Strong decay of hadrons in a semirelativistic quark model

R. Sartor and Fl. Stancu
Université de Liège, Physique Nucléaire Théorique, Institut de Physique au Sart Tilman, Bâtiment B.5, B-4000 Liege 1, Belgium
(Received 19 June 1986)

Pion decay amplitudes of nonstrange baryon resonances are calculated assuming a pseudoscalar single-quark-emission-type model for the decay process. The resonances as well as the nucleon ground state are described by configuration mixings obtained previously in a semirelativistic QCD-inspired quark model. The decay widths are compared to the available experimental data and to other theoretical results.

I. INTRODUCTION

In a previous work[1] we have analyzed the photodecay amplitudes of nonstrange baryon resonances of spins up to $\frac{7}{2}$ for positive and $\frac{5}{2}$ for negative parity. The resonances were obtained[2] as excited states of N or Δ baryons described by a semirelativistic three-quark model. The comparison between the calculated and experimental helicity amplitudes indicated a general good agreement for the sign and the order of magnitude. The photoemission process was described by a standard parameter-free interaction.

A further test of the configuration mixings predicted in Ref. 2 for the resonance wave functions would be the study of the strong-decay processes. The present work is therefore devoted to the calculation of pion decay amplitudes of the same resonances which have been considered in Ref. 1 and for which available data exist.[3]

The first incentive of this work was to check the general assumption made in Ref. 1 for the phase of the amplitude $A_{(1/2)\pi N}$ corresponding to the decay of a nonstrange resonance into a pion and a nucleon. There we have supposed that for each resonance this phase is the same as that obtained by Koniuk and Isgur[4] in a harmonic-oscillator basis. As our theoretical values for the photodecay amplitudes reproduced the sign of the experimental amplitudes in most cases, the above assumption seemed to be reasonable. At the present stage it is useful to check its consistency with the particular wave functions resulting from the model of Ref. 2. In that model the unperturbed Hamiltonian has a relativistic kinetic energy part and an adiabatic potential-energy part derived from a flux-tube potential model.[5] The potential energy can be expressed as a sum of two-body potentials of the form Coulomb + linear confinement and a three-body potential proportional to the tension in the flux tube. The residual interaction is the hyperfine interaction[6] modified to include a finite size for the quark.

In the present study besides the configuration mixings obtained from the diagonalization of the hyperfine interaction the other important ingredient is the pion-quark coupling. Here we consider a pseudoscalar emission model including a recoil-type term.[7] This will allow us to make a detailed and consistent comparison with the work of Koniuk and Isgur[4] where such a model is used in an extensive analysis of strong decays of baryons. In contrast with their work where four parameters have been adjusted we keep free the two parameters appearing in the meson-emission transition operator. Being intended as a test of the baryon structure we use the full configuration mixings predicted in Ref. 2 both for the resonances and the nucleon ground state. In this respect our analysis is different from that of Ref. 4 where only unmixed configurations have been used for the nonstrange sector.

In Sec. II a brief review of the quark model is given. The pion emission model is described in Sec. III and results for the decay widths are presented in Sec. IV. The last section is devoted to a discussion.

II. THE QUARK MODEL OF BARYONS

The semirelativistic quark model used in this work has been extensively presented in Ref. 2 and summarized in Ref. 1. Here we describe its main features. Details of the wave functions are given in Appendix A.

The present analysis deals with nonstrange particles only and is essentially intended to test the validity of the wave functions associated with the positive- and negative-parity spectrum of baryons. The baryon is viewed as a three-quark system described by a semirelativistic Hamiltonian. For the ground state of the unperturbed part of the Hamiltonian we have used the variational wave function of Carlson, Kogut, and Pandharipande[5] which consists of a product of two- and three-body correlation factors related to the specific potential energy derived from a flux-tube model. This energy is the sum of three pair quark-quark potentials and a three-body term proportional to the string tension in the flux tube. The excited states are spanned by a set built orthogonally on the lowest state and containing up to one unit of radial excitation and two units of angular momentum.

The perturbation is represented by the hyperfine interaction[6] containing contact and tensor parts and modified to include the finite size Λ of the quark of mass m. This interaction is diagonalized in a truncated space spanned by the $56(0^+,2^+)$, $56'(0^+)$, $70(0^+,1^-,2^+)$, and $20(1^+)$ SU(6) multiplets. The functions ψ^{μ}_{LM} of Appendix A are used as space parts of these SU(6)-symmetric unperturbed basis states.

The hadronic states to be tested here have been obtained with two different choices for the quark mass and size: namely, set I: $m=360$ MeV, $\Lambda=0.13$ fm; and set II: $m=324$ MeV, $\Lambda=0.09$ fm. Both have been used in Ref. 1 as well, for the analysis of photodecay amplitudes.

III. THE MESON-EMISSION MODEL

As for the photodecay process the strong decay of a resonance is assumed to take place through a single-quark transition. Because of the symmetry of the wave function of three identical quarks the transition operator $\mathscr{H}_S$ can be written as 3 times the contribution of the third quark. We use a pseudoscalar-emission model including a recoil term.[4,7] Then $\mathscr{H}_S$ can be set in the form

$$\mathscr{H}_S=3e^{-i\mathbf{k}\cdot\mathbf{r}^{(3)}}(x\mathbf{k}\cdot\boldsymbol{\sigma}^{(3)}+y\mathbf{p}^{(3)}\cdot\boldsymbol{\sigma}^{(3)})X_M^{(3)}\ , \tag{3.1}$$

where $\mathbf{k}$ is the momentum of the emitted pion, $\mathbf{r}^{(3)}$ the position, $\mathbf{p}^{(3)}$ the momentum, $\frac{1}{2}\boldsymbol{\sigma}^{(3)}$ the spin, and $X_M^{(3)}$ the SU(3)-flavor coupling operator of the third quark. For neutral-pion emission the latter is given by

$$X_{\pi^0}=\lambda_3 \tag{3.2}$$

in terms of the Gell-Mann matrix λ_3. In Eq. (3.1) x and y are free parameters.

The form (3.1) is akin to the interaction used by Mitra and Ross[7] which includes a recoil term ($y\neq0$) on the basis of the Galilean invariance[8] of the pion-quark interaction and to the quark-pair-creation model in the pointlike limit.[9]

This relation is most easily seen in the momentum space. Let us call ψ_R the resonance and ψ_N the nucleon ground-state wave functions and let us write the transition matrix element for the decay process $R\rightarrow N+\pi$ in the form

$$\langle N\,|\,\mathscr{H}_S\,|\,R\,\rangle =3\langle Ne^{i\mathbf{k}\cdot\mathbf{r}^{(3)}}\,|\,(x\boldsymbol{\sigma}^{(3)}\cdot\mathbf{k}+y\boldsymbol{\sigma}^{(3)}\cdot\mathbf{p}^{(3)})X_M^{(3)}\,|\,R\,\rangle\ . \tag{3.3}$$

In terms of the Jacobi momentum coordinates

$$\mathbf{k}_\rho=\frac{1}{\sqrt{2}}(\mathbf{k}_1-\mathbf{k}_2)\ ,\qquad \mathbf{k}_\lambda=\frac{1}{\sqrt{6}}(\mathbf{k}_1+\mathbf{k}_2-2\mathbf{k}_3)\ , \tag{3.4}$$

we have

$$\langle\,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3\,|\,R\,\rangle=\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\psi_R(\mathbf{k}_\rho,\mathbf{k}_\lambda)\ ,\qquad \langle\,\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3\,|\,Ne^{i\mathbf{k}\cdot\mathbf{r}^{(3)}}\rangle=\delta(\mathbf{k}+\mathbf{k}_N)\psi_N[\mathbf{k}_\rho,\mathbf{k}_\lambda+(\tfrac{2}{3})^{1/2}\mathbf{k}]\ , \tag{3.5}$$

where $\mathbf{k}_N$ is the recoil momentum of the third quark after the emission of a pion with momentum $\mathbf{k}$. Then the matrix element of $\mathscr{H}_S$ becomes

$$\langle N\,|\,\mathscr{H}_S\,|\,R\,\rangle=3x\delta(\mathbf{k}+\mathbf{k}_N)\int d^3k_1d^3k_2d^3k_3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\psi_N^*[\mathbf{k}_\rho,\mathbf{k}_\lambda+(\tfrac{2}{3})^{1/2}\mathbf{k}]\left[\boldsymbol{\sigma}^{(3)}\cdot\mathbf{k}+\frac{y}{x}\boldsymbol{\sigma}^{(3)}\cdot\mathbf{k}^{(3)}\right]X_M^{(3)}\psi_R(\mathbf{k}_\rho,\mathbf{k}_\lambda)\ . \tag{3.6}$$

This is proportional to the form taken by Mitra and Ross for the transition matrix elements if

$$\frac{y}{x}=-\frac{E_\pi}{m_q}\ , \tag{3.7}$$

where $E_\pi=(m_\pi{}^2+k^2)^{1/2}$, m_π and m_q being the pion and quark mass, respectively. The second argument of ψ_N is proportional to $\mathbf{k}_3-\frac{2}{3}\mathbf{k}$ and one can write (3.6) as an integral over $\mathbf{k}_\rho$ and $\mathbf{k}_3$.

On the other hand the pointlike limit of the quark-pair creation model[9] is recovered if we take

$$y/x=-1\ . \tag{3.8}$$

The relations (3.7) and (3.8) could be used as constraints, leaving the model with only one free parameter x.

Our calculations have been performed in the configuration space where it is convenient to take $\mathbf{k}$ along the z axis and split $\mathscr{H}_S$ into a sum of four distinct operators. This amounts to writing

$$\langle N\,|\,\mathscr{H}_S\,|\,R\,\rangle=\int d^3\rho d^3\lambda\psi_N^*(\boldsymbol{\rho},\boldsymbol{\lambda})(\mathscr{A}^0S_0^{(3)}+\mathscr{B}^0S_+^{(3)}+\mathscr{C}^0S_-^{(3)}+\mathscr{D}^0S_0^{(3)})X_M^{(3)}\psi_R(\boldsymbol{\rho},\boldsymbol{\lambda})\ , \tag{3.9}$$

where $S_0,S_\pm=S_x\pm iS_y$ are the spin operators and

$$\begin{aligned}
\mathscr{A}^0&=6xke^{i(2/3)^{1/2}\mathbf{k}\cdot\boldsymbol{\lambda}}\ ,\\
\mathscr{B}^0&=i\sqrt{6}ye^{i(2/3)^{1/2}\mathbf{k}\cdot\boldsymbol{\lambda}}\left[\frac{\partial}{\partial\lambda_x}-i\frac{\partial}{\partial\lambda_y}\right]\ ,\\
\mathscr{C}^0&=i\sqrt{6}ye^{i(2/3)^{1/2}\mathbf{k}\cdot\boldsymbol{\lambda}}\left[\frac{\partial}{\partial\lambda_x}+i\frac{\partial}{\partial\lambda_y}\right]\ ,\\
\mathscr{D}^0&=i2\sqrt{6}ye^{i(2/3)^{1/2}\mathbf{k}\cdot\boldsymbol{\lambda}}\frac{\partial}{\partial\lambda_0}\ .
\end{aligned} \tag{3.10}$$

The advantage of this decomposition is that after performing the calculations in the flavor and spin space the matrix element (3.9) reduces to a linear combination of matrix elements of $\mathscr{A}^0$, $\mathscr{B}^0$, $\mathscr{C}^0$, and $\mathscr{D}^0$ from which $\mathscr{A}^0$ and $\mathscr{B}^0$ are proportional to $\mathscr{A}$ and $\mathscr{B}$ used already in Ref. 1. Also the matrix elements of $\mathscr{C}^0$ are related to those of $\mathscr{B}^0$ through the relation

$$\langle\,\psi_{LM}^{\mu}\,|\,\mathscr{C}^0\,|\,\psi_{L'-M'}^{\mu'}\rangle =(-)^{L+M+L'+M'}\langle\,\psi_{L-M}^{\mu}\,|\,\mathscr{B}^0\,|\,\psi_{L'M'}^{\mu'}\rangle^*\ . \tag{3.11}$$

Then in Eq. (3.9) only $\mathscr{D}^0$ is an entirely new operator, the others can be formally related to the photoemission study. As in Ref. 1 the matrix elements of $\mathscr{A}^0$, $\mathscr{B}^0$, $\mathscr{C}^0$, and $\mathscr{D}^0$ can be reduced to three-dimensional integrals (see Appendix B of Ref. 1) over the variables ρ,λ and $x=\rho\cdot\lambda/\rho\lambda$.

IV. THE DECAY WIDTHS

To have a decay width consistent with the semirelativistic quark model we use a relativistic two-body phase space.[10] Then in the rest frame of the resonance the decay width takes the form

$$\Gamma_{N\pi}=\frac{1}{\pi}\frac{|\langle f|\mathscr{H}_S|i\rangle|^2}{2J_R+1}\frac{kE_N}{m_R}(C^{I_N I_\pi I_R}_{I_{3N}I_{3\pi}I_{3R}})^{-2}, \tag{4.1}$$

where J_α, I_α, and $I_{3\alpha}$ are the total angular momentum, the isospin and its projection for $\alpha=N,\pi$ or R, m_R is the resonance mass, E_N is the recoil energy of the nucleon in its ground state, and k (MeV) is the momentum of the emitted pion given by

$$k=\frac{1}{2m_R}[(m_R{}^2-m_N{}^2-m_\pi{}^2)^2-4m_N{}^2m_\pi{}^2]^{1/2}. \tag{4.2}$$

For convenience we take $m_\pi=m_{\pi^0}=134.963$ MeV. The last factor in Eq. (4.1) is the inverse square of the coupling coefficient in the isospin space. The mixing angles for $|i\rangle$ and $|f\rangle$ are given in Appendix A for set II and those for set I can be found in Ref. 2.

By performing the usual operations in the spin and flavor space the helicity amplitudes $\langle f|\mathscr{H}_S|i\rangle$ are expressed as linear combinations of matrix elements of $\mathscr{A}^0$, $\mathscr{B}^0$, and $\mathscr{D}^0$ (see Appendix B).

In Table I we display our numerical results for $\Gamma_{N\pi}^{1/2}$ calculated according to Eq. (4.1) for the 18 resonances whose photodecay has been studied in Ref. 1. All but two are four- and three-star resonances.[3] The fraction averaged square root of the "best guess" experimental width together with the corresponding experimental range extracted from the Particle Data Group[3] are reproduced in the last column of Table I. As in Ref. 1 we have obtained results with two different sets of wave functions associated with set I and set II of parameters m and Λ entering the hyperfine interaction. In order to see the influence of the configuration mixings we have calculated $\Gamma_{N\pi}^{1/2}$ in two versions for each set. In the first, called "no mix," we have neglected the configuration mixings both in $|i\rangle$ and $|f\rangle$ and approximated $|i\rangle$ by the main component given in column 2 of Table I and $|f\rangle$ by $|{}^2N(56,0^+)\frac{1}{2}^+\rangle$. In the second, "all mix," we have considered the full configuration mixings obtained from the diagonalization of the hyperfine splitting and kept the components with $L^\pi=0^+$ in $|f\rangle$ only, the others having a negligible amplitude. For each set the values of x and y of Eq. (3.1) are obtained through a χ^2 fit including all considered resonances in the "all mix" version. These values are

$$\begin{aligned} x&=2.0750\text{ fm},\quad y=-1.0186\text{ fm}\quad\text{for set I},\\ x&=1.0704\text{ fm},\quad y=-0.68001\text{ fm}\quad\text{for set II}. \end{aligned} \tag{4.3}$$

One can note that x and y have opposite signs analogous to the constraints (3.7) or (3.8). For both thus fixed values

TABLE I. Square root of the nonstrange-baryon decay widths, $\Gamma_{N\pi}{}^{1/2}$, in MeV$^{1/2}$. Column 3 (5) and 4 (6): results obtained with set I (II) without and with configuration mixings. Without mixing the resonance is described by the main component given in column 2. Last column reproduces data extracted from Ref. 3.

		Set I		Set II		
Resonance	Main component	No mix	All mix	No mix	All mix	Expt.
P_{11} (1440)	${}^2N(56',0^+)\frac{1}{2}^+$	0.3	0.1	1.2	1.0	$10.9^{+4.8}_{-3.2}$
D_{13} (1520)	${}^2N(70,1^-)\frac{3}{2}^-$	8.4	8.5	5.0	5.0	$8.3^{+0.9}_{-1.2}$
S_{11} (1535)	${}^2N(70,1^-)\frac{1}{2}^-$	20.3	14.9	14.3	9.4	$8.3^{+3.2}_{-2.1}$
S_{11} (1650)	${}^4N(70,1^-)\frac{1}{2}^-$	8.7	12.7	6.0	8.9	$9.5^{+1.9}_{-2.1}$
D_{15} (1675)	${}^4N(70,1^-)\frac{5}{2}^-$	5.0	5.2	3.1	3.4	$7.4^{+1.1}_{-1.4}$
F_{15} (1680)	${}^2N(56,2^+)\frac{5}{2}^+$	7.6	7.9	3.9	3.6	$8.7^{+0.8}_{-0.9}$
D_{13} (1700)	${}^4N(70,1^-)\frac{3}{2}^-$	2.3	3.4	1.5	2.3	$3.2^{+0.6}_{-0.8}$
P_{11} (1710)	${}^2N(70,0^+)\frac{1}{2}^+$	1.4	9.1	0.1	6.0	$4.0^{+1.1}_{-1.0}$
P_{13} (1720)	${}^2N(56,2^+)\frac{3}{2}^+$	0.4	10.8	0.1	8.6	$5.4^{+1.7}_{-1.9}$
F_{17} (1990)	${}^4N(70,2^+)\frac{7}{2}^+$	2.7	1.4	1.5	0.5	
P_{33} (1232)	${}^4\Delta(56,0^+)\frac{3}{2}^+$	10.0	10.5	10.5	10.9	10.7 ± 0.2
P_{33} (1600)	${}^4\Delta(56',0^+)\frac{3}{2}^+$	2.8	14.0	1.8	10.6	
S_{31} (1620)	${}^2\Delta(70,1^-)\frac{1}{2}^-$	6.3	5.8	4.2	3.7	6.5 ± 1.0
D_{33} (1700)	${}^2\Delta(70,1^-)\frac{3}{2}^-$	4.1	4.3	2.7	2.9	$6.1^{+1.6}_{-1.7}$
F_{35} (1905)	${}^2\Delta(70,2^+)\frac{5}{2}^+$	3.4	2.7	2.0	1.1	$5.8^{+1.9}_{-1.3}$
P_{31} (1910)	${}^2\Delta(70,0^+)\frac{1}{2}^+$	0.9	6.1	0.4	6.4	$7.0^{+2.1}_{-0.7}$
P_{33} (1920)	${}^4\Delta(56,2^+)\frac{3}{2}^+$	4.0	0.7	3.4	4.1	$6.5^{+1.2}_{-1.3}$
F_{37} (1950)	${}^4\Delta(56,2^+)\frac{7}{2}^+$	7.1	6.2	4.2	3.6	$9.8^{+2.6}_{-1.4}$

of x and y we find that the configuration mixings play an important role for many resonances. The most outstanding cases are $F_{17}(1990)$, $P_{33}(1920)$, $P_{33}(1600)$, $P_{31}(1910)$, $P_{11}(1710)$, and $P_{13}(1720)$ where $\Gamma_{N\pi}$ changes from 1 up to 4 orders of magnitude by including configuration mixings. The two resonances having the $(20,1^+)$ basis vector as main component acquire a nonzero width due to the SU(6)-symmetry breaking in the resonance state but they are not seen experimentally. The SU(6)-symmetry-breaking effect has also been found important in the study of photodecays.[1,4]

The results from sets I and II are somewhat different. If by analogy to Ref. 11 we take as a serious disagreement criterion

$$|\ln(\Gamma^{\rm calc}/\Gamma^{\rm expt})|\rangle 0.8\ , \tag{4.4}$$

where $\Gamma^{\rm expt}$ is located within the experimental range, we find the following resonances satisfying (4.4): $P_{11}(1440)$, $P_{11}(1710)$, $F_{35}(1905)$, and $P_{33}(1920)$ obtained from set I and $P_{11}(1440)$, $D_{15}(1675)$, $F_{15}(1680)$, $F_{35}(1905)$, and $F_{37}(1950)$ obtained from set II. In this sense the fit with set I looks slightly better than that with set II. For both sets the ${\Gamma_{N\pi}}^{1/2}$ value of the Roper resonance is much smaller than the experimental one. Such a result is not surprising because we had already evidence from the study of the spectrum[2] that this resonance comes out theoretically too high in energy.

The source of difference between the decay widths obtained with sets I and II can be found either in the phase-space factor proportional to kE_N/m_R or in the matrix element $\langle f|\mathscr{H}_S|i\rangle$ or in both. The $P_{33}(1232)$ is the only resonance for which the phase-space factor varies significantly, i.e., by about 40%, when passing from one set to the other. This resonance has the highest weight in the χ^2 fit and as a consequence the change in the phase factor induces a corresponding change in the matrix element $\langle f|\mathscr{H}_S|i\rangle$, resulting in the optimum values (4.3) of x and y. Since for all the other resonances the phase-space factor varies by a few percent only, the differences produced in the decay widths of sets I an II are essentially due to the optimum choice (4.3). These differences are sometimes much more pronounced for "all mix" results than for "no mix," as it is in the case of $P_{11}(1440)$ or $P_{33}(1920)$ resonances.

We have also performed a least-squares fit with the choices (3.7) and (3.8) for the parameters x and y. It turned out that the discrepancies between theory and experiment are larger than for the fit (4.3). The χ^2 for set I (set II) is equal to 1296 (712), 664 (183), and 80 (99) for the optimum parameter values corresponding to Eqs. (3.7), (3.8), and (4.3), respectively. In particular, even the width of the $P_{33}(1232)$ resonance is not well reproduced except when prescription (3.8) is applied to the spectrum given by set II.

V. DISCUSSION

As mentioned in the introduction the first motivation of this study was to check the assumption made in Ref. 1 about the phase of the helicity amplitude $A_{(1/2)N\pi}$. For all resonances we supposed an identity between the phase in our basis and in the harmonic-oscillator basis of Koniuk and Isgur.[4] In the present context this is the phase of the amplitude $\langle f|\mathscr{H}_S|i\rangle$ defined above. It was gratifying to find out that this assumption was entirely correct both for sets I and II and therefore the signs of the photodecay helicity amplitudes predicted in Ref. 1 remain unmodified.

The presently used model of the baryon structure, based on linear confinement, is closer to the QCD ideas than that based on a harmonic-oscillator unperturbed Hamiltonian. Both the unperturbed Hamiltonian (representing the color-electric interaction) and the hyperfine splitting (the color-magnetic interaction) are inspired from the quark-gluon dynamics.[5] Moreover our strong-decay amplitudes involve only two parameters as compared to four parameters in Ref. 4. There the additional parameters appear to mock up the imperfections of the meson-emission model and the absence of configuration mixings because the nonstrange resonances have been treated as pure SU(6) states. Here we have shown the importance of the configuration mixings. With fewer parameters to be fitted we could not have expected a better agreement with the experiment than that found in Ref. 4 but a greater predictive power.

This work also supports the conclusion of Ref. 2 that a better description of the Roper resonance is needed.

Here as a first step we have considered a simple model for the strong-decay process. In this respect it would be interesting to investigate the role of the pair-creation model[9] in the strong decay of baryons. This model has been successfully applied to meson decay[12] and it has the merit of taking explicitly into account the quark-antiquark structure of the emitted particle.

ACKNOWLEDGMENT

We would like to thank J. Paton for a useful discussion.

APPENDIX A

1. Configuration-space wave functions ψ^{μ}_{LM}

Here we provide the functions ψ^{μ}_{L0} which are used to construct the space part of the resonance states we consider in this paper. The index μ defines the symmetry character of the wave functions with respect to permutations ($\mu=\rho$, λ, S, or A). The functions with $M\neq 0$ can be obtained from those having $M=0$ in the usual way. The considered functions are

$$\psi^{S}_{00}=N^{S}_{00}F\ , \tag{A1}$$

$$\psi^{S'}_{00}=N^{S'}_{00}[1-\alpha(\rho^2+\lambda^2)]F\ , \tag{A2}$$

$$\psi^{\rho}_{00}=N^{\rho}_{00}\boldsymbol{\rho}\cdot\boldsymbol{\lambda}F\ , \tag{A3}$$

$$\psi^{\lambda}_{00}=N^{\rho}_{00}\tfrac{1}{2}(\rho^2-\lambda^2)F\ , \tag{A4}$$

$$\psi^{\rho}_{10}=N^{\rho}_{10}\rho_0F\ , \tag{A5}$$

$$\psi^{\lambda}_{10}=N^{\rho}_{10}\lambda_0F\ , \tag{A6}$$

$$\psi^{A}_{10}=N^{A}_{10}(\rho_-\lambda_+-\rho_+\lambda_-)F\ , \tag{A7}$$

$$\psi^{S}_{20}=N^{S}_{20}[3({\rho_0}^2+{\lambda_0}^2)-(\rho^2+\lambda^2)]F\ , \tag{A8}$$

$$\psi_{20}^{\rho}=N_{20}^{\rho}(3\rho_0\lambda_0-\rho\cdot\lambda)F\ , \tag{A9}$$

$$\psi_{20}^{\lambda}=\tfrac{1}{2}N_{20}^{\rho}[3(\rho_0{}^2-\lambda_0{}^2)-(\rho^2-\lambda^2)]F\ . \tag{A10}$$

In the above equations F is the product of two- and three-body correlation factors introduced in Ref. 5, ρ and λ are the Jacobi coordinates

$$\rho=\frac{1}{\sqrt{2}}(\mathbf{r}_1-\mathbf{r}_2)\ , \tag{A11}$$

$$\lambda=\frac{1}{\sqrt{6}}(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3)\ , \tag{A12}$$

while $\rho_\pm$ and $\lambda_\pm$ are given by

$$\rho_\pm=\rho_x\pm i\rho_y\ , \tag{A13}$$

$$\lambda_\pm=\lambda_x\pm i\lambda_y\ . \tag{A14}$$

The normalization constants N_{L0}^{μ} are obtained numerically.

2. Spectrum and mixing angles for set II

In Tables II and III, we give the spectrum and the mixing angles we have obtained by diagonalizing the hyperfine interaction when the quark mass m and the finite-size parameter Λ were chosen equal to 324 MeV and 0.09 fm, respectively. The corresponding information for set I ($m=360$ MeV, $\Lambda=0.13$ fm) has been given in Ref. 2.

APPENDIX B

In this appendix, we provide the explicit expressions for the amplitudes $\langle f\,|\mathcal{H}_S|\,i\rangle$. The initial state $|\,i\rangle$ runs over the 30 basis states considered in Ref. 2 while the final state $|f\rangle$ runs over the first three components of the nucleon ground state. We shall use the notations

$$A(56)=\langle {}^2N(56,0^+)\tfrac{1}{2}^+\,|\mathcal{H}_S|\,i\rangle\ , \tag{B1}$$

$$A(56')=\langle {}^2N(56',0^+)\tfrac{1}{2}^+\,|\mathcal{H}_S|\,i\rangle\ , \tag{B2}$$

$$A(70)=\langle {}^2N(70,0^+)\tfrac{1}{2}^+\,|\mathcal{H}_S|\,i\rangle\ . \tag{B3}$$

For each initial state $|\,i\rangle$, the expression of $A(56')$ can be obtained from that of $A(56)$ by replacing $\langle\psi_{00}^{S}|$ by $\langle\psi_{00}^{S'}|$; hence, in the following we only give $A(56)$ and $A(70)$. To be definite, we consider the decay of resonances of charge $+1$ into a proton and a neutral pion.

TABLE II. Nonstrange baryons of positive parity. Results obtained by diagonalizing the hyperfine interaction with $m=324$ MeV and $\Lambda=0.09$ fm.

State	Mass (MeV)	Mixing angles				
${}^4N(70,2^+)\frac{7}{2}^+$	1980	(1)				
${}^4\Delta(56,2^+)\frac{7}{2}^+$	1952	(1)				
${}^2N(56,2^+)\frac{5}{2}^+$	1754	0.833	−0.553	1.7×10^{-4}		
${}^2N(70,2^+)\frac{5}{2}^+$	1970	−0.544	−0.821	−0.174		
${}^4N(70,2^+)\frac{5}{2}^+$	2033	−0.096	−0.145	0.985		
${}^4\Delta(56,2^+)\frac{5}{2}^+$	1962	0.408	0.913			
${}^2\Delta(70,2^+)\frac{5}{2}^+$	1985	0.913	−0.408			
${}^4N(70,0^+)\frac{3}{2}^+$	1752	0.098	−0.824	0.558	−0.013	-1.8×10^{-3}
${}^2N(56,2^+)\frac{3}{2}^+$	1914	−0.760	−0.298	−0.296	0.469	0.164
${}^2N(70,2^+)\frac{3}{2}^+$	1979	0.614	−0.309	−0.556	0.332	0.329
${}^4N(70,2^+)\frac{3}{2}^+$	1985	−0.082	−0.359	−0.530	−0.491	−0.585
${}^2N(20,1^+)\frac{3}{2}^+$	2046	−0.173	−0.084	−0.108	−0.655	0.723
${}^4\Delta(56,0^+)\frac{3}{2}^+$	1285	0.977	−0.185	−0.088	0.058	
${}^4\Delta(56',0^+)\frac{3}{2}^+$	1904	0.126	0.902	−0.319	0.261	
${}^4\Delta(56,2^+)\frac{3}{2}^+$	1964	0.169	0.388	0.643	−0.638	
${}^2\Delta(70,2^+)\frac{3}{2}^+$	1979	−0.025	−0.031	−0.691	−0.722	
${}^2N(56,0^+)\frac{1}{2}^+$	941	0.969	0.174	−0.172	−0.034	1.6×10^{-3}
${}^2N(56',0^+)\frac{1}{2}^+$	1607	0.155	−0.980	−0.124	0.014	-1.7×10^{-3}
${}^2N(70,0^+)\frac{1}{2}^+$	1795	0.172	−0.095	0.934	−0.290	0.066
${}^4N(70,2^+)\frac{1}{2}^+$	1930	0.080	-9.7×10^{-3}	0.274	0.825	−0.488
${}^2N(20,1^+)\frac{1}{2}^+$	2042	−0.030	4.7×10^{-4}	−0.083	−0.484	−0.870
${}^2\Delta(70,0^+)\frac{1}{2}^+$	1910	0.908	0.419			
${}^4\Delta(56,2^+)\frac{1}{2}^+$	1935	−0.419	0.908			

TABLE III. Same as Table II, for nonstrange baryons of negative parity.

State	Mass (MeV)	Mixing angles	
$^4N(70,1^-)\frac{5}{2}^-$	1653	(1)	
$^2N(70,1^-)\frac{3}{2}^-$	1496	0.997	0.079
$^4N(70,1^-)\frac{3}{2}^-$	1714	−0.079	0.997
$^2\Delta(70,1^-)\frac{3}{2}^-$	1631	(1)	
$^2N(70,1^-)\frac{1}{2}^-$	1475	0.923	−0.384
$^4N(70,1^-)\frac{1}{2}^-$	1627	0.384	0.923
$^2\Delta(70,1^-)\frac{1}{2}^-$	1631	(1)	

(1) $|i\rangle = |^4N(70,2^+)\frac{7}{2}^+\rangle$:

$$A(56)=\frac{\sqrt{35}}{105}\langle\psi_{00}^S|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\lambda\rangle+\frac{2\sqrt{210}}{315}\langle\psi_{00}^S|\mathcal{B}^0|\psi_{21}^\lambda\rangle\,, \tag{B4}$$

$$A(70)=-\frac{\sqrt{70}}{210}\langle\psi_{00}^\lambda|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\lambda\rangle-\frac{\sqrt{70}}{70}\langle\psi_{00}^\rho|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\rho\rangle-\frac{2\sqrt{105}}{315}\langle\psi_{00}^\lambda|\mathcal{B}^0|\psi_{21}^\lambda\rangle-\frac{2\sqrt{105}}{105}\langle\psi_{00}^\rho|\mathcal{B}^0|\psi_{21}^\rho\rangle\,. \tag{B5}$$

(2) $|i\rangle = |^4\Delta(56,2^+)\frac{7}{2}^+\rangle$:

$$A(56)=\frac{4\sqrt{35}}{105}\langle\psi_{00}^S|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^S\rangle+\frac{8\sqrt{210}}{315}\langle\psi_{00}^S|\mathcal{B}^0|\psi_{21}^S\rangle\,, \tag{B6}$$

$$A(70)=-\frac{2\sqrt{70}}{105}\langle\psi_{00}^\lambda|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^S\rangle-\frac{8\sqrt{105}}{315}\langle\psi_{00}^\lambda|\mathcal{B}^0|\psi_{21}^S\rangle\,. \tag{B7}$$

(3) $|i\rangle = |^2N(56,2^+)\frac{5}{2}^+\rangle$:

$$A(56)=\frac{\sqrt{15}}{18}\langle\psi_{00}^S|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^S\rangle+\frac{\sqrt{10}}{9}\langle\psi_{00}^S|\mathcal{B}^0|\psi_{21}^S\rangle\,, \tag{B8}$$

$$A(70)=\frac{\sqrt{30}}{45}\langle\psi_{00}^\lambda|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^S\rangle+\frac{4\sqrt{5}}{45}\langle\psi_{00}^\lambda|\mathcal{B}^0|\psi_{21}^S\rangle\,. \tag{B9}$$

(4) $|i\rangle = |^2N(70,2^+)\frac{5}{2}^+\rangle$:

$$A(56)=\frac{\sqrt{30}}{45}\langle\psi_{00}^S|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\lambda\rangle+\frac{4\sqrt{5}}{45}\langle\psi_{00}^s|\mathcal{B}^0|\psi_{21}^\lambda\rangle\,, \tag{B10}$$

$$A(70)=\frac{\sqrt{15}}{36}\langle\psi_{00}^\lambda|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\lambda\rangle-\frac{\sqrt{15}}{60}\langle\psi_{00}^\rho|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\rho\rangle+\frac{\sqrt{10}}{18}\langle\psi_{00}^\lambda|\mathcal{B}^0|\psi_{21}^\lambda\rangle-\frac{\sqrt{10}}{30}\langle\psi_{00}^\rho|\mathcal{B}^0|\psi_{21}^\rho\rangle\,. \tag{B11}$$

(5) $|i\rangle = |^4N(70,2^+)\frac{5}{2}^+\rangle$:

$$A(56)=-\frac{\sqrt{210}}{630}\langle\psi_{00}^S|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\lambda\rangle-\frac{2\sqrt{35}}{315}\langle\psi_{00}^S|\mathcal{B}^0|\psi_{21}^\lambda\rangle\,, \tag{B12}$$

$$A(70)=\frac{\sqrt{105}}{630}\langle\psi_{00}^\lambda|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\lambda\rangle+\frac{\sqrt{105}}{210}\langle\psi_{00}^\rho|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\rho\rangle+\frac{\sqrt{70}}{315}\langle\psi_{00}^\lambda|\mathcal{B}^0|\psi_{21}^\lambda\rangle+\frac{\sqrt{70}}{105}\langle\psi_{00}^\rho|\mathcal{B}^0|\psi_{21}^\rho\rangle\,. \tag{B13}$$

(6) $|i\rangle = |^4\Delta(56,2^+)\frac{5}{2}^+\rangle$:

$$A(56)=-\frac{2\sqrt{210}}{315}\langle\psi_{00}^S|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^S\rangle-\frac{8\sqrt{35}}{315}\langle\psi_{00}^S|\mathcal{B}^0|\psi_{21}^S\rangle\,, \tag{B14}$$

$$A(70)=\frac{2\sqrt{105}}{315}\langle\psi_{00}^\lambda|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^S\rangle+\frac{4\sqrt{70}}{315}\langle\psi_{00}^\lambda|\mathcal{B}^0|\psi_{21}^S\rangle\,. \tag{B15}$$

(7) $|i\rangle = |^2\Delta(70,2^+)\frac{5}{2}^+\rangle$:

$$A(56)=\frac{\sqrt{30}}{90}\langle\psi_{00}^S|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\lambda\rangle+\frac{2\sqrt{5}}{45}\langle\psi_{00}^S|\mathcal{B}^0|\psi_{21}^\lambda\rangle\,, \tag{B16}$$

$$A(70)=-\frac{\sqrt{15}}{90}\langle\psi_{00}^\lambda|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\lambda\rangle-\frac{\sqrt{15}}{30}\langle\psi_{00}^\rho|\mathcal{A}^0+\mathcal{D}^0|\psi_{20}^\rho\rangle-\frac{\sqrt{10}}{45}\langle\psi_{00}^\lambda|\mathcal{B}^0|\psi_{21}^\lambda\rangle-\frac{\sqrt{10}}{15}\langle\psi_{00}^\rho|\mathcal{B}^0|\psi_{21}^\rho\rangle\,. \tag{B17}$$

(8) $|i\rangle = |^4N(70,0^+)\frac{3}{2}^+\rangle$:

$$A(56)=\frac{\sqrt{2}}{18}\langle\psi_{00}^S|\mathcal{A}^0+\mathcal{D}^0|\psi_{00}^\lambda\rangle\,, \tag{B18}$$

$$A(70)=-\tfrac{1}{18}\langle\psi_{00}^\lambda|\mathcal{A}^0+\mathcal{D}^0|\psi_{00}^\lambda\rangle-\tfrac{1}{6}\langle\psi_{00}^\rho|\mathcal{A}^0+\mathcal{D}^0|\psi_{00}^\rho\rangle\,. \tag{B19}$$

(9) $|i\rangle = |^2N(56,2^+)\frac{3}{2}^+\rangle$:

$$A(56)=-\frac{\sqrt{10}}{18}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{S}\rangle+\frac{\sqrt{15}}{9}\langle\psi_{00}^{S}|\mathscr{B}^0|\psi_{21}^{S}\rangle\,, \tag{B20}$$

$$A(70)=-\frac{2\sqrt{5}}{45}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{S}\rangle+\frac{2\sqrt{30}}{45}\langle\psi_{00}^{\lambda}|\mathscr{B}^0|\psi_{21}^{S}\rangle\,. \tag{B21}$$

(10) $|i\rangle=|{}^2N(70,2^+)\frac{3}{2}^+\rangle$:

$$A(56)=-\frac{2\sqrt{5}}{45}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\lambda}\rangle+\frac{2\sqrt{30}}{45}\langle\psi_{00}^{S}|\mathscr{B}^0|\psi_{21}^{\lambda}\rangle\,, \tag{B22}$$

$$A(70)=-\frac{\sqrt{10}}{36}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\lambda}\rangle+\frac{\sqrt{10}}{60}\langle\psi_{00}^{\rho}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\rho}\rangle+\frac{\sqrt{15}}{18}\langle\psi_{00}^{\lambda}|\mathscr{B}^0|\psi_{21}^{\lambda}\rangle-\frac{\sqrt{15}}{30}\langle\psi_{00}^{\rho}|\mathscr{B}^0|\psi_{21}^{\rho}\rangle\,. \tag{B23}$$

(11) $|i\rangle=|{}^4N(70,2^+)\frac{3}{2}^+\rangle$:

$$A(56)=-\frac{\sqrt{10}}{90}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\lambda}\rangle+\frac{\sqrt{15}}{45}\langle\psi_{00}^{S}|\mathscr{B}^0|\psi_{21}^{\lambda}\rangle\,, \tag{B24}$$

$$A(70)=\frac{\sqrt{5}}{90}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\lambda}\rangle+\frac{\sqrt{5}}{30}\langle\psi_{00}^{\rho}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\rho}\rangle-\frac{\sqrt{30}}{90}\langle\psi_{00}^{\lambda}|\mathscr{B}^0|\psi_{21}^{\lambda}\rangle-\frac{\sqrt{30}}{30}\langle\psi_{00}^{\rho}|\mathscr{B}^0|\psi_{21}^{\rho}\rangle\,. \tag{B25}$$

(12) $|i\rangle=|{}^2N(20,1^+)\frac{3}{2}^+\rangle$:

$$A(56)=0\,, \tag{B26}$$

$$A(70)=0\,. \tag{B27}$$

(13) $|i\rangle=|{}^4\Delta(56,0^+)\frac{3}{2}^+\rangle$:

$$A(56)=\frac{2\sqrt{2}}{9}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{S}\rangle\,, \tag{B28}$$

$$A(70)=-\tfrac{2}{9}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{S}\rangle\,. \tag{B29}$$

(14) $|i\rangle=|{}^4\Delta(56',0^+)\frac{3}{2}^+\rangle$:

$$A(56)=\frac{2\sqrt{2}}{9}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{S'}\rangle\,, \tag{B30}$$

$$A(70)=-\tfrac{2}{9}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{S'}\rangle\,. \tag{B31}$$

(15) $|i\rangle=|{}^4\Delta(56,2^+)\frac{3}{2}^+\rangle$:

$$A(56)=-\frac{2\sqrt{10}}{45}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{S}\rangle+\frac{4\sqrt{15}}{45}\langle\psi_{00}^{S}|\mathscr{B}^0|\psi_{21}^{S}\rangle\,, \tag{B32}$$

$$A(70)=\frac{2\sqrt{5}}{45}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{S}\rangle-\frac{2\sqrt{30}}{45}\langle\psi_{00}^{\lambda}|\mathscr{B}^0|\psi_{21}^{S}\rangle\,. \tag{B33}$$

(16) $|i\rangle=|{}^2\Delta(70,2^+)\frac{3}{2}^+\rangle$:

$$A(56)=-\frac{\sqrt{5}}{45}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\lambda}\rangle+\frac{\sqrt{30}}{45}\langle\psi_{00}^{S}|\mathscr{B}^0|\psi_{21}^{\lambda}\rangle\,, \tag{B34}$$

$$A(70)=\frac{\sqrt{10}}{90}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\lambda}\rangle+\frac{\sqrt{10}}{30}\langle\psi_{00}^{\rho}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\rho}\rangle-\frac{\sqrt{15}}{45}\langle\psi_{00}^{\lambda}|\mathscr{B}^0|\psi_{21}^{\lambda}\rangle-\frac{\sqrt{15}}{15}\langle\psi_{00}^{\rho}|\mathscr{B}^0|\psi_{21}^{\rho}\rangle\,. \tag{B35}$$

(17) $|i\rangle=|{}^2N(56,0^+)\frac{1}{2}^+\rangle$:

$$A(56)=\tfrac{5}{18}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{S}\rangle\,, \tag{B36}$$

$$A(70)=\frac{\sqrt{2}}{9}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{S}\rangle\,. \tag{B37}$$

(18) $|i\rangle=|{}^2N(56',0^+)\frac{1}{2}^+\rangle$:

$$A(56)=\tfrac{5}{18}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{S'}\rangle\,, \tag{B38}$$

$$A(70)=\frac{\sqrt{2}}{9}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{S'}\rangle\,. \tag{B39}$$

(19) $|i\rangle=|{}^2\mathrm{N}(70,0^+)\frac{1}{2}^+\rangle$:

$$A(56)=\frac{\sqrt{2}}{9}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{\lambda}\rangle\,, \tag{B40}$$

$$A(70)=\tfrac{5}{36}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{\lambda}\rangle-\tfrac{1}{12}\langle\psi_{00}^{\rho}|\mathscr{A}^0+\mathscr{D}^0|\psi_{00}^{\rho}\rangle\,. \tag{B41}$$

(20) $|i\rangle=|{}^4\mathrm{N}(70,2^+)\frac{1}{2}^+\rangle$:

$$A(56)=\frac{\sqrt{10}}{90}\langle\psi_{00}^{S}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\lambda}\rangle-\frac{\sqrt{15}}{45}\langle\psi_{00}^{S}|\mathscr{B}^0|\psi_{21}^{\lambda}\rangle\,, \tag{B42}$$

$$A(70)=-\frac{\sqrt{5}}{90}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\lambda}\rangle-\frac{\sqrt{5}}{30}\langle\psi_{00}^{\rho}|\mathscr{A}^0+\mathscr{D}^0|\psi_{20}^{\rho}\rangle+\frac{\sqrt{30}}{90}\langle\psi_{00}^{\lambda}|\mathscr{B}^0|\psi_{21}^{\lambda}\rangle+\frac{\sqrt{30}}{30}\langle\psi_{00}^{\rho}|\mathscr{B}^0|\psi_{21}^{\rho}\rangle\,. \tag{B43}$$

(21) $|i\rangle=|{}^2N(20,1^+)\frac{1}{2}^+\rangle$:

$$A(56)=0\ , \tag{B44}$$

$$A(70)=0\ . \tag{B45}$$

(22) $|\,i\,\rangle=|\,^2\Delta(70,0^+)\tfrac{1}{2}^+\rangle$:

$$A(56)=\frac{\sqrt{2}}{18}\langle\psi^S_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{00}\rangle\ , \tag{B46}$$

$$A(70)=-\tfrac{1}{18}\langle\psi^\lambda_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{00}\rangle -\tfrac{1}{6}\langle\psi^\rho_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\rho_{00}\rangle\ . \tag{B47}$$

(23) $|\,i\,\rangle=|\,^4\Delta(56,2^+)\tfrac{1}{2}^+\rangle$:

$$A(56)=\frac{2\sqrt{10}}{45}\langle\psi^S_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^S_{20}\rangle -\frac{4\sqrt{15}}{45}\langle\psi^S_{00}|\mathscr{B}^0|\psi^S_{21}\rangle\ , \tag{B48}$$

$$A(70)=-\frac{2\sqrt{5}}{45}\langle\psi^\lambda_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^S_{20}\rangle +\frac{2\sqrt{30}}{45}\langle\psi^\lambda_{00}|\mathscr{B}^0|\psi^S_{21}\rangle\ . \tag{B49}$$

(24) $|\,i\,\rangle=|\,^4N(70,1^-)\tfrac{5}{2}^-\rangle$:

$$A(56)=\frac{\sqrt{30}}{90}\langle\psi^S_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle +\frac{\sqrt{15}}{45}\langle\psi^S_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle\ , \tag{B50}$$

$$A(70)=-\frac{\sqrt{15}}{90}\langle\psi^\lambda_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle -\frac{\sqrt{15}}{30}\langle\psi^\rho_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\rho_{10}\rangle -\frac{\sqrt{30}}{90}\langle\psi^\lambda_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle-\frac{\sqrt{30}}{30}\langle\psi^\rho_{00}|\mathscr{B}^0|\psi^\rho_{11}\rangle\ . \tag{B51}$$

(25) $|\,i\,\rangle=|\,^2N(70,1^-)\tfrac{3}{2}^-\rangle$:

$$A(56)=\frac{2\sqrt{3}}{27}\langle\psi^S_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle +\frac{2\sqrt{6}}{27}\langle\psi^S_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle\ , \tag{B52}$$

$$A(70)=\frac{5\sqrt{6}}{108}\langle\psi^\lambda_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle -\frac{\sqrt{6}}{36}\langle\psi^\rho_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\rho_{10}\rangle +\frac{5\sqrt{3}}{54}\langle\psi^\lambda_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle-\frac{\sqrt{3}}{18}\langle\psi^\rho_{00}|\mathscr{B}^0|\psi^\rho_{11}\rangle\ . \tag{B53}$$

(26) $|\,i\,\rangle=|\,^4N(70,1^-)\tfrac{3}{2}^-\rangle$:

$$A(56)=-\frac{\sqrt{30}}{270}\langle\psi^S_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle -\frac{\sqrt{15}}{135}\langle\psi^S_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle\ , \tag{B54}$$

$$A(70)=\frac{\sqrt{15}}{270}\langle\psi^\lambda_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle +\frac{\sqrt{15}}{90}\langle\psi^\rho_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\rho_{10}\rangle +\frac{\sqrt{30}}{270}\langle\psi^\lambda_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle+\frac{\sqrt{30}}{90}\langle\psi^\rho_{00}|\mathscr{B}^0|\psi^\rho_{11}\rangle\ . \tag{B55}$$

(27) $|\,i\,\rangle=|\,^2\Delta(70,1^-)\tfrac{3}{2}^-\rangle$:

$$A(56)=\frac{\sqrt{3}}{27}\langle\psi^S_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle+\frac{\sqrt{6}}{27}\langle\psi^S_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle\ , \tag{B56}$$

$$A(70)=-\frac{\sqrt{6}}{54}\langle\psi^\lambda_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle -\frac{\sqrt{6}}{18}\langle\psi^\rho_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\rho_{10}\rangle -\frac{\sqrt{3}}{27}\langle\psi^\lambda_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle-\frac{\sqrt{3}}{9}\langle\psi^\rho_{00}|\mathscr{B}^0|\psi^\rho_{11}\rangle\ . \tag{B57}$$

(28) $|\,i\,\rangle=|\,^2N(70,1^-)\tfrac{1}{2}^-\rangle$:

$$A(56)=-\frac{\sqrt{6}}{27}\langle\psi^S_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle +\frac{4\sqrt{3}}{27}\langle\psi^S_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle\ , \tag{B58}$$

$$A(70)=-\frac{5\sqrt{3}}{108}\langle\psi^\lambda_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle +\frac{\sqrt{3}}{36}\langle\psi^\rho_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\rho_{10}\rangle +\frac{5\sqrt{6}}{54}\langle\psi^\lambda_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle-\frac{\sqrt{6}}{18}\langle\psi^\rho_{00}|\mathscr{B}^0|\psi^\rho_{11}\rangle\ . \tag{B59}$$

(29) $|\,i\,\rangle=|\,^4N(70,1^-)\tfrac{1}{2}^-\rangle$:

$$A(56)=-\frac{\sqrt{6}}{54}\langle\psi^S_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle +\frac{2\sqrt{3}}{27}\langle\psi^S_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle\ , \tag{B60}$$

$$A(70)=\frac{\sqrt{3}}{54}\langle\psi^\lambda_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle +\frac{\sqrt{3}}{18}\langle\psi^\rho_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\rho_{10}\rangle -\frac{\sqrt{6}}{27}\langle\psi^\lambda_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle-\frac{\sqrt{6}}{9}\langle\psi^\rho_{00}|\mathscr{B}^0|\psi^\rho_{11}\rangle\ . \tag{B61}$$

(30) $|\,i\,\rangle=|\,^2\Delta(70,1^-)\tfrac{1}{2}^-\rangle$:

$$A(56)=-\frac{\sqrt{6}}{54}\langle\psi^S_{00}|\mathscr{A}^0+\mathscr{D}^0|\psi^\lambda_{10}\rangle +\frac{2\sqrt{3}}{27}\langle\psi^S_{00}|\mathscr{B}^0|\psi^\lambda_{11}\rangle\ , \tag{B62}$$

$$A(70)=\frac{\sqrt{3}}{54}\langle\psi_{00}^{\lambda}|\mathscr{A}^0+\mathscr{D}^0|\psi_{10}^{\lambda}\rangle+\frac{\sqrt{3}}{18}\langle\psi_{00}^{\rho}|\mathscr{A}^0+\mathscr{D}^0|\psi_{10}^{\rho}\rangle-\frac{\sqrt{6}}{27}\langle\psi_{00}^{\lambda}|\mathscr{B}^0|\psi_{11}^{\lambda}\rangle-\frac{\sqrt{6}}{9}\langle\psi_{00}^{\rho}|\mathscr{B}^0|\psi_{11}^{\rho}\rangle\,. \tag{B63}$$

The above expressions as well as their reduction to three-dimensional integrals have been obtained by means of the REDUCE symbolic-manipulation program.[13] They are very general and can be particularized to, for example, a harmonic-oscillator basis by defining F and N^{μ}_{L0} accordingly.

[1]R. Sartor and Fl. Stancu, Phys. Rev. D 33, 727 (1986).
[2]R. Sartor and Fl. Stancu, Phys. Rev. D 31, 128 (1985).
[3]Particle Data Group, Rev. Mod. Phys. 56, S25 (1984).
[4]R. Koniuk and N. Isgur, Phys. Rev. D 21, 1868 (1980).
[5]J. Carlson, J. Kogut, and V. R. Pandharipande, Phys. Rev. D **27**, 233 (1983).
[6]A. de Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).
[7]A. N. Mitra and M. Ross, Phys. Rev. **158**, 1630 (1967).
[8]This is the analog of the Galilean-invariant pion-nucleon interaction; see M. Bolsterli *et al.*, Phys. Rev. C 10, 1225 (1974). For a review, see H. W. Ho, M. Alberg, and E. M. Henley, *ibid.* 12, 217 (1975), and references quoted therein.
[9]L. Micu, Nucl. Phys. **B10**, 521 (1969); R. Carlitz, and M. Kislinger, Phys. Rev. D 2, 336 (1970); E. W. Colglazier and J. L. Rosner, Nucl. Phys. **B27**, 349 (1971); W. P. Peterson and J. L. Rosner, Phys. Rev. D **6**, 820 (1972); **7**, 747 (1973); A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, *ibid.* **8**, 2223 (1973); **9**, 1415 (1974); **11**, 1272 (1975).
[10]See, e.g., S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1976), p. 142.
[11]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **3**, 2706 (1971).
[12]R. Kokoski, Ph.D. thesis, University of Toronto, 1984.
[13]A. C. Hearn, REDUCE version 3.2, The Rand Corporation, Santa Monica, CA 90406.