuds)+(usdusd)+(dsudsu)-2(ussudd)-2(dssdud)-2(usudsd)] \quad (8)$$

and so must be the H. We suggest computation of the normalization factor as an exercise for the reader. Let us again raise the issue of wave-function uniqueness, but in an extended and ultimately more instructive form.

In the $(abcdef)$ construction, the spins of a,b and of d,e are each paired to spin zero. So are the spins of c,e. An alternative spin-coupling procedure would be to pair a,b and d,e each as spin one entities and then to couple them together to spin zero. Quarks c,e would be treated as before. The resulting configuration, denoted by $[abcdef]$, has the form

$$[abcdef]=\epsilon^{\alpha\beta\gamma}\epsilon^{\eta\sigma\rho}\epsilon^{mn}a^\dagger_{\alpha p}(\sigma i\sigma_2)_{pq}b^\dagger_{\beta q}c^\dagger_{\gamma m}d^\dagger_{\eta r}\times(\sigma i\sigma_2)_{rs}e^\dagger_{\sigma s}f^\dagger_{\rho n}\ . \quad (9)$$

The quantity $[abcdef]$ obeys the replacement rule

$$[abcdef]=[defabc]\ , \quad (10)$$

and there are twelve distinct spin terms in each such configuration. We can express the H dibaryon in terms of these bracketed entities to obtain

$$|H\rangle=\tfrac{1}{216}[[udsuds]+[usdusd]+[dsudsu]-[uusdds]-[ddussu]-[uudssd]-[suusdd]-[duudss]-[udduss]+[uussdd]+[ddssuu]+[ssuudd]+[uuddss]+[dduuss]+[ssdduu]-[dsuusd]-[sduuds]-[suddus]]\ . \quad (11)$$

The equivalence of the wave functions appearing in Eqs. (8) and (11) is proved as before. The absolute value of their inner product equals the product of their norms. This lack of uniqueness in writing down six-quark wave functions has its bright side, of course. It is far easier to work with the more compact form of Eq. (8) than it is with the cumbersome object in Eq. (11).

There is one more six-quark configuration which is worth studying. Suppose we attempt to obtain a color-singlet six-quark composite as a consequence of coupling two color octets. We could then define the construction

$$\{abcdef\}=\epsilon^{\alpha\beta\gamma}\epsilon^{\eta\sigma\rho}\lambda^A_{\gamma\delta}\lambda^A_{\rho\phi}\epsilon^{mn}\epsilon^{pq}\epsilon^{st}\times a^\dagger_{\alpha m}b^\dagger_{\beta n}c^\dagger_{\delta s}d^\dagger_{\eta p}e^\dagger_{\sigma q}f^\dagger_{\phi t}\ . \quad (12)$$

However, in view of the identity

$$\lambda^A_{\gamma\delta}\lambda^A_{\rho\phi}=-\tfrac{2}{3}\delta_{\gamma\delta}\delta_{\rho\phi}+2\delta_{\gamma\phi}\delta_{\delta\rho} \quad (13)$$

we see that

$$\{abcdef\}=-\tfrac{2}{3}(abcdef)+\langle abcdef\rangle\ , \quad (14)$$

where

$$\langle abcdef\rangle=\epsilon^{\alpha\beta\gamma}\epsilon^{\eta\sigma\rho}\epsilon^{mn}\epsilon^{pq}\epsilon^{st}\times a^\dagger_{\alpha m}b^\dagger_{\beta n}c^\dagger_{\rho s}d^\dagger_{\eta p}e^\dagger_{\sigma q}f^\dagger_{\gamma t}\ . \quad (15)$$

Observe that the colors of the quarks c,f have been interchanged relative to their values in the $(abcdef)$ state of Eq. (8). However this "twisted color" configuration $\langle abcdef\rangle$ has the same spin content as $(abcdef)$. The appearance of the twisted color configuration in Eq. (14) suggests that the $\{abcdef\}$ construction in which two color octets are coupled gives rise to yet a third form [along with Eqs. (8) and (11)] for the H wave function. Such is not the case. Upon taking the linear combination of $\langle abcdef\rangle$ as given in the H wave function of Eq. (8), we obtain a state of zero norm, i.e., the null state. Therefore the constructions $\{abcdef\}$ and $(abcdef)$ lead to identical H wave functions.

This concludes our analysis of six-quark wave functions. We have developed a compact and powerful notation for dealing with such states, and have successfully addressed the issue of uniqueness. There is a lesson to be learned here. We have seen how changes can be made in the flavor, spin, or color couplings of the six quarks such as to yield a different appearing, yet equivalent, wave function. Thus it is hard to see how to attach physical significance to such concepts as "hidden color" or the like for the six-quark states.

The weak Hamiltonian which stimulates the H into undergoing a transition is itself describable in the quark model.[6] In the following we define the relevant weak operators. First however consider a technical point regarding the $\Delta S=2$ decay amplitude:

$$M^{\Delta S=2}=M^{\Delta S=2}_{\rm box}+M^{\Delta S=2}_{\rm disp}\ , \quad (16)$$

where $M^{\Delta S=2}_{\rm box}$ describes short-range physics ("box" diagram) and $M^{\Delta S=2}_{\rm disp}$ describes the action of two $\Delta S=1$ interactions with a low-energy intermediate state ("dispersive" contribution) between them. We shall provide an estimate for $M^{\Delta S=2}_{\rm disp}$ in Sec. VI. To obtain a measure of the $M^{\Delta S=2}_{\rm box}$ and $M^{\Delta S=1}$ amplitudes, we compute the appropriate weak matrix elements of the initial-state H dibaryon.

The $\Delta S=1$ nonleptonic weak Hamiltonian is conventionally written as[6]

$$H^{\Delta S=1}=\frac{G_F\cos\theta_C\sin\theta_C}{2\sqrt{2}}\sum_{i=1}^{6}c_iO_i\ , \quad (17)$$

where c_i are numerical coefficients and the $\{O_i\}$ are chiral four-quark operators. Because of the flavor-SU(3)-symmetry structure of the six-quark states, two parts of Eq. (17) with special relevance here are the 27-plet operators,

$$H_{27}^{\Delta S=1}=\frac{G_F\cos\theta_C\sin\theta_C}{2\sqrt{2}}(c_3O_3+c_4O_4)\ , \tag{18}$$

where $c_3\simeq 0.084$, $c_4\simeq 0.42$, and

$$\begin{aligned} O_3&=(\bar{d}u)(\bar{u}s)+(\bar{d}s)(\bar{u}u)+2(\bar{d}s)(\bar{d}d)-3(\bar{d}s)(\bar{s}s)\ ,\\ O_4&=(\bar{d}u)(\bar{u}s)+(\bar{d}s)(\bar{u}u)-(\bar{d}s)(\bar{d}d)\ . \end{aligned} \tag{19}$$

The above shorthand notation for quark bilinears in Eq. (19) is defined as

$$(\bar{a}b)(\bar{c}d)=\bar{a}_i\gamma^{\mu}(1+\gamma_5)b_i\bar{c}_j\gamma_{\mu}(1+\gamma_5)d_j\ . \tag{20}$$

For anyone who has done research on nonleptonic weak interactions, the presence of Eq. (18) is highly ironic. In kaon and hyperon decays, it is usually neglected in favor of the octet four-quark operators. However the SU(3) structure of six-quark states forces $H_{27}^{\Delta S=1}$ into the spotlight.

For $\Delta S=2$ transitions we employ

$$H^{\Delta S=2}=\frac{G_F{}^2}{16\pi^2}m_C{}^2\cos^2\theta_C\sin^2\theta_C\eta(\bar{d}s)(\bar{d}s)\ , \tag{21}$$

where $\eta\simeq 0.7$, and m_c is the charmed-quark mass.

The net effect of the weak Hamiltonians $H_{27}^{\Delta S=1}$ and $H^{\Delta S=2}$ is to convert, respectively, one and two s quarks into d quarks. The resulting states are bag-confined configurations with content *uuddds* and *uudddd,* respectively. In the following sections we detail the decay amplitudes and describe how to relate the bag-confined six-quark composites to the baryon-baryon plane-wave states seen experimentally.

III. P-MATRIX FORMALSIM

Each of the H decay modes mentioned thus far involves a weak nonleptonic transition to a pair of baryons in the continuum. However, in quark models, such as the bag model, one cannot treat a continuum state directly since the six-quark state representing the baryon pair is permanently confined as a result of the imposed boundary conditions. The P matrix represents a rigorous way to connect this artificially confined six-quark state with the real strongly interacting two-baryon final state.

In order to see how this is achieved, imagine a pair of baryons each of mass m artificially confined to a spherical well of radius b. Even if the baryons do not interact with each other there is a series of eigenfrequencies corresponding to the bound-state energies. When interactions are present, these energies are shifted. The P matrix relates the position and residues of these poles to the interactions present in the scattering matrix. For the $l=0$, $s=0$ configuration which corresponds to a $^1S_0$ final state, Jaffe and Low[7] showed that the relation is of the form

$$S=-e^{-ikb}\frac{1-i\bar{P}}{1+i\bar{P}}e^{-ikb}\ , \tag{22}$$

where $k=\frac{1}{2}(s-4m^2)^{1/2}$ is the particle momentum in the center-of-mass frame and S is the usual scattering matrix. The matrix $\bar{P}$ is the P matrix (the overbar indicates a different normalization from that of Ref. 7). In the elastic-scattering region we have

$$S=\exp(2i\delta) \tag{23}$$

and

$$\bar{P}=\cot(kb+\delta)\ , \tag{24}$$

so that even in the absence of interaction between the particles (i.e., $\delta=0$) there exists a sequence of $\bar{P}$-matrix poles

$$\bar{P}\,|_{\delta=0}=\cot(kb)=\frac{1}{kb}+\frac{2k}{b}\sum_{n=1}^{\infty}\frac{1}{k^2-n^2\dfrac{\pi^2}{b^2}} \tag{25}$$

which are termed "compensation poles" by Jaffe and Low. In the presence of interactions the location and residue of the poles should change somewhat and these can be fit for the lowest-lying resonances. Thus for the case of a single pole one parametrizes the pole with $n=1$ by

$$\bar{P}(s,b)\simeq c_0+\frac{r_0}{s-s_0}+\cot(kb)-\frac{2\pi/b^2}{k^2-\pi^2/b^2} \tag{26}$$

and fits to experiment for assumed values of the matching radius b, the residue r_0, and pole location s_0. This procedure has been carried out by Mulders, for example, in the case of the $^1S_0$ NN system.[8] For a reasonable matching radius of $b=6.5$ GeV$^{-1}\approx 1.3$ fm, a detailed fit determines the lowest NN "bound" state to have mass $\sqrt{s_0}=2.25$ GeV with residue $r_0=1.35$ GeV^2. This can be compared with values calculated in various confining models. Thus, for example, a quark bag calculation of the $^3S_0$ NN mass gives $(s_0{}^{\text{theory}})^{1/2}=2.23$ GeV, in good agreement with the fitted value. A corresponding bag calculation of the residue is not as successful, yielding a value only 20% of the experimentally determined parameter. However, this may well be associated with strong color correlations. In any event all we shall require is the empirically fitted values.

In the case that a narrow state such as the H is coupled to this channel, it will also appear as a $\bar{P}$ matrix pole of the form

$$\bar{P}=\bar{c}+\frac{\bar{r}}{s-s_0}+\cdots \tag{27}$$

in the vicinity of the singularity. The corresponding form of the S matrix is the well-known Breit-Wigner resonant shape

$$S=1+\frac{2im\Gamma}{s-m^2-im\Gamma} \tag{28}$$

which has the property

$$\begin{aligned} S\,|_{s=m^2}&=-1\ ,\\ \left.\frac{\partial S}{\partial s}\right|_{s=m^2}&=-2\frac{i}{m\Gamma}\ . \end{aligned} \tag{29}$$

Then using the form of the S matrix given in Eq. (22),

$$\frac{\partial S}{\partial s}\approx\frac{2i}{\bar{r}}S\left[\frac{\bar{r}^2}{(s-s_0)^2}\frac{1}{1+\bar{P}^2}\right]\to -2i\frac{1}{\bar{r}} \tag{30}$$

near the $\bar{P}$-matrix pole. Thus in the narrow resonance limit we can identify the width in terms of the residue as

$$\Gamma = \frac{\bar{r}}{m} \tag{31}$$

which is the relationship we have been seeking.

In order to predict the lifetime of the H we need to determine the residue of the corresponding P-matrix pole in that channel. This may be cast in terms of a simple mixing problem. That is, if we denote the confined two-baryon state as $|B_1B_2\rangle$, then the effective Hamiltonian of the coupled HB_1B_2 system can be written as

$$H_{\text{eff}} = \begin{bmatrix} m_H & \langle B_1B_2|H_w|H\rangle \\ \langle B_1B_2|H_w|H\rangle & m_{B_1B_2} \end{bmatrix} \tag{32}$$

in an obvious notation. The first-order mixing between H and B_1B_2 channels induced by the nonleptonic weak interaction H_w then gives the perturbed H state as

$$|\psi_H\rangle \simeq |H\rangle + \frac{1}{m_H - m_{B_1B_2}}|B_1B_2\rangle \tag{33}$$

whence the residue becomes

$$\bar{r}_H = \left|\frac{\langle B_1B_2|H_w|H\rangle}{m_H - m_{B_1B_2}}\right|^2 \bar{r}_{B_1B_2} , \tag{34}$$

where $\bar{r}_{B_1B_2}$ is the residue determined by fitting the empirical phase shifts in the B_1B_2 channel. In the next sections we apply these techniques in order to calculate rates for decays of the H dibaryon.

IV. THE $\Delta S = 1$ DECAY $H \to B_1B_2$

If the H lies in the mass range

$$m_\Lambda + m_n < m_H < 2m_\Lambda , \tag{35}$$

then its decay will be via $H_w^{\Delta S=1}$ [Eq. (18)] into $n\Lambda^0$, $n\Sigma^0$, or $p\Sigma^-$ systems. Both ${}^1S_0$ or ${}^3P_0$ final-state channels are available. Consider first then the parity-conserving S-wave decay, for which the P-matrix methods described previously may by applied. By Jaffe's theorem[1] the two-baryon state must be in a 27-plet and so only O_3 and O_4 can contribute. Thus we have a unique situation wherein the octet component of $H_w^{\Delta S=1}$ is unable to contribute so that we may study the 27-plet without the interference of an accompanying enhanced octet term.

The quark-model calculation of the matrix element of the weak Hamiltonian proceeds in two parts.[6] The spin and color Clebsch-Gordan coefficients, which come from summing all the allowed contributions, are model independent in that any version of the quark model would give the same results. The remaining part of the calculation, the spatial wave function overlap, is more model dependent. We use the spatial wave functions of the MIT bag model, as they give a good account of the size and shape of hadrons.[6] Our answer is expressed in terms of the upper and lower components of the quark's Dirac wave function [denoted by $u(r)$ and $l(r)$]. A prime indicates the wave function of the strange quark. The calculation is straightforward and we find

$$\begin{aligned} \langle 27,\tfrac{3}{2}|H_w^{\Delta S=1}|H\rangle &= (\tfrac{3}{2})^{1/2}Ac_4 , \\ \langle 27,\tfrac{1}{2}|H_w^{\Delta S=1}|H\rangle &= (\tfrac{15}{2})^{1/2}Ac_3 , \end{aligned} \tag{36}$$

where

$$A = \frac{G}{2\sqrt{2}}\cos\theta_C\sin\theta_C 2\sqrt{10} \times \int d^3r(uu' + ll')(u^2 + l^2) . \tag{37}$$

To see how specific final states are produced, we project the 27-plet states onto the two-baryon channels. In our phase convention we write

$$\begin{aligned} |27,\tfrac{3}{2}\rangle &= (\tfrac{1}{3})^{1/2}|p\Sigma^-\rangle + (\tfrac{2}{3})^{1/2}|n\Sigma^0\rangle , \\ |27,\tfrac{1}{2}\rangle &= (\tfrac{1}{15})^{1/2}|p\Sigma^-\rangle - (\tfrac{1}{30})^{1/2}|n\Sigma^0\rangle - \frac{3}{\sqrt{10}}|n\Lambda\rangle . \end{aligned} \tag{38}$$

The relative amplitudes are then

$$\begin{aligned} \langle\Lambda n|H_w|H\rangle &= -\frac{3\sqrt{3}}{2}c_3A , \\ \langle\Sigma^0 n|H_w|H\rangle &= (c_4 - \tfrac{1}{2}c_3)A , \\ \langle\Sigma^- p|H_w|H\rangle &= \frac{1}{\sqrt{2}}(c_4 + c_3)A . \end{aligned} \tag{39}$$

Ratios of these amplitudes depend only on general quark-model properties, and so it is worthwhile to comment on this aspect. The usual octet rule for H_w would predict ratios

$$\Sigma^- p : \Sigma^0 n : \Lambda n :: 1 : -\frac{1}{\sqrt{2}} : -(\tfrac{1}{6})^{1/2} . \tag{40}$$

Instead we find ratios

$$\Sigma^- P : \Sigma^0 n : \Lambda n :: 1. : 1.06 : -0.61 . \tag{41}$$

Note that this is the only situation in hadronic weak decays where the $\Delta I = \frac{3}{2}$ amplitude is expected to dominate its $\Delta I = \frac{1}{2}$ counterpart. The reason is simple and model independent: The Jaffe theorem only allows a 27-plet final state and in the weak Hamiltonian c_4, the coefficient of the 27-plet $\Delta I = \frac{3}{2}$ term is larger than c_3, the $\Delta I = \frac{1}{2}$ 27-plet coefficient.

The S-wave contribution to the H decay rate and branching ratios may now be determined. We take the P-matrix pole in the $S = -1$ ${}^1S_0$ channel to occur at 2.44 GeV. This is scaled from the observed nn pole[8] by adding 190 MeV to account for the replacement of a nonstrange quark by a strange quark. The residue is assumed to be the same as in nn except for an obvious phase-space factor. Starting then with the decay amplitude evaluation

$$\langle\Sigma^- p|H_w|H\rangle \simeq 2.2\times 10^{-9}\ \text{GeV} \tag{42}$$

and employing the ratios of Eq. (41), we can use the P-matrix Eqs. (31) and (34) to obtain $\Gamma_H^{(S\text{ wave})}(\text{tot})$. For example, at an H mass of $2M_\Lambda$ we obtain

$$\Gamma_H^{(S)}(\text{tot}) \simeq 1.24\times 10^{-16}\ \text{GeV} . \tag{43}$$

Before using the S-wave decay rates to provide a lifetime estimate for the H, we must next consider the P-wave contributions. In our previous work on the $\Delta S=2$ interaction[5] we argued that the effect of the P wave was expected to be small, say, at the 5% level of the S-wave amplitude. That is not the case here. Several conflicting factors must be taken into account. First, P-wave decays are suppressed by three effects: (1) P-wave phase space, (2) a smaller wave-function overlap

$$\frac{\int d^3x(u\tilde{u}-l\tilde{l})(uu'+ll')}{\int d^3x(u^2+l^2)(uu'+ll')}=0.29 , \tag{44}$$

where the tilde denotes a P-wave wave function, and (3) the location of the $1P$ primitives is further from the H mass and hence the energy denominator is larger. We estimate that the combined effect yields a factor of about 5 suppression in amplitude. However this is compensated in part by the fact that the P waves can be in an SU(3)-octet state, and hence can be enhanced in amplitude by the QCD Wilson coefficients in the weak Hamiltonian, viz.,

$$c_1/c_4=\frac{2.5}{0.4}\simeq 6 . \tag{45}$$

Thus in $\Delta S=1$ decay the S and P waves are expected to be roughly comparable. The classification of six-quark states for the P wave has not been worked out and the experimental information on the P-matrix poles and residues is missing. Thus we feel that we are limited in predictive power to this very rough estimate. It will introduce a factor of 2 uncertainty in our lifetime estimates.

Since a precise value for Γ_H (P wave) cannot be given it might appear that it will be difficult to really learn much from $\Delta S=1$ H decays. However, that is not entirely the case. Although we cannot separate S waves from P waves by the usual technique of measuring a parity-violating correlation (the H is a scalar and cannot therefore be polarized; only final-state $\epsilon_{\alpha\beta\gamma\delta}S_1^\alpha S_2^\beta P_1^\gamma P_2^\delta$ correlations are possible), it is in principle possible to attempt a separation by measuring branching ratios for $\Delta S=1$ decay. The

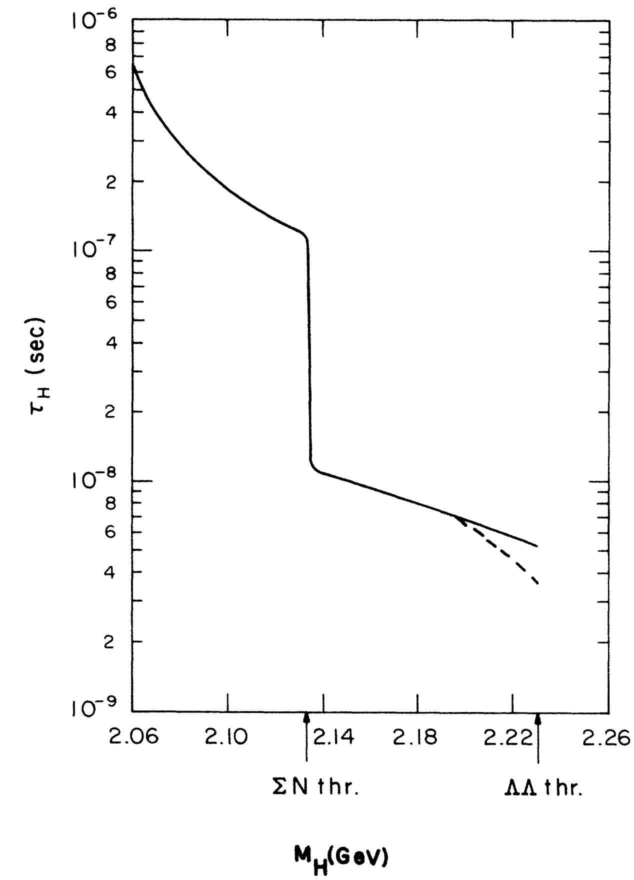

FIG. 1. S-wave $\Delta S=1$ $H\to B_1B_2$ lifetime as a function of H mass. The dashed curve displays the effect of $H\to\Lambda N\pi$, and thr means threshold.

point is that the P-wave final state must be antisymmetric in flavor and must come from the octet component of $H_w^{\Delta S=1}$. The relevant final-state wave function is then

$$|8_A\rangle=(\tfrac{1}{2})^{1/2}n\Lambda-(\tfrac{1}{6})^{1/2}n\Sigma^0+(\tfrac{1}{3})^{1/2}p\Sigma^- . \tag{46}$$

Thus we expect, for the P-wave rates ($\Gamma_H^{(P)}$)

$$\Gamma_H^{(P)}(\Sigma^-p):\Gamma_H^{(P)}(\Sigma^0n):\Gamma_H^{(P)}(\Lambda n)::1:\tfrac{1}{2}\left[\frac{k(\Sigma^0n)}{k(\Sigma^-p)}\right]^3:\frac{3}{2}\left[\frac{k(\Lambda n)}{k(\Sigma^-p)}\right]^3 , \tag{47}$$

as compared to the S-wave rates ($\Gamma_H^{(S)}$),

$$\Gamma_H^{(S)}(\Sigma^-p):\Gamma_H^{(S)}(\Sigma^0n):\Gamma_H^{(S)}(\Lambda n)::1:1.12\left[\frac{k(\Sigma^0n)}{k(\Sigma^-p)}\right]:0.37\left[\frac{k(\Lambda n)}{K(\Sigma^-p)}\right] , \tag{48}$$

where $k(B_1B_2)$ is the decay momentum for the B_1B_2 channel, and we have referred to Eq. (41) in writing Eq. (48). If the decay momenta were known, it would be straightforward using Eqs. (47) and (48) to disentangle the S-wave and P-wave decay rate contributions.

An interesting possibility arises if the H mass lies in the range $M_\Lambda+M_n<M_H<M_\Sigma+M_N$ in which case only decay to the Λn channel will be allowed. The ΛN final state can only be $I=\frac{1}{2}$ and hence the S-wave amplitude is suppressed by the QCD coefficients. The lifetime will then be somewhat longer than one might otherwise expect.

The H lifetime is given by

$$\tau_H = \frac{\hbar}{\Gamma_H(\text{tot})} \tag{49}$$

with

$$\Gamma_H(\text{tot}) = \Gamma_H^{(S)} + \Gamma_H^{(P)} + \cdots \ . \tag{50}$$

The ellipsis in Eq. (50) refers to modes with final states other than B_1B_2 such as $H \to \Lambda N\pi$ (see Sec. V). Our overall results, for all masses relevant to $\Delta S = 1$ $H \to B_1B_2$ decay, are summarized in Fig. 1. Only S-wave contributions are plotted there, so that we must remind the reader of the factor of 2 decrease in τ which is possible due to the P wave. We see a rather long lifetime, somewhat comparable to the K^+ decay, in the range of $\tau_H \sim 10^{-8}$ sec. This long lifetime is important for those experiments which search for the H by attempting to see its decay. The detector must be large, or the H very low energy, in order that the decay be visible.

V. THE $\Delta S = 1$ DECAY $H \to \Lambda N\pi$

If the H exists at the mass predicted by Jaffe it is below the threshold for decay by pion emission. Only in the 38-MeV range of H mass from $\Lambda\Lambda$ threshold down to 2.193 GeV is the decay $\Lambda N\pi$ kinematically allowed (the $\Sigma N\pi$ channel is always closed). Given such limited phase space we would ordinarily expect that the pionic mode would make only a small correction to the lifetime obtained from $H \to B_1B_2$. However there is a symmetry consideration which can make the pionic mode important when energetically allowed. Previously we have argued that for S waves $H \to B_1B_2$ proceeds only through the (suppressed) 27-plet portion of H_w. The decay $H \to \Lambda N\pi$ can occur via the (enhanced) octet operator in H_w and hence could be large. In this section we provide an estimate of the octet contribution to $H \to \Lambda N\pi$.

When dealing with pions it is useful to employ the soft-pion theorem which relates a pionic amplitude to one with the pion removed:

$$\lim_{q \to 0} \langle \beta\pi^i(q) \,|\, 0 \,|\, \alpha \rangle = \frac{-i}{F_\pi} \langle \beta \,|\, [F_5^i, 0] \,|\, \alpha \rangle + \cdots \ . \tag{51}$$

In the case of the weak Hamiltonian the commutator of Eq. (52) leaves its chiral structure unaltered:

$$[F_5^3, H_w] = \tfrac{1}{2} H_w \ . \tag{52}$$

Thus in the soft-pion limit the pionic amplitude is related to one calculated in the previous section:

$$\lim_{q_\pi \to 0} \langle \Lambda N\pi^0 \,|\, H_w \,|\, H \rangle = \frac{i}{2F_\pi} \langle \Lambda N \,|\, H_w \,|\, H \rangle \ . \tag{53}$$

We have seen that the latter amplitude proceeds only using the 27-plet. This shows that in the soft-pion limit there is no octet contribution from the commutator. However in standard usage of PCAC (partial conservation of axial-vector current) the commutator term must be supplemented by dibaryon poles, where the H emits a pion via the strong interaction to become an $I = 1$ SU(3)-octet dibaryon, which later makes a weak transition to ΛN. When the weak and strong processes occur in the reverse order, the $H \to$dibaryon transition must use the 27-plet operator, and we will drop this contribution. A study of Jaffe's dibaryon states (see Table I) shows that the only state which can occur is the $J = 1$ octet state. Thus the only pole terms which are important have the pion in a P-wave state. It is these which we must estimate.

The overall transition amplitude for $H \to \Lambda N\pi$ is beyond the technical capability of present quark-model methods due to the number of particles in the final state. Likewise the strong pion vertex is not presently calculable. For our estimate we will use a method that utilizes the theory of P-wave hyperon decays. The P-wave amplitudes for $B \to B'\pi$ are themselves treated by pole diagrams,[6] as we wish to do for the H. We will treat the H as an SU(3)-singlet combination of B_1B_2 and then use the *experimental* $B_1 \to \Lambda\pi$ or $B_2 \to N\pi$ amplitudes as our pole amplitudes. The other baryon is treated as a "spectator." Of course, P-wave phase space will be included to appropriately account for the various possible H masses. For the decay $H \to \Lambda N\pi$ there are only two B_1B_2 combinations which are relevant, viz., $B_1 = \Xi$ (with $\Xi \to \Lambda\pi$), $B_2 = N$, and $B_1 = \Lambda$ (with $\Lambda \to N\pi$), $B_2 = \Lambda$. In this approach, then, we have

$$\begin{aligned}\Gamma(H \to \Lambda p\pi^-) = {} & 2C(\Lambda\Lambda)\Gamma_P(\Lambda \to p\pi^-) \times (\text{phase space}) \\ & + C(\Xi^- p)\Gamma_P(\Xi^- \to \Lambda\pi^-) \\ & \times (\text{phase space}) \ ,\end{aligned} \tag{54}$$

where $C(\Lambda\Lambda)$ and $C(\Xi^- p)$ are the probabilities of finding $\Lambda\Lambda$ and $\Xi^0 n$ in the H and Γ_P is the rate due to the P wave only. The amplitudes do not interfere because the two decays populate different regions on the Dalitz plot. An SU(3) singlet has the composition

$$1 = \frac{1}{\sqrt{14}} (\Lambda\Lambda + \Sigma^0\Sigma^0 + 2\Sigma^+\Sigma^- + 2p\Xi^- + 2n\Xi^0) \ . \tag{55}$$

This leads us to use $C(\Lambda\Lambda) = \frac{1}{14}$, $C(\Xi^- p) = \frac{2}{7}$. Phase space scales as $(q/q_{\max})^3$ due to its P-wave character. Let us first give an estimate of the decay rate at the largest possible mass, i.e., at $\Lambda\Lambda$ threshold. Using the experimental P-wave amplitudes we find

$$\Gamma(H \to \Lambda p\pi^-) = 3.9 \times 10^{-14} \text{ MeV} \tag{56}$$

which would be equivalent by itself to a lifetime of 1.7×10^{-8} sec. Thus the pionic mode does not overwhelm the B_1B_2 channel calculated in the last section. To complete the calculation one needs to add in the $\Lambda n\pi^0$ mode (a 50% increase of Γ) and account for phase space:

$$\Gamma(H \to \Lambda N\pi) = (6 \times 10^{-14} \text{ MeV}) \left[\frac{q}{100 \text{ MeV}} \right]^3 \ . \tag{57}$$

This contribution is displayed by the dotted curve in Fig. 1.

VI. $\Delta S = 2$ DECAYS

If the H mass were below $m_n + m_\Lambda$, all $\Delta S = 1$ decay channels would be kinematically forbidden and the H would need to decay via $\Delta S = 2$ to an nn final state. This possibility seems unlikely, but it has been raised recently as a possible way to explain the unusual events associated

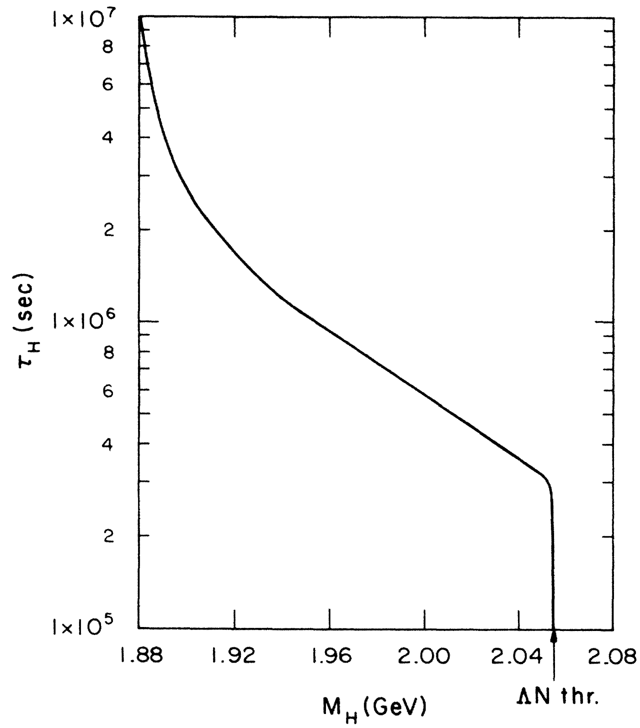

FIG. 2. S-wave $\Delta S=2$ $H\to NN$ lifetime as a function of H mass.

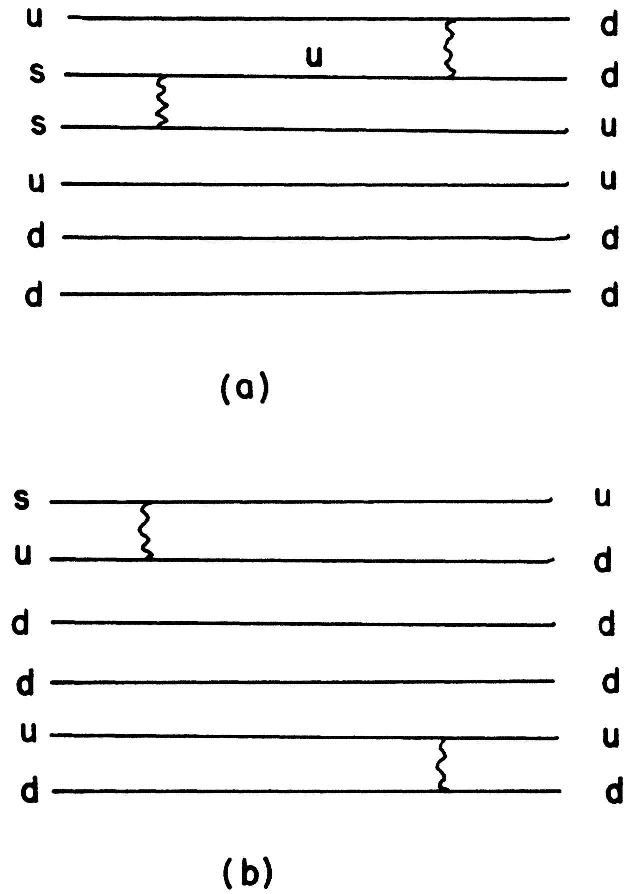

FIG. 3. Dispersive contributions to the $\Delta S=2$ $H\to NN$ amplitude.

with radiation from Cygnus X-3 (Ref. 2). We have previously published a calculation of the short-distance $\Delta S=2$ transition due to the box diagram. After a brief summary of this result, we address the possible long-range dispersive component which makes use of two $\Delta S=1$ transitions.

Because it has hypercharge $Y=2$, the nn state is automatically a member of the 27-plet. The $\Delta S=2$ Hamiltonian is also in a 27-plet, and is a partner of the $\Delta S=1$ operators O_3 and O_4. The hadronic matrix element can then be obtained from Eq. (39) by SU(3) Clebsch-Gordan factors or by direct computation. The result is

$$\langle nn\,|\,H_w^{\Delta S=2}\,|\,H\rangle=\frac{G_F{}^2}{16\pi^2}m_c{}^2\cos^2\theta_C\sin^2\theta_C\eta A'$$
$$=3.3\times10^{-16}\ \text{GeV} \tag{58}$$

with

$$A'=2\sqrt{10}\int d^3x(uu'+ll')^2\ . \tag{59}$$

When combined with P-matrix techniques, this yields the lifetimes displayed in Fig. 2.

We turn now to the dispersive component. In a six-quark bag picture there will exist two types of diagrams as shown in Fig. 3, which differ as to whether the W exchange is between two separate pairs of quarks 3(b) or among just three quarks 3(a). Here the wiggly line represents an effective $\Delta S=1$ Hamiltonian.

Consider first Fig. 3(b). In this case the intermediate six-quark state must by Jaffe's theorem by an SU(3) 27-plet. Denoting this positive-parity configuration as 27^+,

$$\langle nn\,|\,H_w^{\Delta S=2}(\text{disp})\,|\,H\rangle^{3b}$$
$$\simeq\frac{\langle nn\,|\,H_w^{\Delta S=1}\,|\,27^+\rangle\langle 27^+\,|\,H_w^{\Delta S=1}\,|\,H\rangle}{m_H-m_{27^+}}\ . \tag{60}$$

Of course, since a sum over a complete set of intermediate states is implied here a precise evaluation is impossible. However, we can estimate this contribution by calculating

$$\langle 27^+\,|\,H_w^{\Delta S=1}\,|\,H\rangle\frac{G_F}{2\sqrt{2}}\cos\theta_C\sin\theta_C c_4\langle 27^+\,|\,O_4\,|\,H\rangle$$
$$\simeq c_4 6\times10^{-9}\ \text{GeV}\ . \tag{61}$$

Thus only the 27-component of $H_w^{\Delta S=1}$ is involved so this contribution is somewhat suppressed. However, the connection of the **27** state to nn can proceed by the dominant octet transition, i.e.,

$$\langle nn\,|\,H_w^{\Delta S=1}\,|\,27^+\rangle\simeq\frac{G_F}{2\sqrt{2}}\cos\theta_C\sin\theta_C c_1\langle nn\,|\,O_1\,|\,27^+\rangle$$
$$\simeq\frac{c_1}{c_4}\langle 27^+\,|\,H_w^{\Delta S=1}\,|\,H\rangle\ . \tag{62}$$

Also we estimate

$$m_H - m_{27^+} \sim E_s^{1S} - E_u^{1S} \sim 150 \text{ MeV}$$

so that

$$\langle nn \mid H_w^{\Delta S=2}(\text{disp}) \mid H \rangle^{3b} \sim 2.4 \times 10^{-16} \text{ GeV} \tag{63}$$

which is comparable to the box-diagram contribution. Likewise from Fig. 3(a) we have a contribution from a class of six-quark intermediate states with all quarks in the $1S_{1/2}$ spatial levels. By Jaffe's theorem these states must be members of the SU(3) **27**, and our results from the previous section [Eq (39)] should apply. However, it is also possible for an intermediate state wherein one quark is promoted to the $1P_{1/2}$ level. In this case we have negative-parity octet intermediate states (denoted as 8^-) so

$$\langle nn \mid H_w^{\Delta S=2}(\text{disp}) \mid H \rangle^{2a}_{P\text{ wave}} \simeq \frac{\langle nn \mid H_w^{\Delta S=1} \mid 8^- \rangle \langle 8^- \mid H_w^{\Delta S=1} \mid H \rangle}{m_H - m_{8^-}} \tag{64}$$

and there is no $H_w^{\Delta S=1}$ (27) suppression. However, P-wave bag-model overlaps tend to be somewhat smaller than their S-wave counterparts so we take [see Eq. (44)]

$$\frac{\langle nn \mid H_w^{\Delta S=1} \mid 8^- \rangle}{\langle nn \mid H_w^{\Delta S=1} \mid 27^+ \rangle} \sim 0.3 , \tag{65}$$

$$\frac{\langle 8^- \mid H_w^{\Delta S=1} \mid H \rangle}{\langle 27^+ \mid H_w^{\Delta S=1} \mid H \rangle} \sim \frac{c_1}{c_4} \times 0.3 \sim 2 . \tag{66}$$

Finally we estimate

$$m_H - m_{8^-} \sim E_s^{1S} - E_u^{1P} \sim -200 \text{ MeV} . \tag{67}$$

Thus we have

$$\langle nn \mid H_w^{\Delta S=2}(\text{disp}) \mid H \rangle^{3a}_{P\text{ wave}} \sim 0.4 \langle nn \mid H_w^{\Delta S=2}(\text{disp}) \mid H \rangle^{3b} . \tag{68}$$

While the sign change in Eq. (67) associated with the P-wave excitation energy suggests the possibility of cancellation between dispersive contributions from Figs. 3(a) and 3(b), the very rough nature of our estimates forbids us from concluding more than that the size of the dispersive component may well be comparable to that of the box diagram. Both the short-distance and dispersive effects have characteristic lifetimes of a few$\times 10^5$ sec $\sim$ a few days. Given the uncertainty of our methods this is about as well as we can do at present. However it is clear that a lifetime as long as ten years cannot be accommodated in the $\Delta S = 2$ decays of the H (unless its mass is so close to NN threshold to render it almost stable). This poses difficulties for the H explanation of the Cygnus X-3 events.[2]

VII. SUMMARY

In this paper we developed a methodology for constructing quark-model states in the six-quark sector and then applied it to compute weak decays of the H particle. We found that to attribute the six-quark wave function with "intrinsic" properties such as hidden color is a misguided practice. A given six-quark quantum state can take on several very different looking forms. The key to properly characterizing such states is with symmetry (as in Table I) which alone respects the central role played by the Pauli principle.

If discovered, the H particle considered here, a genuine six-quark bound state, would be of considerable interest to particle physics. Its mass would reveal to us a significant clue regarding how quarks bind with gluons. Its lifetime and branching ratios would test our understanding both of the nonleptonic weak Hamiltonian and also of the symmetry structure of the multiquark states.

Although we were forced to make rough estimates at several junctures (especially for the P-wave $\Delta S = 1$ amplitudes and the dispersive contribution to the $\Delta S = 2$ decay), our results were in fact quite decisive.

(i) A very tightly bound H ($M_H < M_n + M_\Lambda$) has a lifetime on the order of days rather than of years. It is therefore not likely to be associated with the hadronic component, if any, in the emission spectrum of Cygnus X-3.

(ii) A less tightly bound H ($M_H > M_n + M_\Lambda$) probably decays with a lifetime (depending on its mass) in the 10^{-8}-sec range, which is rather longer than a naive estimate based on Λ decay of about 10^{-10} sec. Moreover we anticipate a major violation of the $\Delta I = \frac{1}{2}$ rule in the S-wave amplitudes. Needless to say, this phenomenon alone would attract substantial attention.

It is hard to imagine doing much better on the thorny issues of either P-wave $\Delta S = 1$ decay or the dispersive $\Delta S = 2$ amplitude. A proper P-wave calculation would first entail a group-theoretical analysis of six-quark states in which one of the quarks is excited. This should prove a lengthy exercise. As for the $\Delta S = 2$ dispersive amplitude, it is well known in mesonic systems (especially K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$) that a knowledge of low-energy particle interactions greater than that now available is required.

We indicated in Sec. I that for all H masses except those very near the $\Lambda\Lambda$ threshold, we expect our description of a true six-quark bound state to be valid. There is another issue of potential concern, viz., effects of SU(3)-symmetry breaking which result in configuration mixing. In principle such configuration mixing could drastically affect our conclusion regarding S-wave decay of the H by allowing the octet part of the weak Hamiltonian to contribute. However this is not likely to be important. The point is that an SU(3) singlet and 27-plet would need to mix. However, the dominant part of SU(3) breaking is octet in nature, so configuration mixing occurs to second order in symmetry breaking. It is estimated to be small.[9]

We can only hope that our paper succeeds in motivating experimental searches for the H. Some theoretical work exists on formation mechanisms,[10] which together with our analysis of weak decays should provide the experimentalist with useful advice on how to find this elusive yet fascinating particle.

ACKNOWLEDGMENT

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