Continuum limit of an SU(2) gauge theory with a scalar doublet

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The only known fixed point of the SU(2) gauge-Higgs model is the Gaussian fixed point. Approaching this point in the Higgs phase defines a cutoff-independent field theory of massive vector bosons and a scalar. This field theory turns out to be free, however. In the vicinity of the Gaussian point effective interacting models can be defined with a large but finite cutoff. These effective models can be investigated in detail, as it is possible to control both the ultraviolet and the infrared behavior in perturbation theory.

I. INTRODUCTION, SUMMARY OF RESULTS

Motivated by the scalar sector of the Weinberg-Salam model, several numerical studies dealt with SU(2) gauge-Higgs systems.[1] These Monte Carlo (MC) calculations indicate a first-order phase transition between the symmetric and Higgs phase. In the parameter space investigated until now there is no sign of a candidate, non-Gaussian fixed point (FP), where a cutoff-independent quantum field theory could be defined. In this situation increasing attention has been devoted to the Gaussian FP (the FP at zero gauge and scalar self-coupling). The question was raised[2] whether following a procedure similar to that used in QCD, an interesting, cutoff-independent theory can be defined on this FP. The physical idea behind this suggestion is that although the bare coupling is tuned towards zero, the physical coupling is nonzero and can create an appropriate effective potential for the scalar fields. Several numerical calculations turned toward this problem recently.[2,3]

However, the behavior of the field theory in the vicinity of the Gaussian FP is a perturbative problem. One should be able to find out the basic properties of this continuum limit with the help of perturbation theory and renormalization-group considerations.

In this paper we summarize the results of an analysis of this kind. It will be shown that the continuum theory defined on the Gaussian FP is a free-field theory of massive gauge bosons and a massive scalar with an arbitrary ratio $R=m_H/m_W$ (if the Gaussian FP is approached in the Higgs phase). Unlike in QCD, not only the ultraviolet but the infrared behavior is calculable perturbatively for small couplings. At any point (r,u,g^2) of the parameter space in the Higgs phase (here r and u are the parameters of the scalar potential, g^2 is the gauge coupling) one can determine the physical couplings u_p and $g_p{}^2$, the scalar field expectation value M, and the gauge-boson and Higgs-boson masses, m_W and m_H, respectively.

Although a cutoff-independent, interacting theory cannot be defined this way, there are parts of the parameter space (namely those which are close to the weak first-order phase transition surface) where the physical couplings are nonzero (the model is interacting) and the cutoff is very large. In these effective theories $R=m_H/m_W$ is essentially free. There exists only a lower bound on R (Ref. 4) (saturated on the singular surface) and, presumably, an upper bound,[5] which is, however, outside the range of perturbation theory. The apparently constant mass ratio R found in recent MC studies[3] is presumably due to the very limited parameter space explored in these simulations.

The results mentioned above are, of course, independent of the regularization used. Most of the formulas of this paper refer to dimensional regularization. The specific modifications due to lattice regularization will be discussed at the end.

II. DEFINITIONS AND NOTATIONS

The Lagrangian has the form ($d=4$, Euclidean space)

$$\mathscr{L}=\tfrac{1}{4}\sum_{a=1}^{3}F^a_{\mu\nu}F^a_{\mu\nu}+\sum_{i=1}^{2}(D_\nu\Phi)_i^*(D_\nu\Phi)_i+\tfrac{1}{2}r_0\sum_{\alpha=1}^{4}\phi_\alpha{}^2+\frac{u_0}{4!}\left[\sum_{\alpha=1}^{4}\phi_\alpha{}^2\right]^2, \quad (1)$$

where

$$\Phi=\frac{1}{\sqrt{2}}\begin{bmatrix}\phi_1+i\phi_2\\ \phi_3+i\phi_4\end{bmatrix}$$

is the scalar doublet, while the covariant derivative is defined as

$$D_\nu=\partial_\nu\mathbb{1}-ig_0A^a_\nu\frac{\tau^a}{2}\ .$$

We shall work in the Landau gauge and use dimensional regularization. The renormalization mass is denoted by

Λ, the corresponding dimensionless parameters are r $[=r(\Lambda)]$, u, and g^2. The dimensions are carried by the arbitrary scale Λ. In the following it will be convenient to consider Λ as the largest scale of the system ($\sim$ cutoff), which, in a correctly defined theory, should go to infinity. In this case r, u, and g^2 can be considered as bare parameters, in a specific regularization scheme. This language is close to that usually used on the lattice and makes the comparison with this case easier.

We shall call the parameters on the scale of the W mass "physical," and denote them by $r_p[=r(m_W)]$, u_p, and $g_p{}^2$. On the tree level we have $m_W{}^2=\frac{1}{4}g^2M^2$, where M is the scalar field expectation value.

III. EFFECTIVE POTENTIAL AND THE RENORMALIZATION-GROUP EQUATIONS

Since we are interested in the phase structure of the theory, let us calculate the scalar effective potential.[6] A simple one-loop calculation gives[7]

$$V_{\rm eff}(x)=\tfrac{1}{2}rx^2+\frac{1}{4!}ux^4+\frac{1}{(8\pi)^2}\{(r+\tfrac{1}{2}ux^2)^2[\ln(r+\tfrac{1}{2}ux^2)-\alpha]+3(r+\tfrac{1}{6}ux^2)^2[\ln(r+\tfrac{1}{6}ux^2)-\alpha]$$
$$+\tfrac{9}{16}g^4x^4[\ln(\tfrac{1}{4}g^2x^2)-\alpha]\}+\text{const}\ . \qquad (2)$$

The constant is fixed by the convention $V_{\rm eff}(0)=0$ while α depends on the scheme chosen. [The minimal-subtraction scheme gives $\frac{3}{2}+\ln 4\pi+\Gamma'(1)$, for example.] In the following we shall choose the scheme where $\alpha=\frac{1}{2}$, which leads to simple expressions.

The problem with $V_{\rm eff}(x)$ in Eq. (2) is the following. Just in the interesting parameter regions, where the radiative corrections are large enough to influence the behavior of the system, the higher loop corrections become important and the one-loop result is not reliable.

Renormalization-group considerations help to control the "dangerous logarithms" of Eq. (2) (Ref. 8). Under the change of scale

$$\Lambda\to e^{-t}\Lambda,\quad t>0 \qquad (3)$$

the effective potential transforms as

$$V_{\rm eff}(x;r,u,g^2)=e^{-4t}V_{\rm eff}(\bar{x}(t);\bar{r}(t),\bar{u}(t),\bar{g}^2(t)) \qquad (4)$$

where[9]

$$\frac{d\bar{g}^2}{dt}=\beta_0\bar{g}^4,\quad \beta_0=\frac{1}{16\pi^2}\frac{86}{6}\ , \qquad (5)$$

$$\frac{d\bar{u}}{dt}=-\frac{1}{16\pi^2}(4\bar{u}^2-9\bar{u}\,\bar{g}^2+\tfrac{27}{4}\bar{g}^4)\ , \qquad (6)$$

$$\frac{d\bar{r}}{dt}=\left[2-\frac{2}{16\pi^2}\bar{u}+\frac{1}{16\pi^2}\frac{9}{2}\bar{g}^2\right]\bar{r}\ , \qquad (7)$$

$$\frac{d\bar{x}^2}{dt}=\left[2-\frac{1}{16\pi^2}\frac{9}{2}\bar{g}^2\right]\bar{x}^2\ , \qquad (8)$$

and $\bar{r}(0)=r,\ldots$ We cannot expect to control $V_{\rm eff}(x)$ in perturbation theory for arbitrary x values. It should be determined reliably, however, at least in the region $x\le M$, where M is the second, nontrivial minimum (if it exists). The idea is to calculate $V_{\rm eff}(x;r,u,g^2)$ via the right-hand side of Eq. (4) by choosing an appropriate t value, where the "dangerous logarithms" become small in this region. At this t value the radiative corrections in $V_{\rm eff}(\bar{x};\bar{r},\bar{u},\bar{g}^2)$ are small and even the tree-level part should give reasonable results.

We shall take $t=t^*$, where

$$\ln[\tfrac{1}{4}\bar{g}^2(t^*)\bar{M}^2(t^*)]=0\ . \qquad (9)$$

The scale $\Lambda=e^{-t^*}\Lambda$ is just the scale of the W mass [according to Eq. (9) the dimensionless $\bar{m}_W{}^2=\frac{1}{4}\bar{g}^2\bar{M}^2=1$]; therefore the corresponding couplings are the physical couplings and will be indexed by p in the following $[\bar{g}^2(t^*)\equiv g_p{}^2,\ldots]$.

IV. SOLUTION OF THE RENORMALIZATION-GROUP EQUATIONS

Equations (5)–(8) are easy to integrate. One obtains

$$g_p{}^2=\frac{g^2}{1-\beta_0g^2t^*}\ , \qquad (10)$$

$$u_p=g_p{}^2\left[\frac{a}{2}\tan\left[\frac{a}{b}\ln(1-\beta_0g^2t^*)+\delta\right]-\frac{2}{3}\right]\ , \qquad (11)$$

$$r_p=r\exp\left[2t^*-\frac{35}{12}\frac{1}{b}\ln(1-\beta_0g^2t^*)-\frac{1}{2}\ln\left[\frac{\cos[(a/b)\ln(1-\beta_0g^2t^*)+\delta]}{\cos\delta}\right]\right]\ , \qquad (12)$$

$$\bar{M}^2(t^*)=M^2\exp\left[2t^*+\frac{9}{4}\frac{1}{b}\ln(1-\beta_0g^2t^*)\right]\ , \qquad (13)$$

where

$$\delta=\delta(u,g^2)=\arctan\left[\frac{2u/g^2+\frac{4}{3}}{a}\right]\ , \qquad (14)$$

and the constants a and b are given by

$$a=(\tfrac{27}{4}-\tfrac{16}{3})^{1/2}\ ,\qquad b=8\pi^2\beta_0\ . \qquad (15)$$

For given parameters (r,u,g^2) the five equations (9)–(13) contain six unknowns: r_p, u_p, $g_p{}^2$, $\bar{M}(t^*)$, M, and t^*. The missing equation is provided by $V_{\rm eff}(\bar{x};r_p,u_p,g_p{}^2)$.

V. ANALYSIS OF $V_{\rm eff}(\bar{x},r_p,u_p,g_p^2)$

If there exists a nontrivial minimum at $\overline{M}$, we have

$$\left.\frac{dV_{\rm eff}}{d\bar{x}}\right|_{\bar{x}=\overline{M}}=0 .$$

Using Eqs. (2) and (9) we obtain

$$r_p=-\frac{2}{3}\frac{u_p}{g_p^2}+O(u) . \quad (16)$$

Now we can solve the system of algebraic equations (9)–(13) and (16). Equations (11), (12), and (16) give

$$r\exp\left[2t^*-\frac{35}{12}\frac{1}{b}\ln(1-\beta_0 g^2 t^*)-\frac{1}{2}\ln\left[\frac{\cos[(a/b)\ln(1-\beta_0 g^2 t^*)+\delta]}{\cos\delta}\right]\right]=-\frac{2}{3}\left[\frac{a}{2}\tan\left[\frac{a}{b}\ln(1-\beta_0 g^2 t^*)+\delta\right]-\frac{2}{3}\right] , \quad (17)$$

which determines $t^*=t^*(r,u,g^2)$. Having t^*, we obtain immediately g_p^2, u_p, r_p, $\overline{M}^2$, and M^2 as the function of r,u,g^2 from Eqs (10)–(12), (9), and (13), respectively. Before discussing these results, let us check under what conditions the system will be in the Higgs phase. The condition reads

$$V_{\rm eff}(\overline{M})\le V_{\rm eff}(0) , \quad (18)$$

giving

$$u_p\ge -\frac{27}{256\pi^2}g_p^4 . \quad (19)$$

Since u_p and g_p^2 can be expressed in terms of the original (bare) parameters (r,u,g^2), Eq. (19) gives the equation of the two-dimensional singular surface $r^c=r^c(u,g^2)$ separating the two phases. We shall see that $\overline{M}(t^*)=4/g_p^2$ implies finite M^2 [via Eq. (13)] even on the singular surface. Therefore the phase transition is of first order.

As we already discussed, the W mass is given by

$$m_W=e^{-t^*}\Lambda , \quad (20)$$

while the Higgs-boson mass is obtained from the second derivative of the effective potential at M. For the ratio we get

$$R^2\equiv\frac{m_H^2}{m_W^2}=\frac{4u_p}{3g_p^2}+\frac{9}{32\pi^2}g_p^2 , \quad (21)$$

where we kept the radiative corrections proportional to g^2 (which is important when u_p is small, say, $u_p\sim g_p^4$), but we suppressed the $O(u_p)$ corrections, which are always smaller than the first term. Equation (19) implies then a lower bound for R^2:

$$R_{\rm min}^2=\frac{9}{64\pi^2}g_p^2 . \quad (22)$$

The ratio R takes its minimum value on the singular surface. The bound Eq. (22) is well known.[4] ($R_{\rm min}^2$ is smaller by a factor of 2 than the Coleman-Weinberg value,[6] which is just one of the possible values of R^2 in our context.) Using a modified condition $\ln(\frac{1}{4}\bar{g}^2\overline{m}^2)=c$ $(c\sim 1)$ in Eq. (9), or a different α value in Eq. (2) (different scheme) the physical conclusions [like Eq. (22)] remain unchanged.

VI. BEHAVIOR OF THE SINGULAR SURFACE

On the singular surface Eq. (19) gives

$$\frac{u_p}{g_p^2}=-\frac{27}{256\pi^2}g_p^2 , \quad (23)$$

which, with the help of Eqs. (10) and (11) implies

$$\frac{a}{2}\tan\left[\delta(u,g^2)-\frac{a}{b}\ln z\right]-\frac{2}{3}=-\frac{27}{256\pi^2}g^2 z , \quad (24)$$

where

$$z=\frac{1}{1-\beta_0 g^2 t^*} . \quad (25)$$

The solution of Eq. (24) can be written as

$$z=z_0+O(g^2) , \quad (26)$$

where

$$z_0(u,g^2)=\exp\left[\frac{b}{a}\left[\delta(u,g^2)-\arctan\frac{4}{3a}\right]\right] , \quad (27)$$

since δ is bounded by $\pi/2$, z_0 is finite, $O(1)$. Therefore

$$\lim_{g^2\to 0}(g_p^2/g^2)=\lim_{g^2\to 0} z =z_0\le\exp\left[\frac{b}{a}\left[\frac{\pi}{2}-\arctan\frac{4}{3a}\right]\right]\simeq 27.6 , \quad (28)$$

which implies that $g_p^2\to 0$ as g^2 is tuned towards the Gaussian point along the singular surface. Inside the Higgs phase, away from the singular surface, the left-hand side of Eq. (24) is larger than the right-hand side [see Eq. (19)]; therefore, the solution z becomes smaller. It follows that in the $g^2\to 0$ limit the physical gauge couplings g_p^2 become zero, no gauge interaction remains. Without gauge interaction the scalar field cannot sustain nonzero self-interaction u_p either. These expected results follow from the equations. Really, finite u_p would require $u_p/g_p^2\to\pm\infty$. The case of $-\infty$ is excluded by Eq. (19), while $+\infty$ is excluded by Eq. (11), if we remember that t^* is positive.

Equations (25) and (26) give

$$t^*=\frac{1-1/z_0}{\beta_0 g^2}+O(1) , \quad (29)$$

and we get, from Eqs. (20), (12), and (13),

$$m_W \sim \Lambda \exp\left[-\frac{1-1/z_0}{\beta_0 g^2}\right],$$
$$r = r^{\text{crit}} \sim \Lambda^2 g^2 \exp\left[-2\frac{1-1/z_0}{\beta_0 g^2}\right], \quad (30)$$
$$M^2 \sim \Lambda^2 \frac{1}{g^2}\exp\left[-2\frac{1-1/z_0}{\beta_0 g^2}\right],$$

where we restored the dimensions. The way the dimensionless masses go to zero in the $g^2 \to 0$ limit resembles the corresponding behavior in a pure gauge theory. The big difference is the occurrence of a finite z_0 in Eq. (30), due to which the dimensionless masses drop to zero less fast, and $g_p{}^2 \to 0$ according to Eq. (28). This is the reason also that perturbation theory is applicable to these infrared problems. The $O(1)$ contribution in Eq. (29) and the proportionality factors of Eq. (30) can be fixed on the two-loop level.

VII. SU(2) GAUGE-HIGGS MODEL AS AN EFFECTIVE THEORY

Although the cutoff-independent theory in the $g^2 \to 0$ limit is a massive free-field theory, one can define interacting models with a finite cutoff. Depending on the value of the physical coupling this cutoff can be very large, thus defining an excellent effective theory.[10]

Consider, for example, the effective theories defined on the singular surface. If we fix the physical gauge coupling $g_p{}^2$ to some finite value, Eqs. (25) and (26) imply

$$g^2 = \frac{1}{z_0} g_p{}^2 ; \quad (31)$$

from Eq. (29) we get

$$t^* = \frac{z_0 - 1}{\beta_0 g_p{}^2},$$

giving

$$\Lambda / m_W \sim e^{(z_0-1)/\beta_0 g_p{}^2} . \quad (32)$$

Since $z_0 |_{\max} \approx 27.6$, the cutoff can be extremely large. On the singular surface $R^2 = R_{\min}{}^2$. Moving away from the singular surface R^2 is increasing and, at the same time, the cutoff is decreasing. For illustration, a few results are collected in Table I by solving the algebraic equations Eqs. (9)–(13) and (16) numerically. For fixed physical gauge couplings $g_p{}^2 = 0.4$ and for three different cutoff values ($\Lambda = e^{t^*} m_W$, $t^* = 1.0$, 10.0, and 50) u_p and R are given for different u values. As can be seem from this table, by increasing the cutoff it becomes more difficult to get large R values. Actually, for any fixed cutoff and $g_p{}^2$ one obtains an upper bound $R_{\max}$, which is saturated at $u = \infty$. This is, however, clearly outside the range of perturbation theory.

VIII. MODIFICATIONS DUE TO LATTICE REGULARIZATION

Using a hypercubic lattice instead of dimensional regularization, makes rather little difference and, of course,

TABLE I. In effective models with different cutoff values (first column) the physical self-coupling u_p and $R = m_H / m_W$ are given for different bare coupling u values. The physical gauge coupling is fixed to be $g_p{}^2 = 0.4$ everywhere. ($u/4!$ enters the action.)

Λ^{cut}/m_W	u	u_p	R
$e^{1.0}$	1.0	0.99	1.82
	5.0	4.53	3.89
	10.0	8.14	5.21
	15.0	11.08	6.08
	20.0	13.52	6.71
	25.0	15.58	7.21
	30.0	17.34	7.60
$e^{10.0}$	1.0	0.91	1.74
	5.0	2.51	2.89
	10.0	3.19	3.26
	15.0	3.50	3.42
	20.0	3.68	3.50
	25.0	3.80	3.56
	30.0	3.88	3.60
$e^{50.0}$	1.0	0.62	1.44
	5.0	0.94	1.78
	10.0	1.00	1.83
	15.0	1.02	1.85
	20.0	1.036	1.86
	25.0	1.042	1.868
	30.0	1.047	1.87

the main conclusion remains unchanged. The renormalization-group equation for the mass parameter r [Eq. (7)] is modified. In Eq. (7) the right-hand side is proportional to r, which is a specific property of dimensional regularization, where certain quadratic divergences are defined to be zero. The modified equations on the lattice have the form

$$a\frac{d\overline{r}}{da} = \left[2 - \frac{2}{16\pi^2}\overline{u} + \frac{1}{16\pi^2}\frac{9}{2}\overline{g}^2\right]\overline{r} + \gamma(2\overline{u} + \tfrac{9}{2}\overline{g}^2) , \quad (33)$$

where

$$\gamma = \int\int\int\int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{1}{4\sum_{\nu=1}^{4}\sin^2\frac{k_\nu}{2}} \approx 0.1549 .$$

According to Eq. (29), t^* is large on the singular surface and, as a consequence, the dimensionless $r^c(u,g^2)$ is very close to zero in case of dimensional regularization [Eq. (30)]. On the lattice Eq. (33) gives

$$r^c_{\text{latt}}(u,g^2) \approx -\frac{\gamma}{2}(2u + \tfrac{9}{2}g^2) . \quad (34)$$

In lattice MC calculations the hopping parameter κ and a somewhat differently defined λ are used:

$$u = 6\frac{\lambda}{\kappa^2} ,$$
$$r = \frac{-2\lambda + 1 - 8\kappa}{\kappa} . \quad (35)$$

In terms of these variables the equation of the singular surface Eq. (34) has the form

$$\kappa_c(\lambda,g^2)\approx \tfrac{1}{8}+0.68\lambda+0.0055g^2\ , \qquad (36)$$

which is valid for small λ and g^2 values.

The form of the effective potential in Eq. (2) is modified also. The value of the constant α is given by

$$\alpha\approx 1+\ln\pi^2+\frac{(8\pi)^2}{4}0.0158\ , \qquad (37)$$

and there is an extra radiative correction of the form

$$\frac{1}{(8\pi)^2}W[(r+\tfrac{1}{2}ux^2)+3(r+\tfrac{1}{6}ux^2)+\tfrac{9}{4}g^2x^2]\ , \qquad (38)$$

where

$$W=2\pi^2+\frac{(8\pi)^2}{2}(\gamma-\tfrac{1}{16})\ . \qquad (39)$$

The results, including the lower bound Eq. (22) on R, remain unchanged. Since the steps are identical to those followed before, we do not repeat this analysis here.

IX. REMARKS ON THE MODEL IN THE LIMIT $g^2\to 0$, u IS NOT SMALL

This case is outside perturbation theory applied in this paper. One can observe, however, that the question of the existence of a non-Gaussian fixed point in this limit can be answered within the scalar sector (without gauge interactions.) There are many indications, that no such fixed points exist.[10,11] On the other hand, as we discussed, the Gaussian FP defines a free field theory. Therefore, one cannot expect an interacting, cutoff-independent field theory in the $g^2\to 0$ limit, independently of the value of the self-coupling u.

Note added in proof: It has been shown by Mack[12] that the $\lambda\to\infty$, $g^2\to 0$, $\kappa\to\kappa_c$ limit defines a massive free field theory. This result is consistent with the remarks of Sec. IX. We are indebted to Professor Mack for discussion.

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