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Brief Reports

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Test of Newton's second law at small accelerations

Alex Abramovici and Zeev Vager
Department of Nuclear Physics, Weizmann Institute of Science, Rehovot 76100, Israel

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We present the results of a test of Newton's second law for small accelerations. It is found that within experimental errors $F=ma$ holds down to the lowest accelerations measurable with our apparatus, $\sim 3\times 10^{-9}$ cm/s^2.

It has been suggested recently that the flat velocity curves of disk galaxies may be understood without the need to assume the presence of hidden (dark) matter, provided Newton's second law is replaced by $F=m_g\mu(a/a_0)a$ (Refs. 1 and 2). In the latter equation, m_g is the gravitational mass of a system which is accelerated by a force F with an acceleration a, while a_0 is an acceleration constant which takes values in the range $(2-6)\times 10^{-8}$ cm/s^2. It is assumed that $\mu(a/a_0)\sim 1$ for $a>a_0$ and $\mu(a/a_0)\sim a/a_0$ for $a<a_0$ (Ref. 1).

This modified dynamics seems to be consistent with many observed features of galaxies and galaxy clusters.[2,3] Observational aspects of open clusters in the solar neighborhood indicate that Newtonian behavior is reestablished once the system moves in an external acceleration field larger than a_0, even if the system's internal movements proceed at accelerations smaller than a_0 (Ref. 1), which amounts to a violation of the strong equivalence principle (SEP) (for a definition of the SEP, see, e.g., Ref. 4). As a consequence, no deviation from Newton's second law would be detectable in earthbound laboratory experiments, since the apparatus is immersed in the strong gravitational field of Earth. If, on the other hand, the partial restoration of Newtonian behavior in the neighboring open clusters is due to some, as yet, unsuspected mechanism, peculiar to these clusters, while the strong equivalence principle is preserved, laboratory experiments involving accelerations smaller than a_0 should display deviations from $F=ma$.

It is the aim of this Brief Report to describe an experiment in which Newton's second law has been tested for accelerations as low as 3×10^{-9} cm/s^2. No deviation has been detected. The measurements have been carried out with the prototype active cavity gravitational-radiation detector in our laboratory.[5]

The displacement sensor of our gravitational-radiation detector consists of two similar $\lambda=632.8$-nm He-Ne lasers, operated in a heterodyne scheme, as shown in Fig. 1. The laser beams combine at the beam splitter and provide, after photodetection, a beat signal of frequency $\omega_B=\omega_2-\omega_1$, where ω_1,ω_2 are the frequencies of the two laser beams. When the mirror spacings in the two lasers change by ΔL_2 and ΔL_1, respectively, the beat frequency changes by an amount $\Delta\omega_B=\omega(\Delta L_2-\Delta L_1)/L$ where ω is the mean value of ω_1,ω_2, while L is the mean value of L_1,L_2, the distances between the mirrors in the two lasers. Changes in ω_B are monitored by frequency demodulation. The laser mirrors are rigidly mounted on cylindrical stainless-steel test masses which are loosely suspended from a very rigid structure built around a central aluminum block. The central block also houses the laser tube. The two lasers and the beam splitter are assembled on a single $65\times 65\times 5$-cm^3 aluminum board, which is further suspended from four steel wires attached to rubber-steel stacks supported by a steel structure on top of an optical table. The suspension, the stacks, and the

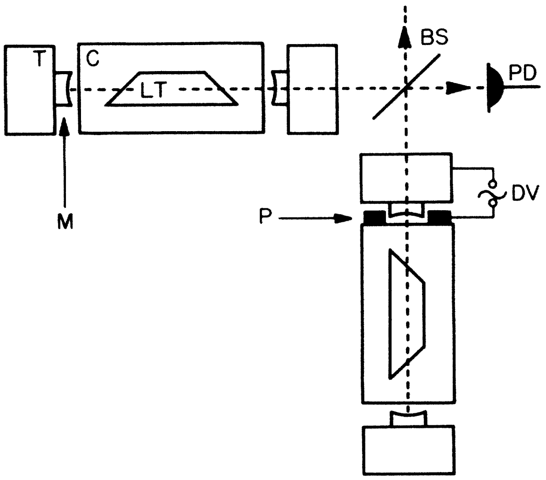

FIG. 1. Layout of the experimental setup. T, test mass; M, mirror; C, central block; P, brass plate; DV, driving voltage; LT, laser tube; BS, beam splitter; PD, photodiode.

optical table provide very efficient isolation from floor vibrations. Acoustic isolation is obtained by confining the detector and the suspension within an evacuated container.

One of the test masses was driven by a small electric force obtained by applying an alternating voltage between the test mass, on one hand, and two flat brass plates attached to the central block in positions facing the mass, on the other (see Fig. 1). The mass and the brass plates formed a plane capacitor. The small oscillatory accelerations which were induced by the driving force were measured by feeding the frequency demodulated output of the active cavity interferometer into a lock-in amplifier. This arrangement has been used in the past for measuring the response function of the detector.[5]

When an ac voltage of amplitude V and circular frequency Ω is applied to a plane capacitor, the force acting between the capacitor plates consists of a dc term and of an ac term with frequency 2Ω and amplitude proportional to V^2. We locked the lock-in amplifier onto the 2Ω component of the frequency demodulated interferometer signal and checked whether the output, which is proportional to the 2Ω component of the test mass acceleration, is still linear in V^2 for $a < a_0$. Note that a dc term superimposed on the driving voltage, e.g., due to contact voltages, will add a dc term and a term with frequency Ω to the interferometer output, but will make no contribution at 2Ω. Moreover, direct pick-up of the driving voltage by the output circuits will not contribute a 2Ω component.

The accelerations measured for various values of the driving voltage are shown in Fig. 2, with the mass driven at a frequency of 430 Hz. The errors are due mainly to laser frequency fluctuations caused by spontaneous emission.[5] The standard deviation for the lowest point is 1.2×10^{-9} cm/s^2. Since less statistics were accumulated for higher driving voltages, the error bars shown in Fig. 2 are the greater, the larger the driving voltage. As the number of samples N_i corresponding to a given driving voltage V_i increased, we noted that the standard deviation of the average acceleration a_i decreased in a way consistent with $1\sqrt{N_i}$ behavior. This indicates that the parameters of the system were stable throughout the experiment. As a check of the data presented in Fig. 2, a smaller data set (not shown) was collected for a driving frequency of 900 Hz. The same linear dependence on V^2 was found and, moreover, the accelerations measured at 900 Hz coincided within experimental errors with those measured at 430 Hz for the same driving level. Also, we calculated the force acting between the capacitor plates for the various driving levels and the corresponding expected accelerations, according to $F=ma$. The calculated and measured accelerations agreed up to an uncertainty of a few % due to the limited accuracy with which the distance between the capacitor plates ($\sim$0.4 mm) could be measured.

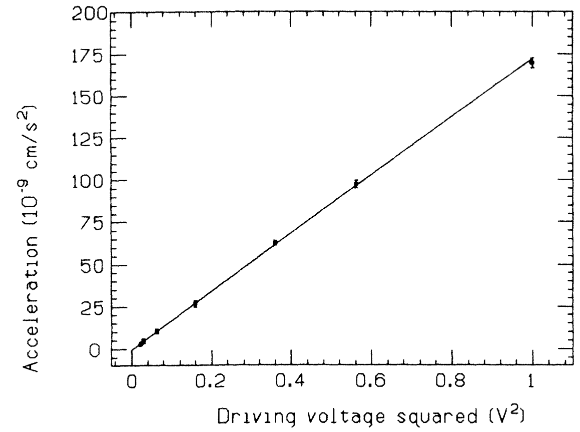

FIG. 2. Test-mass acceleration vs driving voltage squared. The solid line is a linear least-squares fit to the data.

An unconstrained least-squares fit of the data to the function $a=\alpha+\beta V^2$ yielded $\alpha=(-0.52\pm0.71)\times10^{-9}$ cm/s^2 and $\beta=(172.6\pm2.5)\times10^{-9}$ cm/(s^{2}V^2), at a 90.1% *goodness-of-fit* level. This provides strong evidence for purely Newtonian behavior in our experiment, around $(2-6)\times10^{-8}$ cm/s^2 as well as for smaller accelerations.

In conclusion, we found that $F=ma$ is valid around and below a_0 in a laboratory environment where a loosely suspended test mass was driven by a small electric force. Since the modified dynamics has been inspired by and applied to the behavior of large astrophysical objects where gravity is the only relevant force, it seems of interest to perform an experiment similar to the one we report here, but which will employ a gravitational driving force.

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