On the stability of the vacuum in supersymmetric QCD

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Instanton—anti-instanton contribution to the vacuum energy is used to analyze the stability of the vacuum in massless supersymmetric QCD. It is pointed out that if this contribution is not canceled out by other nonperturbative effects the vacuum energy has a stable minimum, which fixes the vacuum expectation value of the scalar fields. This vacuum energy, though, signals an explicit breaking of supersymmetry in the Higgs phase of the model.

Supersymmetry breaking has been studied extensively in the past few years.[1–4] Whereas perturbative effects cannot induce such a breaking due to the nonrenormalization theorem, it may happen that nonperturbative effects can induce it. To that end the role of instantons in supersymmetric theories has been studied. However, because of the fermionic zero modes of the Dirac operator in a topologically nontrivial background, instantons cannot induce supersymmetry breaking. Indeed, correlation functions which were calculated in a background of instantons do not show signs of supersymmetry breaking.[3,4]

An attempt to study the nonperturbative contribution of topologically trivial configurations, such as an instanton—anti-instanton, in supersymmetric QCD (SQCD) was made.[2] The contribution of an instanton to the F term in a superpotential, or equivalently the contribution of an instanton—anti-instanton to the potential, was claimed to have been found. This superpotential, though, turns out to be zero in the massive theory because an infinite action configuration was used to induce the tunneling. (The divergence is due to the infinite contribution of the mass term of the classically nontrivial scalar fields.) A finite action configuration, defined by zero classical scalar fields in a background of instantons, does not help either as it does not generate a superpotential. This was pointed out recently.[5] In the massless theory, on the other hand, the superpotential of Ref. 2 does not reproduce correctly the anomalous commutation relation for composite operators,[6] as was pointed out in Ref. 7(a). Thus, one cannot rely on the claims[2] that supersymmetry is not broken in SQCD with a vacuum at infinite expectation value of the scalar fields in the massless theory (or finite expectation value in the massive one).[7(b)]

A direct analysis of the instanton—anti-instanton contribution to the path integral in supersymmetric Yang-Mills (SYM) theory and in SQCD was then carried out.[8–10] It turns out that a negative vacuum energy is induced, which signals an explicit breaking of supersymmetry if it is not canceled out by other nonperturbative effects. However, it is only in massless SQCD that there is an infrared-finite contribution induced by instanton—anti-instanton and nonzero classical scalar fields. In SYM theory and in massive SQCD the scalar fields are necessarily zero in a background of an instanton—anti-instanton, thus making the contribution to the vacuum energy infrared divergent. (Such an infrared-divergent contribution can be found also in the massless theory.) As a result, only when the infrared divergence is properly understood can the pure instanton—anti-instanton contribution (with zero scalar fields) to the vacuum energy be unambiguously determined.

We concentrate in this paper on the infrared-finite nonperturbative contribution to the vacuum energy in massless SQCD. In particular we point out that if this contribution is not canceled out by other nonperturbative effects, a stable minimum in the vacuum energy can be found, which fixes the vacuum expectation value of the scalar fields. The theory can then be defined around this vacuum even though not supersymmetrically. Furthermore, the gauge symmetry-breaking scale in the Higgs phase of this model is tied to the fact that supersymmetry is explicitly broken by this contribution. The other infrared-divergent contribution may not be relevant to the analysis of supersymmetry breaking nor to the Higgs phase of the model. (The scalar fields have zero vacuum expectation value.)

We start by defining the model. It is an SU(2) gauge theory with one matter and one antimatter supermultiplet transforming under the fundamental representation of the gauge group. The Lagrangian is defined by

$$\mathscr{L}=\mathscr{L}_{\rm SYM}+\mathscr{L}_{\rm matter}\,, \tag{1}$$

where $\mathscr{L}_{\rm SYM}$ is the super-Yang-Mills Lagrangian given in the Wess-Zumino gauge by

$$\mathscr{L}_{\rm SYM}=-\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}+\bar\lambda^a iD_\mu\bar\Sigma^\mu\lambda^a \tag{1a}$$

with

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g\epsilon^{abc}A^b_\mu A^c_\nu\,,$$
$$D^{ac}_\mu=\delta^{ac}+g\epsilon^{abc}A^b_\mu\,;$$

A^a_μ are the vector potentials and λ^a are Weyl spinors. They are expressed in the Weyl basis with Dirac matrices being

$$\gamma_\mu=\begin{bmatrix}0 & \Sigma_\mu\\ \bar\Sigma_\mu & 0\end{bmatrix},\quad \Sigma_\mu=\bar\Sigma^\mu=(\mathbb{1},\sigma_i)$$

and $\mathrm{Tr}(\Sigma_\mu\bar\Sigma_\nu)=2g_{\mu\nu}$ with the Minkowskian metric

$g_{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$. $\mathscr{L}_{\text{matter}}$ is the matter-field Lagrangian

$$\begin{aligned}\mathscr{L}_{\text{matter}}=&(\mathscr{D}_\mu\phi_1)^\dagger\mathscr{D}^\mu\phi_1+(\mathscr{D}_\mu^*\phi_2)^\dagger\mathscr{D}^{\mu*}\phi_2\\&+\bar\psi_1^Ti\mathscr{D}_\mu\Sigma^\mu\psi_1+\bar\psi_2^Ti\mathscr{D}_\mu^*\bar\Sigma^\mu\psi_2\\&+\frac{ig}{\sqrt{2}}(\phi_1^\dagger\tau^a\lambda^a\psi_1+\phi_2^T\tau^a\bar\lambda^a\bar\psi_2)\\&-\frac{g^2}{32}(\phi_1^\dagger\tau^a\phi_1-\phi_2^T\tau^a\phi_2^*)^2+\text{H.c.}\end{aligned}\tag{1b}$$

Here $\{\psi_i,\phi_i\}$ $(i=1,2)$ form the matter supermultiplets, $\mathscr{D}_\mu=\partial_\mu\mathbb{1}+igA_\mu^a\tau^a/2$, and $\tau^a/2$ are the SU(2) generators in the fundamental representation.

The vacuum of this model is given by[5]

$$|\theta\rangle=\sum_{n=1}^{\infty}e^{in\theta}|n\rangle\,,$$

where n is the Pontryagin index. Tunneling between vacua differing by one unit of topological charge is mediated by instantons (anti-instantons), which are finite action configurations in Euclidean space-time:

$$\begin{aligned}A_\mu^{Ia}&=\frac{2}{g}\frac{\eta_{a\mu\nu}(x-x_1)_\nu}{(x-x_1)^2+\rho_1{}^2}\,,\\A_\mu^{\bar{I}a}&=\frac{2}{g}\frac{\bar\eta_{a\mu\nu}(x-x_2)_\nu}{(x-x_2)^2+\rho_2{}^2}\,.\end{aligned}\tag{2}$$

Here $\eta_{a\mu\nu},\bar\eta_{a\mu\nu}$ are 't Hooft symbols; x_1,x_2 and ρ_1,ρ_2 are the location and size of the instanton and the anti-instanton, respectively.

As to the scalar fields there are two possible configurations which can induce the tunneling, depending on the topology of the vacuum in (scalar) field space.

(i) The vacuum is topologically trivial; then the tunneling is induced by instantons (anti-instantons) and

$$\phi_1=\phi_2=0\,.\tag{3a}$$

(ii) The vacuum is topologically nontrivial; then the scalar fields are characterized by the same index as the gauge potentials (Pontryagin index),[5] and tunneling is induced by instantons (anti-instantons) accompanied by

$$\begin{aligned}\phi_{1I}=\phi_{2I}^*&=\frac{(x-x_1)_\mu\bar\tau_\mu}{[(x-x_1)^2+\rho_1{}^2]^{1/2}}v\,,\\\phi_{1\bar I}=\phi_{2\bar I}^*&=\frac{(x-x_2)_\mu\tau_\mu}{[(x-x_2)^2+\rho_2{}^2]^{1/2}}v\,,\end{aligned}\tag{3b}$$

where $\tau_\mu=\bar\tau_\mu^\dagger=(\mathbb{1},i\tau_i)$ and v is a constant two-vector.

The action in both cases is finite:

$$S_E=\frac{8\pi^2}{g^2(\rho)}+4\pi^2v^2\rho^2\,.\tag{4}$$

The second term in Eq. (4) is the contribution of the topologically nontrivial scalar fields in case (ii) whereas it is zero in case (i).

This analysis is valid for the massless theory only. If a mass term is added to the Lagrangian, the topologically nontrivial configuration has an infinite action due to the divergence of the mass term. Therefore, only the topologically trivial configuration of case (i) can induce the tunneling[5] (in a background of an instanton—anti-instanton).

As was mentioned above instantons do not contribute to the path integral when there are massless fermions due to the fermionic zero modes of the Dirac operator in a topologically nontrivial background. Therefore, in the supersymmetric model (which contains massless fermions) vacuum energy stays at zero and supersymmetry is not broken by instantons (or any other topologically nontrivial configurations). For nonperturbative but topologically trivial configurations, such as the instanton—anti-instanton, there are no fermionic zero modes,[11] and a nonzero contribution to the path integral is possible. Indeed, it was shown in previous publications[8–10] that these contributions are not zero and the induced vacuum energy is negative. However, it is infrared divergent in case (i) above and in SYM theory because the integrations over ρ_1,ρ_2 do not converge, thereby introducing an infrared cutoff ρ_c. In case (ii) on the other hand, the contribution of the scalar fields to the classical action introduces a Gaussian factor which makes the integration over ρ_1,ρ_2 finite. To avoid the problems resulting from the infrared-divergent contribution of case (i) we concentrate here on case (ii) which is valid for the massless theory only.

The density vacuum of the energy as given in Ref. 10 is bounded by

$$E(\theta)\le-\frac{3}{4480\pi^8}\frac{1}{(\kappa+1)^3}\left[\frac{8\pi^2}{g^2(v)}\right]^6\left[\frac{\Lambda}{v}\right]^6\Lambda^4\,,\tag{5}$$

which is the contribution of a widely separated instanton—anti-instanton configuration, represented by an instanton in half of the space and an anti-instanton in the other half [Eq. (7) in Ref. 10]. Here Λ is the renormalization-group-invariant scale:

$$\Lambda^5=\mu^5\exp\left[-\frac{8\pi^2}{g^2(\mu)}\right],\tag{6}$$

with μ being the renormalization point and κ is a measure of the minimal distance, Δ_0, between the instanton and the anti-instanton where the approximation is valid; $\Delta_0{}^2=\kappa(\rho_1{}^2+\rho_2{}^2)$. To get this bound the integrations were done to the leading order in g using the largest value of the interaction action between the instanton and the anti-instanton (within the range of validity of the approximation). Thus κ has effectively the value $(\kappa+1)^2=8\pi^2/g^2(v)$.

However, to get a better estimate of the contribution to the vacuum energy it is important to notice that the average of the interaction action in group space is zero, so one can take κ to be the point where the instanton—anti-instanton interaction action introduces a hard-core repulsive potential. This was estimated in Ref. 12 to be $\kappa=3.37$. With this we can replace the inequality in Eq. (5) by an approximate equality, and we find that the contribution of far separated instanton—anti-instanton to the vacuum energy has a stable minimum given by

$$v_{\min}=e\Lambda\,,\tag{7}$$

where the renormalization-group equation

$$\frac{8\pi^2}{g^2(v)}=\frac{8\pi^2}{g^2(\mu)}-5\ln\frac{\mu}{v}\ , \tag{8}$$

is used to get $v_{\min}$. The minimal contribution to the density of the vacuum energy is then

$$E(\theta)\simeq-\frac{3}{4480\pi^8}\frac{1}{(\kappa+1)^3}\left[\frac{5}{e}\right]^6\Lambda^4\ . \tag{9}$$

This contribution, though, makes sense only if it is within the range of the validity of the approximation. Here we have two kinds of higher-order corrections: (i) perturbative corrections resulting from the Feynman diagrams in a background of an instanton—anti-instanton; (ii) higher-order corrections in Δ^{-1}, where Δ is the distance between the instanton and the anti-instanton. These were proven to be equivalent to higher-order corrections in the coupling constant when the interaction between the instanton and the anti-instanton is properly included.[9]

For both corrections the effective expansion parameter is $g^2/16\pi^2$, so the approximation is valid provided $g^2/16\pi^2<1$. Indeed, the minimum of the induced vacuum energy is within this range:

$$\frac{g^2(v_{\min})}{16\pi^2}=\tfrac{1}{10}<1\ .$$

We can thus conclude that if the contribution of an instanton—anti-instanton to the vacuum energy is not canceled out by other nonperturbative effects, the induced vacuum energy has a stable minimum (within the range of validity of the approximation), which fixes the scale of gauge symmetry breaking. This scale turns out to be $v_{\min}\simeq e\Lambda$ for SQCD with gauge group SU(2).

If this picture turns out to be valid, the minimum can be used to define the theory even though not supersymmetrically. This minimum links the Higgs phase of the model to the breaking of supersymmetry whose scale, $\Delta m\simeq c\Lambda$ ($c\simeq10^{-2}$), is much less than the scale of masses generated by breaking the gauge symmetry, $v_{\min}\simeq e\Lambda\gg\Delta m$. Furthermore a gauge hierarchy protected by supersymmetry is expected not to be violated by such a supersymmetry-breaking mechanism. This makes supersymmetry an approximate symmetry, and it opens the way for the study of phenomenological models based upon global supersymmetry.

We would like to comment now about the validity of such a picture. Other known nonperturbative effects which could contribute to the vacuum energy are the torons. However, a detailed analysis of such contributions in SYM theory shows[13] that, even though there is a nonzero contribution to the path integral, it does not shift the vacuum energy, simply because the torons contribution to the path integral is not proportional to space-time volume (unlike the contribution of instantons). Therefore, they cannot induce supersymmetry breaking. Whether such contributions exist in a theory such as SQCD is not known, but even if they do exist they are unable to contribute to the vacuum energy, and they certainly cannot cancel the contribution of the instanton—anti-instanton. Thus, unless some other nonperturbative effects in Yang-Mills theory are found, the instanton—anti-instanton contribution to the vacuum energy in SQCD cannot be canceled out.

Finally, one could use the same analysis for the infrared-divergent contribution found in SYM theory, massive SQCD, and in case (i) above, when a cutoff ρ_c is introduced. The minimum of the induced vacuum energy will fix the value of the cutoff. However, since there is an arbitrariness in the way such a cutoff is introduced, the result is not very meaningful. Therefore, we will not elaborate on this point anymore.

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