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Relationship between longitudinally polarized vector bosons and their unphysical scalar partners

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We present a new and simple proof of the Lee-Quigg-Thacker theorem concerning the equivalence of longitudinally polarized W and Z bosons and their would-be-Goldstone partners. The proof rests on the Becchi-Rouet-Stora invariance of the gauge theory.

The Higgs sector comprises that part of the standard model of electroweak interactions which has neither been tested experimentally nor really understood theoretically. Therefore, it will be an important task to look for the actual existence of scalar particles with the properties of Higgs bosons. In case such states will be found with low masses (<500 GeV) the case of electroweak interactions may be essentially closed. But if the Higgs particle will not be found below 1 TeV then many new options will be opened. If one continues to stick to the orthodoxy of the standard model, one has to believe in the existence of a heavy and (consequently) very broad Higgs particle which probably will never be identified unambiguously. Furthermore, the Higgs-boson self-coupling constants (both H^4 and H^3) necessarily become large such that lowest-order calculations of amplitudes involving Higgs particles cease to be reliable. This fact would not be too disturbing at energies below 1 TeV (if one neglects higher-order contributions to nonscalar amplitudes), were it not for the fact that strong-interaction effects should also show up in the interactions of (longitudinal) vector bosons.[1,2] It is the "mixing" of (*a priori* massless) gauge bosons to the (nonphysical) Higgs particles which besides giving masses to the vector particles induces the strong interaction of the scalars into the vector sector.

Mathematically this effect can most clearly be formulated within the R_ξ gauge[3] in which case a one-to-one correspondence between vector particles and the corresponding unphysical scalars (which are "eaten up" by the vectors finally) is inferred from the beginning by the specific gauge-fixing functions:

$$F_a[W,\varphi]=\begin{cases}\partial^\mu W_\mu^\pm-\xi M_W\varphi^\pm,\\ \partial^\mu Z_\mu-\xi M_Z\chi,\\ \partial^\mu A_\mu\,.\end{cases} \tag{1}$$

Here the complex Higgs doublet has been parametrized in the form

$$\Phi=\begin{bmatrix}\Phi^+\\ \Phi^\square\end{bmatrix}=\begin{bmatrix}-i\varphi^+\\ \dfrac{1}{\sqrt{2}}(v+H+i\chi)\end{bmatrix}.$$

[H denotes the physical Higgs particle and $(1/\sqrt{2})v=\langle\Phi^\square\rangle$.]

The physical relationship between longitudinal vector particles and the corresponding scalar (would-be Goldstone) bosons can then be expressed by means of the so-called equivalence theorem, which roughly says that matrix elements describing high-energy processes involving longitudinal vector bosons can be calculated (within the R_ξ gauge) by replacing the vector particles by their respective scalar partners.[3] This theorem which in essence has been known already by Cornwall, Levin, and Tiktopoulos[4] and Vayonakis[5], has been first formulated in a nonperturbative way, together with a sketch of a proof, by Lee, Quigg, and Thacker[2] (LQT). A complete but very complicated proof has been presented recently by Chanowitz and Gaillard.[6] Since the theorem will play an important role in understanding the strong-interaction regime of (longitudinaly polarized) W and Z bosons, particularly if the Higgs particle turns out to be very heavy, it may be worthwhile to present here another proof of this theorem which is much simpler than the one of Ref. 6.

The basic ingredient of the LQT theorem is contained in the identity

$$\langle A,\text{out}\,|\,T[F_{a_1}(x_1)\cdots F_{a_n}(x_n)]\,|\,B,\text{in}\rangle_{\text{con}}=0\,, \tag{2}$$

where $F_a(x)$ is given by (1), and $|B,\text{in}\rangle$, $|A,\text{out}\rangle$ are physical states. By physical state we mean here a state which (asymptotically) contains only physical fermions and Higgs particles, as well as gauge bosons with physical polarizations; i.e., no Faddeev-Popov (FP) ghosts are contained in $|B,\text{in}\rangle$ or $|A,\text{out}\rangle$.

Before proving (2) we would like to make clear why it directly leads to the equivalence theorem. We replace F_a in (2) by (1) and transfer the resulting matrix element into S-matrix elements. Thereby use can be made of the fact that $\partial_\mu W_a^\mu$ satisfies the Klein-Gordon equation

$$(\square+\xi M_a{}^2)\partial_\mu W_a^\mu=-\xi\partial_\mu J_a^\mu \tag{3}$$

which includes the same mass as the unphysical Goldstone boson φ_a. J_a^μ denotes the total current to which W_a

is coupled (both matter and self-coupling). In this way one ends up with a relation which connects S-matrix elements involving longitudinal W's and "S-matrix elements" where the W's are replaced by their corresponding would-be Goldstone bosons carrying the same momenta (therefore eventually being off-shell). It is exactly the relation (2.1) of Ref. 6 from which those authors could easily derive the equivalence theorem.

We are not going to write the general relation down nor repeat the further derivative steps, but only sketch here the case $n=1$, mainly for demonstrative reasons. In this case we get, from (2) immediately,

$$-i\frac{p^\mu}{M_a}\hat{S}[B\to A+W_a(p,\lambda)]=S[B\to A+\varphi_a(p)] . \tag{4}$$

Here, $\epsilon^\mu(p,\lambda)\hat{S}[B\to A+W_a(p,\lambda)]$ denotes the S-matrix element for the indicated process $B\to A+W_a(p,\lambda)$, whereas $S[B\to A+\varphi_a(p)]$ represents the "S-matrix element" for the connected reaction where the vector boson is replaced by the corresponding unphysical scalar boson with the same momenta. For a longitudinal vector particle the polarization vector has the form

$$\epsilon^\mu(p,\lambda=L)=\frac{p^\mu}{M}+v^\mu(p) ,$$

where the components of $v^\mu(p)$ are of order M/E. Therefore, at sufficiently high energies we get from (4)

$$(-i)S[B\to A+W_a(p,\lambda=L)] = S[B\to A+\varphi_a(p)]+O\left[\frac{M}{E}\right] . \tag{5}$$

Equation (5) exhibits the equivalence theorem for the case of one longitudinal vector particle.

Having indicated how (2) leads to the LQT theorem we return to the proof of equation (2). It is based on the symmetry of any quantized gauge theory under Becchi-Rouet-Stora (BRS) transformations.[7,8] In particular we need the following properties of the generator S_{BRS} of the BRS transformations:[7]

$$S_{\mathrm{BRS}}(\,|A,{}^{\mathrm{in}}_{\mathrm{out}}\rangle)=0 . \tag{6}$$

The validity of (6) is based on the fact that if the field Φ represents a physical particle (with physical polarization) then the propagator

$$\langle 0\,|\,T[S_{\mathrm{BRS}}(\Phi)\bar{S}_{\mathrm{BRS}}(\Phi)]\,|\,0\rangle \tag{7}$$

does not have a pole at the mass of any such physical particle. [The anti-BRS operator $\bar{S}_{\mathrm{BRS}}$ is described in Ref. 7. When applied to physical fields it has the same effect as S_{BRS} with the ghost field c_a replaced by the corresponding antighost $\bar{c}_a$. Consequently, S_{BRS} ($\bar{S}_{\mathrm{BRS}}$) changes the ghost number by $+1$ (-1).] In fact, Eq. (6) is used as a definition of a physical state $|A,{}^{\mathrm{in}}_{\mathrm{out}}\rangle$, in the canonical quantization of non-Abelian gauge theories.[9,10] It represents a natural generalization of the usual (Lorentz gauge) Abelian condition:

$$\partial^\mu A_\mu^{(+)}\,|\,A\rangle=0 .$$

Notice that the matrix element of any operator (string) with nonzero ghost number between physical states vanishes identically.

The BRS symmetry of a non-Abelian gauge theory, together with (6), implies that

$$0=\sum_{k=1}^{n}\langle A,\mathrm{out}\,|\,T(O_1(x_n)\cdots O_{k-1}(x_{k-1})\{S_{\mathrm{BRS}}[O_k(x_k)]\}\cdots O_n(x_n))\,|\,B,\mathrm{in}\rangle \tag{8}$$

for any string of local operators $O_i(x_i)$, where $|A,\mathrm{out}\rangle$, $|B,\mathrm{in}\rangle$ are physical states as defined above. This relation will be the starting point for the proof of (2).

We will utilize (8) for cases where the operators $O_i(x)$ either denote the antighost fields $\bar{c}_a(x)$ or the gauge-fixing field combinations $F_a(x)$, their respective BRS transformations being given by

$$S_{\mathrm{BRS}}[\bar{c}_a(x)]=-\frac{1}{\xi}F_a(x) \tag{9}$$

and

$$S_{\mathrm{BRS}}[F_a(x)]=-\frac{\partial L}{\partial\bar{c}_a(x)} =-\left[\frac{\partial L}{\partial\bar{c}_a}-\partial_\mu\frac{\partial L}{\partial(\partial_\mu\bar{c}_a)}\right]=-L_{\bar{c}_a} . \tag{10}$$

The last relation expresses the well-known fact that the BRS transform of any gauge-fixing function is proportional to the equation of motion of the associated antighost.[7,8]

Finally, we also need the equation of motion for a Green's function[11] written in the form (notice the convention

$$L_\Phi\equiv\frac{\partial L}{\partial\Phi}-\partial_\mu[\partial L/\partial(\partial_\mu\Phi)]) ,$$

$$\langle A,\mathrm{out}\,|\,T[L_\Phi(x)O]\,|\,B,\mathrm{in}\rangle =i\left\langle A,\mathrm{out}\left|T\left[\frac{\partial O}{\partial\Phi(x)}\right]\right|B,\mathrm{in}\right\rangle , \tag{11}$$

which is valid provided that the asymptotic states $|A,\mathrm{out}\rangle$, $|B,\mathrm{in}\rangle$ do not contain the particle carried by the field Φ.

Equipped with this information we present now a simple proof of (2) using induction.

(i) $n=1$. We choose $O_1(x_1)=\bar{c}_a(x_1)$. Then (8) and (9) imply

$$\langle A,\mathrm{out}\,|\,F_a(x_1)\,|\,B,\mathrm{in}\rangle=0 . \tag{12}$$

This together with (1) gives

$$\langle A,\mathrm{out}\,|\,F_a(x_1)\,|\,B,\mathrm{in}\rangle_{\mathrm{con}}=0\ . \tag{13}$$

(ii) $n=2$. Now we choose

$$O_1(x_1)=F_{a_1}(x_1),\quad O_2(x_2)=\bar{c}_{a_2}(x_2)\ .$$

If inserted in (8) we get, using (9) and (10),

$$-\frac{1}{\xi}\langle A\,|\,T[F_{a_1}(x_1)F_{a_2}(x_2)]\,|\,B\rangle - \langle A\,|\,T[L_{\bar{c}_{a_1}}(x_1)\bar{c}_{a_2}(x_2)]\,|\,B\rangle=0\ . \tag{14}$$

Since no antighost field appears in the physical states $|\,A\rangle$, $|\,B\rangle$ we can apply (11) to the second term

$$\langle A\,|\,T[L_{\bar{c}_{a_1}}(x_1)\bar{c}_{a_2}(x_2)]\,|\,B\rangle=+i\delta(x_1-x_2)\delta_{a_1,a_2}\langle A\,|\,B\rangle\ ,$$

which leads to

$$\frac{1}{\xi}\langle A\,|\,T[F_{a_1}(x_1)F_{a_2}(x_2)]\,|\,B\rangle + i\delta(x_1-x_2)\delta_{a_1,a_2}\langle A\,|\,B\rangle=0\ . \tag{15}$$

In turn, (15) immediately implies

$$\langle 0\,|\,T[F_{a_1}(x_1)F_{a_2}(x_2)]\,|\,0\rangle=-i\xi\delta(x_1-x_2)\delta_{a_1,a_2} \tag{16a}$$

and

$$\langle A,\mathrm{out}\,|\,T[F_a(x_1)F_a(x_2)]\,|\,B,\mathrm{in}\rangle_{\mathrm{con}}=0\ . \tag{16b}$$

(iii) The induction proof is finally completed by assuming (2) for $n\le N-1$ ($N\ge 2$), and proving it for $n=N$. Indeed using

$$\left.\begin{aligned}&O_i(x_i)=F_{a_i}(x_i),\quad i=1,2,\ldots,N-1\ ,\\ &O_N(x_N)=\bar{c}_{a_1}(x_N)\end{aligned}\right. \tag{17}$$

together with (8)—(11) we get

$$i\langle A\,|\,T[F_{a_2}(x_2)\cdots F_{a_{N-1}}(x_{N-1})]\,|\,B\rangle\delta_{a_1a_N}\delta(x_1-x_N)+\cdots + i\langle A\,|\,T[F_{a_1}(x_1)\cdots F_{a_{N-2}}(x_{N-2})]\,|\,B\rangle\delta_{a_{N-1}a_N}\delta(x_{N-1}-x_N)+\frac{1}{\xi}\langle A\,|\,T[F_{a_1}(x_1)\cdots F_{a_N}(x_N)]\,|\,B\rangle=0\ . \tag{18}$$

Since (2) is already assumed to be valid for $n\le N-1$, (18) implies

$$\langle A,\mathrm{out}\,|\,T[F_{a_1}(x_1)\cdots F_{a_N}(x_N)]\,|\,B,\mathrm{in}\rangle_{\mathrm{con}}=0\ ,$$

thereby completing the proof of (2).

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[11]See, e.g., J. C. Collins, *Renormalization* (Cambridge University Press, Cambridge, England, 1984), Chap. 2.13.