

Systematic study of large CP violations in decays of neutral b -flavored mesons

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CP -violation effects in partial-decay-rate asymmetries of B_d^0 and B_s^0 systems are examined within the framework of the Kobayashi-Maskawa model. We concentrate on those hadronic final states into which both B^0 and \bar{B}^0 can decay. The rephasing-invariant formalism is used in our calculation. We find a quite large asymmetry (~ 0.6) in some decay modes, such as $B_d^0 \rightarrow D^+ \pi^-$, $B_s^0 \rightarrow D^+ \pi^-$, $D^0 \phi$, etc., but we still need 10^6 – 10^7 $b\bar{b}$ pairs for testing these effects for 3σ signature. In addition, the contribution of the penguin diagrams and the problem of strong-interaction phases of final states are also briefly discussed. We find that for testing CP violations with only the penguin contribution the best decays are $B_d^0 \rightarrow \phi K_S$, $B_s^0 \rightarrow \phi K_S$, which need, for 3σ signature, 3×10^8 $b\bar{b}$ pairs.

I. INTRODUCTION

Until now CP violation has been observed only in the K^0 - \bar{K}^0 complex.¹ As of yet, this effect could be entirely indirect CP violation coming solely from mixing. Only experimental upper bounds exist for a direct CP -violation effect (the ϵ'/ϵ parameter).

There are many speculations and estimates for large CP -violating effects in the B_d^0 - \bar{B}_d^0 and B_s^0 - \bar{B}_s^0 systems. As we know, the charge asymmetry in semileptonic decays, the same-sign dilepton asymmetry,² is predicted to be very small ($< 10^{-2}$). But in nonleptonic decays, the asymmetry may be large due to the interplay of mixing and amplitude interference. Bigi, Carter, and Sanda³ were the first to discuss this problem in general. They restricted themselves to the hadronic final states being CP eigenstates (i.e., $f = \bar{f}$, here f denotes the final hadronic state, \bar{f} is the CP -conjugate state of f , namely, $|\bar{f}\rangle = CP|f\rangle$). Chau and Cheng⁴ discussed the case for $f \neq \bar{f}$ under the condition $B^0 \rightarrow f$, $\bar{B}^0 \not\rightarrow f$, so they have to calculate the decay amplitudes explicitly. We should avoid this because we do not know how to calculate the nonleptonic decay amplitude reliably. Sachs⁵ was the first to discuss the asymmetry of B_d^0 , $\bar{B}_d^0 \rightarrow D^\pm \pi^\mp$ without calculating the decay amplitudes. There are several advantages of Sachs' idea.

(a) The decay amplitudes have a very simple dependence on Kobayashi-Maskawa (KM) matrix elements.

(b) The decay amplitudes cancel out approximately in the expression of the partial-decay-rate asymmetries, so the estimated asymmetries are not sensitive to the hardly measurable strong-interaction amplitudes and phases.

(c) The amplitude interference will lead to the asymmetries even when the $\eta \equiv |p/q| \approx 1$ (see the text).

Following Sachs' idea, we studied systematically the asymmetries in nonleptonic decays of B_d^0 - \bar{B}_d^0 and B_s^0 - \bar{B}_s^0 systems. We discuss both cases for $f = \bar{f}$ and $f \neq \bar{f}$ and

avoid calculating the decay amplitudes. Especially we analyzed carefully the possible two-body nonleptonic decay channels. We find that the most promising decay channels for large asymmetry are $B_d^0 \rightarrow D^+ \pi^-$, $B_s^0 \rightarrow D^+ \pi^-$, $B_s^0 \rightarrow D^0 \phi$. The penguin contributions are also examined. We discuss the two extreme cases: the penguin-dominant case and the one in which penguins are negligible. We find that $B_{d,s}^0 \rightarrow \phi K_S$ are the best candidates for testing CP violation with only the penguin contribution. In our calculation, the rephasing invariants⁶ of the KM matrix are extensively used. That is the only way that the KM elements can enter into physical calculations. We want to stress the fact that as long as we limit ourselves to final states which are strong-interaction eigenstates, the final-state strong-interaction phases cancel out, and we are left with phases coming solely and intrinsically from the KM matrix. This defines a clean and useful testing ground for the three-generation standard model.

The outline of this article is as follows. In Sec. II we give the rephasing-invariant formalism. In Sec. III we discuss partial-decay-rate asymmetries and the exclusive two-body nonleptonic decays. A different tagging, other than the leptonic tagging of Bigi, Carter, and Sanda³ will be advocated. In Sec. IV we discuss the penguin-diagram contributions for the extreme cases. Section V is devoted to the discussion and conclusion.

II. REPHASING-INVARIANT FORMALISM

As we know, all the physical quantities must be particle-phase independent. In the standard model the KM matrix⁷ appears in the charged current

$$\bar{\psi}_u V \psi_d,$$

where

$$\psi_u^T = (u, c, t), \quad \psi_d^T = (d, s, b),$$

and the KM matrix is

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (1)$$

In the standard KM parametrization we have

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}. \quad (2)$$

However, our quark fields are defined up to arbitrary phases. If we make a phase transformation

$$\psi_u \rightarrow \psi'_u = U^\dagger \psi_u, \quad \psi_d \rightarrow \psi'_d = D \psi_d,$$

where

$$U = \begin{pmatrix} U_1 & & \\ & U_2 & \\ & & U_3 \end{pmatrix}, \quad D = \begin{pmatrix} D_1 & & \\ & D_2 & \\ & & D_3 \end{pmatrix},$$

and $|U_i| = |D_i| = 1$. Then $V' = UVD$ will appear as our new KM matrix. Any physical quantity will not depend on our phase convention. As proved by Jarlskog and Wu,⁶ the rephasing invariants we can construct from the KM elements are the nine absolute values of KM matrix elements $|V_{ij}|$; and the nine $\Delta_{i\alpha}$ defined by

$$\Delta_{i\alpha} = V_{j\beta} V_{k\gamma} (V_{j\gamma}^* V_{k\beta}^*) \quad (i, j, k \text{ and } \alpha, \beta, \gamma \text{ co-cyclic}). \quad (3)$$

In Appendix A we list all the nine independent quantities $\Delta_{i\alpha}$. As a trivial consequence we obtain a rephasing-invariant proof that $\text{Im} \Delta_{i\alpha} = t$, where t is a unique CP -gaugeing parameter independent of i and α (Refs. 6 and 8). All physical quantities should be expressed by means of these rephasing invariants.

Take the phase convention as

$$CP |B^0\rangle = |\bar{B}^0\rangle \quad (4)$$

and assume CPT invariance, then the physical eigenstates are

$$|B_L\rangle = p |B^0\rangle + q |\bar{B}^0\rangle, \quad (5a)$$

$$|B_H\rangle = p |B^0\rangle - q |\bar{B}^0\rangle, \quad (5b)$$

with eigenvalues $\lambda_{H,L} = m_{H,L} - i\gamma_{H,L}/2$ where the subscript indicates heavy or light, respectively. According to Ref. 9,

$$\frac{p}{q} = \left[\frac{M_{12} - \frac{i}{2} \Gamma_{12}}{M_{12}^* - \frac{i}{2} \Gamma_{12}^*} \right]^{1/2}. \quad (6)$$

Define $B_\alpha^0 \equiv \bar{b}\alpha$, $\bar{B}_\alpha^0 \equiv b\bar{\alpha}$, where $\alpha = d$ or s . Using the box-diagram dominance¹⁰⁻¹³ and assuming $(m_t/M_W)^2 \ll 1$ we get

$$M_{12} = -B_M \frac{G_F^2 f_B^2 m_B}{12\pi^2} \{ \zeta_t^2 [m_t^2 + O(m_b^2)] + \zeta_t \zeta_c O(m_c^2) \}, \quad (7)$$

$$\Gamma_{12} = B_\Gamma \frac{G_F^2 f_B^2 m_B}{8\pi} [\zeta_t^2 m_b^2 + \frac{8}{3} \zeta_c \zeta_t m_c^2 + O(m_c^4/m_b^2)], \quad (8)$$

where $\zeta_t = V_{tb} V_{t\alpha}^*$ and $\alpha = d$ or s for B_d or B_s . $B_M = B_\Gamma = 1$ under the assumption of vacuum insertion. Actually, for B_d^0 and B_s^0 systems,

$$\Delta m \approx 2 |M_{12}|, \quad (9)$$

$$\Delta\gamma \approx -2 |\Gamma_{12}|. \quad (10)$$

By spectator dominance¹⁴⁻¹⁷ we get

$$\gamma = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \rho, \quad (11)$$

where $\rho \approx 3.2$ in which we include phase-space and QCD corrections.

Therefore

$$z_{B_s} \equiv \left[\frac{\Delta m}{\gamma} \right]_{B_s} = 0.8 B_M^s \left[\frac{f_{B_s}}{100 \text{ MeV}} \right]^2 \left[\frac{m_t}{40 \text{ GeV}} \right]^2, \quad (12)$$

$$z_{B_d} \equiv \left[\frac{\Delta m}{\gamma} \right]_{B_d} = 0.8 B_M^d \left[\frac{f_{B_d}}{100 \text{ MeV}} \right]^2 \left[\frac{m_t}{40 \text{ GeV}} \right]^2 \left| \frac{V_{31}^*}{V_{23}} \right|^2. \quad (13)$$

The uncertainties in determining our mixing parameter are the decay constant $f_B = 50-200$ MeV, bag parameter ~ 1 , top-quark mass, and for the B_d system an added uncertainty of KM elements. Using¹⁸ the b lifetime 1.06 ± 0.17 psec and the upper limit on $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c) < 4\%$ (Ref. 19) we obtain for the KM phase convention that

$$s_2 = 0.05 \pm 0.02, \quad s_3 < 0.04. \quad (14)$$

In a later calculation, we shall use, for definiteness,

$$m_t \sim 40 \text{ GeV}, \quad s_1 \sim 0.231, \quad (15)$$

$$s_2 \sim s_1^2, \quad s_3 \sim 0.5s_2, \quad s_8 \sim 1.$$

Our physical states evolve in time by $e^{-imt} |B_{L,H}\rangle$ with m being the mass matrix. A pure B^0 at $t=0$ evolves in time as

$$|B^0_{\text{phys}}(t)\rangle = f_+(t) |B^0\rangle + \frac{q}{p} f_-(t) |\bar{B}^0\rangle, \quad (16)$$

whereas a pure \bar{B}^0 at $t=0$ evolves as

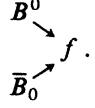
$$|\bar{B}^0_{\text{phys}}(t)\rangle = \frac{p}{q} f_-(t) |B^0\rangle + f_+(t) |\bar{B}^0\rangle, \quad (17)$$

where

$$f_+(t) = \frac{1}{2}(e^{-i\lambda_L t} + e^{-i\lambda_H t}), \quad (18a)$$

$$f_-(t) = \frac{1}{2}(e^{-i\lambda_L t} - e^{-i\lambda_H t}). \quad (18b)$$

We limit ourselves to the final states that both the pure B^0 and \bar{B}^0 can decay into



Unlike Bigi, Carter, and Sanda, we do not restrict f to be a CP eigenstate. Denote the CP -conjugate state of f by \bar{f} :

$$|\bar{f}\rangle = CP|f\rangle. \quad (19)$$

Define

$$x = \frac{\bar{A}(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} \equiv \frac{\bar{A}(f)}{A(f)}, \quad \bar{x} = \frac{A(B^0 \rightarrow \bar{f})}{\bar{A}(\bar{B}^0 \rightarrow \bar{f})} \equiv \frac{A(\bar{f})}{\bar{A}(\bar{f})}. \quad (20)$$

It is easy to see that $(q/p)x$ and $(p/q)\bar{x}$ are particle rephasing invariants. [Since $|B_H\rangle, |B_L\rangle$ in Eq. (5) are physical eigenstates, rephasing⁶ B^0, \bar{B}^0 must not change our physical eigenstates. Suppose we rephase $|B^0\rangle \rightarrow e^{i\beta}|B^0\rangle, |\bar{B}^0\rangle \rightarrow e^{i\beta}|\bar{B}^0\rangle$. Then we must simultaneously rephase $p \rightarrow e^{-i\beta}p, q \rightarrow e^{-i\beta}q$. Also $x \rightarrow e^{i(\beta-\beta)}x$. Therefore $(q/p)x \rightarrow (q/p)x$. We still have an overall phase ambiguity of $|B_H\rangle$ to $|B_L\rangle$, but that does not concern us here.] Hence even the phase of $(q/p)x$ and $(p/q)\bar{x}$ has physical relevance.

Now we are in a position to discuss the partial-decay-rate asymmetries.

III. PARTIAL-DECAY-RATE ASYMMETRY

Since the neutral b -flavored mesons have a short lifetime, we will be primarily interested in time-integrated effects. Our CP -violating asymmetry is defined as

$$C_f = \frac{\Gamma(B_{\text{phys}}^0 \rightarrow f) - \Gamma(\bar{B}_{\text{phys}}^0 \rightarrow \bar{f})}{\Gamma(B_{\text{phys}}^0 \rightarrow f) + \Gamma(\bar{B}_{\text{phys}}^0 \rightarrow \bar{f})}, \quad (21)$$

where $\Gamma(B_{\text{phys}}^0 \rightarrow f)$ is the time-integrated partial decay rate of a time-evolved $B^0(t=0)$ into the specified final state f .

We will consider in this paper mainly those decays where there is no direct CP violation in magnitude in the pure B^0 and \bar{B}^0 . Namely, that

$$|A(B^0 \rightarrow f)| = |\bar{A}(\bar{B}^0 \rightarrow \bar{f})|. \quad (22)$$

It is readily realized how to accomplish requirement (22). Take^{3,6}

$$A(f) = G_1 a e^{i\alpha} + G_2 b e^{i\beta}, \quad (23)$$

where G_1, G_2 are multiplication of two KM elements, α, β are the strong-interaction final-state phases, and a, b denote real amplitudes. Then the CP -conjugated amplitude is

$$\bar{A}(\bar{f}) = G_1^* a e^{i\alpha} + G_2^* b e^{i\beta}. \quad (24)$$

It is clear that to obtain $|A(f)| \neq |\bar{A}(\bar{f})|$ we need to have two different strong channels ($\alpha \neq \beta$) and furthermore $G_1 \neq G_2$ must happen too. Therefore as long as we limit ourselves to decay amplitudes where only one KM combination appears; or alternatively limit ourselves to a decay amplitude where only one strong channel is available we are guaranteed that no direct magnitudinal CP violations occur in the pure amplitudes. Assuming Eq. (22) holds, we get, for our time-integrated asymmetry (21),

$$C_f = \frac{\left(\left| \frac{q}{p}x \right|^2 - \left| \frac{p}{q}\bar{x} \right|^2 \right) (1-a) + 2y \operatorname{Re} \left[\frac{q}{p}x - \frac{p}{q}\bar{x} \right] - 2az \operatorname{Im} \left[\frac{q}{p}x - \frac{p}{q}\bar{x} \right]}{2(1+a) + \left(\left| \frac{q}{p}x \right|^2 + \left| \frac{p}{q}\bar{x} \right|^2 \right) (1-a) + 2y \operatorname{Re} \left[\frac{q}{p}x + \frac{p}{q}\bar{x} \right] - 2az \operatorname{Im} \left[\frac{q}{p}x + \frac{p}{q}\bar{x} \right]}, \quad (25)$$

where

$$z \equiv \frac{\Delta m}{\gamma}, \quad y \equiv \frac{\Delta \gamma}{2\gamma}, \quad a \equiv \frac{1-y^2}{1+z^2}, \quad \omega \equiv \frac{\Delta \gamma}{\Delta m}. \quad (26)$$

Assuming box-diagram dominance we obtain^{5,11-13} $|\omega| \ll 1$ and^{3,5,11-13}

$$\frac{q}{p} \simeq \frac{\xi_t^*}{\xi_t} \left[1 + O \left[\left(\frac{m_c}{m_t} \right)^2 \right] \right]. \quad (27)$$

Now if we limit ourselves further to hadronic final states which can proceed only through one strong-interaction channel we get

$$\bar{x} = x^*,$$

since the final-state strong-interaction phases cancel out and we are left only with the complex conjugation of our weak phases (KM elements).

After some lengthy arithmetic we obtain for our asymmetry assuming $|A(f)| = |\bar{A}(\bar{f})|$, $\bar{x} = x^*$, and box-diagram dominance

$$C_f = \frac{M[|x|^2(1-a) + y \operatorname{Re} \lambda] - 2az \operatorname{Im} \lambda}{1+a + |x|^2(1-a) + 2y \operatorname{Re} \lambda - Maz \operatorname{Im} \lambda}, \quad (28)$$

where

$$M = 4\pi \left(\frac{m_c}{m_t} \right)^2 \operatorname{Im} \frac{\xi_c}{\xi_t}, \quad \lambda \equiv \frac{\xi_t^*}{\xi_t} x, \quad (29)$$

and ξ_c, ξ_t are defined in Eqs. (7) and (8). We notice that

ξ_c/ξ_t is rephasing invariant, and the $\text{Im}(\xi_c/\xi_t)$ part in the standard KM parametrization is $\text{Im}(\xi_c/\xi_t) \approx -s_1^2(s_3/s_2)s_\delta$ for B_s , and $\text{Im}(\xi_c/\xi_t) = (s_3/s_2)s_\delta \lesssim 1$ for B_d , so that M is small for B_s and B_d cases:

$$M|_{B_d} \approx 10^{-2}, \quad M|_{B_s} \approx (-)5 \times 10^{-4}.$$

Assuming $\omega^2 \equiv (\Delta\gamma/\Delta m)^2 \ll 1$ in addition to all the assumptions needed for obtaining Eq. (28), we get

$$C_f = \frac{z\{M[|x|^2z + \frac{1}{2}\omega(1+z^2)\text{Re}\lambda] - 2(1 - \frac{1}{4}\omega^2z^2)\text{Im}\lambda\}}{2+z^2+z^2|x|^2 + \omega z(1+z^2)\text{Re}\lambda - Mz(1 - \frac{1}{4}\omega^2z^2)\text{Im}\lambda}. \quad (30)$$

For the large asymmetries we will encounter below, the M terms can be neglected up to a few percent. That is to say $q/p \approx \xi_t^*/\xi_t$.

The accessible final state f in quark content via both a B^0 and \bar{B}^0 are presented in Table I for B_d and B_s systems. For convenience of later calculation, we list in Table II all the expressions for $|x|^2$, $\text{Im}\lambda$, and $\text{Re}\lambda$ in the KM phase convention.

The conventional wisdom is to tag onto the primary lepton of the accompanying B_{phys}^0 (Ref. 3). Hence the time-integrated asymmetry³ to be measured is

$$C_{fl} \equiv \frac{N(fl^-) - N(\bar{f}l^+)}{N(fl^-) + N(\bar{f}l^+)}, \quad (31)$$

where $N(fl^-)$ denotes the number of $B_{\text{phys}}^0 \bar{B}_{\text{phys}}^0 \rightarrow fl^-$ events integrated over time. The $B^0 \bar{B}^0$ wave function can be charge-conjugation parity $(-)^c$ even or odd, and is given by³

$$|B_{\text{phys}}^0, \bar{B}_{\text{phys}}^0\rangle = |B_{\text{phys}}^0, \mathbf{K}\rangle |\bar{B}_{\text{phys}}^0, -\mathbf{K}\rangle + (-)^c |\bar{B}_{\text{phys}}^0, \mathbf{K}\rangle |B_{\text{phys}}^0, -\mathbf{K}\rangle. \quad (32)$$

For primary leptons (not from cascade decay) we assume that^{3,2,31} a pure $B^0 (\equiv \bar{b}\alpha) \rightarrow l^+ X$ only, a pure $\bar{B}^0 (\equiv b\bar{\alpha}) \rightarrow l^- X$ only, that $|A(l^+)| = |\bar{A}(l^-)|$ and that Eq. (22) is satisfied, namely, $|\bar{x}| = |x|$. Then we obtain, for our time-integrated lepton-tagging asymmetry,

$$C_{fl} = \frac{N}{D}, \quad (33a)$$

$$N = \left[\left| \frac{q}{p}x \right|^2 - \left| \frac{p}{q}\bar{x} \right|^2 \right] [1 - a^2 + (-)^c(y^2 + z^2a^2)] + [1 + (-)^c] 2 \left[y \text{Re} \left[\frac{q}{p}x - \frac{p}{q}\bar{x} \right] - za^2 \text{Im} \left[\frac{q}{p}x - \frac{p}{q}\bar{x} \right] \right], \quad (33b)$$

$$D = 2(1 + a^2) + 2(-)^c(y^2 - z^2a^2) + \left[\left| \frac{q}{p}x \right|^2 + \left| \frac{p}{q}\bar{x} \right|^2 \right] [1 - a^2 + (-)^c(y^2 + z^2a^2)] + [1 + (-)^c] 2 \left[y \text{Re} \left[\frac{q}{p}x + \frac{p}{q}\bar{x} \right] - za^2 \text{Im} \left[\frac{q}{p}x + \frac{p}{q}\bar{x} \right] \right]. \quad (33c)$$

For C odd $B^0 \bar{B}^0$ state, $|q/p| \neq 1$ will lead to nonzero asymmetry C_{fl} . In addition to the assumptions leading to Eq. (33) we require also $\bar{x} = x^*$ and box-diagram dominance including $\omega^2 \ll 1$ (Refs. 5, 11–13, 10); we get, for the C even case,

$$C_{fl} = \frac{N_{fl}}{D_{fl}}, \quad (34a)$$

$$N_{fl} \approx zM[|x|^2z(3+z^2 + \frac{1}{4}\omega^2z^4) + \omega(1+z^2)^2\text{Re}\lambda] - 4z(1 - \frac{1}{2}\omega^2z^2 + \frac{1}{16}\omega^4z^4)\text{Im}\lambda, \quad (34b)$$

$$D_{fl} \approx 2+z^2+z^4 + \frac{1}{4}\omega^2z^6 + |x|^2z^2(3+z^2 + \frac{1}{4}\omega^2z^4) + 2\omega z(1+z^2)^2\text{Re}\lambda - M2z(1 - \frac{1}{2}\omega^2z^2 + \frac{1}{16}\omega^4z^4)\text{Im}\lambda. \quad (34c)$$

At a high-energy e^+e^- collider, for example, CERN LEP, we have copious production of Z^0 . Sitting on the Z^0 resonance, $b\bar{b}$ pairs will be produced and hadronized into $B_{d,s}^0 B_u^-$ and $\bar{B}_{d,s}^0 B_u^+$. Here, the creation of b -flavored mesons is incoherent, then we can tag on the accompanying charged b -flavored mesons B_u^\pm . Observing the charge of B_u^+ (B_u^-) would confirm the observed decayed neutral b -flavored meson to be $\bar{B}_d^0 = b\bar{d}$ ($B_d^0 = \bar{b}d$) or $\bar{B}_s^0 = b\bar{s}$ ($B_s^0 = \bar{b}s$) structure at $t=0$. In that case, we can use the asymmetry defined by Eq. (21).

Assuming the ratio of the probabilities of creating $q\bar{q}$ pairs from the vacuum

$$u\bar{u}:d\bar{d}:s\bar{s} \approx 2:2:1, \quad (35)$$

the probabilities³ for producing $B_s B_u^-$, $\bar{B}_s B_u^+$, according to Eq. (35) are $\sigma(B_s B_u^\pm) = \frac{2}{25}$, and for $B_d B_u^-$, $\bar{B}_d B_u^+$, $\sigma(B_d B_u^\pm) = \frac{4}{25}$. In general, having an asymmetry

TABLE I. The accessible final states f in quark content via both a B^0 and \bar{B}^0 are presented for B_d and \bar{B}_s systems. The parentheses adjacent to each final state gives the possible decay mechanisms into this particular f . We employ Chau's (Ref. 1) weak-decay classification: spec for spectator; ex for exchange; pen for penguin. The subscript attached to the exchange and penguin diagrams tells you what vacuum pair has been created. For example, ex_u is the exchange diagram with $u\bar{u}$ pair creation from the vacuum. We also list the corresponding parameters x and λ . Note that λ is expressed by KM phase invariants and $q_K p_K$ are the corresponding parameters q and p for $K^0-\bar{K}^0$ system.

Case (quark decay)	f	$x = \frac{\bar{A}(f)}{A(f)}$	λ
$B_d^0-\bar{B}_d^0$ system			
1. $\bar{b} \rightarrow \bar{u}ud$	$\bar{u}u\bar{u}(ex_u), \bar{u}u\bar{s}(ex_s)$	$\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}$	$\frac{\Delta_{22}^*}{\Delta_{22}}$
2. $\bar{b} \rightarrow \bar{u}c\bar{d}$	$\bar{u}c\bar{d}d(\text{spec}, ex_d), \bar{u}c\bar{u}u(ex_u),$ $\bar{u}c\bar{s}(ex_s), \bar{u}c\bar{c}(ex_c)$	$\frac{V_{cb}V_{ud}^*}{V_{ub}^*V_{cd}}$	$\left \frac{V_{23}}{V_{13}} \right ^2 \frac{\Delta_{22}^*}{\Delta_{12}^*}$
3. $\bar{b} \rightarrow \bar{c}u\bar{d}$	$\bar{c}u\bar{d}d(\text{spec}, ex_d), \bar{c}u\bar{u}u(ex_u)$ $\bar{c}u\bar{s}(ex_s), \bar{c}u\bar{c}(ex_c)$	$\frac{V_{ub}V_{cd}^*}{V_{cb}^*V_{ud}}$	$\left \frac{V_{13}}{V_{23}} \right ^2 \frac{\Delta_{12}}{\Delta_{22}}$
4. $\bar{b} \rightarrow \bar{c}c\bar{d}$	$\bar{c}c\bar{s}(ex_s)$	$\frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}}$	$\frac{\Delta_{12}}{\Delta_{12}^*}$
5. $\bar{b} \rightarrow \bar{u}u\bar{s}$	$\bar{u}uK_S(\text{spec}, \text{pen})$	$\frac{V_{ub}V_{us}^*}{V_{ub}^*V_{us}} \left[\frac{q_K}{p_K} \right]^*$	$\frac{\Delta_{22}^*}{\Delta_{22}}$
6. $\bar{b} \rightarrow \bar{u}c\bar{s}$	$D^0K_S(\text{spec})$	$\frac{V_{cb}V_{us}^*}{V_{ub}^*V_{cs}} \left[\frac{q_K}{p_K} \right]^*$	$(-)\frac{\Delta_{12}}{\Delta_{22}}$
7. $\bar{b} \rightarrow \bar{c}u\bar{s}$	$\bar{D}^0K_S(\text{spec})$	$\frac{V_{ub}V_{cs}^*}{V_{cb}^*V_{us}} \left[\frac{q_K}{p_K} \right]^*$	$(-)\frac{\Delta_{22}^*}{\Delta_{12}^*}$
8. $\bar{b} \rightarrow \bar{c}c\bar{s}$	$\psi K_S(\text{spec}, \text{pen})$	$\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}} \left[\frac{q_K}{p_K} \right]^*$	$\frac{\Delta_{12}}{\Delta_{12}^*}$
9. $\bar{b} \rightarrow \bar{s}$	$\phi K_S(\text{pen}_s), \eta K_S^*(\text{pen}, \text{spec})$	$\frac{V_{tb}V_{ts}^*}{V_{tb}^*V_{ts}} \left[\frac{q_K}{p_K} \right]^*$	$\frac{\Delta_{23}}{\Delta_{23}^*}$
10. $\bar{b} \rightarrow \bar{d}$	$K^0\bar{K}^0$	$\frac{V_{tb}V_{td}^*}{V_{tb}^*V_{td}}$	1
$B_s^0-\bar{B}_s^0$ system			
1. $\bar{b} \rightarrow \bar{u}u\bar{s}$	$\bar{u}u\bar{u}(ex_u), \bar{u}u\bar{d}(ex_d)$	$\frac{V_{ub}V_{us}^*}{V_{ub}^*V_{us}}$	$\frac{\Delta_{21}}{\Delta_{21}^*}$
2. $\bar{b} \rightarrow \bar{u}c\bar{s}$	$\bar{u}c\bar{s}(ex_s), \bar{u}c\bar{u}u(ex_u),$ $\bar{u}c\bar{d}d(ex_d), \bar{u}c\bar{c}(ex_c)$	$\frac{V_{cb}V_{us}^*}{V_{ub}^*V_{cs}}$	$\left \frac{V_{23}}{V_{13}} \right ^2 \frac{\Delta_{21}}{\Delta_{11}}$
3. $\bar{b} \rightarrow \bar{c}u\bar{s}$	$\bar{c}u\bar{s}(ex_s), \bar{c}u\bar{u}u(ex_u)$ $\bar{c}u\bar{d}d(ex_d), \bar{c}u\bar{c}(ex_c)$	$\frac{V_{ub}V_{cs}^*}{V_{cb}^*V_{us}}$	$\left \frac{V_{13}}{V_{23}} \right ^2 \frac{\Delta_{11}^*}{\Delta_{21}^*}$
4. $\bar{b} \rightarrow \bar{c}c\bar{s}$	$\bar{c}c\bar{d}(ex_d)$	$\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}$	$\frac{\Delta_{11}^*}{\Delta_{11}}$
5. $\bar{b} \rightarrow \bar{u}u\bar{d}$	$\bar{u}uK_S(\text{spec}, \text{pen})$	$\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}} \left[\frac{p_K}{q_K} \right]^*$	$\frac{\Delta_{21}}{\Delta_{21}^*}$
6. $\bar{b} \rightarrow \bar{u}c\bar{d}$	$D^0K_S(\text{spec})$	$\frac{V_{cb}V_{ud}^*}{V_{ub}^*V_{cd}} \left[\frac{p_K}{q_K} \right]^*$	$(-)\frac{\Delta_{11}^*}{\Delta_{21}^*}$
7. $\bar{b} \rightarrow \bar{c}u\bar{d}$	$\bar{D}^0K_S(\text{spec})$	$\frac{V_{ub}V_{cd}^*}{V_{cb}^*V_{ud}} \left[\frac{p_K}{q_K} \right]^*$	$(-)\frac{\Delta_{21}}{\Delta_{11}}$

TABLE I. (Continued).

Case (quark decay)	f	$x = \frac{\bar{A}(f)}{A(f)}$	λ
8. $\bar{b} \rightarrow \bar{c}d\bar{d}$	ψK_S (spec, pen)	$\frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}} \left(\frac{p_K}{q_K} \right)^*$	$\frac{\Delta_{11}^*}{\Delta_{11}}$
9. $\bar{b} \rightarrow \bar{d}$	ϕK_S (pen _s)	$\frac{V_{tb} V_{td}^*}{V_{tb}^* V_{td}} \left(\frac{p_K}{q_K} \right)^*$	$\frac{\Delta_{23}^*}{\Delta_{23}}$
10. $\bar{b} \rightarrow \bar{s}$	$\phi\phi(K^0\bar{K}^0)$ (pen) $\phi\eta$ (pen, spec)	$\frac{V_{tb} V_{ts}^*}{V_{tb}^* V_{ts}}$	1

^aIf η has some glue in it, the penguin will by far dominate the spectator diagram.

TABLE II. Expressions for $|x|^2$, $\text{Im}\lambda$, and $\text{Re}\lambda$.

Case	$ x ^2$	$\text{Im}\lambda$	$\text{Re}\lambda$
$B_d^0 - \bar{B}_d^0$ system			
1. $\bar{b} \rightarrow \bar{u}u\bar{d}, \bar{u}u\bar{s}$	1	$-\sin 2\delta + 2s_2s_3s_8$	$\cos 2\delta - 2s_2s_3c_8$
2. $\bar{b} \rightarrow \bar{u}c\bar{d}$	$\frac{s_2^2 + s_3^2 + 2s_2s_3c_8}{s_1^4s_3^2}$	$\frac{(s_2 + 2s_3c_8)s_8}{s_1^2s_3}$	$(-)\frac{(s_2c_8 + s_3\cos 2\delta)}{s_1^2s_3}$
3. $\bar{b} \rightarrow \bar{c}u\bar{d}$	$\frac{s_1^4s_3^2}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$\frac{s_1^2s_3[s_2 + 2s_3c_8]s_8}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$(-)\frac{s_1^2s_3(s_2c_8 + s_3\cos 2\delta)}{s_2^2 + s_3^2 + 2s_2s_3c_8}$
4. $\bar{b} \rightarrow \bar{c}c\bar{d}, \bar{c}c\bar{s}, \bar{s}$	1	$(-)\frac{2s_3(s_2 + s_3c_8)s_8}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$\frac{s_2^2 + s_3^2\cos 2\delta + 2s_2s_3c_8}{s_2^2 + s_3^2 + 2s_2s_3c_8}$
$\bar{b} \rightarrow \bar{u}c\bar{s}$	$\frac{s_2^2 + s_3^2 + 2s_2s_3c_8}{s_3^2}$	$(-)\left(\frac{s_2}{s_3} + 2c_8\right)s_8$	$\frac{s_2}{s_3}c_8 + \cos 2\delta$
$\bar{b} \rightarrow \bar{c}u\bar{s}$	$\frac{s_3^2}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$(-)\frac{s_3(s_2 + 2s_3c_8)s_8}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$\frac{s_3(s_3\cos 2\delta + s_2c_8)}{s_2^2 + s_3^2 + 2s_2s_3c_8}$
$\bar{b} \rightarrow \bar{d}$	1	0	1
$B_s^0 - \bar{B}_s^0$ system			
1. $\bar{b} \rightarrow \bar{u}u\bar{d}, \bar{u}u\bar{s}$	1	$(-)\frac{2s_2s_8(s_3 + s_2c_8)}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$\frac{s_2^2\cos 2\delta + s_3^2 + 2s_2s_3c_8}{s_2^2 + s_3^2 + 2s_2s_3c_8}$
2. $\bar{b} \rightarrow \bar{u}c\bar{s}$	$\frac{s_2^2 + s_3^2 + 2s_2s_3c_8}{s_3^2}$	$(-)\frac{s_2}{s_3}s_8$	$\frac{s_2c_8 + s_3}{s_3}$
3. $\bar{b} \rightarrow \bar{c}u\bar{s}$	$\frac{s_3^2}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$(-)\frac{s_2s_3s_8}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$\frac{s_3(s_2c_8 + s_3)}{s_2^2 + s_3^2 + 2s_2s_3c_8}$
4. $\bar{b} \rightarrow \bar{c}c\bar{s}, \bar{c}c\bar{d}$	1	$\frac{2s_1^2s_2s_3s_8}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	1
$\bar{b} \rightarrow \bar{u}c\bar{d}$	$\frac{s_2^2 + s_3^2 + 2s_2s_3c_8}{s_1^4s_3^2}$	$\frac{s_2s_8}{s_1^2s_3}$	$(-)\frac{s_3 + s_2c_8}{s_1^2s_3}$
$\bar{b} \rightarrow \bar{c}u\bar{d}$	$\frac{s_1^4s_3^2}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$\frac{s_1^2s_2s_3s_8}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$(-)\frac{s_1^2s_3(s_3 + s_2c_8)}{s_2^2 + s_3^2 + 2s_2s_3c_8}$
$\bar{b} \rightarrow \bar{d}$	1	$\frac{2s_3(s_2 + s_3c_8)s_8}{s_2^2 + s_3^2 + 2s_2s_3c_8}$	$\frac{s_2^2 + s_3^2\cos 2\delta + 2s_2s_3c_8}{s_2^2 + s_3^2 + 2s_2s_3c_8}$
$\bar{b} \rightarrow \bar{s}$	1	0	1

$$A = \frac{n_+ - n_-}{n_+ + n_-}$$

and desiring a 3σ signature ($\delta A = \frac{1}{3}A$), we need

$$n_+ + n_- = 9 \frac{1 - A^2}{A^2}.$$

So, if

$$C_f = \frac{\Gamma(B_{\alpha\text{phys}}^0 \rightarrow f) - \Gamma(\bar{B}_{\alpha\text{phys}}^0 \rightarrow \bar{f})}{\Gamma(B_{\alpha\text{phys}}^0 \rightarrow f) + \Gamma(\bar{B}_{\alpha\text{phys}}^0 \rightarrow \bar{f})} \quad (36)$$

is the asymmetry, for 3σ accuracy, we need the number of b, \bar{b} pairs

$$N_{b\bar{b}} = 9 \frac{1 - C_f^2}{C_f^2} \frac{1}{\sigma(B_{\alpha} B_{\bar{u}}^{\pm}) B(f + \bar{f})} \epsilon, \quad (37)$$

where $\alpha = d$ or s , ϵ is the inverse of our detection efficiency, and

$$B(f + \bar{f}) \equiv B(B_{\alpha\text{phys}}^0 \rightarrow f) + B(\bar{B}_{\alpha\text{phys}}^0 \rightarrow \bar{f}) \cong B(B_{\alpha\text{pure}}^0 \rightarrow f) \frac{4 + 2z^2 + 2z^2 |x|^2 + 2\omega z(1 + z^2)\text{Re}\lambda}{2(1 - \frac{1}{4}\omega^2 z^2)(1 + z^2)}. \quad (38)$$

As a challenge to experimentalists and therefore being optimistic, we assume the detection efficiency of a B^{\pm} to be 50%. However for the other particles we are realistic and their efficiencies can be read off Table III.

The $B_{\alpha\text{pure}}^0 \rightarrow f$ branching ratios are whenever possible taken directly from experiment:

$$B(B_d^0 \rightarrow D^- \pi^+) \sim 2\% \text{ (Ref. 21)}, \quad (39a)$$

$$B(B_d^0 \rightarrow \psi K^0) \sim 0.1\% \text{ (Ref. 22)}. \quad (39b)$$

If not yet available we extract them from Eq. (39) with the help of KM elements. Whenever only the exchange diagram leads to the final state we assume the exchange to be significant and thereby get an estimate of this branching ratio again with the help of Eq. (39). Whenever the internal W -emission diagram is being encountered, we do not color suppress it. All that is not unreasonable extrapolating from our accumulating knowledge of D^{\pm}, D^0, \bar{D}^0 decays from Mark III data.²³

Now we make numerical estimates for exclusive two-body nonleptonic decays.

A good approximation is setting $M=0$, $\omega=0$. Then

TABLE III. Detection efficiency for various particles.

Particle	Detection efficiency (%)
B^{\pm}	50
D^{\pm}, D^0, \bar{D}^0	10
K^0	33
π^0, π^{\pm}	100
K^{\pm}	100
η	40
F^{\pm}	1
ϕ	50
ψ	14

the form of $C_f, B(f + \bar{f})$, and C_{fl} are considerably simplified:

$$C_f = \frac{-2z \text{Im}\lambda}{2 + z^2 + z^2 |x|^2}, \quad (40)$$

$$B(f + \bar{f}) \approx B(B_{\alpha\text{pure}}^0 \rightarrow f) \frac{2 + z^2 + z^2 |x|^2}{1 + z^2}, \quad (41)$$

for C even,

$$C_{fl} = \frac{-4z \text{Im}\lambda}{2 + z^2 + z^4 + |x|^2 z^2 (3 + z^2)}, \quad (42)$$

where $\text{Im}\lambda, |x|^2$ can be read off Table II, and z can be calculated by use of Eqs. (12) and (13). Thus from Eqs. (40), (41), and (37), we can estimate the number of $b\bar{b}$ pairs $N_{b\bar{b}}$ for exclusive two-body nonleptonic decays. We list them in Table IV. In these tables we also present some possible two-body hadronic final states, C_f, z , and the branching ratios. Note however that final-state phases have been entirely omitted in constructing these tables. Also, we take the values of the parameters given in Eq. (15). The only exception is for $\bar{b} \rightarrow \bar{u}ud, \bar{u}u\bar{s}$ processes for the $B_d - \bar{B}_d$ system, where, we take $\delta = 45^\circ$ to increase the asymmetry. All the values of $|x|^2, \text{Im}\lambda$, and $\text{Re}\lambda$ are shown in Table II only for quark processes. For final physical states, care must be taken. If the final state is a CP -odd eigenstate, an additional minus sign should be added to $\lambda, x, \text{Im}\lambda, \text{Re}\lambda$ in Table II. Also, the relative sign²⁴ of λ for a $B^0 \rightarrow P_1 P_2$ ($P_{1,2}$ pseudoscalar) decay versus $B^0 \rightarrow V_1 P_2$ (V_1 the excited vector state of P_1) is, neglecting differences in matrix structures,

$$\lambda_{B^0 \rightarrow V_1 P_2} = -\lambda_{B^0 \rightarrow P_1 P_2}.$$

For example,

$$\lambda_{B_s \rightarrow \phi D^0} = \lambda_{B_s \rightarrow D^* + \pi^-} = -\lambda_{B_s \rightarrow D + \pi^-}$$

when neglecting final-state phases. The sign of $\lambda_{B^0 \rightarrow D^0 K_s}$

and $\lambda_{B^0 \rightarrow D^0 K_L}$ are opposite. However, an open question is how to fix the overall sign of the λ 's. For completeness, we assume that $B^0 \rightarrow D^0 K_S$ and $B^0 \rightarrow \bar{D}^0 K_S$ are described by the λ 's in Tables I and II. Then all the sign of λ 's for other physical final states will be fixed. In Table IV we still use the λ 's in Tables I and II. Thus, for individual exclusive two-body decays, a minus sign might be needed to multiply the λ . This essentially will cause a sign flip of

C_f , but not change the number of $b\bar{b}$ pairs needed. For details see Appendix B.

Notice that the B_s system has the best asymmetry for $B_s \rightarrow D^0 \phi$ with $\sim 10^7$ $b\bar{b}$ pairs needed. If exchange diagrams prove to be important, $B_s \rightarrow D^+ \pi^-$ has also a pure $\Delta I = \frac{1}{2}$ transition and only needs $\sim 7 \times 10^6$ $b\bar{b}$ pairs. For the B_d case, even if nature is so kind as to provide us with a large mixing parameter $(\Delta m / \gamma)_{B_d} \sim 0.1$, still $B_d \rightarrow \psi K_S$

TABLE IV. Some possible two-body hadronic final states, corresponding decay diagrams, $z = \Delta m / \gamma$, C_f , branching ratios, and numbers of $b\bar{b}$ pairs.

Case	$B_d \text{ phys} \rightarrow f$	$z = \Delta m / \gamma$	Asymmetry C_f	$B(B_d^0 \text{ pure} \rightarrow f)$	$N_{b\bar{b}}$		
1. $\bar{b} \rightarrow \bar{u}ud$ $\bar{u}u\bar{s}$	$\pi^+ \pi^-$	0.1	0.1	$< 10^{-4}$	$> 5.6 \times 10^7$		
	$\pi^0 K_S$			1.8×10^{-6}	4.7×10^9		
	$K^+ K^-$			$< 10^{-4}$	$> 5.6 \times 10^7$		
	ηK_S			6.1×10^{-7}	3.5×10^{10}		
2. $\bar{b} \rightarrow \bar{u}c\bar{d}$	$D^+ \pi^-$	0.044	-0.61	10^{-5}	3.5×10^7		
	$F^+ K^-$			10^{-5}	3.5×10^8		
	ψD^0			5×10^{-6}	0.5×10^9		
	$D^0 \pi^0$			5×10^{-6}	7×10^7		
3. $\bar{b} \rightarrow \bar{c}u\bar{d}$	$D^- \pi^+$	0.1	-2×10^{-3}	2×10^{-2}	0.5×10^{10}		
	$F^- K^+$			2×10^{-2}	0.5×10^{11}		
	$\psi \bar{D}^0$			10^{-2}	1×10^{11}		
	$\bar{D}^0 \pi^0$			10^{-2}	1.4×10^{10}		
4. $\bar{b} \rightarrow \bar{c}c\bar{s}$, $\bar{c}c\bar{d}$, \bar{s}	$F^+ F^-$	0.1	0.08	10^{-4}	8.8×10^{11}		
	$\psi \phi$			10^{-5}	1.3×10^{10}		
	ψK_S			5×10^{-4}	2×10^8		
	ϕK_S			5×10^{-5}	5.3×10^8		
	$D^+ D^-$			10^{-3}	8.8×10^8		
	$\pi^0 K_S$			2.5×10^{-5}	5.3×10^8		
	ηK_S			10^{-4}	3.3×10^8		
	$\bar{b} \rightarrow \bar{u}c\bar{s}$			0.1	0.19	10^{-4}	2.2×10^8
	$\bar{b} \rightarrow \bar{c}u\bar{s}$			0.1	0.04	5×10^{-4}	1×10^9
	Case			$B_s \text{ phys} \rightarrow f$	$z = \Delta m / \gamma$	Asymmetry C_f	$B(B_s^0 \text{ pure} \rightarrow f)$
1. $\bar{b} \rightarrow \bar{u}ud$ $\bar{u}u\bar{s}$	$\pi^+ \pi^-$	0.8	0.38	10^{-5}	6.7×10^7		
	$K^+ K^-$			10^{-5}	6.7×10^7		
2. $\bar{u}c\bar{s}$	$\pi^0 K_S$	0.8	0.56	5×10^{-5}	2.0×10^7		
	$D^0 \phi$			2×10^{-4}	1.4×10^7		
	$F^+ K^-$			2×10^{-4}	7×10^7		
	$D^+ \pi^-$			2×10^{-4}	7×10^6		
3. $\bar{b} \rightarrow \bar{c}u\bar{s}$	$D^0 \pi^0$	0.8	0.23	10^{-4}	1.4×10^7		
	$\bar{D}^0 \phi$			10^{-3}	5×10^7		
	$F^- K^+$			10^{-3}	2.4×10^8		
	$D^- \pi^+$			10^{-3}	2.4×10^7		
4. $\bar{b} \rightarrow \bar{c}c\bar{s}$ $\bar{c}c\bar{d}$	$\psi \phi$	0.8	-0.022	5×10^{-4}	4.8×10^7		
	ψK_S			3×10^{-3}	1.1×10^9		
				2.7×10^{-5}	9×10^{10}		
	$D^- D^+$			5×10^{-3}	4.7×10^9		
$\bar{b} \rightarrow \bar{u}c\bar{d}$ $\bar{b} \rightarrow \bar{c}u\bar{d}$ $\bar{b} \rightarrow \bar{d}$	$F^- F^+$	0.8	-0.054	2×10^{-2}	1.2×10^{11}		
	$D^0 K_S$			5.7×10^{-6}	3×10^8		
	$\bar{D}^0 K_S$			10^{-2}	1.2×10^9		
	ϕK_S			2.1×10^{-6}	8.4×10^8		

requires 2×10^8 $b\bar{b}$'s (with leptonic tagging we encounter here constructive interference; we could increase our asymmetry by a factor of 2). However, most estimates abound predicting smaller^{3,11-13,15} mixing parameters and for $(\Delta m/\gamma)_{B_d} \sim 10^{-2} - 5 \times 10^{-2}$ we require $\sim 4 \times 10^7$ $b\bar{b}$ for $B_d \rightarrow D^+ \pi^-$. (This decay however is plagued with two final-state isospin phases.) Or, again, if exchange is important $B_d \rightarrow F^+ K^- \sim 4 \times 10^8$ $b\bar{b}$ pairs are needed. (The detection efficiency of F^+ is 10% relative to that for D^+ .)

In Table V we show the maximal value of C_{fl} and C_f and the corresponding mixing parameter for various processes. The maximal value of C_f (C_{fl}) is reached for a

mixing parameter typically of order 1 (0.5). However, the $\bar{b} \rightarrow \bar{u}c\bar{d}$ process is an exception, due to the large ratio of amplitudes $|x|^2 = 1756$ the maximum is reached at 3×10^{-2} (2×10^{-2}).

If nature chooses a tiny mixing for $B^0 - \bar{B}^0$, the highly Cabibbo-suppressed process $\bar{b} \rightarrow \bar{u}c\bar{d}$ might be a good choice to observe CP violation.

For mixing parameter of order 1, the processes $\bar{b} \rightarrow \bar{u}u\bar{d}, \bar{u}u\bar{s}, \bar{c}c\bar{s}, \bar{c}c\bar{d}, \bar{s}, \bar{u}c\bar{s}, \bar{c}u\bar{s}$, lead to large asymmetries for B_d system (see Table V). And $\bar{b} \rightarrow \bar{u}u\bar{d}, \bar{u}u\bar{s}, \bar{u}c\bar{s}, \bar{c}u\bar{s}, \bar{d}$, lead to large asymmetries for B_s system. If mixing in the B_s system is large ($z \geq 5$), C_{fl} is unsuitable due to the z^4 dependence in the denominator of Eq. (42), and C_f must

TABLE V. The maximal values of C_{fl} and C_f and the corresponding mixing parameter z . Here $s_1 = 0.231$, $s_2 = 0.05$, $s_3 = \frac{1}{2}s_2$, $\delta = 90^\circ$ but for $\bar{b} \rightarrow \bar{u}u\bar{d}, \bar{u}u\bar{s}$ in B_d decays, $\delta = 45^\circ$.

$B_d - \bar{B}_d$ system		
Process	$z \equiv \frac{\Delta m}{\gamma} (\text{max})$	$C_{fl} (\text{max})$
$\bar{b} \rightarrow (\bar{u}u\bar{d}, \bar{u}u\bar{s})$	0.58	0.65
$\bar{b} \rightarrow \bar{u}c\bar{d}$	1.9×10^{-2}	-0.736
$\bar{b} \rightarrow \bar{c}u\bar{d}$	0.81	-2.20×10^{-2}
$\bar{b} \rightarrow (\bar{c}c\bar{s}, \bar{c}c\bar{d}, \bar{s})$	0.58	0.531
$\bar{b} \rightarrow \bar{u}c\bar{s}$	0.33	0.686
$\bar{b} \rightarrow \bar{c}u\bar{s}$	0.74	0.362
$B_s - \bar{B}_s$ system		
Process	$\frac{\Delta m}{\gamma} (\text{max})$	$C_{fl} (\text{max})$
$\bar{b} \rightarrow (\bar{u}u\bar{d}, \bar{u}u\bar{s})$	0.57	0.509
$\bar{b} \rightarrow \bar{u}c\bar{s}$	0.33	0.700
$\bar{b} \rightarrow \bar{c}u\bar{s}$	0.75	0.370
$\bar{b} \rightarrow (\bar{c}c\bar{s}, \bar{c}c\bar{d})$	0.59	-2.90×10^{-2}
$\bar{b} \rightarrow \bar{u}c\bar{d}$	2.0×10^{-2}	-0.723
$\bar{b} \rightarrow \bar{c}u\bar{d}$	0.81	-2.20×10^{-2}
$\bar{b} \rightarrow \bar{d}$	0.58	-0.529
$B_d - \bar{B}_d$ system		
Process	$z (\text{max})$	$C_f (\text{max})$
$\bar{b} \rightarrow \bar{u}u\bar{d}, \bar{u}u\bar{s}$	1.00	0.500
$\bar{b} \rightarrow \bar{u}c\bar{d}$	3.3×10^{-2}	-0.635
$\bar{b} \rightarrow \bar{c}u\bar{d}$	1.41	-1.48×10^{-2}
$\bar{b} \rightarrow \bar{c}c\bar{s}, \bar{c}c\bar{d}, \bar{s}$	1.02	0.408
$\bar{b} \rightarrow \bar{u}c\bar{s}$	0.58	0.573
$\bar{b} \rightarrow \bar{c}u\bar{s}$	1.28	0.256
$B_s - \bar{B}_s$ system		
Process	$z (\text{max})$	$C_f (\text{max})$
$\bar{b} \rightarrow \bar{u}u\bar{d}, \bar{u}u\bar{s}$	0.98	0.393
$\bar{b} \rightarrow \bar{u}c\bar{s}$	0.58	0.583
$\bar{b} \rightarrow \bar{c}u\bar{s}$	1.30	0.260
$\bar{b} \rightarrow \bar{c}c\bar{s}, \bar{c}c\bar{d}$	1.03	-2.22×10^{-2}
$\bar{b} \rightarrow \bar{u}c\bar{d}$	3.4×10^{-2}	-0.627
$\bar{b} \rightarrow \bar{c}u\bar{d}$	1.41	-1.48×10^{-2}
$\bar{b} \rightarrow \bar{d}$	1.02	-0.407

be used.

For large B_s - \bar{B}_s mixing, $\bar{b} \rightarrow \bar{c}u\bar{s}$ due to the soft variation on z is particularly promising, less so are the $\bar{b} \rightarrow \bar{d}, \bar{u}u\bar{d}, \bar{u}u\bar{s}$ processes, and even less promising is $\bar{b} \rightarrow \bar{u}c\bar{s}$.

A copious supply of $b\bar{b}$ is expected from the Cornell Electron Storage Ring, LEP, and the Stanford Linear Collider. There 10^5 – 10^6 $e^+e^- \rightarrow b\bar{b}$ per year seem not impossible. So, we hope experimentalists will search for these asymmetries.

IV. PENGUIN-DIAGRAM CONTRIBUTIONS

We follow the analysis of penguins of Guberina, Peccei, and Rückl²⁵ and neglect the absorptive part of the penguin diagrams. It will be shown that we can neglect the penguin contribution safely for $B \rightarrow \psi K_S$ decay.^{3,11} For $B_d \rightarrow \pi^0 K_S$, the penguin will be shown to dominate over the spectator diagram, however, not extremely. So, this will be a bad CP testing ground. For the extreme case $\Gamma_{\text{pen}}(B_d \rightarrow \pi^0 K_S) \gg \Gamma_{O_{\pm}}(B_d \rightarrow \pi^0 K_S)$, we obtain

$$\lambda_{B_d \rightarrow \pi^0 K_S} = \frac{\Delta_{23}}{\Delta_{23}^*}, \quad (43)$$

where pen denotes penguin contribution, O_{\pm} stands for ordinary-diagram contributions defined in Ref. 26.

Guberina, Peccei, and Rückl got for the penguin $b \rightarrow s$ decay the additional effective Hamiltonian

$$\begin{aligned} H_W^{\text{pen}} = & C_P \frac{G_F}{\sqrt{2}} V_{ub} V_{ts}^* \gamma_{\mu} (1 - \gamma_5) \lambda_a b (\bar{c} \gamma^{\mu} \lambda_a c + \bar{u} \gamma^{\mu} \lambda_a u \\ & + \bar{d} \gamma^{\mu} \lambda_a d + \bar{s} \gamma^{\mu} \lambda_a s) \\ & + \text{H.c.}, \end{aligned} \quad (44)$$

where

$$C_P = \frac{\alpha_s(K^2)}{12\pi} \ln \left[\frac{m_t^2}{K^2} \right]. \quad (45)$$

In the above, K^2 is the momentum transfer carried by the gluon and we assume $K^2 \approx m_b^2$. The mass correction term $\ln[K^2/(K^2 + m_c^2)]$ has been neglected and $C_P \sim (2-5) \times 10^{-2}$. We take $C_P \sim 0.03$ in later estimation. Different λ_a, γ^{μ} structure of the penguin operators as compared to O_{\pm} operators provides an enhancement in the inclusive rates^{26,25} over the small penguin coefficient C_P .

The ratio of the partial widths for the decay $B_d \rightarrow K + n\pi$ generated by penguins and O_{\pm} operators, respectively, is given by

$$\frac{\Gamma_{\text{pen}}(B_d \rightarrow K + n\pi)}{\Gamma_{O_{\pm}}(B_d \rightarrow K + n\pi)} \approx 5C_P^2 \frac{|V_{tb} V_{ts}|^2}{|V_{ub} V_{us}|^2} \gtrsim 4. \quad (46)$$

In the above the inequality arises from the experimental limit (14). We have used the KM parameters in Eq. (15) for definiteness.

Applying those ratios (46) to exclusive two-body decays we see

$$\frac{\Gamma_{\text{pen}}(B_d \rightarrow \pi^0 K_S)}{\Gamma_{O_{\pm}}(B_d \rightarrow \pi^0 K_S)} \sim 6, \quad (47a)$$

$$\frac{\Gamma_{\text{pen}}(B_d \rightarrow \pi\pi)}{\Gamma_{O_{\pm}}(B_d \rightarrow \pi\pi)} \approx 5C_P^2 \left| \frac{V_{tb} V_{td}}{V_{ub} V_{ud}} \right|^2 \approx 2 \times 10^{-2}, \quad (47b)$$

$$\begin{aligned} \frac{\Gamma_{\text{pen}}(B_d \rightarrow \psi K_S)}{\Gamma_{O_{\pm}}(B_d \rightarrow \psi K_S)} & \approx 5C_P^2 \left| \frac{V_{tb} V_{ts}}{V_{cb} V_{cs}} \right|^2 P(\text{vac} \rightarrow c\bar{c}) T(\text{ZF}) \\ & \approx 5 \times 10^{-3} P(\text{vac} \rightarrow c\bar{c}) T(\text{ZF}), \end{aligned} \quad (47c)$$

and for B_s system

$$\frac{\Gamma_{\text{pen}}(B_s \rightarrow \pi^0 K_S)}{\Gamma_{O_{\pm}}(B_s \rightarrow \pi^0 K_S)} \approx 5C_P^2 \left| \frac{V_{tb} V_{td}}{V_{ub} V_{ud}} \right|^2 \approx 2 \times 10^{-2}, \quad (48a)$$

$$\begin{aligned} \frac{\Gamma_{\text{pen}}(B_s \rightarrow \psi K_S)}{\Gamma_{O_{\pm}}(B_s \rightarrow \psi K_S)} & \approx 5C_P^2 \left| \frac{V_{tb} V_{td}}{V_{cb} V_{cd}} \right|^2 P(\text{vac} \rightarrow c\bar{c}) T(\text{ZF}) \\ & \approx 5 \times 10^{-3} P(\text{vac} \rightarrow c\bar{c}) T(\text{ZF}), \end{aligned} \quad (48b)$$

$$\begin{aligned} \frac{\Gamma_{\text{pen}}(B_s \rightarrow F^- F^+)}{\Gamma_{O_{\pm}}(B_s \rightarrow F^- F^+)} & \approx 5C_P^2 \left| \frac{V_{tb} V_{ts}}{V_{cb} V_{cs}} \right|^2 P(\text{vac} \rightarrow c\bar{c}) \\ & \approx 5 \times 10^{-3} P(\text{vac} \rightarrow c\bar{c}), \end{aligned} \quad (48c)$$

$$\frac{\Gamma_{\text{pen}}(B_s \rightarrow \psi\phi)}{\Gamma_{O_{\pm}}(B_s \rightarrow \psi\phi)} \approx 5 \times 10^{-3} P(\text{vac} \rightarrow c\bar{c}) T(\text{ZF}). \quad (48d)$$

In the above $P(\text{vac} \rightarrow c\bar{c})$ denotes the probability of creating a $c\bar{c}$ pair out of the vacuum [$P(\text{vac} \rightarrow c\bar{c}) < 1$] and $T(\text{ZF})$ stands for Zweig-rule-forbidden penguin transition [$T(\text{ZF}) < 1$]. We take the square roots of the above rate ratios to obtain the relative strength between the penguin amplitudes (with KM combination factor) and the O_{\pm} operators (with different KM combination factor).

However, for $B_d \rightarrow \pi^0 K_S$, we cannot claim any knowledge of asymmetry. Taking the amusing limit of infinite penguin dominance, we get

$$\frac{\Gamma_{\text{pen}}(B_d \rightarrow \pi^0 K_S)}{\Gamma_{O_{\pm}}(B_d \rightarrow \pi^0 K_S)} \gg 1 \quad (49)$$

then

$$\lambda_{B_d \rightarrow \pi^0 K_S} = \frac{\Delta_{23}}{\Delta_{23}^*}. \quad (50)$$

In KM phase convention, this means

$$\text{Im} \lambda_{B_d \rightarrow \pi^0 K_S} = \frac{-2s_3 s_8 (s_2 + s_3 c_8)}{s_2^2 + s_3^2 + 2s_2 s_3 c_8}, \quad (51a)$$

$$\text{Re} \lambda_{B_d \rightarrow \pi^0 K_S} = \frac{s_2^2 + s_3^2 \cos 2\delta + 2s_2 s_3 c_8}{s_2^2 + s_3^2 + 2s_2 s_3 c_8}. \quad (51b)$$

Even in this extreme case, care must be taken to see how the mass effects of m_c^2 versus K^2 enter. They give rise to

a $V_{cb}V_{cs}^*$ term. For details consult Ref. 25. Back to reality, for $B_d \rightarrow \pi^0 K_S$, we obtain comparable amplitudes for penguins and the spectator diagram, and hence even the assumption of $|A(B_d^0 \rightarrow \pi^0 K_S)| = |\bar{A}(\bar{B}_d^0 \rightarrow \pi^0 K_S)|$ does not hold due to penguins [$G_1 \neq G_2$, see Eqs. (23) and (24)].

The estimates (46) and (48) have been derived from perturbative QCD calculations. See Ref. 27 for a discussion of experimentally isolating penguin contributions through exclusive B -meson decays, that will test the predicted penguin strength.

In this presentation we look at final states where the penguin diagram does not contribute to the asymmetry or at worst, if it is present we can safely neglect it. Our approach can be contrasted with Chau and Cheng's analysis⁴ of CP violation in the neutral B mesons. First, they consider final states that cannot be fed simultaneously from B^0 and \bar{B}^0 . So, in that case no interference of $A(B \rightarrow f)$ and $\bar{A}(\bar{B} \rightarrow f)$ will occur. Such cases we do not consider at all in our paper. Second, they consider final states in which both B^0 and \bar{B}^0 can decay. However they constrain themselves to the case $f = \bar{f}$, and they do not look at final states which arise solely via penguin operators. We, however, discuss later large CP asymmetries predicted in $B_{d,s} \rightarrow \phi K_S$ for which only penguins contribute.

It is our belief that to estimate the penguins involves more theoretical uncertainties. And to extract their relative strengths from experiments in the years to come is much more problematic (see Ref. 27) than to obtain the strengths of the spectator and/or the exchange diagrams. The strength of spectator and exchange suffices for the large asymmetries that we obtain. Therefore we avoided the penguin diagrams in this paper or we looked at decays where we can safely neglect them, or we looked at decays where only penguins contribute.

Note that the result for $B_s \rightarrow K^+ K^-$ in Ref. 4 is

$$\text{Im}\lambda_{B_s \rightarrow K^+ K^-} = 2s_2 s_3 s_8 .$$

Let us check this. We know that

$$\frac{\Gamma_{\text{pen}}(B_s \rightarrow K^+ K^-)}{\Gamma_{O_{\pm}}(B_s \rightarrow K^+ K^-)} \approx 5C_P^2 \left| \frac{V_{tb}V_{ts}}{V_{ub}V_{us}} \right|^2 \gtrsim 4 . \quad (52)$$

Taking the representative values Eq. (15) we get for the ratio in Eq. (52):

$$\frac{\Gamma_{\text{pen}}(B_s \rightarrow K^+ K^-)}{\Gamma_{O_{\pm}}(B_s \rightarrow K^+ K^-)} \sim 6 .$$

Hence, the penguins and O_{\pm} amplitudes have the same order of magnitude. If Chau and Cheng still have $|A(f)| = |\bar{A}(\bar{f})|$, assuming penguin dominance they should obtain, up to some mass corrections which introduce $V_{cb}^*V_{cs}$ in penguin amplitude,

$$x = \frac{\bar{A}(K^+ K^-)}{A(K^+ K^-)} = \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}} .$$

Hence,

$$\lambda = \frac{\xi_t^*}{\xi_t} x = \frac{V_{tb}^* V_{ts}^*}{V_{tb}^* V_{ts}} \frac{V_{tb}^* V_{ts}}{V_{tb}^* V_{ts}} = 1 + \text{mass correction} .$$

So, $\text{Im}\lambda = 0$. This is in good agreement with Chau and Cheng's original result because $\text{Im}\lambda = 2s_2 s_3 s_8$ is very small.

We must emphasize that for some processes, such as $B_{d,s} \rightarrow \phi K_S$, only penguins contribute. We can obtain potentially large asymmetries. We take only the leading term $\ln(m_t^2/K^2)$ of Eq. (44) and neglect the mass-correction term $\ln[K^2/(K^2 + m_c^2)]$. The asymmetry for $B_d \rightarrow \phi K_S$ involves penguin transition $b \rightarrow s$ and can reach -0.1 . While the asymmetry for $B_s \rightarrow \phi K_S$ invokes $b \rightarrow d$ penguin transition and can reach up to 0.4. In both cases 5×10^8 $b\bar{b}$ pair is needed for testing these asymmetries. All these results are listed in Table IV.

Now we estimate the branching ratios:

$$\frac{\Gamma_{\text{pen}}(B_d \rightarrow \phi K_S)}{\Gamma_{O_{\pm}}(B_d \rightarrow D^- \pi^+)} \sim 5C_P^2 \frac{1}{2} \left| \frac{V_{tb}V_{ts}}{V_{cb}V_{ud}} \right|^2 F_{\text{ps}} F_{\text{mtp}} p(\text{vac} \rightarrow s\bar{s}) ,$$

where Eq. (46) has been used. The factor $\frac{1}{2}$ arises from $|\langle K_S | K^0 \rangle|^2$. F_{ps} is the phase space, F_{mtp} is the multiplicity factor, $p(\text{vac} \rightarrow s\bar{s})$ gives the probability creating an $s\bar{s}$ pair from vacuum. We put them all together as $F_{\text{ps}} F_{\text{mtp}} p(\text{vac} \rightarrow s\bar{s}) \sim 1$ to obtain²⁷

$$B(B_d \rightarrow \phi K_S) \sim 5 \times 10^{-5} ,$$

$$B(B_s \rightarrow \phi K_S) \sim 2.5 \times 10^{-6} .$$

The last number 2.5×10^{-6} is obtained from

$$\frac{B(B_s \rightarrow \phi K_S)}{B(B_d \rightarrow \phi K_S)} \approx \left| \frac{V_{tb}V_{td}}{V_{tb}V_{ts}} \right|^2 .$$

These branching ratios are also listed in Table IV.

V. DISCUSSION AND CONCLUSION

We have used

$$|A(f)| = |\bar{A}(\bar{f})| , \quad (53)$$

$$\bar{x} = x^* , \quad (54)$$

to derive our asymmetries.

Now, criterion (53) is always satisfied when no penguin diagram contributes to the final state [see discussion around Eq. (22)]. It is also satisfied quite accurately for those processes where penguins are negligible or dominant. For $B_d \rightarrow \pi^0 K_S$ Eq. (53) cannot be justified. But Eq. (54) is not always satisfied even when Eq. (53) holds true. The hurdle is the problem of final-state phases. For example, consider $B_d^0 \rightarrow D^- \pi^+$ and $\bar{B}_d^0 \rightarrow D^+ \pi^-$. After a lengthy analysis^{5,28} we have

$$x_{D^- \pi^+} = \frac{V_{ub}V_{cd}^* a_{3/2}(D\pi)e^{i\delta} - \sqrt{2}a_{1/2}(D\pi)}{V_{ud}V_{cb}^* a_{3/2}(\bar{D}\pi)e^{i\delta} + \sqrt{2}a_{1/2}(\bar{D}\pi)} . \quad (55)$$

Similarly we get

$$\bar{x}_{D^+ \pi^-} = \frac{V_{ub}^*V_{cd} a_{3/2}(D\pi)e^{i\delta} - \sqrt{2}a_{1/2}(D\pi)}{V_{cb}V_{ud}^* a_{3/2}(\bar{D}\pi)e^{i\delta} + \sqrt{2}a_{1/2}(\bar{D}\pi)} , \quad (56)$$

where

$$\delta = \delta_{3/2} - \delta_{1/2}, \quad (57)$$

$$x_{D^-\pi^+} = \frac{\bar{A}(D^-\pi^+)}{A(D^-\pi^+)}, \quad \bar{x}_{D^+\pi^-} = \frac{A(D^+\pi^-)}{\bar{A}(D^+\pi^-)}.$$

For $\bar{x}_{D^+\pi^-} = x_{D^-\pi^+}^*$ to hold, we need the KM stripped from the right-hand side of Eqs. (55) and (56) to be real. That does not happen in general. So in general

$$\bar{x}_{D^+\pi^-} \neq x_{D^-\pi^+}^*. \quad (58)$$

The decay $B_s^0 \rightarrow \pi^+\pi^-$ seems promising, since the weak effective Hamiltonian allows only $I=0$ and $I=1$ channels. Because of Bose statistics this final state can proceed through $I=0$ channel only. Unfortunately, the branching ratio for this decay mode is very small ($< 10^{-5}$). For analogous reasoning the decay $B_d^0 \rightarrow K^+K^-$ should also have a pure isospin channel. But the rescattering through the multipion intermediate states involves $I=1$ channels, so makes Eq. (54) problematic. The decay $B_s \rightarrow D^-\pi^+$ or $B_s \rightarrow D^+\pi^-$ has only $I=\frac{1}{2}$ channel. These two decays seem promising. The only open question is how much do those virtual inclusive states “ $D^-\pi^+$ (pairs of π)” or “ $D^+\pi^-$ (pairs of π)” introduce phases to our rescattering.³² The only drawback is that the branching ratio might be by a factor of 10 smaller than what we optimistically assumed—a scaling argument for exchange diagrams from the observed D decays.²⁶

$B_s \rightarrow D^0\phi$ suffers from the final-state phases. The eigenvectors of the S matrix is a linear combination of $D^0\phi$ and F^+K^- and therefore

$$A(B_s \rightarrow D^0\phi) = V_{cs} V_{ub}^* (ae^{i\delta_a} + be^{i\delta_b}), \quad (59)$$

δ_a, δ_b being final-state phases.

The decay $B_d^0 \rightarrow \psi K_S$ does not suffer from final-state phases because of Eq. (B7), although there is a rescattering of $D^-F^+ \rightleftharpoons \psi K_S$. For non- CP eigenstates our quantitative argument based solely on KM angles turns sadly into just a qualitative one. Further work is definitely needed to elucidate final-state phases. An especially promising way to study CP violation is via inclusive or semi-inclusive channels, due to their larger branching ratios.³² However, progress is also hampered due to our lack of understanding of final-state phases.

We want to point out that if the mixing of $B_d^0 \rightarrow \bar{B}_d^0$ is extremely small (even smaller than to observe reasonable $B_d^0 \rightarrow D^+\pi^-$ asymmetry, which needs $z \geq 10^{-2}$ for good event number $N_{b\bar{b}}$), we still could hope for asymmetries in the pure decay rate $B_d^0 \rightarrow \pi^0 K_S$. Here penguin and O_{\pm} operators have the same order of magnitude but different KM structure. Therefore assuming different final-state strong-interaction phases, a large decay-rate asymmetry $\Gamma(B_{d\text{pure}} \rightarrow \pi^0 K_S)$ versus $\Gamma(\bar{B}_{d\text{pure}} \rightarrow \pi^0 K_S)$ could result. So we have CP violation in magnitude. The same remark might apply to $B_s \rightarrow \pi^+\pi^-$. In general, whenever we have two decay channels with different weak and strong phases this remark might apply.

Probably, the best approach, in the event of negligible $B^0\text{-}\bar{B}^0$ mixing, would be to look for partial-decay-rate

differences of the charged b -flavored mesons (i.e., $B_{u,c}^+ \rightarrow f$ versus $B_{u,c}^- \rightarrow \bar{f}$) (Refs. 29, 30, and 31).

Now we come to our conclusion.

(a) CP -violation effects in nonleptonic decays of B^0 meson can be quite large owing to the mixing and amplitude interference. The best decay modes for testing CP violation are $B_d^0 \rightarrow D^+\pi^-$, $B_s^0 \rightarrow D^+\pi^-$, and $B_s^0 \rightarrow D^0\phi$. They need 4×10^7 , 7×10^6 , and 10^7 $b\bar{b}$ pairs for 3σ signature, respectively. For testing CP violation with only penguin contribution, the best modes are $B_{d,s}^0 \rightarrow \phi K_S$ which need 5×10^8 $b\bar{b}$ for 3σ signature.

(b) The problem of the strong-interaction phases of the final states is very difficult and subtle. It needs further investigation.

(c) According to the prediction of the standard KM model, the Cabibbo-Kobayashi-Maskawa-favored decays, $\bar{b} \rightarrow \bar{c}c\bar{s}$ (e.g., $B_s \rightarrow \psi\phi$, F^+F^- , etc.), $\bar{b} \rightarrow \bar{c}u\bar{d}$ (e.g., $B_s \rightarrow \bar{D}^0 K_S$) for B_s decays and $\bar{b} \rightarrow \bar{c}u\bar{d}$ (e.g., $B_d \rightarrow D^-\pi^+$, etc.), $\bar{b} \rightarrow \bar{c}c\bar{s}$ ($B_d \rightarrow \psi K_S$) for B_d decays have very large decay rates (so branching ratios) but very small asymmetries. The only exception is the $B_d^0 \rightarrow \psi K_S$ where the asymmetry could be large, however, probably suppressed by small mixing. If we find a large asymmetry in all the processes mentioned above, new physics will emerge. So it is worthwhile to make the efforts.

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APPENDIX A

We list the invariants of the KM matrix:

$$\begin{aligned} \Delta_{11} &= V_{22} V_{33} (V_{23} V_{32})^*, \\ \text{Re}\Delta_{11} &\approx -c_1 s_2 s_3 c_8 (1 + c_1^2) - c_1^2 (s_2^2 + s_3^2); \\ \Delta_{22} &= V_{33} V_{11} (V_{31} V_{13})^*, \\ \text{Re}\Delta_{22} &\approx c_1 s_1^2 s_2 s_3 (c_8 - c_1 s_2 s_3); \\ \Delta_{33} &= V_{11} V_{22} (V_{12} V_{21})^*, \quad \text{Re}\Delta_{33} \approx -c_1^2 s_1^2; \\ \Delta_{12} &= V_{23} V_{31} (V_{21} V_{33})^*, \\ \text{Re}\Delta_{12} &\approx -s_1^2 s_2 (s_2 + c_1 c_8 s_3); \\ \Delta_{13} &= V_{21} V_{32} (V_{22} V_{31})^*, \\ \text{Re}\Delta_{13} &\approx c_1 s_1^2 s_2 (c_1 s_2 + c_8 s_3); \\ \Delta_{21} &= V_{32} V_{13} (V_{33} V_{12})^*, \\ \text{Re}\Delta_{21} &\approx -s_1^2 s_3 (s_3 + c_1 c_8 s_2); \end{aligned}$$

$$\begin{aligned}\Delta_{23} &= V_{31} V_{12} (V_{32} V_{11})^* , \\ \text{Re}\Delta_{23} &\approx -c_1 s_1^2 s_2 (c_1 s_2 + c_8 s_3) ; \\ \Delta_{31} &= V_{12} V_{23} (V_{13} V_{22})^* , \\ \text{Re}\Delta_{31} &\approx s_1^2 s_3 (c_1^2 s_3 + c_1 c_8 s_2) ; \\ \Delta_{32} &= V_{13} V_{21} (V_{11} V_{23})^* , \\ \text{Re}\Delta_{32} &\approx -c_1 s_1^2 s_3 (c_1 s_3 + c_8 s_2) ; \\ \text{Im}\Delta_{i\alpha} &= c_1 c_2 c_3 s_1^2 s_2 s_3 s_8 \\ &\approx c_1 s_1^2 s_2 s_3 s_8 \text{ for all } i, \alpha\end{aligned}$$

(where we have taken $c_2 \sim c_3 \sim 1, s_2, s_3 \ll s_1$).

APPENDIX B

When dealing with final states that are *CP* eigenstates, special care must be taken. Assume for simplicity that only one weak phase contributes to our process. Then the claim is that for *CP*-odd states we obtain $-\lambda$ (not λ), for *CP*-even state we obtain λ .

Proof. We have a couple of different strong eigenchannels labeled by α . Define

$$\text{out}\langle f, \alpha | B^0 \rangle_{\text{in}} = a_\alpha e^{i\delta_\alpha} , \quad (\text{B1a})$$

$$\text{out}\langle f, \alpha | \bar{B}^0 \rangle_{\text{in}} = \bar{a}_\alpha e^{i\delta_\alpha} . \quad (\text{B1b})$$

Put

$$CP | f \rangle = \pm | f \rangle , \quad (\text{B2})$$

i.e., $+$ for *CP* even $| f \rangle$, $-$ for *CP* odd $| f \rangle$, for instance, $(+)$ for $f = D^+ D^-$, $(-)$ for $f = \psi K_S$.

Choose the phase convention

$$CP | B^0 \rangle = | \bar{B}^0 \rangle . \quad (\text{B3})$$

Applying *CPT* onto (B1a) and (B1b), we obtain

$$\bar{a}_\alpha = \pm a_\alpha^* . \quad (\text{B4})$$

In the above, the sign \pm merely reflects whether we deal with *CP*-even ($+$) or *CP*-odd ($-$) eigenstates f .

In reality our decay may proceed via several strong eigenchannels with unknown final-state strong-interaction phases. Now

$$x = \frac{\text{out}\langle f | \bar{B}^0 \rangle_{\text{in}}}{\text{out}\langle f | B^0 \rangle_{\text{in}}} = \frac{\sum_{\alpha \text{out}} \langle f | f, \alpha \rangle_{\text{out}} \bar{a}_\alpha e^{i\delta_\alpha}}{\sum_{\alpha \text{out}} \langle f | f, \alpha \rangle_{\text{out}} a_\alpha e^{i\delta_\alpha}} . \quad (\text{B5})$$

Assume that only one weak phase enters

$$a_\alpha = |a_\alpha| e^{i\phi_{\omega k}} , \quad (\text{B6})$$

where $\phi_{\omega k}$ does not depend on α . Then

$$\begin{aligned}x &= \frac{\pm e^{-i\phi_{\omega k}} \sum_{\alpha \text{out}} \langle f | f, \alpha \rangle_{\text{out}} |a_\alpha| e^{i\delta_\alpha}}{e^{i\phi_{\omega k}} \sum_{\alpha \text{out}} \langle f | f, \alpha \rangle_{\text{out}} |a_\alpha| e^{i\delta_\alpha}} \\ &= \frac{\pm e^{-i\phi_{\omega k}}}{e^{i\phi_{\omega k}}} , \quad (\text{B7})\end{aligned}$$

Eq. (B7) means that x will be essentially a ratio of KM combinations (note here we only have one weak phase, i.e., one KM combination) and the \pm sign reflects what *CP* eigenstate we deal with. Because x changes sign for *CP*-odd eigenstates, λ does also. That completes our proof.

For final states that are not *CP* eigenstates, for instance, $D^+ \pi^-$, $D^{*+} \pi^-$, etc., we have also a sign ambiguity in λ . In general, for $B^0 \rightarrow P_1 P_2$ and $B^0 \rightarrow V_1 P_2$, owing to the odd relative *CP* parity, we should have²⁴

$$\lambda_{P_1 P_2} = -\lambda_{V_1 P_2} , \quad (\text{B8})$$

where, we have neglected the difference of the strong-interaction phases and the kinematical considerations, and P_1, P_2 are pseudoscalars, V_1 is just the excited vector counterpart of P_1 .

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