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Correlation and screening in finite-temperature SU(2) gauge theory

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We study the temperature dependence of the correlation length in SU(2) gauge theory around the deconfinement point, using high-statistics Monte Carlo simulation on large lattices.

Statistical QCD predicts a deconfinement transition, in which hadronic matter is transformed into a quark-gluon plasma.[1,2] It has been suggested[3] that the screening of the color charge is the physical mechanism responsible for this transition: the presence of many other color charges in dense matter shields the long-range part of the potential felt by any given charge and thereby dissolves its binding.

The aim of this paper is to study the temperature dependence of the screening phenomenon in SU(2) gauge theory. The general thermodynamics of this system has been studied extensively for some years.[2] The question of screening, however, requires high-statistics simulations on large lattices; except for some rather qualitative considerations,[4] it has therefore been addressed only quite recently.[5-7] In particular we want to see how rapidly the range of the potential of a static quark decreases beyond the deconfinement point, and whether its behavior in the transition region is in accord with the form predicted by universality considerations.[8]

Our starting point is the connected correlation function for two Polyakov loops separated by a spatial distance $|\mathbf{x}|=r$,

$$\Gamma(r,T)=\langle L(0)L(r)\rangle-\langle L\rangle^2\ , \tag{1}$$

where

$$L(\mathbf{x})=\tfrac{1}{2}\,\mathrm{tr}\prod_{x_0=1}^{N_\tau} U_{\mathbf{x},x_0}\ . \tag{2}$$

We work on a lattice of size $N_\sigma^{\,3}\times N_\tau$, with isotropic spacing a; hence $T=1/(N_\tau a)$. To obtain $\Gamma(r,T)$, we calculate

$$\langle L(\mathbf{x})L(\mathbf{y})\rangle = \left[\int\prod_{\text{links}} dU\,e^{-S(U)}L(\mathbf{x})L(\mathbf{y})\right]\Big/\left[\int\prod_{\text{links}} dU\,e^{-S(U)}\right] \tag{3}$$

with the Wilson action

$$S(U)=\frac{4}{g^2}\sum_{\text{plaquettes}}(1-\tfrac{1}{2}\,\mathrm{Tr}UUUU)\ , \tag{4}$$

and then average over all results for $|\mathbf{x}-\mathbf{y}|=r$; for simplicity, we consider only separations parallel to the coordinate axes. The average Polyakov loop $\langle L\rangle$ is analogously defined. To avoid cancellations due to finite lattice size (spin flips), however, we first calculate the average $\bar{L}$ over a given lattice configuration, then average $|\bar{L}|$ over successive configurations. Hence $\langle L\rangle$ will here always mean $\langle|\bar{L}|\rangle$; we shall return to this point later on.

A Polyakov loop in a pure gauge theory corresponds to a static quark (fundamental representation of the gauge group) in an environment containing only gluons (adjoint representation). We can therefore not have a direct screening of the force between the two static quarks in $\langle L(\mathbf{x})L(\mathbf{y})\rangle$. At low temperatures, the gluon-gluon interaction contracts the force field between $\mathbf{x}$ and $\mathbf{y}$ into an essentially one-dimensional confining flux tube. With increasing T, the color screening of the gluon-gluon interaction, due to the increased gluon density, weakens this contraction; eventually, at $T=T_c$, it leads to a nonconfining three-dimensional force field. Above T_c, it is thus the Coulomb-type part of the static quark potential which experiences the screening effect of the gauge field environment. We expect this screening—which corresponds to a three-dimensional field of effectively massive gluons—to be very similar to that between the gluons themselves or to that between dynamical quarks.

The correlation function $\Gamma(r,T)$ is expected to decrease exponentially with (large) r both above and below the transition point:

$$\Gamma(r,T)\sim e^{-r/\xi(T)}\ , \tag{5}$$

where $\xi(T)$ is the correlation length. Let us see how it is related to the interaction parameters of the static quark system.

Below T_c, in the confinement regime, $\langle L\rangle$ vanishes, apart from finite lattice size effects; hence here

$$\Gamma(r,T)=\langle L(0)L(r)\rangle\sim e^{-V(r,T)/T}\ , \tag{6}$$

where $V(r,T)$ denotes the potential between the two static quarks. At $T=0$, the predicted form for large r is[9]

$$V(r,0)=\sigma r-\frac{c}{r}\ , \tag{7}$$

with σ for the string tension. For the $1/r$ dependence, string theory gives[10] $c_{\text{string}}=\pi/12$. At small r, the Coulomb potential also gives a $1/r$ dependence, and for a given intermediate r, it may be difficult to decide which mechanism is more important. For $T>0$, the form (7) becomes modified; in particular, when $rT \gg 1$, the leading string terms are[9,11]

$$V(r,T)=\left[\sigma(0)-\frac{\pi}{3}T^2+O(T^3)\right]r+T\ln 2rT , \tag{8}$$

so that we now have

$$\Gamma(r,T)=N(T)\left[\frac{e^{-\sigma(T)r/T}}{r}\right] , \tag{9}$$

where we have defined $\sigma(T)$ as the temperature-dependent string tension. In Eq. (9), $N(T)$ denotes the T-dependent overall normalization. Below T_c, we thus get

$$\xi(T)=T/\sigma(T) . \tag{10}$$

As T increases, color screening between the gluons will decrease $\sigma(T)$, and as $T\rightarrow T_c$, $\sigma(T)\rightarrow 0$; hence $\xi(T)$ diverges at T_c. For small or intermediate r it may be meaningful to add a Coulombic $1/r$ term to the string form (9).

For $T>T_c$, in the deconfinement region, there are no more string effects, and we expect

$$\Gamma(r,T)=\langle L\rangle^2(e^{-V(r,T)/T}-1) \tag{11}$$

with

$$V(r,T)=-\frac{c(T)}{r}e^{-r/r_D(T)} . \tag{12}$$

Here $r_D(T)$ denotes the effective color screening radius as felt by the static quarks. The $1/r$ term now is due to the Coulomb potential; hence in the perturbative high-temperature regime, $c(T)=\alpha_{\text{eff}}(T)$, the temperature-dependent running coupling constant,[12] and $r_D{}^{-1}(T)$ becomes the effective gluon mass. For large r we thus again obtain the form

$$\Gamma(r,T)\simeq N(T)\left[\frac{e^{-r/r_D(T)}}{r}\right] , \tag{13}$$

where the correlation length $\xi(T)=r_D(T)$ now measures the color screening radius. At large T,

$$N(T)=\langle L\rangle^2\alpha_{\text{eff}}(T)/T \tag{14}$$

determines the overall normalization.

The average Polyakov loop $\langle L\rangle\sim\exp(-F/T)$ measures the free energy F of an isolated static quark, with the relation

$$\lim_{r\rightarrow\infty}\langle L(0)L(r)\rangle=\langle L\rangle^2 , \tag{15}$$

which we have used in fixing the normalization of Eq. (11). For $T>T_c$, $\langle L\rangle>0$; for $T\le T_c$, $\langle L\rangle$ vanishes. Below T_c, the confining string tension makes F diverge. Above T_c, where $r_D(T)$ is finite, F also remains finite and hence $\langle L\rangle>0$. To have F diverge at T_c, as $T\rightarrow T_c$ from above, $r_D(T)$ must diverge there: at T_c, the static quarks feel an effectively unscreened three-dimensional Coulomb potential.

Thus we expect $\xi(T)$ to diverge as $T\rightarrow T_c$ from either above or below. The universality conjecture,[8] which relates the critical behavior in finite-temperature SU(2) gauge theory to that of the Ising model of the same spatial dimensionality, predicts

$$\xi(T)=A\,|T-T_c|^{-\nu}(1+B\,|T-T_c|^{\theta}) , \tag{16}$$

where we have included a correction to the leading term, since the range of validity of the latter is not known. The three-dimensional Ising model yields[13]

$$\nu\simeq 0.63,\quad \theta\simeq 0.5 , \tag{17}$$

for the exponents in Eq. (16). At $T=T_c$, we have, from Eqs. (10) and (13),

$$\Gamma(r,T_c)=N(T_c)/r . \tag{18}$$

This form neglects the "anomalous dimension" exponent, which gives $r^{-(1+\eta)}$ instead of r^{-1}. However, the three-dimensional Ising model gives $\eta\simeq 0.04$, which is in any case beyond the accuracy of our numerical work.

We have calculated $\Gamma(r,T)$ in a high-statistics (20 000–60 000 updates per point) Monte Carlo evaluation on lattices with $N_\sigma=18$, $N_\tau=3$, 4, and 5, with additional results for $N_\sigma=16$, $N_\tau=4$, 5, and 6 in the region below T_c; for further details of this evaluation, see Ref. 7. We have used periodic boundary conditions also in the spatial directions; this means that $\langle L(0)L(r)\rangle$ contains the effect of the interaction at the separation $(N_\sigma a-r)$ as well as r, so that $\Gamma(r,T)$ can be determined only up to $r_{\max}=N_\sigma a/2$.

In Fig. 1 we show the behavior of $\langle L(0)L(r)\rangle$ as a

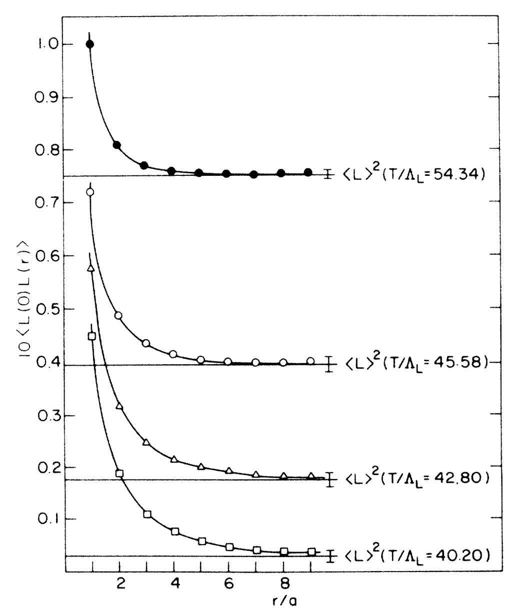

FIG. 1. $\langle L(0)L(r)\rangle$ vs r for different temperatures, compared to the corresponding $\langle L\rangle^2$; some typical statistical errors are indicated. The curves are the fits to the correlation function $\Gamma(r,T)$ as defined in Eq. (20).

function of r, calculated for several different temperatures on the $18^3\times4$ lattice. We include in this figure the corresponding values of $\langle L\rangle^2$, which, as mentioned, were calculated from the configuration average $\langle\,|\bar{L}|\,\rangle$ of the absolute value of the lattice average $\bar{L}$. As r increases, $\langle L(0)L(r)\rangle$ is seen to converge to this point, above as well as below T_c. The physically meaningful $\langle L\rangle$ is really defined by Eq. (15), and in the configuration average of $L(0)L(r)$, overall flips of the entire system from one configuration to another in fact have no effect. Hence our result is to be expected, and it is the quantity

$$\Gamma(r,T)=\langle L(0)L(r)\rangle-\langle L(0)L(r_{\max})\rangle \simeq\langle L(0)L(r)\rangle-\langle\,|\bar{L}|\,\rangle^2 \qquad (19)$$

which best approximates the thermodynamic limit on a large but finite lattice.

It is clear from Fig. 1 that the interaction range drops rapidly as we move out of the transition region. To study this more quantitatively, we fit our results with the form

$$\Gamma(r,T)=N(T)\left[\frac{e^{-r/\xi(T)}}{r}+\frac{e^{-(N_\sigma a-r)/\xi(T)}}{(N_\sigma a-r)}\right]; \qquad (20)$$

the second term takes into account the mentioned periodicity of the lattice in the space directions.

In Figs. 2–4, we show the resulting behavior of the correlation length as function of the temperature, for $T>T_c$. To obtain these quantities in units of the lattice scale Λ_L, we have made use of the SU(2) renormalization-group relation

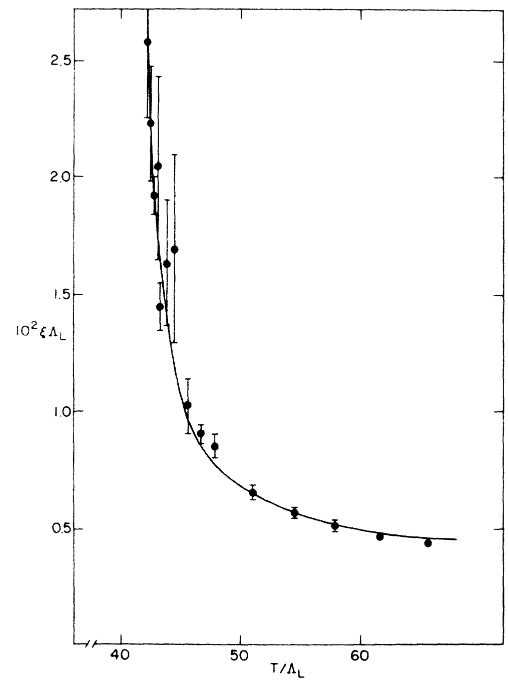

FIG. 3. As Fig. 2, for the $18^3\times4$ lattice, with $T_c/\Lambda_L=41.89$, from Ref. 7.

$$a\Lambda_L=\exp\left\{\frac{3\pi^2}{11}\left[\frac{4}{g^2}\right]-\frac{51}{121}\ln\left[\frac{6\pi^2}{11}\left[\frac{4}{g^2}\right]\right]\right\}. \qquad (21)$$

The errors shown in the figures were obtained by requir-

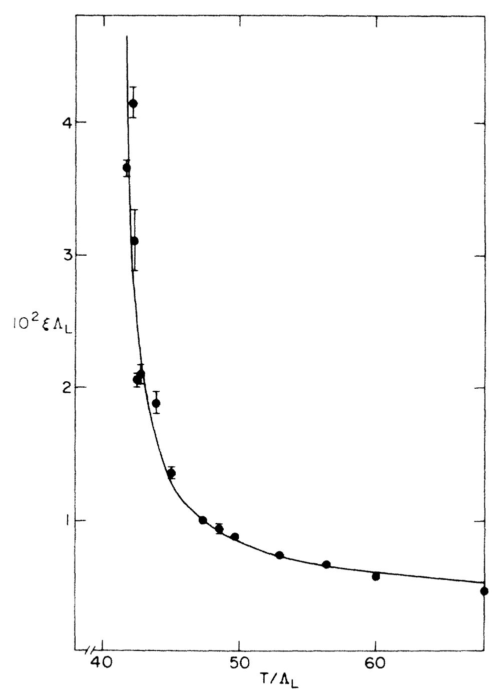

FIG. 2. Correlation length vs temperature, from the $18^3\times3$ lattice; the curve is the fit to the universality form (16), with $T_c/\Lambda_L=41.40$, from Ref. 7.

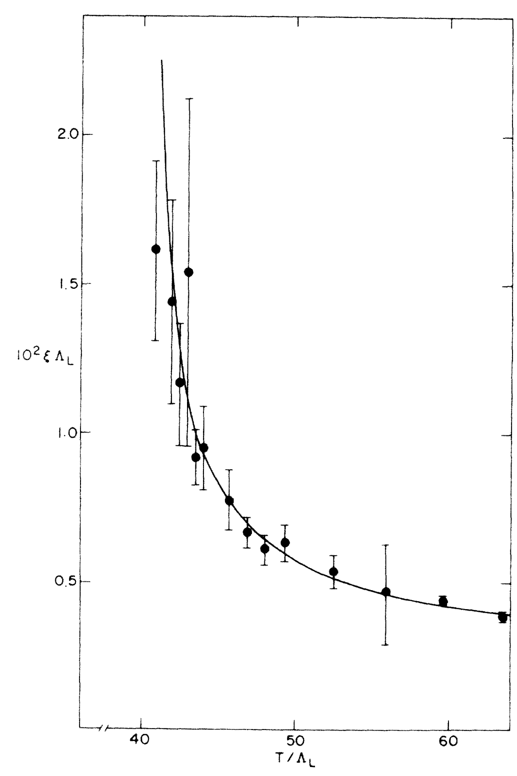

FIG. 4. As Fig. 2, for the $18^3\times5$ lattice, with $T_c/\Lambda_L=40.58$, from Ref. 7.

ing fits with a 95% confidence level, as measured by χ^2 values normalized to the number of degrees of freedom. For the range of couplings involved in our study $(2.1 \le 4/g^2 \le 2.6)$, we still expect some deviations from scaling, and in fact we do note a shift by about one to two units of Λ_L between the results at different N_τ. To have some assurance that the ξ values we get by fitting $\Gamma(r,T)$ with Eq. (20) indeed reflect its large r behavior, we have repeated our fits using only $r \ge 3a$; this does not lead to a significant change of the results.

The behavior for $T < T_c$ is shown in Fig. 5. Since $\sigma(T) = T/\xi(T)$ is nonzero at $T=0$, the coefficient A in Eq. (16) must contain a factor T; this follows also from relations (6) and (7). Since we cover here a fairly large range of T, we show our results for $\sigma(T)$ rather than for $\xi(T)$. The corresponding universal form then becomes

$$[\sigma(T)]^{-1} = A' \mid T - T_c \mid^{-\nu}(1 + B \mid T - T_c \mid^{\theta}) , \qquad (22)$$

in place of Eq. (16), with $A' = A/T$.

The universality predictions (16) and (22), with the exponents ν and θ fixed to their Ising model values, are included in the figures; $A(A')$ and B were adjusted for best fits. We conclude that our results are fully compatible with the universality prediction.[8]

At sufficiently high temperature, we expect the normalization of the correlation function to be determined by Eq. (14), with $\alpha_{\rm eff}(T) \sim 1/(\ln T)$ for the temperature-dependent running coupling constant. When that is the case, the correlation length should be a reliable measure of the gluonic color screening radius. In Fig. 6, we show our results for

$$\alpha_{\rm eff}(T)\ln(T/\Lambda_L) = \frac{N(T)(T/\Lambda_L)\ln(T/\Lambda_L)}{\langle L \rangle^2} . \qquad (23)$$

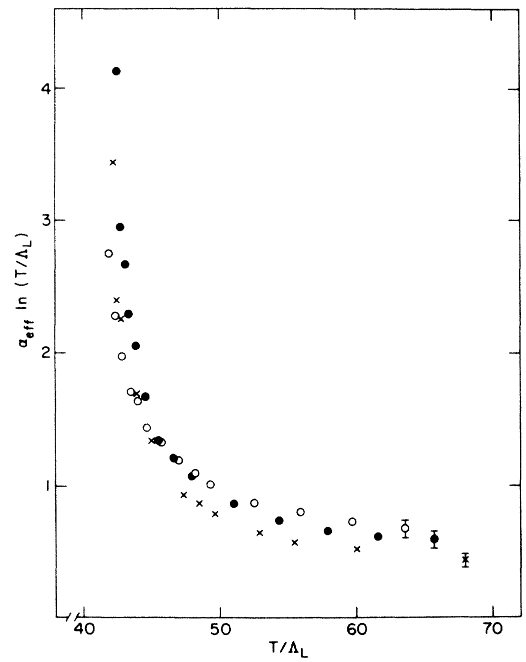

FIG. 6. Temperature dependence of the effective coupling constant $\alpha_{\rm eff}(T)$, from $18^3\times3(\times)$, $18^3\times4(\bullet)$, and $18^3\times5(\circ)$ lattice calculations; some typical statistical errors are indicated.

After a rapid decrease in the transition region, $\alpha_{\rm eff}(T)$ seems indeed to converge to the form $(\ln T)^{-1}$.

We note in Fig. 6 that the results for different N_τ do not coincide completely. On one hand this is due to deviations from the scaling limit (21), as was already mentioned. On the other hand, $\langle L \rangle$ still contains an N_τ-dependent factor due to the self-energy of a point

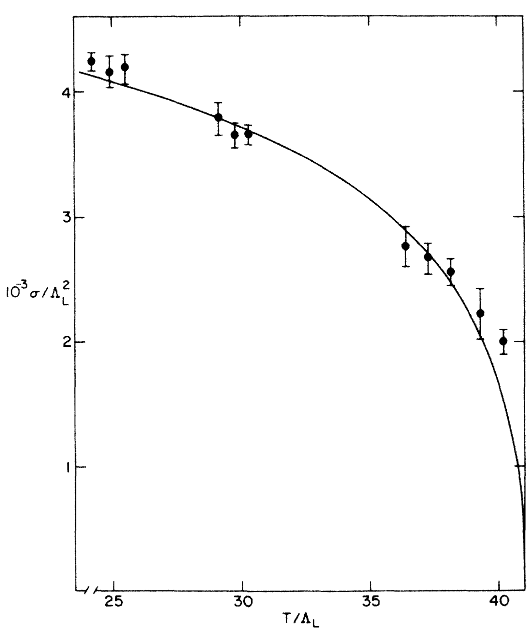

FIG. 5. String tension vs temperature, from the $16^3\times4,5,6$ and the $18^3\times4,5$ lattices; the curve is the fit to the universality form (22), with $T_c/\Lambda_L = 41$.

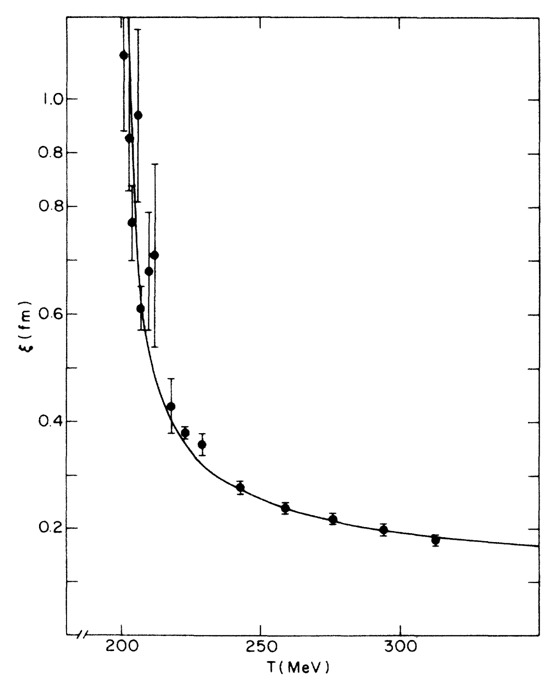

FIG. 7. Correlation length vs temperature, in physical units; from the $18^3\times4$ lattice.

charge.[14] We find, however, in all three cases they converge toward $(\ln T)^{-1}$.

Finally, we would like to get a feeling for the high-temperature behavior of the correlation length in physical units. We have therefore fixed the critical temperature to $T_c = 200$ MeV, as suggested by string tension calculations as well as by phenomenological considerations. In Fig. 7 we show the corresponding $\xi(T)$, together with the universality form (16). It is seen that $\xi(T)$ decreases indeed quite rapidly: at $T/T_c = 1.5$, it is already less than 0.2 F. That is in accord also with results for the SU(3) case.[6] Since at high T the correlation length approaches the gluonic color screening radius, we believe that this information is quite relevant for the screening of interactions in the quark-gluon plasma.[15]

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