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Locally supersymmetric string Jacobian

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A representation of the superstring Jacobian is given for a general choice of the gauge slice $\hat{e}_{a\mu}(x)$, $\hat{\psi}_\mu(x)$, $\hat{A}(x)$. The expression involves an integral over the expected ghost fields $c_a(x)$ and $\lambda(x)$ and traceless antighost fields $b^{ab}(x)$ and $\beta(x)$, and the action is invariant under local supersymmetry and super-Weyl transformations. Conserved currents of the ghost system are calculated directly from this action.

I. INTRODUCTION

In the Polyakov approach[1,2] to the bosonic string one must integrate over all world-sheet metrics $g_{\mu\nu}(x)$ with the effects of diffeomorphisms and Weyl transformations divided out. For this purpose, one chooses a reference metric $\hat{g}_{\mu\nu}(x)$ and calculates the Jacobian of traceless $(h_{\mu\nu})$ and trace $(\tau\hat{g}_{\mu\nu})$ fluctuations with respect to infinitesimal diffeomorphisms generated by vector fields (ξ^μ) and Weyl transformations (σ) using the equations

$$\begin{aligned} h_{\mu\nu} &= \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu - \hat{g}_{\mu\nu} \nabla\cdot\xi \,, \\ 2\tau\hat{g}_{\mu\nu} &= (2\sigma + \nabla\cdot\xi)\hat{g}_{\mu\nu} \,, \end{aligned} \tag{1}$$

where ∇_μ is the covariant derivative in the reference metric. This Jacobian can be represented as an integral over formally real anticommuting ghost fields $c_\mu(x)$ and (traceless symmetric) antighost fields $b^{\mu\nu}(x)$ as

$$J_B(\hat{g}) = \int (db^{\mu\nu})(dc_\rho)\exp\left[\int d^2x\sqrt{-\hat{g}}\, b^{\mu\nu}\nabla_\mu c_\nu\right] . \tag{2}$$

This action is manifestly invariant under reparametrizations of the reference metric combined with standard tensor transformation rules for the ghosts. In treatments where global issues are ignored, one chooses a conformally flat reference metric $\hat{g}_{\mu\nu}(x) = e^{2\hat{\rho}(x)}\delta_{\mu\nu}$ and introduces complex coordinates z and $\bar{z}$. In this basis the independent ghost components b_{zz}, c_z and their conjugates are the ghost fields of the Becchi-Rouet-Stora (BRS) formalism[3] and are important in the covariant second quantization procedure[4] as presently conceived. The ghost stress tensor, whose trace anomaly cancels that of the 26 matter fields, can be derived by conformal methods[5,6] although it is perhaps more straightforward to obtain it by variation of (2) with respect to $\hat{g}_{\mu\nu}$ (see Sec. IV below). One may also hope that the generally covariant action (2) will be relevant in some future formulation of the second quantized theory, where the world-sheet reparametrization invariance plays a more central role.

In this paper we obtain the analog of the Jacobian (2) for the Neveu-Schwarz-Ramond (NSR) superstring. Here, one must integrate over all component supergravity multiplets of frame $e^a_\mu(x)$, gravitino $\psi_\mu(x)$, and auxiliary $A(x)$ fields with the effects of diffeomorphisms, local Lorentz, local supersymmetry, and super-Weyl transformations divided out. One then chooses a reference supergravity multiplet $\hat{e}^a_\mu, \hat{\psi}_\mu, \hat{A}$. Here it is usually convenient to choose a superconformal frame $\hat{e}^a_\mu = e^{\hat{\rho}}\delta^a_\mu$, $\hat{\psi}_\mu = i\hat{\gamma}^\mu\hat{\chi}$, $\hat{A}=0$, and the Jacobian can be expressed as an integral over a real ghost multiplet which consists of an anticommuting vector c_a and commuting spinor λ and a real antighost multiplet containing an anticommuting traceless symmetric tensor b^{ab} and a commuting γ traceless vector spinor β^a. It is in this form that the ghosts have entered in recent treatments of the BRS quantization,[7] the fermion emission vertex,[8] and the second quantization of the NSR superstring.[9] In our work the reference supergravity multiplet is allowed to be arbitrary, and we obtain an expression for the Jacobian:

$$J_{\rm NSR}(\hat{e}^a_\mu, \hat{\psi}_\mu, \hat{A}) = \int (db^{ab})(d\beta^a)(dc_a)(d\lambda)\exp(iS) . \tag{3}$$

The action can be written in several forms which differ by redefinition of the ghost fields. The simplest form is

$$S = \int d^2x\, \hat{e}\left[-ib^{\mu\nu}\nabla_\mu c_\nu - i\bar{\beta}^\mu\nabla_\mu\lambda + (\bar{\hat{\psi}}_\mu - \tfrac{1}{2}\bar{\hat{\psi}}\cdot\hat{\gamma}\hat{\gamma}_\mu)\left[\frac{3}{2}\beta^\nu\nabla_\nu c^\mu - \frac{3}{2}\beta^\mu\nabla\cdot c + \nabla\cdot\beta c^\mu - \nabla_\nu\beta^\mu c^\nu - \frac{i}{2}b^{\mu\nu}\gamma_\nu\lambda\right]\right] \tag{4}$$

in which only the γ-traceless part of $\hat{\psi}_\mu$ enters. This action is invariant under local supersymmetry and super-Weyl transformation rules discussed below. The transformation rules involve the auxiliary field $\hat{A}$ although the action does not. Expressions for the ghost contribution to the stress tensor and supercurrent can be derived straightforwardly from (4), although superconformal methods can also be used.[5,6] The more general form of the ghost action may be useful for further development of the second quantized superstring formalism.

We work throughout with Lorentzian signature world-sheet metrics and emphasize local considerations only. For these reasons we do not discuss moduli and supermoduli which are necessary for a globally correct treatment of the gauge-fixing and the full superstring integration measure. The Jacobian (3) is one important ingredient of this measure, and we expect that the present treatment can be extended to incorporate moduli, perhaps in a more general setting than in other recent work on the bosonic string[10] and superstring.[11]

The representation (3) and (4) of the Jacobian is obtained as follows. In Sec. II we use the transformation rules of the supergravity multiplet to obtain a formal representation of the Jacobian as an integral involving nine antighosts, one for each component of the multiplet, and nine ghosts, one for each component of the supergravity and super-Weyl algebra. The integration over $5+5$ of these fields is trivial and yields an integral over four ghosts c_μ and λ and four constrained antighosts $b^{\mu\nu}$ and β^μ. The fact that the analogous action (2) for the bosonic string is reparametrization invariant and (formally) Weyl invariant suggests that our expression for the superstring determinant is locally supersymmetric and (formally) super-Weyl invariant. To obtain the ghost transformation rules, we start afresh in Sec. III, and consider the coupling to $d=2$ supergravity of the ghost-antighost system starting on flat world sheet with suitable global supersymmetry variations. The action thus obtained differs from that of Sec. II by simple ghost field redefinitions. The conserved currents of the ghost system, such as the stress tensor and supercurrent are obtained in Sec. IV. The algebra of the ghost field transformation rules is not closed and this situation is discussed in Sec. V. A very brief description is given there of a more general super-Weyl-invariant Lagrangian of the superstring ghosts involving quartic ghost couplings.

In the notation used here $\eta^{ab}=(1,-1)$, $\epsilon^{ab}=-\epsilon^{ba}$, and $\epsilon^{01}=1$, while γ matrices are imaginary 2×2 matrices which satisfy $\{\gamma^a,\gamma^b\}=2\eta^{ab}$, and $\gamma_5=\gamma^0\gamma^1$ is real. The covariant derivative D_μ contains Christoffel and spin connections without torsion; $\widetilde{D}_\mu$ includes torsion. When evaluated in the reference configuration $\hat{e}_\mu^{\,a},\hat{\psi}_\mu,\hat{A}$, these same derivatives are denoted by ∇_μ and $\widetilde{\nabla}_\mu$, respectively. In Sec. IV we will briefly use supercovariant derivatives $\mathscr{D}_\mu$ which are defined there.

II. SUPERSTRING JACOBIAN

The local supersymmetry transformation rules of the $N=1$, $d=2$ supergravity multiplet are

$$\begin{aligned}
&\delta_Q(\epsilon)e_\mu^{\,a}=-i\bar{\epsilon}\gamma^a\psi_\mu\ ,\\
&\delta_Q\psi_\mu=2(\widetilde{D}_\mu\epsilon+\tfrac{1}{2}i\gamma_\mu A\epsilon)\ ,\\
&\delta_Q A=\bar{\epsilon}\left[\frac{1}{2}iA\gamma\cdot\psi-\gamma_5\frac{\epsilon^{\rho\lambda}}{e}\widetilde{D}_\rho\psi_\lambda\right]\ ,
\end{aligned}\tag{5}$$

where $\widetilde{D}_\mu$ is the spinor covariant derivative with torsion

$$\begin{aligned}
&\widetilde{D}_\mu=\partial_\mu+\tfrac{1}{2}\widetilde{\omega}_\mu\gamma_5\ ,\\
&\widetilde{\omega}_\mu=\omega_\mu(e)-\frac{i}{4}\frac{\epsilon^{\rho\lambda}}{e}\bar{\psi}_\rho\gamma_\mu\psi_\lambda\ ,\\
&\omega_\mu(e)=-e^{-1}e_\mu^{\,a}\epsilon^{\rho\lambda}\partial_\rho e_{a\lambda}\ .
\end{aligned}\tag{6}$$

The commutator of two variations gives a closed algebra with field-dependent parameters: namely,

$$\begin{aligned}
[\delta_Q(\epsilon_1),\delta_Q(\epsilon_2)]\Phi=&\,\delta_G(\xi)\Phi\\
&+\delta_L(\xi\cdot(\widetilde{\omega}-\omega(e))+2\bar{\epsilon}_1\gamma_5\epsilon_2 A)\Phi\\
&+\delta_Q(-\tfrac{1}{2}\xi\cdot\psi)\Phi
\end{aligned}\tag{7}$$

with diffeomorphism parameter $\xi^\mu=2i\bar{\epsilon}_1\gamma^\mu\epsilon_2$. We use standard local Lorentz transformations and "Lorentz covariantized" diffeomorphisms

$$\delta_L(\Lambda)e_\mu^{\,a}=-\Lambda\epsilon^a{}_b e_\mu^{\,b},\quad \delta_L\psi_\mu=\tfrac{1}{2}\gamma_5\Lambda\psi_\mu,\quad \delta_L A=0\ . \tag{8}$$

$$\begin{aligned}
&\delta_G(\xi)e_\mu^{\,a}=D_\mu\xi^a,\quad \delta_G\psi_\mu=\xi^\rho D_\rho\psi_\mu+D_\mu\xi^\rho\psi_\rho\ ,\\
&\delta_G A=\xi^\rho\partial_\rho A\ .
\end{aligned}\tag{9}$$

The transformation rules of the super-Weyl group are

$$\delta_c(\sigma)e_\mu^{\,a}=\sigma e_\mu^{\,a},\quad \delta_c\psi_\mu=\tfrac{1}{2}\sigma\psi_\mu,\quad \delta_c A=-\sigma A\ . \tag{10}$$

$$\delta_s(\eta)e_\mu^{\,a}=0,\quad \delta_s\psi_\mu=i\gamma_\mu\eta,\quad \delta_s A=\frac{i}{2}\bar{\eta}\gamma\cdot\psi\ . \tag{11}$$

$$\delta_u(u)e_\mu^{\,a}=0,\quad \delta_u\psi_\mu=0,\quad \delta_u A=u\ . \tag{12}$$

Four of the first six of these are standard and are defined so that the superstring matter action[12] is invariant. Then, $\delta_c A$ and $\delta_s A$ are obtained by requiring a uniform algebra. The introduction of the shift transformation δ_u is also necessary for a closed algebra. The matter action is independent of $A(x)$ and thus trivially invariant under $\delta_c A,\delta_s A,\delta_u A$. The algebra of combined supersymmetry and super-Weyl transformation rules is closed with complicated field-dependent parameters. This component representation of the super-Weyl transformations appears to be compatible with the superspace description.[13]

We now have nine fields $\Phi_I(x)$ subject to a gauge algebra with nine parameters $\epsilon_A(x)$. We choose an arbitrary reference configuration $\hat{\Phi}_I(x)$ as the gauge slice, and we need the Jacobian of gauge variations about the slice: namely,

$$J(\hat{\Phi}_I)=\mathrm{sdet}\left[\frac{\delta\Phi_I}{\delta\epsilon_A}\right]\Bigg|_{\Phi_I=\hat{\Phi}_I}\ . \tag{13}$$

For the bosonic string, the analogous Jacobian was calculated[10] after defining local reparametrization-invariant measures on the various function spaces involved. For the superstring, there do not seem to be any local supersymmetric measures which generalize those of the bosonic string (even for the simpler situation of the matter multiplet). Therefore, we will proceed formally (compare Ref. 11) and simply represent $J(\Phi_I)$ as an integral over ghosts $c_A(x)$ and antighosts $b^I(x)$ of the appropriate statistics:

$$\begin{aligned}
&J(\hat{\Phi}_F)=\int(db^I)(dc_A)e^{iS(b^I,c_A,\hat{\Phi}_I)}\ ,\\
&S(b^I,c_A,\hat{\Phi}_i)=-i\int d^2x\,\hat{e}b^I\frac{\delta\Phi_I}{\delta\epsilon_A}c_A
\end{aligned}\tag{14}$$

with "matrix elements" $\delta\Phi_I/\delta\epsilon_A$ obtained directly from the transformation rules (8)—(12). We refer to the antighosts of the bosonic (fermionic) fields as bosonic (fermionic), and to the ghosts of bosonic (fermionic) gauge parameters as bosonic (fermionic). The Bose-Bose terms in the action are real if we take real bosonic ghosts and antighosts. Since Fermi-Bose matrix elements are real, a real action requires that the fermionic antighost is real and we denote it by $-i\tilde{\beta}^\mu$ (since γ^0 is imaginary) with real $\tilde{\beta}^\mu$. Since Bose-Fermi matrix elements are imaginary, a real action requires that the fermionic ghosts are imaginary and we denote them by $\frac{1}{2}i\lambda$ for supersymmetry and $i\eta$ for super-Weyl with real λ and η. The notation then is $c^a,\tilde{\Lambda},\lambda,\tilde{\sigma},\eta,u$ for the ghost of coordinate, Lorentz, supersymmetry, Weyl, super-Weyl, and u transformations and, unconstrained antighosts $\tilde{b}_a^{\ \mu}$, $\tilde{\beta}^\mu$, and s for the fields $e_\mu^{\ a},\psi_\mu,A$. Given these preliminaries, the ghost action can be written as

$$S=-i\int d^2x\,\hat{e}\left\{\tilde{b}_a^{\ \mu}(\nabla_\mu c^a-\epsilon^a{}_b\hat{e}_\mu^{\ b}\tilde{\Lambda}+\hat{e}_\mu^{\ a}\tilde{\sigma}-\tfrac{1}{2}\bar{\psi}_\mu\gamma^a\lambda)\right.$$
$$+\bar{\tilde{\beta}}^\mu\left[-i(\nabla_\nu\psi_\mu+\psi_\nu\nabla_\mu)c^\nu-\frac{i}{2}\gamma_5\psi_\mu\tilde{\Lambda}-\frac{i}{2}\psi_\mu\tilde{\sigma}+\left[\tilde{\nabla}_\mu+\frac{i}{2}\gamma_\mu A\right]\lambda+i\hat{\gamma}_\mu\eta\right]$$
$$\left.+s\left[\partial_\nu Ac^\nu-A\tilde{\sigma}+\frac{i}{2}\left[\frac{\epsilon^{\rho\lambda}}{e}\tilde{\nabla}_\rho\bar{\psi}_\lambda\gamma_5-\frac{i}{2}\bar{\psi}\cdot\gamma A\right]\lambda+\tfrac{1}{2}\bar{\psi}\cdot\hat{\gamma}\eta+u\right]\right\}. \tag{15}$$

The integrals over many of the fields that appear in S are trivial, and we can proceed to integrate them out as follows. The integral over u gives a δ functional of s which can then be integrated immediately. The integral over η gives the constraint $\bar{\tilde{\beta}}^\mu\gamma_\mu=0$, and integrals over $\tilde{\sigma}$ and $\tilde{\Lambda}$ give δ functionals for the constraints $\hat{e}_{a\mu}b^{a\mu}=-(i/2)\bar{\beta}^\mu\psi_\mu$ and $\epsilon_{ab}b^{ab}=(i/2)\bar{\beta}^\mu\gamma_5\psi_\mu$. After integrating over the trace and antisymmetric parts of $\tilde{b}_a^{\ \mu}$ to enforce these constraints, we find the result

$$J(\hat{e}_\mu^{\ a},\hat{\psi}_\mu,\hat{A})=\int(db^{\mu\nu}d\beta^\mu dc_\mu d\lambda)\exp(iS)\,,$$
$$S=\int d^2x\,\hat{e}\left[-ib^{\mu\nu}\nabla_\mu c_\nu-i\bar{\beta}^\mu\tilde{\nabla}_\mu\lambda+\bar{\psi}_\mu\left[-\nabla_\nu(\beta^\mu c^\nu)+\beta^\nu\nabla_\nu c^\mu-\frac{i}{2}b^{\mu\nu}\hat{\gamma}_\nu\lambda-\tfrac{1}{4}\beta^\mu\nabla^\rho c_\rho+\tfrac{1}{4}\gamma_5\beta^\mu\frac{\epsilon^{\rho\lambda}}{e}\nabla_\rho c_\lambda\right]\right.$$
$$\left.+\tfrac{1}{8}\bar{\beta}\cdot\psi\bar{\psi}\cdot\gamma\lambda+\tfrac{1}{8}\bar{\beta}^\mu\gamma_5\psi_\mu\frac{\epsilon^{\rho\lambda}}{e}\bar{\psi}_\rho\gamma_\lambda\lambda\right], \tag{16}$$

where the antighost fields are now constrained, and there are both explicit $O(\psi^2)$ terms and others contained in the torsion.

The action S given above is rather unenlightening. A more useful form results after the field redefinitions

$$\lambda=\lambda'-ic^\mu\psi_\mu\,,$$
$$b^{\mu\nu}=b'^{\mu\nu}-\tfrac{1}{4}i(\bar{\beta}^\mu\psi^\nu-\bar{\beta}^\nu\psi^\mu-g^{\mu\nu}\bar{\beta}\cdot\psi)\,, \tag{17}$$

which gives (after suppressing primes)

$$S=\int d^2x\,\hat{e}\left[-ib^{\mu\nu}\nabla_\mu c_\nu-i\bar{\beta}^\mu\nabla_\mu\lambda+\bar{\psi}_\mu\left[\frac{3}{2}\beta^\nu\nabla_\nu c^\mu-\frac{3}{2}\beta^\mu\nabla\cdot c+\nabla\cdot\beta c^\mu-\nabla_\nu\beta^\mu c^\nu-\frac{i}{2}b^{\mu\nu}\gamma_\nu\lambda\right]\right.$$
$$\left.-\bar{\psi}_\mu\gamma\cdot\psi[\tfrac{3}{4}(\bar{\beta}^\mu\lambda)+\tfrac{1}{2}b^{\mu\nu}c_\nu]\right]. \tag{18}$$

A further field redefinition on (18), namely,

$$\lambda=\lambda'-\frac{i}{2}\gamma\cdot c\gamma\cdot\psi\,,$$
$$b^{\mu\nu}=b'^{\mu\nu}-\tfrac{3}{4}i\bar{\psi}\cdot\gamma\gamma^\mu\beta^\nu \tag{19}$$

brings the action to the form (4). It is for the form (18) of the ghost action that we will establish local supersymmetry in the next section. The manipulations which lead from (16) to (17) to (4) require fierce Fierz rearrangements and extensive use of special identities such as

$$\gamma^\mu\beta^\nu=\gamma^\nu\beta^\mu\,,$$
$$\epsilon^{\mu\nu}\epsilon_{\rho\lambda}=-e^{-2}(\delta^\mu_\rho\delta^\nu_\lambda-\delta^\mu_\lambda\delta^\nu_\rho)\,, \tag{20}$$
$$\bar{\psi}_\rho\gamma_\mu\psi_\lambda=\delta^\mu_\lambda\bar{\psi}_\rho\gamma\cdot\psi-\delta^\mu_\rho\bar{\psi}_\lambda\gamma\cdot\psi\,.$$

We suppress these painful details, but we note that the $O(\psi^3)$ terms generated by the field redefinitions (17) and (19) actually vanish. Note also that the field redefinitions are "triangular" and have trivial Jacobians.

III. LOCAL SUPERSYMMETRY OF THE GHOST ACTION

For the bosonic string, the reparametrization invariance of the ghost action is clear; one must simply take the standard transformation rules for the vector ghost c_ν and the tensor antighost $b^{\mu\nu}$. (The minor complication of the traceless constraint is easily handled.) By analogy, one expects that the superstring ghost action (18) is invariant under supersymmetry transformations (5) of the reference multiplets combined with transformation rules for the ghosts. To find the latter, we use an indirect procedure. Namely, we start on a flat world sheet with a ghost multiplet $c_a(x)$ and $\lambda(x)$ and constrained antighost multiplet $b^{ab}(x)$ and $\beta^a(x)$, and we "invent" transformation rules which leave the action

$$S=-i\int d^2x(b^{ab}\partial_a c_b+\bar{\beta}^a\partial_a\lambda) \tag{21}$$

invariant. These transformation rules are (for constant parameters ϵ)

$$\begin{aligned}&\delta c_a=\bar{\epsilon}\gamma_a\lambda,\quad \delta\lambda=\frac{i}{2}(\gamma^a\partial c_a)\epsilon\ ,\\ &\delta b^{ab}=\tfrac{1}{2}\bar{\epsilon}\partial\gamma^a\beta^b,\quad \delta\beta^a=b^{ab}\gamma_b\epsilon\ .\end{aligned} \tag{22}$$

They are simply the generalizations to Cartesian coordinates and a general Dirac basis of the known[6] transformation rules in the complex chiral basis, and the antighost variations satisfy the required constraints.

The next step is to consider the coupling of this rigid superconformal system to the supergravity multiplet e_μ^a,ψ_μ,A with transformation rules (5) and arbitrary $\epsilon(x)$. We do this using the well-known Noether procedure of supergravity. Thus, this section is logically independent of the previous one; the moment of truth will come when we compare the locally supersymmetric action obtained here with (18) and find that they coincide. We suppress the notational distinction between general and reference values of e_μ^a,ψ_μ,A and restore it at the end of the section.

To start the procedure, we write the covariant version of (21), namely

$$S=-i\int d^2x\, e(b^{a\mu}D_\mu c_a+\bar{\beta}^\mu D_\mu\lambda) \tag{23}$$

and consider its variation using the covariant (i.e., $\partial_a\to D_\mu$) version of (22). There are nonvanishing terms involving $D_\mu\bar{\epsilon}$ and a term of the form $R\bar{\epsilon}\beta_\mu c^\mu$, where R is the Ricci scalar of the world sheet. These terms may be canceled by the $\delta\psi_\mu\sim D_\mu\epsilon$ variations of a Noether-like term which we add to the action: namely,

$$S_N=\int d^2x\, e\bar{\psi}_\mu\left[\frac{3}{2}\beta^\nu\nabla_\nu c^\mu-\frac{3}{2}\beta^\mu\nabla\cdot c+\nabla\cdot\beta c^\mu -\nabla_\nu\beta^\mu c^\nu-\frac{i}{2}b^{\mu\nu}\gamma_\nu\lambda\right]. \tag{24}$$

There are now additional $\delta e_\mu^a\sim\bar{\epsilon}\gamma^a\psi_\mu$ variations of (23) and ghost variations of (24). These are canceled by adding the quadratic term

$$S^{(2)}=(\bar{\psi}_\mu\gamma\cdot\psi)(\tfrac{3}{4}\bar{\beta}^\mu\lambda+\tfrac{1}{2}b^{\mu\nu}c_\nu) \tag{25}$$

to the action and terms linear in ψ_μ and A in $\delta\lambda$ and δb^{ab}. Further variations of order $O(\psi^2)$ are then canceled by $O(\psi^2)$ modifications of $\delta\lambda$ and δb^{ab}, and it is then checked that all $O(\psi^3)$ and $O(\psi^4)$ variations cancel without further modifications. This procedure requires extensive calculations using Fierz rearrangements and two-dimensional identities such as (20) and

$$\begin{aligned}&\epsilon_{\mu\rho}b^\rho_\nu=\epsilon_{\nu\rho}b^\rho_\mu\ ,\\ &\gamma_5\beta^\mu=-\frac{\epsilon^{\mu\nu}}{e}\beta_\nu\ ,\\ &b^{\mu\nu}\gamma^\rho-b^{\rho\nu}\gamma^\mu=g^{\rho\nu}b^{\mu\lambda}\gamma_\lambda-g^{\mu\nu}b^{\rho\lambda}\gamma_\lambda\ ,\\ &a_{\mu\nu}=-\tfrac{1}{2}\epsilon_{\mu\nu}\frac{\epsilon^{\rho\sigma}}{e^2}a_{\rho\sigma}\ ,\end{aligned} \tag{26}$$

where $a_{\mu\nu}$ is any antisymmetric tensor. In the end, the local supersymmetry of (18) is completely verified; no stone has been left unturned.

The ghost transformation rules thus obtained are simplest if ghosts are referred to local frames on the world sheet. They take the form

$$\begin{aligned}&\delta c_a=\bar{\epsilon}\gamma_a\lambda\ ,\\ &\delta\lambda=\frac{i}{2}e_a^\mu(D_\mu c^a-\gamma_5\epsilon^{ab}D_\mu c_b)\epsilon-\frac{i}{4}[(\bar{\psi}_\mu\gamma^\mu\epsilon)\lambda+(\bar{\psi}_\mu\gamma^\mu\gamma_5\epsilon)\gamma_5\lambda]-\gamma\cdot cA\epsilon+\tfrac{1}{2}(\bar{\psi}\cdot\gamma\psi_\mu)(c^\mu-\tfrac{1}{2}\gamma^\mu\gamma\cdot c)\epsilon\ ,\\ &\delta b_{ab}=\frac{i}{2}\bar{\epsilon}\left[D_a\beta_b+D_b\beta_a-\eta_{ab}D\cdot\beta+\tfrac{1}{2}\gamma_c(\psi_a b_b^c+\psi_b b_a^c-\eta_{ab}\psi_d b^{cd})+3i\gamma_a\beta_b A+\frac{3i}{4}(\bar{\psi}\cdot\gamma\psi_\mu)(e_a^\mu\beta_b+e_b^\mu\beta_a-\eta_{ab}\beta^\mu)\right],\\ &\delta\beta_a=b_{ab}\gamma^b\epsilon\ .\end{aligned} \tag{27}$$

The antighost trace constraints are obeyed by δb_{ab} and $\delta\beta_a$.

The total action is the sum of (23)—(25) and it is not difficult to observe that it coincides with (18). Actually, at this stage of the work we observed that the action obtained from the supergravity coupling problem did not coincide with the ghost determinant action (16), but that agreement was obtained after the simple field redefinition (17). Because of the field redefinitions which relate (18) to (16) and to (4), the latter two forms of the action are also supersymmetric, but the transformation rules of λ and $b^{\mu\nu}$ will involve derivatives of ϵ.

It is common in supergravity theories to introduce the notion of supercovariant derivatives for fields which couple to the supergravity multiplet. Generically, the supercovariant derivative of the field ψ is defined so as to transform without derivatives of ϵ. Thus, we have, for example,

$$\mathscr{D}_\mu c_a = \tilde{D}_\mu c_a - \tfrac{1}{2}\bar{\psi}_\mu \gamma_a \lambda \,, \qquad \mathscr{D}_\mu \beta^a = \tilde{D}_\mu \beta^a - \tfrac{1}{2} b^{ab}\gamma_b \psi_\mu \,. \tag{28}$$

Further, the connection with torsion $\tilde{\omega}_\mu$ in (6) is also supercovariant in the same sense. With supercovariant derivatives and torsion, one can simplify the form of all transformation rules, and (27) can be rewritten as

$$\begin{aligned} &\delta c_a = \bar{\epsilon}\gamma_a \lambda \,, \\ &\delta\lambda = \tfrac{1}{2} e_a^\mu (\mathscr{D}_\mu c^a - \gamma_5 \epsilon^{ab}\mathscr{D}_\mu c_b)\epsilon - \gamma\cdot c A \epsilon \,, \\ &\delta b_{ab} = \frac{i}{2}(\mathscr{D}_a \beta_b + \mathscr{D}_b \beta_a - \eta_{ab} e_c^\mu \mathscr{D}_\mu \beta^c) - \tfrac{3}{2}\bar{\epsilon}\gamma_a \beta_b A \,, \\ &\delta\beta_a = b_{ab}\gamma^b \epsilon \,. \end{aligned} \tag{29}$$

It is not easy to incorporate supercovariant derivatives in the action (18), but they do simplify the Euler-Lagrange equations which are

$$\begin{aligned} &\frac{\delta S}{\delta c_A} = i e_b^\mu \mathscr{D}_\mu b^{ab} - \tfrac{3}{2} D_{(\mu}\psi_{\nu)}\beta^\mu \,, \\ &\frac{\delta S}{\delta \bar{\lambda}} = i e_b^\mu \mathscr{D}_\mu \beta^b \,, \\ &\frac{\delta S}{\delta b^{ab}} = -i(e_a^\mu \mathscr{D}_\mu c_b + e_b^\mu \mathscr{D}_\mu c_a - \eta_{ab} e_c^\mu \mathscr{D}_\mu c^c) \,, \\ &\frac{\delta S}{\delta \bar{\beta}^\mu} = \tfrac{1}{2}\gamma^\nu \gamma^\mu (-i\mathscr{D}_\nu \lambda - D_{(\nu}\psi_{\rho)} c^\rho) \,. \end{aligned} \tag{30}$$

The action (18) is also invariant under super-Weyl transformations (10)–(12) for the reference multiplet combined with the following ghost variations:

$$\begin{aligned} &\delta_c c^a = \sigma c^a, \quad \delta_c \lambda = \tfrac{1}{2}\sigma\lambda \,, \\ &\delta_c b_{ab} = -2\sigma b_{ab}, \quad \delta_c \beta^a = -\tfrac{3}{2}\sigma \beta^a \,, \end{aligned} \tag{31}$$

$$\begin{aligned} &\delta_s c^a = 0, \quad \delta_s \lambda = -\gamma\cdot c\eta \,, \\ &\delta_s b_{ab} = -\tfrac{3}{2}(\bar{\eta}\gamma_a \beta_b), \quad \delta_s \beta_a = 0 \,, \end{aligned} \tag{32}$$

$$\delta_u(c_a, b_{ab}, \lambda, \beta_u) = 0 \,. \tag{33}$$

IV. CONSERVED CURRENTS

One advantage of a reparametrization invariant and locally supersymmetric form of the ghost action is that it is straightforward to calculate the stress tensor $T^{\mu\nu}$ and supercurrent S^μ of the ghost system by variation of the action with respect to the frame $\hat{e}_{a\nu}$ and gravitino $\hat{\psi}_\mu$.

For the stress tensor, we take the variation of the simpler action (4) with respect to $\hat{e}_{a\nu}$ keeping local frame components b^{ab}, c_a, β^a, λ, and $\hat{\psi}_a$ fixed (so that there is no contribution to the variation from the antighost constraints). For simplicity, we present the result for a superconformal gravitino $\hat{\psi}_a(x) = i\gamma_a \hat{\eta}(x)$ and for a ghost field which satisfies the equations of motion. By this process one obtains the stress tensor

$$\begin{aligned} T^{\mu\nu} &\equiv \frac{1}{e}\hat{e}_a^{\ \mu}\frac{\delta S}{\delta \hat{e}_{a\nu}} \\ &= i[b^{\rho\mu}\nabla^\nu c_\rho + b^{\rho\nu}\nabla^\mu c_\rho + \nabla^\rho b^{\mu\nu} c_\rho \\ &\quad + \tfrac{3}{4}(\bar{\beta}^\mu \nabla^\nu + \bar{\beta}^\nu \nabla^\mu)\lambda + \tfrac{1}{4}(\nabla^\mu \bar{\beta}^\nu + \nabla^\nu \bar{\beta}^\mu)\lambda] \end{aligned} \tag{34}$$

which is symmetric and conserved and traceless on shell. The stress tensor for the action (18) can be obtained from (34) by making the field redefinition inverse to (19).

Similarly, the supercurrent is obtained from (4) as

$$S^\mu = \frac{1}{e}\frac{\delta S}{\delta \bar{\psi}_\mu} = -\tfrac{3}{2}\beta_\nu \nabla^\mu c^\nu - (\nabla_\nu \beta^\mu)c^\nu - \frac{i}{2} b^{\mu\nu}\gamma_\nu \lambda \,. \tag{35}$$

It is conserved and satisfies $\gamma_\mu S^\mu = 0$ on shell. In a conformal frame $\hat{e}_{a\mu} = e^{\hat{\rho}}\eta_{a\mu}$, the stress tensor and supercurrent obtained here coincide with those obtained earlier, but the present method seems more straightforward than either the method of varying conformal class discussed in Ref. 5 or the operator product expansion method used in Ref. 6.

In addition to $T^{\mu\nu}$ and S^ν, there are two other conserved currents, the ghost number current G^μ and the dual ghost number current $\tilde{G}^\mu$. These are associated with the following global symmetries of the ghost action:

$$\begin{aligned} &\delta c^\mu = g c_\mu, \quad \delta\lambda = g\lambda \,, \\ &\delta b^{\mu\nu} = -g b^{\mu\nu}, \quad \delta\beta^\mu = -g\beta^\mu \,, \end{aligned} \tag{36}$$

$$\begin{aligned} &\delta c^\mu = \tilde{g} e^{\mu\rho} c_\rho, \quad \delta\lambda = -\tilde{g}\gamma_5 \lambda \,, \\ &\delta b^{\mu\nu} = \tfrac{1}{2}\tilde{g}(\epsilon^{\mu\rho} b_\rho^{\ \nu} + \epsilon^{\nu\rho} b_\rho^{\ \mu}), \quad \delta\beta^\mu = -\tilde{g}\gamma_5 \beta^\mu \,, \end{aligned} \tag{37}$$

where g and $\tilde{g}$ are independent parameters. The Noether currents obtained from (18) in an arbitrary (e_μ^a, ψ_μ) background are

$$G^\mu = -i(b^{\mu\nu}c_\nu + \bar{\beta}^\mu \lambda) + \tfrac{1}{2}\bar{\psi}_\nu(\beta^\mu c^\nu - \beta^\nu c^\mu) \,, \tag{38}$$

$$\tilde{G}^\mu = \epsilon_\rho^\mu [i(b^{\rho\nu}c_\nu + \bar{\beta}^{\rho\lambda}) + \tfrac{1}{2}(\bar{\psi}^\rho \beta^\nu)c_\nu] \,. \tag{39}$$

Note that if $\psi_\mu = i\gamma_\mu \eta$, then $G^\nu = \epsilon_\rho^\mu \tilde{G}^\rho$, but that this duality relation fails in a general gravitino background.

V. NONCLOSED ALGEBRA

The algebra of local supersymmetry transformations (27) on the ghost fields does not close; the commutator of two variations contains the terms expected from (7) plus new transformations which vanish when the ghost equations of motion (30) are satisfied. The simplest way to see this is to calculate the commutator of the flat world-sheet variations (22) with constant spinors ϵ_1 and ϵ_2. In addition to the expected space-time translation with parameter $\xi^a = 2i\bar{\epsilon}_1 \gamma^a \epsilon_2$, one finds the transformations

$$\begin{aligned} &\Delta c_a = \xi^b(\partial_a c_b + \partial_b c_a - g_{ab}\partial\cdot c) \,, \\ &\Delta\lambda = \xi^b \partial\!\!\!/ \gamma^b \lambda \,, \\ &\Delta b^{ab} = \xi^a \partial_c b^{bc} + \xi^b \partial_c b^{ac} - \eta^{ab}\xi_c \partial_d b^{cd} \,, \\ &\Delta\beta^a = \xi^b \gamma_b \gamma^a \partial\cdot\beta \,, \end{aligned} \tag{40}$$

under which the action on a flat world sheet (21) is invariant.

The commutator of the transformations (27) on a general gauge slice is very complicated to compute. However, the computation need not really be done; it is inescapable that the result contains the transformations (40) with

$$\xi^a(x)=2i\bar{\epsilon}_1(x)\gamma^a\epsilon_2(x)$$

and the equation of motion operators generalized to those of the action (18) which are given in (30). Possible further terms involving $\xi^a(x)$ and $\bar{\epsilon}_1(x)\gamma_5\epsilon_2(x)$ with other combinations of the equations of motion cannot be ruled out without a more complete calculation, but they will not play a role in the present discussion. Whatever the form taken by (40) for a general gauge slice, it must be a local symmetry of the action (18), since it is obtained by commuting two local symmetries [and because (18) is separately invariant under the field-dependent transformations which appear on the right side of (7)]. One should also note that the fields of the background gauge slice are inert under the off-shell transformations; only the ghost and antighosts transform. Further, the action of (18) is invariant if $\xi^a(x)=2i\bar{\epsilon}_1\gamma^a\epsilon_2$ is replaced in (40) by a general vector gauge parameter $V^a(x)$ unrelated to the diffeomorphism parameter. The nonclosed algebra and off-shell gauge symmetry persist for the heterotic string Jacobian.[11] For the heterotic theory, the ghost representation can be obtained from (18) and (27) by discarding the auxiliary field $A(x)$ and the appropriate chirality components of all spinorial quantities. After this truncation, one readily sees that transformations of the form (40) are still present.

In the rest of this section we will discuss first some properties and the possible significance of these off-shell gauge symmetries, then the question of obtaining a closed algebra by adding auxiliary fields, and finally, but briefly, a new set of ghost field theories with quartic couplings suggested by these considerations.

It is easy to construct off-shell gauge symmetries for many common field theories even when they are not connected with the nonclosure of a gauge algebra. For example, although the algebra of world-sheet diffeomorphisms is closed, the action of the bosonic string Jacobian in (2) has an additional gauge symmetry: namely,

$$\begin{aligned}\delta c_\mu(x)&=V^\nu(x)(\nabla_\mu c_\nu+\nabla_\nu c_\nu-g_{\mu\nu}\nabla\cdot c)\ ,\\ \delta b^{\mu\nu}(x)&=V^\nu(x)\nabla_\rho b^{\rho\nu}+V^\nu(x)\nabla_\rho b^{\rho\mu}-g^{\mu\nu}\nabla_\rho(x)\nabla_\sigma b^{\rho\sigma}\ ,\end{aligned}\tag{41}$$

for an arbitrary vector $V^\mu(x)$. Since this example is related to the general gauge slice version of (40), it will be emphasized in our discussion here. However, another example occurs for any local action $S[\phi^*,\phi]$ for a complex scalar field $\phi(x)$. Namely, we have invariance under $\delta\phi(x)=i\theta(x)\delta S/\delta\phi^*(x)$ for an arbitrary real scalar gauge parameter $\theta(x)$. The free massive Dirac spinor action is invariant under $\delta\psi(x)=\theta(x)(\partial\!\!\!/+im)\psi$, and this can be extended to include interactions. All these "new" symmetries have the feature that they vanish on shell.

The nature of the transformation (41) is best seen in light-cone coordinates $x^\pm=(x^0\pm x^1)/\sqrt{2}$ on a flat world sheet. Then (41) can be rewritten as (an irrelevant factor of 2 is dropped here)

$$\begin{aligned}\delta c_\pm&=V^\pm\partial_\pm c_\pm\ ,\\ \delta b^{\pm\pm}&=V^\pm\partial_\pm b^{\pm\pm}\ .\end{aligned}\tag{42}$$

These are simply separate reparametrizations of x^+ in c_+ and b^{++} and of x^- in c_- and b^{--}. A finite transformation with generator $V^+(x^+,x^-)$ and exponential parameter α can be written as

$$\begin{aligned}c_+(x^+,x^-)&\underset{\alpha,V^+}{\longrightarrow}c_+(\bar{x}^+,x^-)\ ,\\ b^{++}(x^+,x^-)&\underset{\alpha,V^+}{\longrightarrow}b^{++}(\bar{x}^+,x^-)\ ,\end{aligned}\tag{43}$$

where $\bar{x}^+$ is defined implicitly by

$$F(\bar{x}^+)=F(\alpha+F(x^+)),\quad F(x)\equiv\int^x\frac{dz}{V^+(z,x^-)}\ .\tag{44}$$

The fact that $F(x^+)$ and thus $\bar{x}^+$ is a function of both x^+ and x^- is suppressed in the interest of a simpler notation. There is an analogous formula for finite transformations with generator V^- acting on c_- and b^{--}. Equation (44) is simply the standard form of reparametrization transformation on scalar functions of the variable x^+ while the x^- dependence is incidental. A discussion of this standard form has appeared very recently in connection with new work on the conformal group.[14] The transformations (43) are globally well defined if $V^+(x^+,x^-)$ has no zero, and periodic boundary conditions can be imposed consistently. The light-cone action

$$S=-i\int dx^+dx^-(b^{++}\partial_+c_++b^{--}\partial_-c_-)\tag{45}$$

is certainly invariant under (43).

We will now discuss the possible significance of the off-shell gauge symmetry (41) for the representation (2) of the bosonic string Jacobian. At any point in function space of $c_\mu(x)$ and $b^{\mu\nu}(x)$, there is a local vector's worth of directions (i.e., an infinite number) along which the action in (2) is stationary. Thus, one may suspect that the functional integral (2) is not well defined. This surely is not the case for the common field theories of complex scalar and spinor fields discussed above, but perhaps the ghost integrals are special. One difference is that we have a vector gauge parameter for the string Jacobian and a scalar gauge parameter in the other examples.

One way to look for inconsistencies in the path integral (2) is to introduce sources and investigate whether the new gauge invariance can be extended to the source term and whether the appropriate Ward identities are satisfied. Thus, we consider the generating functional

$$W(K_{AB},J^A)=\int db^{AB}dc_A\exp\left[\int dx^+dx^-(b^{++}\partial_+c_++K_{++}b^{++}+c_+J^++\text{similar terms for }b^{--},c_-)\right]\ .\tag{46}$$

This is invariant under (42) provided sources transform as

$$\delta J^{\pm}=\partial_{\pm}(V^{\pm}J^{\pm})\,,\qquad \delta K_{\pm\pm}=\partial_{\pm}(V^{\pm}K_{\pm\pm})\,, \tag{47}$$

and there is a standard Ward identity which $W(K,J)$ should satisfy. If one performs the integral by the standard method of "completing the square," one finds that this identity is satisfied if one uses the "Feynman propagator"

$$\frac{1}{\partial_+}\sim\langle Tb^{++}(x^+),c_+(0)\rangle\sim\frac{x^+}{x^+x^- - i\epsilon}\,,\qquad \frac{1}{\partial_-}\sim\langle Tb^{--}(x^-),c_-(0)\rangle\sim\frac{x^-}{x^+x^- - i\epsilon}\,, \tag{48}$$

which agrees with the ghost two-point function of the conformal field theory approach.[6] Thus the off-shell gauge invariance does not seem to cause a problem.

One may also consider the functional integral (2), without sources on the torus with c_+ and b^{++}, etc., expanded in plane-wave modes:

$$c_+=\sum_{m,n}c_{mn}e^{-i(mx^+ + nx^-)}\,,\qquad b^{++}=\sum_{m,n}b_{mn}e^{-i(mx^+ + nx^-)}\,. \tag{49}$$

There are an infinite number of zero modes of ∂_+, and the det' prescription for the string Jacobian requires that c_{0n} and b_{0n} are excluded from the integral over mode coefficients. Once this is done, the integral appears to require only the expected ultraviolet regularization. Since the function space is now the set of Fourier coefficients c_{mn} and b_{mn} with $m\neq 0$, it seems that allowed gauge parameters $V^+(x^+,x^-)$ should be restricted so that the space of nonzero Fourier coefficients c_{mn} and b_{mn} is mapped into itself. If V_{pq} denotes a Fourier coefficient of V^+, defined as in (49), then from (42) we deduce

$$\delta c_{mn}\sim\sum_{p,q}V_{m-p,n-q}\,p\,c_{pq} \tag{50}$$

with a similar equation for δb_{mn}. Since δc_{0n} must vanish for all choices of c_{pq}, we see that only $V_{0,q}$ can be nonvanishing. This implies that $V^+(x^+)$ is an analytic vector field. Thus, the full gauge invariance of (42) is restricted, but not eliminated after a more careful definition of the functional integral. The product measure over mode coefficients is invariant.

Although we do not claim to understand fully the relation between the off-shell gauge symmetry (42) and the path integral (2), we have not found any indication that the latter is ill defined. Perhaps the restriction on the gauge parameters found in the previous paragraph is significant. Even for analytic vector fields the transformation (41) is different from a conformal transformation, yet it has the same parametric freedom. Since conformal invariance does not lead to an ill-defined functional integral, the restricted off-shell transformations should not either.

We return now to the question of the non-closed algebra (40). Nonclosure occurs in supersymmetry when auxiliary fields are missing, and this is surely the case here for the ghost and antighost multiplets. With this in mind one can go back to the representation (15) of the ghost action which does contain additional fields. We do not have definitive results on supersymmetry transformation rules for (15), but an exhausting study convinced us that the new ghost fields in (15) are not sufficient to give a closed algebra. Undoubtedly, however, a closed algebra can be obtained from the superspace formalism, provided one does not fix the gauge to break up local superfields as seems to have been done in treatments such as Ref. 13 for $N=1$ and Ref. 11 for $N=\frac{1}{2}$. A superspace study is now underway using an independent set of superfield potentials given in Ref. 15. However, it may well be the case that even with a closed algebra the action obtained will still have additional gauge symmetries analogous to (42) for the bosonic string.

To conclude, we discuss briefly a new class of conformal theories motivated by the nonclosed supersymmetry algebra (40). In four-dimensional supergravity, the nonclosed algebra gives rise to quartic ghost couplings in the properly gauge fixed action. Although it is doubtful that such couplings will be required, one can consider, as an independent question, the addition of four-ghost couplings to the actions of (2) and (18). A natural choice for (2) is to take the conserved ghost current $G^{\mu}=-ib^{\mu\nu}c_{\nu}$ for (2) and to form the Lagrangian $\mathscr{L}=b^{\mu\nu}\nabla_{\mu}c_{\nu}+qG^{\mu}G_{\mu}$. The parameter q is a dimensionless coupling, and $\mathscr{L}$ is Weyl invariant. Remarkably, one can formulate a similar quartic ghost extension of (18) which involves the ghost number currents G^{μ} and $\tilde{G}^{\mu}$ of (38) and (39) with local supersymmetry and super-Weyl invariance maintained. The construction and properties of these theories will be discussed elsewhere.[16]

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