Erratum

Erratum: On the relic, cosmic abundance of stable, weakly interacting massive particles [Phys. Rev. D 33, 1585 (1986)]

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In the Appendix of our paper we discussed the relic abundance of a particle (and antiparticle) species with a nonzero chemical potential ($\mu \neq 0$) associated with a conserved quantum number carried by the species. In our analysis of the second limiting case considered in the Appendix, the case in which the effect of $\mu \neq 0$ alters the relic abundance significantly from the $\mu = 0$ result, we have made an error in our analysis. The correct analysis follows below.

The Boltzmann equations which describe the evolution of the relic abundances of the particle and its antiparticle are

$$Y^{\pm\prime} = -\lambda x^{-n-2}(Y^+Y^- - Y_{\rm EQ}{}^2) ,$$

where all quantities are as defined in our paper. For the case at hand, freeze-out occurs when $\xi \equiv \mu/T \gg 1$ (i.e., for $x \gg x_Q$), so that

$$Y^+ \simeq Y^+_{\rm EQ} \simeq (Q/q) , \quad Y^-_{\rm EQ} \simeq Y_{\rm EQ}{}^2/(Q/q) .$$

By using $Y^+ \simeq (Q/q)$, the Boltzmann equation for Y^- becomes

$$Y^{-\prime} \simeq -\lambda x^{-n-2}(QY^-/q - Y_{\rm EQ}{}^2) ,$$

which is easily integrated to give

$$Y^-_\infty = a^2\lambda \int_0^\infty x^{-n+1} \exp\{-2x - \lambda Q x^{-n-1}/[q(n+1)]\} dx .$$

The integral can be evaluated by the method of steepest descent

$$Y^-_\infty \simeq \frac{a^2 q}{Q}\left[\frac{4\pi}{n+2}\right]^{1/2}\left[\frac{\lambda Q}{2q}\right]^{3.5/(n+2)} \exp\left[-\left[\frac{2n+4}{n+1}\right]\left[\frac{\lambda Q}{2q}\right]^{1/(n+2)}\right] .$$

[Using the corrected formula, the estimated relic abundance of antiprotons in the Universe is $Y^-_\infty \simeq 10^{18}\exp(-9\times 10^5)$.]

The relic abundance obtained by correctly integrating the Boltzmann equation for Y^- differs significantly from our estimate which was made by calculating freeze-out using the criterion $\Gamma/H \simeq 1$. From our corrected result it can be seen that the correct freeze-out temperature is about

$$x_f' \simeq (\lambda Q/q)^{1/(n+2)}$$

rather than $(\lambda Q/q)^{1/(n+1)}$. By examining the Boltzmann equation it is clear that freeze-out occurs when $Y^{-\prime}/Y^- \simeq 1$. From the definitions of $\Gamma_{\rm ann}, H, x$ it follows that this corresponds to the criterion $\Gamma_{\rm ann}/xH \simeq 1$. Using this criterion leads to an estimated freeze-out temperature of $x_f' \simeq (\lambda Q/q)^{1/(n+2)}$, which agrees with the result obtained above by integrating the Boltzmann equation. In fact, in general it is true that this criterion is a more appropriate and accurate one than the more familiar $\Gamma/H \simeq 1$. (This point has also been emphasized in Ref. 14.) The error made here by using the criterion $\Gamma/H \simeq 1$ is particularly severe as the final abundance depends exponentially upon the freeze-out temperature, whereas in the $\mu = 0$ case the dependences are only logarithmic, cf. Eq. (8).

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