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Gravitational bags

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Spontaneous compactification is shown to predict a novel type of localized four-dimensional horizon-free objects. These are cores of scalar fields confined by means of a static domain wall. Their surface area is at most $\frac{9}{4}$ the area of an equal-mass black hole. Altogether, they make excellent candidates for Einstein's gravitational bags.

It is amusing to recall[1] that Einstein was actually ready, at a certain point, to abandon his field equations

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=-\kappa T_{\mu\nu}\ , \tag{1}$$

in favor of

$$R_{\mu\nu}-\tfrac{1}{4}g_{\mu\nu}R=-\kappa T_{\mu\nu}\ . \tag{1a}$$

The latter equations exhibit some diverse features, such as the tracelessness of the energy-momentum tensor, and have the advantage of treating the cosmological constant as a constant of integration rather than as a universal quantity. Einstein invoked Eq. (1a) while trying to resolve what he called the problem of matter: namely, the frustrating impotency of Eq. (1) to account for the structure of the electron. He suggested that "in the interior of every elementary corpuscle, where the density of electricity is other than zero, there subsists a negative pressure the fall of which maintains the electromagnetic force in equilibrium." In this Rapid Communication we construct gravitational bags without sacrificing general relativity. In fact, we show that a core of a scalar field, confined by a static domain wall, is predicted by local spontaneous compactification.[2] The surface area of the domain wall is at most $\frac{9}{4}$ the area of an equal-mass black hole. It is quite reasonable that such objects accompany string theories as well.

Our starting point is the Freund-Rubin (FR) mechanism[2] for spontaneous compactification. It requires the introduction of sophisticated potentials $A_{m_1m_2m_3}$ ($m_i=0,1,\ldots,n+3$); the total number of its indices is correlated with the total number of ordinary spacetime dimensions. The associated action is given by

$$S=\int d^{4+n}x\sqrt{-g}\left[\frac{1}{16\pi G}(R-2\Lambda)-\tfrac{1}{48}F^2\right]\ , \tag{2}$$

where

$$F_{mnpq}=\partial_{[m}A_{npq]}\ . \tag{3}$$

It is the vacuum expectation value (VEV) of this antisymmetric rank-4 tensor field which drives compactification of the n extra dimensions. It has not escaped attention that such a mechanism is automatically embedded in the bosonic sector of the 11-dimensional $N=1$ supergravity,[3] only with $\Lambda=0$, of course, as required by supersymmetry. At any rate, making no commitments concerning the origin of the FR mechanism, we momentarily allow for an arbitrary cosmological term.

For the sake of simplicity, the line element of interest is taken to be of the block-diagonal form:

$$ds^2=ds_{(4)}{}^2(x^\mu)+a^2(x^\mu)\frac{dy^\alpha dy^\alpha}{(1+\frac{1}{4}ky^2)^2}\ , \tag{4}$$

with the notations $\mu=0,1,2,3$, and $\alpha=4,\ldots,n+3$. The extra-dimensional space is maximally symmetric, and the four-dimensional piece $ds_{(4)}{}^2$ is consequently y independent. This configuration has an SO$(n+1)$ isometry group if k happens to be positive. It is important to notice that the overall scale of the extra-dimensional manifold is treated as a four-dimensional scalar field. We thus deal with the *local* extension of the FR mechanism.

The generalized Maxwell equations $(1/\sqrt{-g})\partial_m\times(\sqrt{-g}F^{mnpq})=0$ are satisfied by

$$\langle F_{\mu\nu\lambda\sigma}\rangle=\lambda\frac{\sqrt{-g_{(4)}}}{3a^n}\varepsilon_{\mu\nu\lambda\sigma}\ , \tag{5}$$

with the proportionality factor λ being a constant by means of the Bianchi identity. $\varepsilon_{\mu\nu\lambda\sigma}$ denotes the four-dimensional Levi-Civita tensor density, and $g_{(4)}\equiv\det g_{\mu\nu}$. Substituting the above VEV into the energy-momentum tensor one finds $T_{\mu\nu}=-(\lambda^2/a^{2n})g_{\mu\nu}$, and $T_{\alpha\beta}=+(\lambda^2/a^{2n})g_{\alpha\beta}$, with a noticeable opposite sign.

Another technical step is still needed before decoding the *effective* four-dimensional picture. To ensure the recovery of general relativity after the dimensional-reduction procedure, it is crucial to invoke the proper Weyl factor.[4] The dictionary then reads

$$g_{\mu\nu}^{\text{eff}}=a^n g_{\mu\nu} \ , \tag{6}$$

indicating that, in principle, a singularity in $g_{\mu\nu}^{\text{eff}}$ may be the effect of the Weyl factor itself.

Rewriting the Einstein equations such that $R_{\mu\nu}^{\text{eff}} - \frac{1}{2}g_{\mu\nu}^{\text{eff}}R_{\mu\nu}^{\text{eff}}$ stands on the left-hand side, the effective four-dimensional picture becomes clear. It involves a scalar field

$$\phi=\left(\frac{n(n+2)}{2}\right)^{1/2}\ln a \ , \tag{7}$$

subject to a very special potential (which is *not* of the Higgs style)

$$V(\phi)=\lambda^2\exp\left[-3\left(\frac{2n}{n+2}\right)^{1/2}\phi\right]-\tfrac{1}{2}kn(n-1)\exp\left[-2\left(\frac{n+2}{2n}\right)^{1/2}\phi\right]+\Lambda\exp\left[-\left(\frac{2n}{n+2}\right)^{1/2}\phi\right] . \tag{8}$$

The λ^2 term makes the potential bounded from below, allowing for the existence of a *classical* absolute minimum, a physical property which is badly needed in the traditional Kaluza-Klein (KK) scenario.[5] It is this potential which stabilizes the FR mechanism, and furthermore predicts the formation of gravitational bags.

The scalar potential (see, e.g., Fig. 1) is nothing but the effective four-dimensional cosmological "constant", i.e., $V\equiv\Lambda_{\text{eff}}$. For $\Lambda=0$, as a pedagogical example, one faces $V_{\min}<0$, reproducing the well-known supergravity result of an effective anti–de Sitter, rather than flat, vacuum geometry. This is why we hereby constrain the potential to vanish at its absolute minimum, being ready to pay the price of fine-tuning Λ; that is,

$$\Lambda=\tfrac{1}{2}k(n-1)^2\left(\frac{k(n-1)}{2\lambda^2}\right)^{1/(n-1)} . \tag{9}$$

From a cosmological point of view, this would guarantee $\Lambda_{\text{eff}}\to 0$ as the Universe, after undergoing an inflationary era,[6] through oscillations settles in its ground state (we note in passing that the associated cosmic evolution is solitary,[7] driven by the collapse of the extra dimensions). As far as gravitational bags are concerned, on the other hand, Eq. (9) is essential for maintaining asymptotic flatness.

From this point on, we concentrate on the static threefold radially symmetric case. The $r\to\infty$ limit is obviously associated with the absolute minimum of $V(\phi)$, obtained for

$$\phi\to\phi_{\min}=\frac{\sqrt{n(n+2)}}{2\sqrt{2}(n-1)}\ln\frac{2\lambda^2}{(n-1)k} \ ,$$

thus fixing the asymptotic radius of the extra-dimensional sphere already at the classical level. At asymptotic distances, the physics is therefore of the Schwarzschild type perturbed by the Yukawa tail of a scalar field of mass $m^2\equiv\partial^2V/\partial\phi^2|_{\min}$. It is remarkable that, due to Eq. (9), and on dimensional grounds, m^2 also sets the scale of the potential itself. Of particular interest is the local maximum

$$V_{\max}\sim m^2=\frac{8(n-1)}{(n+2)}\lambda^2\left(\frac{k(n-1)}{2\lambda^2}\right)^{3n/2(n-1)} , \tag{10}$$

with the proportionality factor being a decent $O(1)$ function of n.

To analyze the complete r dependence, we first focus attention on the scalar field equation of motion. Using isotropic radial marker, for which

$$ds_{\text{eff}}{}^2=-T^2dt^2+R^2(dr^2+r^2d\Omega^2) \ ,$$

this equation takes the form

$$\frac{1}{R^2}\left[\phi''+\left(\frac{T'}{T}+\frac{R'}{R}+\frac{2}{r}\right)\phi'\right]=\frac{dV}{d\phi} . \tag{11}$$

For the sake of transparency, we momentarily use the Gaussian radial marker ρ, defined via $d\rho/dr=R(r)$, for which Eq. (11) represents a familiar mechanical problem. Treating ρ as "time" and ϕ as "position", Eq. (11) then describes a particle moving in a potential $-V$, with

$$\left(\frac{1}{T}\frac{dT}{d\rho}+\frac{2}{R}\frac{dR}{d\rho}+\frac{2}{R\rho}\right)\frac{d\phi}{d\rho}$$

serving as a friction term. In this Coleman language,[8] we are searching for a solution in which the particle comes to rest at infinite "time" at $\phi_{\min}$ (in which $-V$ is actually maximal). Such a solution always exists, as can be demonstrated numerically by starting at the asymptotic region, where the analytic behavior is well known, and then going backwards with r. In fact, reflecting the asymmetry of the potential with regard to $\phi\to-\phi$, there exist two *inequivalent* nontrivial solutions to the problem. In the asymptotic region where $\phi\sim\phi_{\min}+(s/r)\exp(-mr)$, s being the scalar hypercharge, they correspond to $s>0$ or $s<0$, respectively.

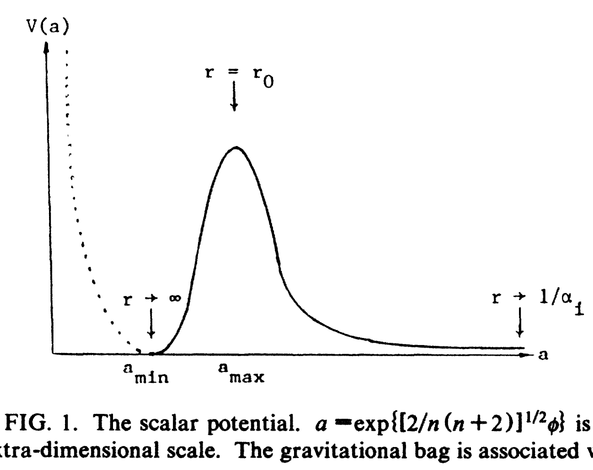

FIG. 1. The scalar potential. $a=\exp\{[2/n(n+2)]^{1/2}\phi\}$ is the extra-dimensional scale. The gravitational bag is associated with $a\geq a_{\min}$.

At this stage we can already have a qualitative description of the object called gravitational bag. It carries a positive hypercharge ($s>0$), so that $\phi(r)\geq\phi_{\min}$ is a monotonically decreasing function of r. With regard to local variations of $\Lambda_{\rm eff}(r)\equiv V(\phi(r))$, the space is now divided into two spherically symmetric regions of negligible $\Lambda_{\rm eff}$, separated by a potential barrier. As $m^2\rightarrow\infty$, and so does $V_{\max}$ by virtue of Eq. (10), this barrier is expected to become a thin domain wall,[9] and the internal structure of the gravitational bag gets significantly simplified. In this approximation it remains to match (it is a four-dimensional matching) (i) *the exterior region* ($r>r_0$), where $\phi(r)=\phi_{\min}$ gives rise to the familiar Schwarzschild geometry, with (ii) *the interior core* ($r<r_0$), where the geometry is dominated by a massless scalar field $\phi(r)>\phi_{\min}$. The two regions are separated by a domain wall of surface tension σ (= surface energy density),[10]

$$\sigma=2\int_{-\Delta/2}^{\Delta/2}V(\phi)dn\simeq 2\Delta V_{\max}\ , \tag{12}$$

where Δ is the normal thickness of the wall. A finite σ, or even $\sigma\sim m$, leading to $\Delta\sim 1/m^2$ or $1/m$, respectively, is fully consistent with the thin-wall approximation.

It has not escaped our attention that, up to an exponentially decaying Yukawa tail, the scalar field is practically confined within the boundaries of the bag. When penetrating the bag, the scalar field grows until reaching its singularity. Using the KK language, the associated gauge coupling g is inversely proportional to the extra-dimensional radius

$$a\sim\exp\left[\left[\frac{2}{n(n+2)}\right]^{1/2}\phi\right]\ ,$$

so that the singular behavior $\phi\rightarrow+\infty$ corresponds to $g\rightarrow 0$. Such a classical short-distance phenomenon highly reminds us of "asymptotic freedom."

The rest of the paper is devoted to probe the consistency of the matching in the thin-wall approximation, and to calculate the size of the gravitational bag. Let $ds_{e(i)}{}^2$ refer to $ds_{\rm eff}{}^2$ in the exterior (interior) region, respectively. Explicitly we have

$$ds_e{}^2=-\left[\frac{\alpha_e r-1}{\alpha_e r+1}\right]^2dt^2+\frac{1}{\alpha_e{}^4r^4}(\alpha_e r+1)^4(dr^2+r^2d\Omega^2)\ , \tag{13a}$$

$$ds_i{}^2=-C_1\left[\frac{\alpha_i r-1}{\alpha_i r+1}\right]^{2p}dt^2+\frac{C_2}{\alpha_i{}^4r^4}\frac{(\alpha_i r+1)^{2(p+1)}}{(\alpha_i r-1)^{2(p-1)}}(dr^2+r^2d\Omega^2)\ . \tag{13b}$$

The only external parameter is the gravitational mass $M_e=2/\alpha_e$, while the internal parameters[11] are analogously the would-have-been gravitational mass $M_i=2p/\alpha_i$, and the scalar hypercharge $s_i{}^2+M_i{}^2=4/\alpha_i{}^2$. The kinematical constraint $-1\leq p\leq 1$ is imposed by the underlying Einstein equations ($p=1$ is the Schwarzschild limit). It is worth pointing out that it is gravity which makes the energy of the gravitational bag finite in spite of the singularity (at $r=1/\alpha_i$) of the scalar field. This would be impossible with a scalar field alone in a flat four-dimensional space-time.[12] Note that this singularity corresponds to a naked singularity of the exact solution. Now, the positive integration constants $C_{1,2}$ can be easily determined by continuity arguments applied in the tangent space. The nontrivial normal matching involves the Gauss-Codazzi formalism[13]

$$\sigma h_{\mu\nu}=-\int_{-\Delta/t}^{\Delta/2}T^{\rm eff}_{\mu\nu}dn={\rm disc}(\Pi_{\mu\nu}-\Pi h_{\mu\nu})\ , \tag{14}$$

where $h_{\mu\nu}$ is the three-metric intrinsic to the domain-wall hypersurface $r=r_0$, and $\Pi_{\mu\nu}\equiv h_\mu{}^\lambda n_{\lambda;\nu}$ denotes the associated extrinsic curvature. n_μ is the normalized ($n_\mu n^\mu=1$) normal vector. Since $h_{\theta\theta}\sim h_{\phi\phi}$, Eq. (14) decomposes into two algebraic equations. The σ-independent combination is the restrictive one, while the other is just used to verify that the positive-energy condition ($\sigma>0$) is not violated.

Consistency is achieved only provided that Eq. (14) admits a solution for which $r_0>1/\alpha_{e,i}$; i.e., the wall is not shielded by a horizon or by a singularity. For this to be the case, our analysis shows that the various parameters must lie inside the "triangle" [in the $(p,\alpha_e/\alpha_i)$ plane]:

$$\begin{aligned}&2+\sqrt{3}>\frac{\alpha_e}{\alpha_i}>1,\\ &\tfrac{1}{2}<p<\tfrac{1}{4}(2-\sqrt{3})\frac{\alpha_e}{\alpha_i}+\tfrac{1}{4}(2+\sqrt{3})\frac{\alpha_i}{\alpha_e}\ ,\end{aligned} \tag{15}$$

with the special point $p=\alpha_e/\alpha_i=1$ resembling the Schwarzschild vertex. The solution itself is given by

$$r_0\alpha_i=\left\{\frac{\alpha_i}{\alpha_e}-\frac{\alpha_e}{\alpha_i}+\left[\left[8p-\frac{\alpha_i}{\alpha_e}-\frac{\alpha_e}{\alpha_i}\right]^2-48(p^2-\tfrac{1}{4})\right]^{1/2}\right\}\Bigg/4\left[1-\frac{\alpha_e}{\alpha_i}p\right]\ , \tag{16}$$

$$\sigma=\frac{2\alpha_e{}^2r_0{}^2}{(\alpha_e r_0+1)^2}\left[\frac{2\alpha_i(\alpha_i r_0-p)}{(\alpha_i{}^2r_0{}^2-1)}-\frac{2\alpha_e}{(\alpha_e r_0+1)}\right]>0\ . \tag{17}$$

For a given gravitational mass $M_e\equiv 2r_{\rm Sc}$, we find a remarkable upper bound for the size of the gravitational bag, namely,

$$r_0^{\max}=(2+\sqrt{3})r_{\rm Sc}\ . \tag{18}$$

The isotropic location $r=\frac{1}{2}(2+\sqrt{3})M_e$ corresponds to a circumferential radius of $3M_e$; hence, *the invariant area of the spherical domain wall is therefore at most* $\frac{9}{4}$ *the area of an equal-mass Schwarzschild black hole.* At any rate, it should be clearly stated that the gravitational bag does not have an event horizon at all. In fact, for $0<p<1$, $r=1/\alpha_i$ is a real singularity for which $ds_i^{\rm eff}$ vanishes completely.

In summary, we have introduced a novel physical object referred to as a gravitational bag. From the effective four-dimensional point of view, it consists of a localized scalar field confined by means of a static spherically symmetric domain wall. The existence of such an object can be regarded as a signature of local spontaneous conpactification, although in principle it may have life of its own. If current conjectures concerning the dilaton potential[14] are to be realized, gravitational bags would accompany superstring theories as well. Moreover, they may contribute or even dominate the missing mass, provided they were abundantly produced in the very early Universe. It may well be, however, that the actual role of gravitational bags is played in the microscopic world. In that case, the non-Abelian characteristics of confinement and asymptotic freedom may find an unexpected gravitational realization. Einstein's conjecture might once again prove correct.

Enlightening discussions with Jacob Bekenstein and David Owen are very much appreciated. After submitting this work we became aware of related work[15] (using however the vacuum Einstein equations, i.e., without the motive of spontaneous compactification) by M. Yoshimura.

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