Spontaneous breaking of global and local symmetries in six-dimensional Einstein-Maxwell-σ theory

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(Received 8 July 1986)

Higher-winding-number solutions of the nonlinear σ field are used to break the global as well as local symmetry of SU(2) in a six-dimensional gravity theory coupled with the U(1) Maxwell and σ field. The masses of three gauge bosons are split and their ratios of masses are obtained for various winding numbers. The low-lying states of this model not of Planck scale are a scalar triplet, the graviton, photon, and low-mass gauge bosons.

I. INTRODUCTION

Theories with higher-dimensional space-time, originated by Kaluza and Klein, have recently attracted much revival interest from the viewpoint of unifying existing gauge interactions with gravity. For such a scenario to be valid in describing the real world one must be able to find a ground state, with the extra space having some specific form of geometry, which allows the extra space to compactify into a size on the order of the Planck length. The four-dimensional gauge symmetries are understood to originate as isometries of the compactified internal space. But what we really need to obtain in four dimensions is not only the gauge symmetry but also a mechanism by which it can be broken. The Kaluza-Klein theories can provide an interesting possibility for the breaking mechanism which might be seen as an alternative to the usual Higgs mechanism.

We have described in a previous paper[1] one possible mechanism which breaks the gauge symmetry by introducing a new field (nonlinear σ field)[2] in the six-dimensional Einstein-Maxwell theory.[3] This scheme is contrary to the conventional wisdom which hopes that the states massless at the tree level would acquire their small masses via the quantum effect.[4] But our electroweak-gravity model has been seen to be more concrete in its predictions as well as have smaller parameters than those of the Higgs scalars. A similar approach has been studied by Sobczyk[5] who uses the conventional scalar fields with tachyonic masses to obtain the symmetry-breaking pattern of $SU(2)\times U(1)\rightarrow U(1)$, the unbroken U(1) being a subgroup of SU(2).

In this paper we will describe a mechanism which breaks the symmetry not only of local SU(2) but also of global SU(2). For this purpose the higher-winding-number solutions of the σ field are used to induce small deformation on the internal manifold. Here the winding number means the homotopic classification $\pi_2(S^2)$ of the mappings from the internal space to the target manifold of the σ field. The case treated in Ref. 1 is just that of $n=1$ winding number.

The symmetry and its breaking are related to the invariances of the background fields.[2,6] The gravitational part is invariant under isometric coordinate transformations on $S^2=SU(2)/U(1)$ induced by a left translation on SU(2) if it is accompanied by a target space rotation. In order to have the corresponding massless gauge vectors these isometries must be invariances of the full background configuration, that is, not only the gravity but also the scalars. The scalar field is invariant under the internal symmetry transformations $\delta\bar{\phi}^{\mu}=\eta_A V_A^{\mu}(\bar{\phi})$, where $V_A^{\mu}(\bar{\phi})$ with A a group index, are Killing vectors corresponding to the isometries of the scalar manifold metric $h_{\mu\nu}$. But η_A is constant and cannot depend on x^i. The scalar field is invariant under global transformation only. Thus the local symmetry is broken and all three gauge bosons acquire equal masses from the remaining global symmetry. Furthermore, the internal manifold deforms slightly for the classical solutions of the σ field with the winding number higher than 1, and the isometry group of the background configuration is reduced to O(2). The remaining SU(2) global symmetry is broken down and the masses of three gauge bosons split into two different masses.

The other low-lying states in this model are the massless graviton and photon and a low-mass scalar triplet. The scalar triplet is originated from the isometry of the scalar manifold metric $h_{\mu\nu}$ and is the characteristic of this model irrespective of special winding number.

Our results are not immediately applicable in understanding electroweak phenomenology. We hope, however, that our model will be of some value in constructing a realistic electroweak-gravity model.

II. SPECIFICATION OF THE MODEL AND BACKGROUND SOLUTION

The phenomenologically interesting Kaluza-Klein models generally contain not only the gravitation but also some additional fields like elementary gauge fields. In this paper we will work with a six-dimensional Einstein-Maxwell σ-field theory as in Ref. 1. The action with a cosmological constant is

$$S=-\int d^6z\sqrt{-g}\left[\frac{1}{\kappa^2}R+\tfrac{1}{4}F_{MN}F^{MN}+\lambda+\frac{1}{2t}g^{MN}\partial_M\phi^{\mu}\partial_N\phi^{\nu}h_{\mu\nu}\right]. \quad (1)$$

The notations are the same as in Ref. 1. Especially the nonlinear σ fields $\phi^\mu(x)$, $\mu=1,2$ are thought of as coordinates of a two-sphere S^2 with metric $h_{\mu\nu}$ (Ref. 2).

The classical equations of motion from the action are

$$R_{MN}-\tfrac{1}{2}g_{MN}R=-\frac{\kappa^2}{2}(T_{MN}-\lambda g_{MN})\,, \tag{2a}$$

$$F^{MN}{}_{;M}=\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}F^{MN})=0\,, \tag{2b}$$

$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}\partial_N\phi^\mu)+\Gamma^\mu_{\nu\delta}\partial_M\phi^\nu\partial_N\phi^\delta g^{MN}=0 \tag{2c}$$

with the energy-momentum tensor

$$T_{MN}=\frac{1}{t}h_{\mu\nu}(\partial_M\phi^\mu\partial_N\phi^\nu-\tfrac{1}{2}g_{MN}g^{PQ}\partial_P\phi^\mu\partial_Q\phi^\nu)+F_{ML}F_N{}^L-\tfrac{1}{4}g_{MN}F^2\,. \tag{3}$$

The topology of the classical solutions of Eqs. (2) is assumed to be $M_4\times B_2$, M_4 being Minkowski space and B_2 being a slightly deformed two-sphere. We take

$$\bar g_{MN}dz^Mdz^N=\eta_{mn}dx^mdx^n+a^2[1+\epsilon f(\theta)]d\theta^2+a^2[1+\epsilon g(\theta)]\sin^2\theta\, d\phi^2\,, \tag{4a}$$

$$\bar A_\theta=\begin{cases}\dfrac{s}{2e}\cos\theta-\dfrac{\epsilon}{2}G(\theta)-1+\dfrac{\epsilon}{2}G(0), & 0\le\theta\le\dfrac{\pi}{2},\\[2mm] \dfrac{S}{2e}\cos\theta-\dfrac{\epsilon}{2}G(\theta)+1+\dfrac{\epsilon}{2}G(\pi), & \dfrac{\pi}{2}\le\theta\le\pi,\end{cases} \tag{4b}$$

$$\bar A_\varphi=0\,,$$

$$\bar\phi^\theta=2\,\mathrm{arccot}\left[\cot^n\frac{\theta}{2}\right], \tag{4c}$$

$$\bar\phi^\phi=n\phi\,,$$

$$h_{\mu\nu}[\phi(y)]d\phi^\mu d\phi^\nu=a^2(d\phi^\theta)^2+a^2\sin^2\phi^\theta(d\phi^\phi)^2\,, \tag{4d}$$

where

$$G(\theta)=\int[f(\theta)+g(\theta)]\sin\theta\, d\theta\,. \tag{5}$$

Equation (4b) describes the monopole configuration of the Maxwell field on a deformed two-sphere. Where the neighborhoods of two patches overlap, say at $\theta=\pi/2$, $0\le\phi\le2\pi$, the two representations of $\bar A_\theta$ must be related by a single-valued gauge transformation requiring s to be

$$\text{integer}\times\left[1-\frac{\epsilon}{4}[G(\pi)-G(0)]\right].$$

The field strength corresponding to (4b) is

$$F_{\theta\varphi}=\frac{-s\sin\theta}{2e}\left[1+\frac{\epsilon f}{2}+\frac{\epsilon g}{2}\right]. \tag{6}$$

The monopole charge calculated from Eq. (6) using the Gauss law is

$$\frac{2\pi}{e}s\left[1+\frac{\epsilon}{4}[G(\pi)-G(0)\right]=\frac{2\pi}{e}\times\text{integer}$$

as expected. This monopole configuration satisfies the equation of motion in (2b) up to $O(\epsilon^2)$.

As the physical coordinate and internal space of the σ field are both described by S^2, it is possible to describe the classical solution of the σ field by homotopic classification $\pi_2(S^2)=Z$ (Ref. 7). The solution in Eq. (4c) has the nontrivial $Z=n$ winding number and satisfies the equation of motion in (2c) in the limit $\epsilon\to0$. Note that the Levi-Civita connection $\Gamma^\mu_{\nu\delta}$ in Eq. (2c) must be obtained from the σ-field internal space metric $h_{\mu\nu}$ of (4d). This solution corresponds to the monopole configuration of the nonlinear O(3) model.[7]

By substituting these vacuum expectation values into (3), we could find the following expression of the energy-momentum tensor:

$$T_{ab}=-\frac{1}{t}\eta_{ab}n^2\frac{\sin^{2n-2}\frac{\theta}{2}\cos^{2n-2}\frac{\theta}{2}}{\left[\sin^{2n}\frac{\theta}{2}+\cos^{2n}\frac{\theta}{2}\right]^2}-\tfrac{1}{4}\eta_{ab}\bar F^2\,,\qquad a,b=0,1,2,3\,, \tag{7a}$$

$$T_{aa}=0\,, \tag{7b}$$

$$T_{\alpha\beta}=\tfrac{1}{4}g_{\alpha\beta}\bar F^2,\quad \alpha,\beta=5,6\,, \tag{7c}$$

where

$$\bar F^2=2F_{\theta\varphi}F^{\theta\varphi}=\frac{s^2}{2e^2a^4}\,. \tag{8}$$

Using these expressions one could obtain the following algebraic equations from (2a) when M,N takes four-dimensional or internal coordinates, respectively:

$$\frac{1}{a^2}\left[1-\frac{\epsilon g''}{2}-\left[\epsilon g'-\frac{\epsilon f'}{2}\right]\cot\theta-\epsilon f\right]=\frac{\kappa^2}{8}\bar F^2+\frac{\kappa^2}{2t}n^2\frac{\sin^{2n-2}\frac{\theta}{2}\cos^{2n-2}\frac{\theta}{2}}{\left[\sin^{2n}\frac{\theta}{2}+\cos^{2n}\frac{\theta}{2}\right]^2}+\frac{\lambda\kappa^2}{2}\,, \tag{9a}$$

$$0=\tfrac{1}{4}\bar F^2-\lambda \tag{9b}$$

We now identify ϵ as $-(\kappa^2n^2/2t)4/\kappa^2\bar F^2$, which is roughly the ratio of the contribution to the energy-momentum tensor from the Maxwell field versus from the σ field. As the σ field acts mainly as the trigger of the spontaneous symmetry breaking while the Maxwell field is responsible for the compactification, and the related two scales are $\sim10^2$ GeV and $\sim10^{19}$ GeV, respectively, the magnitude of $\sqrt{-\epsilon}$ is roughly 10^{-17}. By equating the coefficients of each power of ϵ in Eq. (9) we can obtain the expression for the radius of internal space a and the equations which f and g must satisfy:

$$\frac{1}{a^2}=\frac{\kappa^2\bar F^2}{4}\,, \tag{10a}$$

$$\frac{g''}{2}+\left[g'-\frac{f'}{2}\right]\cot\theta+f=\frac{\sin^{2n-2}\frac{\theta}{2}\cos^{2n-2}\frac{\theta}{2}}{\left[\sin^{2n}\frac{\theta}{2}+\cos^{2n}\frac{\theta}{2}\right]^2}\ . \tag{10b}$$

In summary, we introduce two matter fields into six-dimensional Einstein theory. The Maxwell field is responsible for the dimensional reduction through the compactification and the σ field is responsible for the small deformation of ϵ order on the compactified two-dimensional internal space. The classical solutions of Eqs. (4) represent the consistent correction in ϵ to the non-deformed solutions of Einstein-Maxwell theory without the σ field.

III. FLUCTUATION ANALYSIS AND MASS SPECTRUM

In order to obtain the spectrum one must expand the field around the ground state as

$$g_{MN}=\bar{g}_{MN}+\kappa h_{MN}\ , \tag{11a}$$

$$A_M=\bar{A}_M+V_M\ , \tag{11b}$$

$$\phi^\mu=\bar{\phi}^\mu+z^\mu\ , \tag{11c}$$

where h_{MN}, V_M, and z^μ represent the fluctuations. We next expand the action and retain terms up to those bilinear in the fluctuations. The gravitational and Maxwell parts of the Lagrangian are now standard.[3] The contribution of the scalar after some tedious calculation is given by

$$\begin{aligned}S_{\rm scalar}=-\frac{1}{2t}\int d^6z\sqrt{-g}\,\Bigg[&2\partial_a z_-\partial^a z_+ +2\tilde{\nabla}_+z_+\tilde{\nabla}_-z_- +2\tilde{\nabla}_+z_-\tilde{\nabla}_-z_+ -\frac{2}{a^2}(\partial_\theta\bar{\phi}^\theta)^2 z_+z_-\\&-2\kappa\partial_\theta\bar{\phi}^\theta[e^{i(n-1)\phi}(h^a_+\partial_a z_- +h_{++}\tilde{\nabla}_-z_-)+e^{-i(n-1)\phi}(h^a_-\partial_a z_+ +h_{--}\tilde{\nabla}_+z_+)]\\&+2\kappa\partial_\theta\bar{\phi}^\theta\left[\frac{h}{2}-h_{+-}\right](e^{i(n-1)\phi}\tilde{\nabla}_+z_- +e^{-i(n-1)\phi}\tilde{\nabla}_-z_+)\\&+(\partial_\theta\bar{\phi}^\theta)^2\left[\kappa^2h^\mu_L h^L_\mu-\frac{\kappa^2}{2}hh^\mu_\mu\right]\Bigg]\ ,\end{aligned} \tag{12}$$

where we use $+,-$ helicity eigenstates of Ref. 3 and especially

$$z_\pm=\pm i\frac{a}{\sqrt{2}}\exp(\pm i\bar{\phi}^\phi)(z^\theta\pm i\sin\bar{\phi}^\theta z^\phi)\ . \tag{13}$$

The modified covariant derivatives on $z_\pm$ are

$$\tilde{\nabla}_\pm z_\pm=\frac{e^{\pm i\phi}}{\sqrt{2}a}\left[\pm i\partial_\theta z_\pm-\frac{1}{\sin\theta}\partial_\phi z_\pm\mp\frac{in}{\sin\theta}(\cos\bar{\phi}^\theta-1)z_\pm\right]\ , \tag{14a}$$

$$\tilde{\nabla}_\pm z_\mp=\frac{e^{\pm i\phi}}{\sqrt{2}a}\left[\pm i\partial_\theta z_\mp-\frac{1}{\sin\theta}\partial_\phi z_\mp\pm\frac{in}{\sin\theta}(\cos\bar{\phi}^\theta-1)z_\mp\right]\ . \tag{14b}$$

As can be seen from the above equations, the effective isohelicities of $z_\pm$ are $\pm n$.

At this point we specify to the light-cone gauge.[8] In the light-cone coordinates the scalar product takes the form

$$A\cdot B=A_{\mp}B_{\pm}+A_{\pm}B_{\mp}+A_iB_i,\ \ i=1,2\ . \tag{15}$$

We now put $V_{\pm}=h_{A\pm}=0$. It turns out that the $\mp$ component of V and h_{AB} are nonpropagating fields which may be eliminated by use of their field equations. These procedures are well described in Ref. 8. The Lagrangian $\mathscr{L}$ separates into sectors according to the transverse O(2) quantum number of the fields:

$$\mathscr{L}^{\pm 2}=\tfrac{1}{4}h^T_{ij}\Box h^T_{ij}\ , \tag{16a}$$

$$\mathscr{L}^{\pm 1}=h_{i+}\Box h_{i-}+\frac{\sqrt{2}i}{a}(h_{i+}V_{i;-}-h_{i-}V_{i;+})+\tfrac{1}{2}V_i\Box V_i+\bar{R}_{+-}h_{i+}h_{i-}\ , \tag{16b}$$

$$\begin{aligned}\mathscr{L}^0=&h_{+-}\Box h_{+-}+\tfrac{1}{2}h_{++}\Box h_{--}-\left[\frac{\sqrt{2}i}{a}V_+ +\frac{\kappa}{t}\partial_\theta\bar{\phi}^\theta e^{-i(n-1)\phi}z_+\right]\left[-\frac{\sqrt{2}i}{a}V_- +\frac{\kappa}{t}\partial_\theta\bar{\phi}^\theta e^{i(n-1)\phi}z_-\right]\\&-\frac{\epsilon}{a^2n^2}(\partial_\theta\bar{\phi}^\theta)^2(h_{+-}{}^2-h_{++}h_{--})+\bar{R}_{+-}(h_{+-}{}^2+2h_{++}h_{--})\\&-\frac{\sqrt{2}i}{a}(-h_{++}V_{-;-}+h_{--}V_{+;+}+h_{+-}V_{-;+}-h_{-+}V_{t;-})+V_+\Box V_- +\bar{R}_{+-}V_+V_-\ ,\end{aligned} \tag{16c}$$

$$\mathcal{L}^{0\prime}=-\frac{1}{t}\partial_a z_-\partial_a z_+ -\frac{1}{t}\tilde{\nabla}_+ z_+\tilde{\nabla}_- z_- -\frac{1}{t}\tilde{\nabla}_+ z_-\tilde{\nabla}_- z_+ +\frac{1}{ta^2}(\partial_\theta\bar{\phi}^\theta)^2 z_+ z_-$$
$$+\frac{\kappa}{t}\partial_\theta\bar{\phi}^\theta e^{i(n-1)\phi}(h_{++}\tilde{\nabla}_- z_- + h_{+-}\tilde{\nabla}_+ z_- + h_{++;-}z_- + h_{+-;+}z_-)$$
$$+\frac{\kappa}{t}\partial_\theta\bar{\phi}^\theta e^{-i(n-1)\phi}(h_{--}\tilde{\nabla}_+ z_+ + h_{+-}\tilde{\nabla}_- z_+ + h_{-+;-}z_+ + h_{--;+}z_+)\ , \qquad (16d)$$

where $\Box$ is the "d'Alembertian" on $M^4\times B^2$ and $h_{ij}^T=h_{ij}-\frac{1}{2}\delta_{ij}h_a{}^a$ and

$$\bar{R}_{+-}=\frac{1}{a^2}\left[-1+\frac{\epsilon g''}{2}+\left[\epsilon g'-\frac{\epsilon f'}{2}\right]\cot\theta+\epsilon f\right]\ . \qquad (17)$$

Note that the light-cone coordinates $\mp,\simeq$ are completely absent in the above expressions.

First, we analyze the mass spectrum in the limit $\epsilon\to 0$ or $\kappa^2 a^2/t\to 0$. The various fields are now expanded in harmonics $D^l_{\lambda m}$ of the internal space $S^2=\mathrm{SU}(2)/\mathrm{U}(1)$, i.e.,

$$D^l_{\lambda m}(L_{\theta\phi}{}^{-1})=D^l_{\lambda m}(e^{\mp\phi\theta_3}e^{\theta\theta_2}e^{\phi\theta_3})$$
$$=e^{\mp i\lambda\phi}d^l_{\lambda m}(\theta)e^{im\phi} \qquad (18)$$

in the notation of Wigner. The expansion method in coset space is well discussed by Salam and Strathdee.[9] The fields are decomposed into irreducible representations of the SO(2) rotations, labeled by the "isohelicity" λ. In particular, as noted previously, the isohelicities of the scalar fields are $\pm n$, where n is the winding number of the homotopy classification. The spectrum of the internal Laplacian can be found by algebraic methods if the internal space is a coset space.[9] Using these methods and integrating over the internal space θ,ϕ in Eqs. (16) we can obtain the mass matrix for each Lagrangain in (16). After diagonalizing these matrices we can get the following mass spectrum:

$$\mathcal{L}^{\pm 2}:\ \frac{l(l+1)}{a^2},\ l\geq 0\ ; \qquad (19a)$$

$$\mathcal{L}^{\pm 1}:\ \frac{l(l+1)}{a^2},\ \frac{l(l+1)\pm\sqrt{2l(l+1)}}{a^2},\ l\geq 1\ ,$$
$$\frac{0}{a^2},\ l=0\ ; \qquad (19b)$$

$$\mathcal{L}^{0}:\ \frac{l(l+1)}{a^2},\ \frac{l(l+1)\pm\sqrt{2l(l+1)}}{a^2}\ ,$$
$$\frac{1}{a^2}[l^2+l+\tfrac{1}{2}\pm\tfrac{1}{2}\sqrt{1+12l(l+1)}],\ l\geq 2\ ,$$
$$\frac{2}{a^2},\frac{4}{a^2},\frac{5}{a^2},\ l=1\ ,\ \frac{1}{a^2},\ l=0\ . \qquad (19c)$$

It is difficult to obtain the full spectrum of the $\mathcal{L}^{0\prime}$ part even in the limit $\epsilon\to 0$, because states with different l's no longer decouple. Rather we will concentrate on the zero mass spectrum of $\mathcal{L}^{0\prime}$.

The $\mathcal{L}^{0\prime}$ part can be expressed using conventional internal space Laplacian ∇^2 in the limit $\epsilon\to 0$ as follows:

$$\mathcal{L}^{0\prime}=-z_-\partial^2 z_+ - z_-\nabla^2 z_+$$
$$+\frac{n^2}{a^2\sin^2\theta}(2\cos^2\bar{\phi}^\theta-\cos^2\theta-1)z_+z_-\ . \qquad (20)$$

The zero modes of the spectrum are always related to the symmetry of the classical solution. For the $\mathcal{L}^{0\prime}$ part, it is related to the rotational symmetry of the σ-field target manifold. Thus the zero-mode fluctuations of the scalars can be identified as $z_+\propto D^{l=1}_{1,m}(\bar{\phi}^\theta,\bar{\phi}^\phi)$ with $m=\pm 1,0$. We have explicitly checked that $z_+\propto(1+\cos\bar{\phi})$, $\exp(i\bar{\phi}^\phi)\sin\bar{\phi}^\theta$, $\exp(2i\bar{\phi}^\phi)(1-\cos\bar{\phi}^\theta)$, and $z_-=z_+^*$ are really the zero modes of the $\mathcal{L}^{0\prime}$ part, by substituting these $z_\pm$ into Eq. (16d) and integrating over internal space θ,ϕ. It can be easily shown that $D^{l=1}_{1,m}(\bar{\phi}^\theta,\bar{\phi}^\phi)$ can be expanded in harmonics of isohelicities $\pm n$ as

$$D^{l=1}_{1,m}(\bar{\phi}^\theta,\bar{\phi}^\phi)=\sum_{l\geq n}a_l D^l_{n,nm}(\theta,\phi),\ \ m=0,\pm 1 \qquad (21)$$

for $\bar{\phi}^\theta,\bar{\phi}^\phi$ of Eq. (4c). There are six zero modes corresponding to three $m=\pm 1,0$ states and two degrees of freedom from the complex nature of $z_\pm$ for each m.

We can now understand the mass spectrum of the model in the $\epsilon\to 0$ limit. Each massive graviton of mass $\partial^2=l(l+1)/a^2$ for $l\geq 1$ has five helicity states, helicity ± 2 from $\mathcal{L}^{\pm 2}$, helicity ± 1 from $\mathcal{L}^{\pm 1}$, and the helicity-0 partner of this state is found in the system $\mathcal{L}^0$. The gauge bosons of mass

$$\partial^2=\frac{1}{a^2}[l(l+1)\pm(2l^2+2l)^{1/2}]$$

have similar structure: two transversal helicities ± 1 are found in the $\mathcal{L}^{\pm 1}$ system and one longitudinal helicity-0 state is found in the $\mathcal{L}^0$ system. The massless photon of $l=0$ in the $\mathcal{L}^\pm$ system has no corresponding helicity-0 state in the $\mathcal{L}^0$ system. The massless graviton has only helicity ± 2 states in the $\mathcal{L}^{\pm 2}$ system. The massless gauge bosons of SU(2) from the $\mathcal{L}^{\pm 1}$ system for $l=1$ will be shown to get small masses of order ϵ/a^2 after the ϵ correction. The corresponding helicity-0 state can only be found in the $\mathcal{L}^{0\prime}$ system. This fact has been explicitly checked for the $n=1$ case. Then there remains three massless scalars from the six zero modes, which also acquire small masses of ϵ order. This mass correction should be of nontachyonic nature due to the nontrivial topology of the background solutions. There always occurs three scalars of very small masses due to the above mechanism for the Einstein-Maxwell theory weakly coupled to the nonlinear σ field for any winding number.

We will now calculate the ϵ correction to the mass spectrum. As ϵ is very small the correction to the massive spectrum is rather meaningless. The massless states of

$l=0$, i.e., the graviton and the photon, receive no ϵ corrections as will be seen. The corrections on the massless states from the $\mathscr{L}^{0\prime}$ system are very difficult. Fortunately, the correction on the massless gauge bosons from the $\mathscr{L}^{\pm 1}$ system which is most interesting is rather simple.

We choose a covariant basis $E^{\pm}$ which is suitable for the manifold with the metric of Eq. (4a):

$$E^{\pm}=\pm\frac{a}{i\sqrt{2}}e^{\mp i\phi}\left[\left[1+\frac{\epsilon f}{2}\right]d\theta \mp i\sin\theta\left[1+\frac{\epsilon g}{2}\right]d\phi\right] . \tag{22}$$

The spin connection which satisfies $dE^{\pm}=-\omega^{\pm\mp}\wedge E^{\pm}$ up to $O(\epsilon^2)$ is given by

$$\omega^{+-}=-\omega^{-+}=-i\left[\left[1-\frac{\epsilon f}{2}+\frac{\epsilon g}{2}\right]\cos\theta-1+\frac{\epsilon g'}{2}\sin\theta\right]d\phi \tag{23}$$

which of course modifies the conventional covariant derivatives. We can show that

$$\nabla_{\pm}=\frac{e^{\pm i\phi}}{\sqrt{2}a}\left\{\pm\left[1-\frac{\epsilon f}{2}\right]\partial_\theta-\frac{1-\epsilon g/2}{\sin\theta}\partial_\phi-\frac{i}{\sin\theta}\left[1-\frac{\epsilon g}{2}\right]\left[\left[1-\frac{\epsilon f}{2}+\frac{\epsilon g}{2}\right]\cos\theta-1+\frac{\epsilon g'}{2}\sin\theta\right]\lambda\right\} , \tag{24a}$$

$$\nabla_{\pm}D^1_{01}=\frac{1}{a}\left[1+\left[\mp\frac{\epsilon f}{2}\cos\theta-\frac{\epsilon g}{2}\right]\Big/(1\pm\cos\theta)\right]D^1_{\pm 11} , \tag{24b}$$

$$\nabla^2 D^0_{00}=0 , \tag{24c}$$

$$\nabla^2 D^1_{00}=\frac{1}{a^2}\left[-2(1-\epsilon f)+\frac{\epsilon}{2}(f'-g')\tan\theta\right]D^1_{00} , \tag{24d}$$

$$(\nabla^2+\overline{R}_{+-})D^1_{\pm1\pm1}=\frac{1}{a^2}\left[-2+2\epsilon f+\frac{\epsilon(g-f)}{1+\cos\theta}+\frac{\epsilon f'}{2}\frac{1-2\cos\theta}{\sin\theta}+\frac{\epsilon g'}{2}\frac{1+\cos\theta}{\sin\theta}+\frac{\epsilon}{2}g''\right]D^1_{\pm1\pm1} , \tag{24e}$$

$$(\nabla^2+\overline{R}_{+-})D^1_{\pm1\mp1}=\frac{1}{a^2}\left[-2+2\epsilon f+\frac{\epsilon(g-f)}{1-\cos\theta}-\frac{\epsilon f'}{2}\frac{1+2\cos\theta}{\sin\theta}-\frac{\epsilon g'}{2}\frac{1-\cos\theta}{\sin\theta}+\frac{\epsilon}{2}g''\right]D^1_{\pm1\mp1} , \tag{24f}$$

and similarly for others.

Using Eq. (24c) it is easy to see that the graviton and photon remains massless up to ϵ^2 order. Applying the above formulas to the $l=1$ multiplet of the $\mathscr{L}^{\pm 1}$ system we can get these bilinears into the form, where we suppress the isospin labels,

$$(h^*_{i+}\ \ h^*_{i-}\ \ V^*_i)\begin{bmatrix}-\partial^2+A & 0 & \frac{\sqrt{2}}{a}iD\\ 0 & -\partial^2+B & -\frac{\sqrt{2}}{a}iE\\ -\frac{\sqrt{2}}{a}iD & \frac{\sqrt{2}}{a}iE & -\partial^2+C\end{bmatrix}\begin{bmatrix}h_{i+}\\ h_{i-}\\ V_i\end{bmatrix} , \tag{25}$$

where

$$A=-\int d\theta\, d\phi\sqrt{-\overline{g}}\,D^{1*}_{1m}(\nabla^2+\overline{R}_{+-})D^1_{1m}\Big/\int d\theta\, d\phi\sqrt{-\overline{g}}\,|D^1_{1m}|^2 , \tag{26a}$$

$$C=-\int d\theta\, d\phi\sqrt{-\overline{g}}\,D^{1*}_{0m}\nabla^2 D^1_{0m}\Big/\int d\theta\, d\phi\sqrt{-\overline{g}}\,|D^1_{0m}|^2 , \tag{26b}$$

$$D=\int d\theta\, d\phi\sqrt{-\overline{g}}\,D^{1*}_{-1m}\nabla_- D^1_{0m}\Big/\left[\left[\int d\theta\, d\phi\sqrt{-\overline{g}}\,|D^1_{-1m}|^2\right]\left[\int d\theta\, d\phi\sqrt{-\overline{g}}\,|D^1_{0m}|^2\right]\right]^{1/2} , \tag{26c}$$

and so on, with

$$\overline{g}=g_{\theta\theta}g_{\phi\phi}=a^4(1+\epsilon f+\epsilon g)\sin^2\theta . \tag{27}$$

There seems a large possibility for the forms of f and g which satisfy Eq. (10b). But they are severely constrained for our perturbation scheme to work. The covariant derivatives acting on $D^l_{\lambda m}$ like Eqs. (24) should not introduce any singularity in the region $0\le\theta\le\pi$. The most dangerous point is $\theta=0$ and π. We can choose the following forms for f and g which are free from the above singularities:

$$f(\theta)=\frac{1}{\cos^2\theta}\int_{\pi/2}^{\theta}\frac{-2\sin^{2n-2}\frac{\theta}{2}\cos^{2n-2}\frac{\theta}{2}}{\left[\sin^{2n}\frac{\theta}{2}+\cos^{2n}\frac{\theta}{2}\right]^2}\times\sin\theta\cos\theta\, d\theta , \tag{28a}$$

$$g(\theta)=\text{const}=f(\theta=0)=f(\theta=\pi)\ . \tag{28b}$$

$f(\theta)$ is a solution of the differential equation

$$-\frac{f'}{2}\cot\theta+f=\frac{\sin^{2n-2}\frac{\theta}{2}\cos^{2n-2}\frac{\theta}{2}}{\left[\sin^{2n}\frac{\theta}{2}+\cos^{2n}\frac{\theta}{2}\right]^2}\ . \tag{29}$$

In fact f and g of Eqs. (28) correspond to a new metric:

$$g_{\theta\theta}=\tilde{a}^2[1+\epsilon\tilde{f}(\theta)]\ ,$$
$$g_{\phi\phi}=\tilde{a}^2\sin^2\theta\, d\phi^2 \tag{30}$$

with $\tilde{a}=a[1+\frac{1}{2}\epsilon f(\theta=0)]$ and $\tilde{f}(\theta=0)=\tilde{f}(\theta=\pi)=\tilde{f}'(\theta=0)=\tilde{f}'(\theta=\pi)=0$. It can be easily checked that the covariant derivatives of this new metric do not introduce any singularity when they are applied to every $D^l_{\lambda m}$.

We return now to our mass matrix of Eq. (25). Using Eqs. (28) and (24) we can calculate various quantities in Eqs. (26). For the case of $n=2$, $f(\theta)$ and $g(\theta)$ are

$$f(\theta)=\frac{2}{1+\cos^2\theta}-\frac{1}{\cos^2\theta}\ln(1+\cos^2\theta)\ , \tag{31a}$$

$$g(\theta)=1-\ln 2\ , \tag{31b}$$

and A, B, C, D, and E are calculated to be

$$A=B=C=\frac{2}{a^2}-\frac{3\epsilon}{2a^2}(4-\pi)\ , \tag{32a}$$

$$m=0\ ,$$

$$D=E=\frac{1}{a}[1+\epsilon(\tfrac{3}{8}\pi-\tfrac{3}{2})]\ , \tag{32b}$$

$$A=B=C=\frac{2}{a^2}-\frac{3\epsilon}{4a^2}(\pi-2)\ , \tag{32c}$$

$$m=\pm 1\ .$$

$$D=E=\frac{1}{a}[1-\tfrac{3}{16}\epsilon(\pi-2)]\ , \tag{32d}$$

The simplest case is that of $n=1$:

$$f(\theta)=g(\theta)=1,\ \ A=B=C=\frac{2}{a^2}(1-\epsilon)\ ,$$

$$D=E=\frac{1}{a}\left[1-\frac{\epsilon}{2}\right],\ \ m=0,\pm 1\ . \tag{33}$$

For the other cases we rely on numerical procedures and have found that A, B, and C are equal to each other and D and E are equal for all n. Then the three roots of Eq. (25) are simply $\partial^2=A$ and $\partial^2=A\pm(2/a)D$. The gauge bosons corresponding to $\partial^2=A-(2/a)D$ acquire small masses of order ϵ. The helicity ± 1 partner of the massive graviton has the mass of $\partial^2=A$. Of course, this mass must be equal to that of the helicity ± 2 partner, which is just $\partial^2=C$. Except for the $n=1$ case, the gauge bosons acquire different ϵ corrections according to the quantum number m. This global breaking of SU(2) symmetry is due to the ϵ terms in Eq. (4a). We display in Table I the masses of gauge bosons corresponding to $(\partial^2)^{1/2}=[A-(2/a)D]^{1/2}$ in units of $\sqrt{-\epsilon}$ for the $m=\pm 1$ and $m=0$ sectors and their relative ratio with various winding numbers n.

In a sense, the $m=0$ sector corresponds to the Z boson and the $m=\pm 1$ sector corresponds to the $W^\pm$ boson of the standard model. The analysis for the $n=1$ case can be thoroughly carried out for all $\mathscr{L}^{\pm 2}$, $\mathscr{L}^{\pm 1}$, $\mathscr{L}^0$, and $\mathscr{L}^{0'}$ system and the results are in complete agreement with those of Ref. 1 when we substitute a as $a(1+\epsilon/2)$ and δ as $1+\epsilon/2$ in the results of Ref. 1.

IV. DISCUSSIONS AND CONCLUSIONS

As mentioned before the analysis of the spectrum of the helicity-0 sector is very difficult because states with different values of l no longer decouple. But it is quite certain that the remaining three zero modes of the $\mathscr{L}^{0'}$ system acquire positive mass square of order ϵ after the ϵ correction. This expectation is based on the nontrivial topology of the background solutions of two matter fields.[10] In fact, we can explicitly check this claim for the $n=1$ case as in Ref. 1.

As is well known, it is possible to obtain chiral fermions under the nontrivial background of the Maxwell field in six-dimensional theory.[10] Another interesting property of a six-dimensional theory suitably extended to supergravity coupled to matter fields is the theory is free of both gravitational and Yang-Mills gauge anomalies.[11] We hope that these observations more or less justify our choice of six dimensions.

The initial motivation of Kaluza was to unify the gravitational and electromagnetic forces. From this point of view, the scheme explained here is rather anti—Kaluza-

TABLE I. Masses of the gauge bosons and their relative ratio for various winding numbers n.

Winding number n	Mass in unit of $\sqrt{-\epsilon/a}$: $m=0$ sector	$m=\pm 1$ sector	ratio $=\frac{(m=\pm 1)}{(m=0)}$
1	1	1	1
2	0.8024	0.6543	0.8154
3	0.6801	0.5184	0.7622
4	0.5983	0.4427	0.7400
5	0.5394	0.3931	0.7289
10	0.3857	0.2750	0.7128
20	0.2736	0.1938	0.7085
50	0.1730	0.1224	0.7073

Klein. But it is well known that phenomenologically interesting Kaluza-Klein models generally contain more than the gravitational field. The Maxwell field is necessary for the compactification of the internal space. The nonlinear σ field can be interpreted to be not fundamental but rather an effective field theory for composite scalars.[2] The six dimensions are required to obtain the SU(2) gauge boson. In this sense, our scheme explains economically the phenomenologically successful standard model.

In conclusion, one can break the global SU(2) symmetry as well as the local symmetry, thus splitting the masses of the three gauge bosons. For this purpose, higher-winding-number solutions of the σ fields are utilized to break the rotational symmetry of internal space, reducing its isometry group SU(2) to SO(2). We hope that these results are of some value in constructing some specific physically interesting models, especially for the electroweak theory. Obviously, there remains much to study to make our models realistic: that is, the ratio of masses between the gauge bosons after the renormalization-group effect, the exact content of physical gauge bosons, and couplings to other matter fields such as fermions, etc. The most serious of all is the question that gauge symmetries are indeed obtained by a Kaluza-Klein mechanism.

ACKNOWLEDGMENTS

The author would like to thank J. Sobczyk for showing him his results prior to publication and Professor J. Strathdee and Dr. E. Sezgin for reading the manuscript. He would also like to thank Professor Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. This research was supported in part by the Research Institute for Basic Sciences, Kyung Hee University, Korea, and also by the Korea Science and Engineering Foundation.

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