PHYSICAL REVIEW D VOLUME 34, NUMBER 10 15 NOVEMBER 1986

Candidates for the inflaton field in superstring models

Pierre Binétruy*
Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637
and Department of Physics, University of Florida, Gainesville, Florida 32611

Mary K. Gaillard
Lawrence Berkeley Laboratory, Department of Physics, University of California, Berkeley, California 94720
(Received 1 July 1986)

We consider the flat directions of the potential that play a crucial role in the four-dimensional supersymmetric models that are believed to emerge from the compactification of superstring theories and study the possibility that they give rise to an inflationary scenario. None of the scalar fields present in these models—in particular the dilaton field connected with supersymmetry breaking and the SU(5)-singlet scalars in the matter sector—seem, however, to be good candidates for the inflaton, the scalar field whose cosmological evolution leads to an inflationary expansion of the Universe.

I. INTRODUCTION

Among the theories where inflation[1] can be implemented, the best candidates have proven to be supersymmetric models.[2–8] A major bonus gained by incorporating supersymmetry into an inflationary scenario is the absence of vacuum renormalization. If supersymmetry remained unbroken, this would allow one to fine-tune the shape of the potential in order to obtain the right amount of inflation. Of course supersymmetry is broken (if for no better reason, because inflation demands a nonzero vacuum energy) and fine-tuning there must be, although to a lesser extent than in the standard nonsupersymmetric models.

The supersymmetric versions of inflation involve a gauge-singlet scalar field (inflaton) placed in the hidden sector[9] of the theory: both requirements—gauge singlet and hidden sector—are such that the inflaton field couples as weakly as possible (i.e., gravitationally) and hence that the quantum corrections are kept to a minimum.

Although the number of problems to which inflation is supposed to give a clue—or at least avoid—give rise to numerous constraints[10–12] on the parameters of the models, one is still left with several possible candidates, reflecting the richness of theories coupled to $N=1$ supergravity. On that ground, the possibility that four-dimensional field theories with one supersymmetry unbroken originate from the compactification[13] of ten-dimensional superstring theories[14–16] is a very welcome one. It is interesting to see, in that respect, how the new constraints that are imposed on our four-dimensional field theory models by superstrings compare with the constraints that were derived previously from inflationary considerations. In other words, since superstring theories are supposed to provide a framework for grand unified theories coupled to $N=1$ supergravity, do they incorporate inflation along the same lines?

It seems reasonable to think that the cosmology of superstring models will involve some period of inflationary expansion. The reason is the presence in the massless sector of a scalar field: the dilaton. The "flat direction" of the potential associated with its masslessness is precisely the necessary ingredient for a "new inflationary" scenario.[1] On the other hand, the scale of the dilaton field determines the magnitude of the coupling constant. Hence, as long as it remains massless, the model is not fully determined. Some sort of dynamics has to fix the scale. In other words, since a ground state with a zero cosmological constant has to be singled out, the flat region of the effective potential that determines the ground state has presumably a nonzero vacuum energy. This could account for the occurrence of an inflationary era during the cosmic evolution of such systems. (It has actually been shown that the nonvanishing dilaton tadpole found at the one-loop level in the bosonic string[17] is compensated by introducing a nontrivial background of the de Sitter type.[18])

Let us note in particular the close connection of these questions with the breakdown of supersymmetry. Flat directions of the potential have been studied in that respect because one can prove that there is no way to lift the corresponding degeneracy, to any finite order in perturbation theory, as long as supersymmetry remains unbroken.[19] This therefore calls for a breakdown of supersymmetry at the suitable scale and for nonperturbative effects, possibly one triggering the other. This was discussed at length by Affleck, Dine, and Seiberg[20] in the context of dynamical supersymmetry breaking. They found in particular that whenever a nonzero potential is generated, it slopes to zero at infinity. This in turn is a serious problem for superstring models and, in what is of more immediate concern to us, it will eventually have to be dealt with, if an inflaton field is to be associated with one of these flat directions. We will return to this question below.

There is however one obstacle to the study of inflation at the level of superstrings: string theories have until recently been considered in backgrounds of zero energy.[14] This is one of the reasons why we will confine ourselves to the field-theory limit of these theories, allowing for possible interactions arising from higher orders in the string theory (increasingly complicated world-sheet topology) or in the two-dimensional nonlinear σ model. If some energy is stored in the vacuum of the superstring theory for some time during the evolution of the Universe, this should also show up at the field-theory level, in the potential energy of some scalar field. (A possible loophole in this argument might lie in the fact that the one-loop cosmological constant of string theory does not seem to be equal to the sum of the one-loop contributions of the individual particles.[17])

This scalar field is the inflaton field common to all grand unified models that purport to give rise to an inflationary epoch. From the study of these models, we know what kind of constraints such a field—its potential—must satisfy. What we intend to do in the following is to determine which of the scalars present in superstring theories satisfy this set of constraints, in other words which of them are candidates for the inflaton field.

The constraints that models must satisfy in order to yield a successful inflationary scenario have been reviewed elsewhere.[10–12] Two parameters of particular relevance are $V_0 \equiv \mu_0{}^4$, the energy stored in the vacuum during inflation, and σ, the ground-state value of the inflaton field, reached at the end of inflation.

If inflation is to solve the two basic problems that it was devised for[1]—flatness and horizon problems—the number N_e of e-foldings that the cosmic scale factor must undergo during that period must be large enough. More qualitatively, this reads in terms of μ_0 and σ

$$N_e = O\left[\frac{\sigma^m}{M_{\rm Pl}{}^{m-n}\mu_0{}^n}\right] \gtrsim 65 , \tag{1.1}$$

where, throughout this paper, $M_{\rm Pl}$ is the reduced Planck scale (the Planck scale divided by $\sqrt{8\pi}$: $M_{\rm Pl} \sim 2.4\times 10^{18}$ GeV), and m and n are in most cases positive integers.[12] In models coupled to $N=1$ supergravity, a more stringent constraint[21] comes from the requirement that fluctuation densities in the Universe are predicted with the right size, on a range that goes from the galaxy scale to the scale of the background radiation. The constraint on the density fluctuations at the time t_f when they reenter the horizon reads, typically,

$$\frac{\delta\rho}{\rho}(t_f) \sim O\left[\frac{M_{\rm Pl}{}^{m'-n'}\mu_0{}^{n'}}{\sigma^{m'}}\right] \sim 10^{-5}\text{–}10^{-4} , \tag{1.2}$$

where, once again, in most models m' and n' are positive integers[12] (for example, $n'=2$, $m'=3$).

The constraints (1.1) and (1.2) are usually satisfied by letting σ be of the order of the Planck scale $M_{\rm Pl}$ and μ_0 a few orders of magnitude below it. An indication that this is the right solution comes from another constraint: the production of gravitons.[22] If the inflation scale is too close to $M_{\rm Pl}$, the energy density of gravitons produced during inflation will not be negligible and will give rise to distortions in the radiation background. More quantitatively, limits on the large-scale anisotropy $\Delta T/T$ yield[22]

$$\mu_0/M_{\rm Pl} \lesssim 10^{-2} . \tag{1.3}$$

The question of how this small hierarchy of scales arises is usually not addressed in supersymmetric models (since the inflaton field is taken to be a gauge singlet, there is no reason to choose μ_0 equal to the grand unification scale $M_{\rm GUT}$). In the case of superstring models, such a hierarchy seems difficult to implement if the dilaton is the inflaton field: as we will discuss below, all relevant scales seem to be of the same order.

To come back to the constraint (1.2) on density fluctuations, let us note that in these models density fluctuations might have an origin other than quantum fluctuations in the de Sitter phase. It is well known that superstring theories predict cosmic strings,[23] which seem to be good candidates to provide an origin for density fluctuations (in particular because these fluctuations are originally uncoupled to matter[24]). Of course the amplitude of these fluctuations has to come out right (their spectrum is scale invariant as previously) and the constraint (1.2) turns into a constraint on the linear mass density of the cosmic string.[24] But this could account for the origin of density fluctuations only in the case where quantum fluctuations in the de Sitter phase yield a value of $\delta\rho/\rho$ lower than the one required by the constraint (1.2).

The next constraint is one about initial conditions: when inflation starts, that is, when the energy density of the Universe becomes of the order of V_0, the inflaton field—more precisely its space-averaged value $\langle\phi\rangle$—should be located in the plateau region of its potential $[V(\langle\phi\rangle)\sim V_0]$. The most common way to account for this is to require that its temperature-dependent effective potential has a global minimum there, for $T \gg \mu_0$. This is the so-called thermal constraint.[4] Let us note however that this does not mean that one takes seriously the shape of the one-loop effective potential in its full details; this is only a way to require the existence of a metastable state with nonzero vacuum energy V_0 (Ref. 12).

Since one of the effects of inflation is to dilute away any baryon number present in the Universe, a successful scenario must provide a source for baryon number posterior to the de Sitter phase of exponential expansion. However, the general question of baryon-number-violating interactions is very dependent on the model (in particular the low-energy gauge symmetry[25]) and we will therefore defer a further study of that question in the case of inflation.

Finally, although inflation also dilutes away primordial gravitinos, new sources of gravitinos appear during the reheating phase that follows (decay of the inflaton coherent oscillations). In order to address this question—the so-called gravitino problem—we will first have to discuss the issue of supersymmetry breaking.

As mentioned earlier, we consider models where the compactification process preserves one supersymmetry: to be precise, a compactification on a six-dimensional compact manifold K of SU(3) holonomy, where the spin connection is embedded in the Yang-Mills gauge group[13] (taken to be $E_8\times E_8$). In particular, the dilaton field ϕ originating from superstring theory—or rather its massless mode in four-dimensional Minkowski space M_4—is a part of the scalar component of a chiral superfluid. It turns out[26,27] that it actually resolved into two chiral superfields that also incorporate the "breathing mode" e^σ, which corresponds to a fluctuation of the overall size of the internal compact manifold K. Since these two fields are real, we need two pseudoscalars to complete the scalar components. One of them is obtained from the gauge-invariant field strength $H_{\mu\nu\rho}$ of the two-form $B_{\mu\nu}$, through a duality transformation (in four-dimensional space),

$$H_{\mu\nu\rho}=\phi^{3/2}e^{-6\sigma}\epsilon_{\mu\nu\rho\sigma}\partial^{\sigma}h \ , \tag{1.4}$$

where from now on, unless specified, we will take $M_{\rm Pl}=1$. The other one is obtained in the decomposition of the two-form on the harmonic forms of type (1,1) (the number of independent such forms is the topological number $b_{1,1}$):

$$B_{i\bar{j}}(x,y)=\sum_{n=1}^{b_{1,1}}\beta_{(k)}(x)\omega_{i\bar{j}}^{(k)}(y) \ , \tag{1.5}$$

where x (y) refer to M_4 (K) coordinates, i ($\bar{j}$) are holomorphic (antiholomorphic) indices, and the $\omega_{i\bar{j}}^{(k)}$ are a basis of harmonic forms of type (1,1). The $\beta^{(k)}(x)$ are therefore the massless modes originating from the two-form. We can now write the two complex scalar fields which involve the dilaton ϕ (Ref. 26):

$$S=\phi^{-3/4}e^{3\sigma}+3i\sqrt{2}h, \quad T=\phi^{3/4}e^{\sigma}-i\sqrt{2}\beta \ , \tag{1.6}$$

where $\beta\equiv\beta^{(1)}(x)$.

Similarly,[28] the remaining pseudoscalars $\beta^{(k)}$, $k=2,\dots,b_{1,1}$, will combine under supersymmetry with $b_{1,1}$ scalars $\alpha^{(k)}$ that characterize the choice of Kähler structure (among these we have already singled out the "breathing mode"; hence only $b_{1,1}-1$ remain):

$$T^{(k)}=\alpha^{(k)}+i\beta^{(k)}, \quad k=2,\dots,b_{1,1} \ . \tag{1.7}$$

To be complete and introduce all the structure-related fields, the choice of complex structure depends on $b_{2,1}$ complex numbers, which gives rise to $b_{2,1}$ massless chiral (gauge-singlet) supermultiplets $C^{(\alpha)}$, $\alpha=1,\dots,b_{2,1}$, in four dimensions.

Finally, matter multiplets—which include quarks, leptons, Higgs fields, and their supersymmetric partners—are in representations **27** and $\overline{\mathbf{27}}$ of E_6: to be precise, $b_{2,1}\overline{\mathbf{27}}$ and $b_{1,1}\mathbf{27}$. We will denote them generically by y_i. None of them is a gauge singlet under the low-energy gauge symmetry.

In what follows we review how each of these fields stands as a candidate for the inflaton. Since our analysis was prompted by the fact that some energy might be stored in the dilaton field ϕ, we first consider in Sec. II the two fields which are related to ϕ: S and T. We also comment on the possibility of using the other structure-related fields $T^{(k)}$ and $C^{(\alpha)}$. In Sec. III we determine the possible candidates among the matter superfields (y_i fields). Let us note that this was the only case considered in no-scale models.[8] The situation is however somewhat different here because none of these fields is a gauge singlet. Finally in Sec. IV we give our conclusions.

II. THE DILATON AVATARS: S AND T FIELDS

The complete expression of the S and T fields in terms of the dilaton and the "breathing mode" involves also the matter fields [compare with (1.6)]

$$\begin{aligned} S&=\phi^{-3/4}e^{3\sigma}+3i\sqrt{2}h \ , \\ T&=\phi^{3/4}e^{\sigma}-i\sqrt{2}\beta+\sum_i |y_i|^2 \ . \end{aligned} \tag{2.1}$$

In order to write the complete supersymmetric theory, one only needs to know[29] the Kähler potential $\mathcal{G}$ and the normalization of the gauge fields kinetic terms $f_{\alpha\beta}$. If one trusts the results of a simple truncation model, they read[26]

$$\begin{aligned} \mathcal{G}&=-\ln(S+S^*)+G+\ln|W|^2 \ , \\ G&=-3\ln\left[T+T^*-k\sum_i |y_i|^2\right] \ , \\ W&=\tilde{W}(y_i), \quad f_{\alpha\beta}=\delta_{\alpha\beta}S \ , \end{aligned} \tag{2.2}$$

where $\tilde{W}(y_i)$ is the superpotential, cubic in the y_i fields, and k is a normalization constant. This yields a theory of the no-scale type.[30] The corresponding potential is[29]

$$\begin{aligned} V&=e^{\mathcal{G}}+\hat{V}+\mathcal{D}, \quad e^{\mathcal{G}}=e^{G}|W|^2/(S+S^*) \ , \\ \hat{V}&=\frac{1}{3k}e^{2G/3}\overline{W}_i W^i/(S+S^*), \quad W^i=\frac{\delta W}{\delta y_i}=(\overline{W}_i)^* \ , \\ \mathcal{D}&=\frac{1}{S+S^*}D^{\alpha}D^{\alpha}, \quad D^{\alpha}=3ke^{G/3}\overline{y}^i T_i^{\alpha j}y_j \ , \end{aligned} \tag{2.3}$$

where the matrices T^{α} represent the generators of the gauge group. The global minimum is for $y_i=0$ and the vacuum expectation values (VEV's) of both S and T remain undetermined at the tree level.

There are two known ways to break supersymmetry in these models: either give the field strength H_{mnp} (m,n,p are indices of the compact manifold K) a nonzero VEV (Ref. 31) or let the gauginos of the hidden sector (E_8') condense.[32] Both of them yield a nonzero cosmological constant but it turns out that the two contributions cancel when one combines them:[33] the point is that the new terms in the potential make up a perfect square whose minimum value is therefore zero. If we look at the effective theory below the scale Λ_c of gaugino condensation (i.e., the scale where the hidden sector gauge group E_8' or one of its subgroups Q becomes strong), the effective superpotential reads[33]

$$W=c+he^{-3S/2b_0}+\tilde{W}(y_i) \ , \tag{2.4}$$

where b_0 determines the one-loop β function of the gauge group Q [for $Q=E_8'$, $b_0=90/(16\pi^2)$]. This yields the potential

$$\begin{aligned} V&=U+\hat{V}+\mathcal{D} \ , \\ U&=\frac{e^{G}}{(S+S^*)}|\tilde{W}(y_i)+c+h(1+\alpha)e^{-\alpha/2}e^{-i\beta/2}|^2 \ , \end{aligned} \tag{2.5}$$

where

$$\alpha=3\frac{\mathrm{Re}S}{b_0},\quad \beta=3\frac{\mathrm{Im}S}{b_0} \tag{2.6}$$

(not to be confused with $\alpha^{(k)},\beta^{(k)}$ introduced earlier).

The global minimum (α_0,β_0) is obtained for $y_i=0$ (which ensures that $\hat{V}=\mathscr{D}=0$) and $U=0$:

$$c+h(1+\alpha_0)e^{-\alpha_0/2}e^{-i\beta_0/2}=0\ , \tag{2.7}$$

which determines the VEV of S.

It is worth pointing out that the VEV of H does not have an *a priori* value that is canceled at a later stage by gaugino condensation. This would break supersymmetry at scales above gaugino condensation. As discussed by Rohm and Witten,[28] after gaugino condensation has occurred, the VEV of H follows by tunneling to a nonzero value that corresponds to the global minimum where the cosmological constant is zero.

At this stage, the VEV of T remains undetermined at the tree level and hence so is the gravitino mass:

$$m_{3/2}{}^2=e^{\mathscr{G}}=\frac{e^G}{(S+S^*)}\,|W|^2\ . \tag{2.8}$$

The situation is somewhat reminiscent of what happens in no-scale models.[30] There, the degeneracy is lifted by the low-energy radiative corrections at the one-loop level (therefore allowing for the possibility of a gravitino mass of the order of the weak-gauge-boson mass M_W). It was shown in Refs. 34 and 35 that this is not quite true in the case of superstring models: the point is that radiative corrections induced by the field S (not present in standard no-scale model) already lift the degeneracy. We computed in Ref. 35 the quadratically and logarithmically divergent contributions to the effective potential for T, in the direction where S and the y_i fields are kept at their ground-state values [given by Eq. (2.7) and $y_i=0$]. It reads in terms of $u=(T+T^*)^{-2}=e^{2G/3}$:

$$\begin{aligned}\overline{V}_{\mathrm{eff}}(u)&=\frac{1}{32\pi^2}\left[\Lambda_c{}^2\,\mathrm{STr}M^2+\frac{1}{2}\mathrm{STr}M^4\ln\frac{M^2}{\Lambda_c{}^2}\right]\\&=\frac{9|h|^2}{(4\pi)^2 4b_0}e^{-\alpha_0}\{u^2\mathscr{F}(\alpha_0)\\&\qquad+u^3|h|^2[\mathscr{C}_1(\alpha_0)\ln u\\&\qquad+\mathscr{C}_2(\alpha_0)]\}\ ,\end{aligned} \tag{2.9}$$

where $\mathscr{F}$, $\mathscr{C}_1$, and $\mathscr{C}_2$ are calculable functions of α_0 (Ref. 35). The stability of this potential requires $\mathscr{C}_1(\alpha_0)>0$ which, to a good approximation, simply reads $\alpha_0{}^2>\frac{1}{6}$. This in turn places a restriction on the gauge coupling constant at the unification scale [inferred from $f_{\alpha\beta}$ in Eq. (2.2)]:

$$\alpha_{\mathrm{GUT}}=\frac{1}{4\pi\,\mathrm{Re}S}=\frac{3}{4\pi b_0\alpha_0}\ . \tag{2.10}$$

Once the minimum u_0 of the potential (2.9) is obtained, all the scales of the effective theory are determined in terms of the single variable α_0 [or equivalently α_{GUT}, using (2.10)]:

$$\begin{aligned}\frac{M_{\mathrm{GUT}}}{M_{\mathrm{Pl}}}&=\left[\frac{6}{b_0}\right]^{1/2}\alpha_0{}^{-1/2}u_0{}^{1/4}\ ,\\ \frac{m_{3/2}}{M_{\mathrm{Pl}}}&=\left[\frac{3|h|^2}{2b_0}\right]^{1/2}\alpha_0{}^{1/2}e^{-\alpha_0/2}u_0{}^{3/4}\ ,\\ \frac{\Lambda_c}{M_{\mathrm{Pl}}}&=\left[\frac{6}{b_0}\right]^{1/2}\alpha_0{}^{-1/2}e^{-\alpha_0/6}u_0{}^{1/4}\ .\end{aligned} \tag{2.11}$$

The condition that the unification scale M_{GUT} remain smaller than the Planck scale constrains α_0, or equivalently α_{GUT}. This, in turn, imposes restrictions on the scale of supersymmetry breaking, $m_{3/2}$ or Λ_c. One finds the bounds[35]

$$m_{3/2}/M_{\mathrm{Pl}}>1.5\times10^{-3},\quad \Lambda_c/M_{\mathrm{Pl}}>0.05\ . \tag{2.12}$$

In view of these results, it has been recently argued[36,37] that, since the mass scales involved are of the order of the physical cutoff Λ_c, one should actually include the complete cutoff dependence in the effective potential; i.e.,

$$\begin{aligned}V_{\mathrm{eff}}=\frac{1}{64\pi^2}&\left[\Lambda_c{}^2\,\mathrm{STr}M^2-\mathrm{STr}M^4\ln\left[1+\frac{\Lambda_c{}^2}{M^2}\right]\right.\\&\left.+\Lambda_c{}^4\,\mathrm{STr}\ln\left[1+\frac{M^2}{\Lambda_c{}^2}\right]\right]\ .\end{aligned} \tag{2.13}$$

When doing so, one realizes that the stabilization of $\overline{V}_{\mathrm{eff}}$ is an artifact of our keeping only the divergent terms. Actually, the expression (2.13) is unbounded from below,[36,37] as one can check from its large-u behavior:

$$V_{\mathrm{eff}}(u)\sim-\frac{9}{4\pi^2 b_0{}^2}\alpha_0{}^{-2}e^{-2\alpha_0/3}u\ln u\ . \tag{2.14}$$

The expression that led to the bound (2.12) represents therefore at best a simplified modelization of what should happen at the scale of supersymmetry breaking (Λ_c). But a precise determination of the scales will require a precise understanding of the loop corrections in the models considered. [For example, radiative corrections at the two-loop level would make contributions to $\mathscr{C}_1$ and $\mathscr{C}_2$ in (2.9) or equivalently to the last two terms in (2.13); but, at this stage, no one would take these models seriously enough to compute these corrections and/or trust their results.]

One indication that the result (2.12) is going in the right direction is the observation made by Dine and Seiberg[38] and Kaplunovsky[39] that the different scales of superstring models are all of the same order. As stressed by these authors, this implies that the string theory is strongly coupled, which casts considerable doubt on the semiclassical analysis used to derive the four-dimensional field theory that we are using. Although this remains a serious problem that endangers the whole analysis, the result of Eq. (2.12) suggests that there might be more truth to the four-dimensional version obtained than could have been expected.

However, at least one scale—the scale of electroweak symmetry breaking—has to come out of the model many orders of magnitude below the Planck scale: this is the standard hierarchy problem. A result obtained in Refs. 34 and 35 which goes precisely in that direction is the fact that scalars remain massless at the one-loop level: their squared-mass (which triggers gauge symmetry breaking by becoming negative) is therefore several orders of magnitude below the Planck scale. One has to distinguish between the scale of supersymmetry breaking in the hidden sector which is given by $m_{3/2}$ and the scale of supersymmetry breaking in the observable sector (y_i fields) which could (must) be several orders of magnitude smaller. A gravitino mass much larger than 1 TeV may thus not be a problem since, as in standard no-scale models and contrary to most supersymmetric models,[40] the scalar masses are protected from the gravitino mass by some symmetry (at least at the one-loop level). It has been shown recently[41] that the leading gravitational contributions to the gaugino masses are of order $m_{3/2}{}^3/M_{\rm Pl}{}^2$; in this case, through coupled renormalization-group equations, the scalar masses should be of the same order. An analysis similar to the one in no-scale models[30] (where, in contrast to the analysis of Ref. 35 described above, the scale is eventually determined by low-energy radiative corrections) would typically give[41] $m_{3/2}\sim 10^{13}$ GeV together with scalar masses of order 1 TeV.

To finally come back to inflation, such a large mass for the gravitino is welcome to avoid the standard gravitino problems. Any primordial gravitino density is diluted away during the inflationary era and gravitinos are too heavy to be produced during or after reheating.

We are now in a position to study the inflationary properties of the potential. Since the determination of the T field potential by radiative corrections is not well understood, we will restrict our analysis to the field S whose potential is determined at the tree level (below the gaugino condensation scale Λ_c) and is given in (2.5). We will therefore consider S alone and put the other fields at their ground states. This means in particular that $y_i=0$ and that we will fix $\langle T\rangle$ by taking a given value for $m_{3/2}$ [cf. Eq. (2.8)]. We will suppose for simplicity that c is real and negative, in which case the ground state is given by (2.7)

$$\begin{aligned}&\hat{c}\equiv -c/h=(1+\alpha_0)e^{-\alpha_0/2}\ ,\\&\beta_0=4n\pi,\ \ n\in Z\ .\end{aligned}\tag{2.15}$$

[There is a term $\beta_0 F\tilde{F}$ in the Lagrangian.[29] For $c>0$, $\beta_0=2\pi+4n\pi$ and the ground state is always CP violating. The choice $c<0$, on the other hand, allows us to choose a CP-conserving minimum ($\beta_0=0$).] And the potential reads from (2.5) and (2.11)

$$V(\alpha,\beta)=U=\mu^4\frac{\alpha_0}{\alpha}\left[1+\frac{1}{\hat{c}^2}(1+\alpha)^2e^{-\alpha}-\frac{2}{\hat{c}}(1+\alpha)e^{-\alpha/2}\cos\frac{\beta}{2}\right]\tag{2.16}$$

with

$$\frac{\mu}{M_{\rm Pl}}=\left[\frac{3|h|^2}{2b_0}u_0{}^{3/2}\frac{(1+\alpha_0)^2}{\alpha_0}e^{-\alpha_0}\right]^{1/4}=\left[\frac{m_{3/2}}{M_{\rm Pl}}\frac{1+\alpha_0}{\alpha_0}\right]^{1/2}.\tag{2.17}$$

One could determine the value of μ following the same lines of reasoning that led to Eq. (2.12), that is, using the "truncated" one-loop effective potential of Eq. (2.9). The lowest possible value for μ is then obtained when E_8' remains unbroken, which yields the bound

$$\mu/M_{\rm Pl}>0.04\ .\tag{2.18}$$

The fact that the scale μ is found close to the Planck sale should come as no surprise in this case: in the (S,T) sector, there is basically no scale available apart from $M_{\rm Pl}$; this was already the reason why we found earlier $M_{\rm GUT}\sim m_{3/2}\sim\Lambda_c\sim M_{\rm Pl}$. But it seems difficult to reconcile this bound (2.18) with the bound arising from the production of gravitons, Eq. (1.3).

On the other hand, a gravitino mass $m_{3/2}$ smaller than $M_{\rm PL}$ by several orders of magnitude (as would be the case if it was determined by low-energy radiative corrections[41]) would yield a scale μ for inflation compatible with Eq. (1.3). For instance, choosing $\alpha_{\rm GUT}=0.28$ and $m_{3/2}=10^{12}$ GeV, which yields [using (2.10) and (2.11)] the not unreasonable[25] value $M_{\rm GUT}=4.9\times10^{16}$ GeV for the grand unified scale, one obtains, from (2.17),

$$\frac{\mu}{M_{\rm Pl}}=8.4\times10^{-4}\ .\tag{2.19}$$

This gives the kind of hierarchy between the inflation scale and the Planck scale that is needed in supersymmetric models.

Before investigating this question further, we have to discuss the issue of the initial conditions. From a temperature of the order of the compactification scale ($T\sim M_{\rm Pl}\sim M_{\rm GUT}$) down to a temperature $T\sim\Lambda_c$, the potential for S (and T, not to be confused with the temperature) is flat, the field H_{mnp} has a zero VEV, and supersymmetry remains unbroken. When the temperature reaches Λ_c, the gauginos in the invisible sector (E_8' gauginos) start condensing, which breaks supersymmetry. The ground state for the system formed by these gauginos and H_{mnp} (or, rather, the set of four-dimensional fields that it represents) corresponds now to a VEV $c\neq0$ for H_{mnp}. Since the presence of the Lorentz and Yang-Mills Chern-Simons three-forms in H appears to allow for the possibility of tunneling between two different vacua,[28] some bubbles of true ($c\neq0$) vacuum will start to develop. (Let us note here that a different attitude is adopted in a recent analysis by Ellis, Enqvist, Nanopoulos, and Quirós. In Ref. 42, they assume that gaugino condensation occurs while H has already acquired a nonzero VEV; whereas we suppose here that gaugino condensation triggers a nonzero VEV for H.) It is doubtful that any substantial amount of entropy will be released during that process because an inflationary period taking place then would presumably prevent these bubbles from coalescing, much in the same way as in old inflationary scenarios.[43] The tunneling process should therefore be sufficiently rapid in order that at

$T \leq \Lambda_c$ all our observed Universe should be contained in a single bubble of true vacuum [$\langle H_{mnp}\rangle = c \neq 0$ given by (2.7)]. Within this bubble we can compute the one-loop temperature corrections to the potential for S.

They have the form[44,45]

$$\Delta V_T = \frac{1}{24}\frac{T^2}{M_{\rm Pl}{}^2}(\,\mathrm{STr}M_0{}^2 - \tfrac{1}{2}\,\mathrm{STr}M_{1/2}{}^2 + \mathrm{STr}M_1{}^2 - \tfrac{1}{2}\mathrm{STr}M_{3/2}{}^2)\,, \tag{2.20}$$

where

$$\mathrm{STr}M_J{}^2 = (-1)^{2J}(2J+1)\mathrm{Tr}M_J{}^2,\quad J=0,\tfrac{1}{2},1\,,$$
$$\mathrm{STr}M_{3/2}{}^2 = 4m_{3/2}{}^2\,. \tag{2.21}$$

The relevant traces have been computed in Ref. 35 (there is a misprint in Eq. (21) of Ref. 35: the last term in $\mathrm{Tr}M_F{}^2$ should read $(3\,|\,h\,|^2/2b_0)e^G\alpha^3e^{-\alpha}$ [cf. $\mathrm{Tr}M_{1/2}{}^2$ in (2.22)] and $N_G/4$ should be replaced by N_G) in terms of U and $e^{\mathscr{G}}$ defined in (2.3) and (2.5) ($\hat{V},\mathscr{D}$ do not contribute when we put the y_i to zero):

$$\mathrm{Tr}M_0{}^2 = 2(N+6)U - 2(U-e^{\mathscr{G}}) + \frac{3\,|\,h\,|^2}{b_0}e^G\alpha^3e^{-\alpha}\,,$$
$$\mathrm{Tr}M_{1/2}{}^2 = (10+N_G)U - 4(U-e^{\mathscr{G}}) + \frac{3\,|\,h\,|^2}{2b_0}e^G\alpha^3e^{-\alpha}\,, \tag{2.22}$$
$$\mathrm{Tr}M_1{}^2 = 0\,,$$

where N is the number of matter chiral multiplets (the y_i's) and N_G the effective number of gauginos. The temperature correction therefore reads

$$\Delta V_T = \frac{1}{24}\frac{T^2}{M_{\rm Pl}{}^2}\left[(2N+N_G+20)U - 4(U-e^{\mathscr{G}}) + \frac{9\,|\,h\,|^2}{2b_0}e^G\alpha^3e^{-\alpha}\right]. \tag{2.23}$$

It is usually sufficient in grand-unified supersymmetric models[44,45] to study the leading N behavior of the temperature corrections, since the number of chiral fields is large (at least $3\times 27 = 81$ in our case). However, in such an approximation, it is clear that the ground state is not displaced at high temperatures since the corresponding part in ΔV_T is proportional to U and hence to the zero-temperature potential [Eq. (2.5)]. We therefore have to include the nonleading part, but this shows already that the displacement of the ground state due to temperature corrections will be minute.

Using Eqs. (2.3), (2.4), and (2.16) to express U and $e^{\mathscr{G}}$ one obtains for the full temperature correction:

$$\Delta V_T(\alpha,\beta) = \frac{1}{24}\frac{T^2}{M_{\rm Pl}{}^2}\mu^4\frac{\alpha_0}{\alpha}\left[(2N+N_G+20) + \frac{1}{\hat{c}^2}e^{-\alpha}[(2N+N_G+16)(1+\alpha)^2+4+3\alpha^4] - \frac{2}{\hat{c}}e^{-\alpha/2}[(2N+N_G+16)(1+\alpha)+4]\cos\frac{\beta}{2}\right]. \tag{2.24}$$

Since the coefficient of the $\cos\beta/2$ term is always negative, it is obvious that at high temperature the ground state remains at $\beta_0 = 4n\pi$ (n integer). Let us note moreover that even if some sort of initial conditions yielded a positive coefficient [the axion-type symmetry $\beta\to\beta+4n\pi$ remains presumably unbroken, which ensures that the potential is of the form $f(\alpha)+g(\alpha)\cos\beta/2$], this would not be sufficient to provide the right amount of inflation. Indeed the potential in the β direction reads [we take $\alpha=\alpha_0$ in (2.16)]

$$V(\alpha_0,\beta) = 4\mu^4\sin^2\frac{\beta}{4}\,. \tag{2.25}$$

One has to normalize the fields properly since their kinetic term is not in the canonical form

$$\mathscr{L}_{\rm kin} = \frac{1}{(S+S^*)^2}\partial^\mu S^*\partial_\mu S = \frac{1}{4\alpha^2}(\partial^\mu\alpha\partial_\mu\alpha + \partial^\mu\beta\partial_\mu\beta)\,. \tag{2.26}$$

Therefore the potential for the properly normalized field $\hat{\beta} = \beta/\alpha_0\sqrt{2}$ is

$$V(\hat{\beta}) = 4\mu^4\sin^2\left[\frac{\alpha_0}{2\sqrt{2}}\hat{\beta}\right]. \tag{2.27}$$

If, because of some specific initial conditions other than high temperatures, the field $\hat{\beta}$ started around the maximum of the potential $\hat{\beta} = \pi\sqrt{2}/\alpha_0$, the number of e-foldings undergone by the cosmic scale factor during the rollover would be

$$N_e = -\int_{\hat{\beta}_0}^{\hat{\beta}_e}\frac{3H^2(\hat{\beta})M_{\rm Pl}{}^2}{V'(\hat{\beta})}d\hat{\beta} \sim -\int_{\hat{\beta}_0}^{\hat{\beta}_e}\frac{V(\hat{\beta})}{V'(\hat{\beta})}d\hat{\beta}\,, \tag{2.28}$$

where $\hat{\beta}_0$ is the value of $\hat{\beta}$ for which one can start treating classically the evolution of the field in the potential, and $\hat{\beta}_e$ its value at the end of the inflation era; H is the Hubble constant [$H^2(\hat{\beta}) \sim V(\hat{\beta})/(3M_{\rm Pl}{}^2)$]. It turns out that the dominant contribution comes from the $\hat{\beta}_0$ end of the integral. Taking[46]

$$\left[\frac{\pi\sqrt{2}}{\alpha_0} - \hat{\beta}_0\right] \sim H_0/M_{\rm Pl} = (2/\sqrt{3})\mu^2/M_{\rm Pl}{}^2\,,$$

we obtain

$$N_e \sim -\frac{4}{\alpha_0{}^2}\ln\sin\left[\frac{\alpha_0}{\sqrt{6}}\frac{\mu^2}{M_{\rm Pl}{}^2}\right] \ll 65 \qquad (2.29)$$

for any reasonable value of $\mu/M_{\rm Pl}$. Therefore even if initial conditions yielded the most favorable case (and temperature corrections do not), the evolution of the field S in its imaginary direction (β) does not lead to any substantial amount of inflation.

From now on we will fix β at its ground-state value $\beta_0=0$. The potential at $T=0$ then reads

$$V(\alpha)=\mu^4\frac{\alpha_0}{\alpha}\left[1-\frac{1}{\hat{c}}(1+\alpha)e^{-\alpha/2}\right]^2 . \qquad (2.30)$$

Its shape depends strongly on the value of $\hat{c}$: it is given schematically in Fig. 1 for different ranges of values of $\hat{c}$ [or equivalently α_0, using (2.15)]. The number of ground states can easily be found by studying the number of solutions of the equation $f(\alpha)\equiv(1+\alpha)e^{-\alpha/2}=\hat{c}$. The function f has a maximum for $\alpha=1$: $f(1)=2e^{-1/2}$; also $f(0)=1$. Therefore, the equation has no solution for $\hat{c}>2e^{-1/2}\sim1.21$, two solutions for $1\le\hat{c}<2e^{-1/2}$, and only one (positive) solution for $\hat{c}\le1$. Hence the curves in Fig. 1. For $\hat{c}>2e^{-1/2}$, there is no minimum with a zero cosmological constant.

The temperature corrections to this potential are given by (2.24) with $\beta=0$,

$$\Delta V_T(\alpha)=\frac{1}{24}\frac{T^2}{M_{\rm Pl}{}^2}\mu^4[(2N+N_G+16)v_1+v_2+v_3] , \qquad (2.31)$$

where

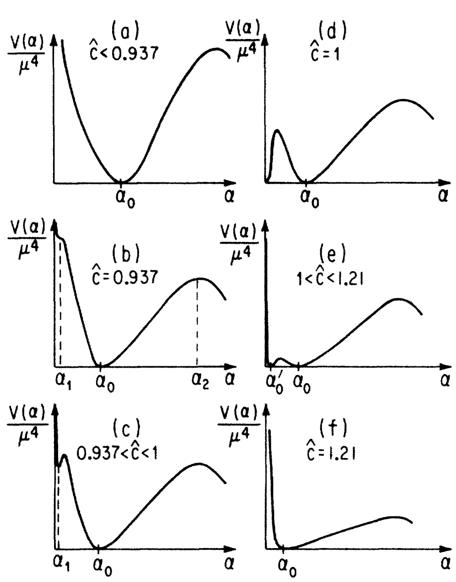

FIG. 1. Shape of the potential $V(\alpha)/\mu^4=(\alpha_0/\alpha)[1-(1/\hat{c})(1+\alpha)e^{-\alpha/2}]^2$ for different values of α_0 or equivalently $\hat{c}=(1+\alpha_0)e^{-\alpha_0/2}$: (a) $\hat{c}<0.937$, $\alpha_0>2.8$; (b) $\hat{c}=0.937$, $\alpha_0=2.8$; (c) $0.937<\hat{c}<1$, $2.51<\alpha_0<2.8$; (d) $\hat{c}=1$, $\alpha_0=2.5129$; (e) $1<\hat{c}<1.21$, $\alpha_0<2.51$, and $\alpha_0\neq1$; (f) $\hat{c}=1.21$, $\alpha_0=1$.

$$v_1(\alpha)=\frac{U}{\mu^4}=\frac{\alpha_0}{\alpha}\left[1-\frac{1}{\hat{c}}(1+\alpha)e^{-\alpha/2}\right]^2 ,$$
$$v_2(\alpha)=\frac{4e^{\mathcal{G}}}{\mu^4}=4\frac{\alpha_0}{\alpha}\left[1-\frac{1}{\hat{c}}e^{-\alpha/2}\right]^2 , \qquad (2.32)$$
$$v_3(\alpha)=\alpha_0\frac{3}{\hat{c}^2}e^{-\alpha}\alpha^3 .$$

Up to a factor μ^4, v_1 is the potential at zero temperature and is given by Fig. 1: it is minimal at α_0 and (possibly) α_0'. On the other hand, v_2 has a (local) maximum at $\alpha=\alpha_0$. However, as stressed earlier, because the factor $2N+N_G+16$ is large ($N>81, N_G>8+3+1+1=13$), the contribution of v_2,v_3 is negligible and the shape of the potential is basically not changed at high temperature. It is only when $v_1(\alpha)=0$, that is $\alpha\sim\alpha_0,\alpha_0'$, that the extra terms play a role. Indeed when $\alpha_0\le2.51$, α_0' is a local maximum of v_2 adjacent to the local maximum α_0 and $v_2(\alpha_0')<v_2(\alpha_0)$; similarly $v_3(\alpha_0')<v_3(\alpha_0)$. Therefore the ground state at high temperature is α_0' (or very close to it). In the limiting case $\alpha_0=2.51(\hat{c}=1)$, $v_1(0)=v_2(0)=v_3(0)=0$, and $\alpha_0'=0$ is a global minimum with zero cosmological constant even at high temperature.

None of this however helps very much with respect to inflation: whether α_0 or α_0' is the ground state, the minimum at high temperature does not seem to be one where (nonthermal) energy would be stored.

We will pause for a moment and consider the contribution $\overline{\Delta V}_T$, arising from fluctuations of the S and T fields alone, to the full effective potential ΔV_T. We have noted in the Introduction that S and T originate from the dilaton and structure-related fields. It seems of interest to study their behavior independently of the matter sector of the theory where we have to answer such questions as which mechanism for compactification, which choice for the gauge group, etc. The properties of the dilaton are, on the other hand, known since this field is present at the superstring level and we do not have to rely on the details of the structure-related sector: only the properties of the simplest such field, the breathing mode, are needed.

It turns out that, by setting $y_i=0$ in (2.22), we have gotten rid of most of the fluctuations in the y_i's. The only terms left in (2.22) which come from the y_i are the terms of order N (arising from differentiating twice with respect to the same field y_iy^i) and of order N_G for gauginos. The temperature corrections therefore read, when limited to fluctuations in S and T [compare with (2.23)]

$$\overline{\Delta V}_T(\alpha)=\frac{1}{24}\frac{T^2}{M_{\rm Pl}{}^2}\left[20U-4(U-e^{\mathcal{G}})+\frac{9|h|^2}{2b_0}e^{G}\alpha^3e^{-\alpha}\right] \qquad (2.33)$$

or

$$\overline{\Delta V}_T(\alpha)=\frac{1}{24}\frac{T^2}{M_{\rm Pl}{}^2}[16v_1(\alpha)+v_2(\alpha)+v_3(\alpha)] , \qquad (2.34)$$

where v_1,v_2,v_3 are given in (2.32). These corrections are given in Figs. 2 and 3 by plotting $V+\overline{\Delta V}_{T=M_{\rm Pl}}$ for the

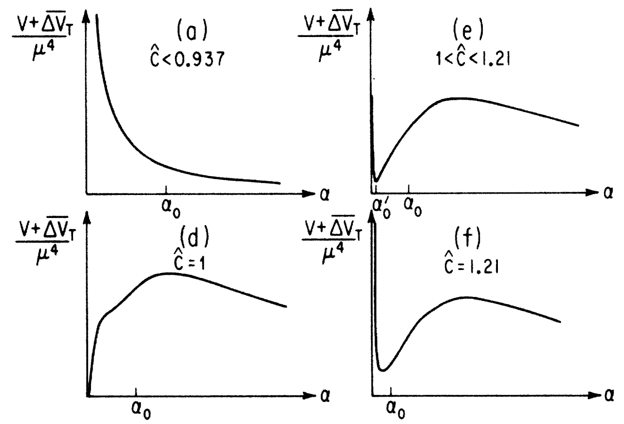

FIG. 2. Potential $(V+\overline{\Delta V}_T)(\alpha)/\mu^4$ at $T=M_{\rm Pl}$, where $\overline{\Delta V}_T(\alpha)$ are the temperature corrections contributed by S and T at the one-loop level and are given in Eq. (2.34). [Strictly speaking, these corrections are valid only for $T<\Lambda_c$, and the appropriate rescaling of $\overline{\Delta V}_T$ by a factor $(T/M_{\rm Pl})^2$ should be performed for realistic temperatures.] *The letters refer to the cases of Fig. 1:* (a) $\hat{c}<0.937$, $\alpha_0>2.8$; (d) $\hat{c}=1$, $\alpha_0=2.5129$; (e) $1<\hat{c}<1.21$, $\alpha_0<2.51$, and $\alpha_0\neq 1$; (f) $\hat{c}=1.21$, $\alpha_0=1$.

different cases [(a)—(f)] of Fig. 1. For cases (a), (d), (e), and (f) of Fig. 2, the situation is not very different from the one discussed above with the full temperature corrections ΔV_T. When $\alpha_0>2.8$ (a), the function is monotonously decreasing and the ground state is reached only for α infinite. In the presence of two zero-temperature global minima α_0' and α_0 [(e) and the two limiting cases (d) and (f)], the corrections $\overline{\Delta V}_T$ tend to favor a minimum close to α_0'. The cases where there is a local minimum α_1 with nonzero vacuum energy [(c) and the limiting case (b)] are more interesting. Figure 3 shows that, in these two cases,

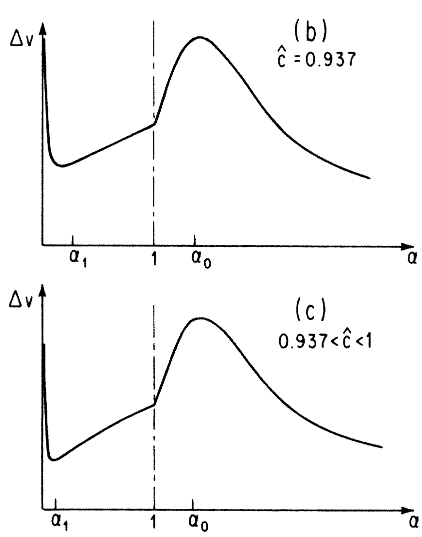

FIG. 3. Shape of the "truncated" temperature corrections [(b) and (c) of Fig. 1] [contribution of S and T only, Eq. (2.34)] $\Delta v=\overline{\Delta V}_T/[\frac{1}{24}(T^2/M_{\rm Pl}{}^2)\mu^4]$ at $T=M$, for (b) $\hat{c}=0.937$, $\alpha_0=2.8$; (c) $0.937<\hat{c}<1$, $2.51<\alpha_0<2.8$.

this local minimum is precisely the one preferred by the truncated temperature corrections $\overline{\Delta V}_T$. Therefore if, for some reason [one could note here that S and T decouple thermally from the rest of matter, down to temperatures much smaller than the Planck sale. As noted by Holman, Ramond, and Ross,[5] the same remark applies actually to any candidate for an inflaton field in supersymmetric theories; it is not clear however that this is a good enough reason to discard matter (y_i and gauginos) when computing the one-loop effective potential. Even if S and T fields form a decoupled phase which is at a temperature T' different from the one of matter, fluctuations will create matter fields at temperature T' and this is precisely what contributes to the effective potential (at least this argument makes sense in a nonexpanding universe, although, if there was no expansion, there would be no decoupling)] ordinary matter does not contribute to the temperature corrections for the potential of α, and if, for some other reason, $2.51\lesssim\alpha_0\lesssim 2.8$ (this in turn corresponds to $0.937\lesssim\hat{c}\leq 1$ which, since the value of c is quantized and a geometrical constant depending on the shape of the underlying compact manifold,[28] should correspond to a narrow set of manifolds) the minimum favored at high temperature is a local minimum $\alpha_1[V(\alpha_1)\neq 0]$ which is separated from the ground state by a small barrier at zero temperature.

Apart from temperature corrections, it has been noted recently[47,42] that a role could be played in the initial conditions by the Peccei-Quinn-type symmetry associated with translations of ImS: $h\to h+$const. For nonzero values of the corresponding charge Q, the h kinetic energy provides an energy barrier that prevents α from running away to infinity. Therefore if Q balls[48] were formed in the early stages, this would provide a way to constrain the field α in the region $\alpha<\alpha_0$.

Let us now consider in more detail the potential $V(\alpha)$ in the case where there is a local minimum α_1 with nonzero vacuum energy: $0.937\leq\hat{c}<1$ [(b) and (c) in Fig. 1]. We will determine whether the initial condition $\alpha=\alpha_1$ would yield an inflationary evolution that satisfies the constraints discussed in the Introduction.

Let us start with the limiting case $\hat{c}=\hat{c}_0=0.937$ [Fig. 1(b)] where $\alpha_1(=2-\sqrt{3}\equiv A)$, an inflection point, is not separated from the ground state α_0 by any barrier. The classical equation of motion of α in the potential V during the slow rollover phase reads

$$3H\dot{\alpha}=-2\alpha^2V'(\alpha)/M_{\rm Pl}{}^2 . \tag{2.35}$$

The unfamiliar form of the right-hand side is due to the noncanonical form of the kinetic term for α [cf. Eq. (2.26); one can check (2.35) by normalizing α: $\alpha=e^{-x\sqrt{2}}$].

This gives for the number of e-foldings that the cosmic scale factor undergoes during inflation:

$$N_e=\int_{\alpha_i}^{\alpha_e}-\frac{3H^2(\alpha)}{2\alpha^2V'(\alpha)M_{\rm Pl}{}^2}d\alpha$$

$$\sim\frac{V(A)}{A^2|V'''(A)|}\int_{\alpha_i}^{\alpha_e}\frac{d\alpha}{(\alpha-A)^2} , \tag{2.36}$$

where $H^2=V/3M_{\rm Pl}^2$ and we have used the fact that the dominant contribution to the integral comes from the lower bound $\alpha\sim\alpha_i\sim A$. Using $(\alpha_i-A)\sim H/M_{\rm Pl}$, one obtains $[V(A)=\mu^4\alpha_0/8,\ V'''(A)=-\mu^4(\sqrt{6}/16)\alpha_0 A^{-5/2}]$

$$N_e\sim M_{\rm Pl}{}^2\sqrt{3}\frac{\sqrt{V(A)}}{A^2|V'''(A)|}=4\alpha_0{}^{-1/2}A^{1/2}\left[\frac{M_{\rm Pl}}{\mu}\right]^2 \tag{2.37}$$

and the constraint $N_e>65$ [Eq. (1.1)] reads

$$\frac{\mu}{M_{\rm Pl}}<(\tfrac{4}{65}\alpha_0{}^{-1/2}A^{1/2})^{1/2}\sim 0.14\ , \tag{2.38}$$

which is not too stringent a bound (as stressed earlier, a stronger bound comes from the scale of density fluctuations).

What happens when we let $\hat{c}$ increase so that a barrier separates α_1 from the ground state α_0 [Fig. 1(c)]? Let us recall first the corresponding analysis of Hawking and Moss[46,49] for a scalar field of mass m whose self-interactions are described by a coupling λ; the value of the Hubble constant at the false vacuum is H. (i) For $m^2<\lambda^{1/2}H^2$, the field basically does not see the barrier and evolves classically in the potential as soon as it reaches the point at which nonlinear effects due to interactions become important. The situation is identical to the one just discussed ($\hat{c}=\hat{c}_0$). (ii) for $m^2>2H^2$, the field has to tunnel nonperturbatively through the barrier. Bubbles of the true vacuum materialize and expand. This is nothing else but the "old" inflationary scenario and it suffers the same shortcomings, in particular, large inhomogeneities.[1] Finally, (iii) in the intermediate region $\lambda^{1/2}H^2<m^2<2H^2$, the radius of the bubbles at the time they materialize is equal to the horizon H^{-1} and tunneling occurs homogeneously over any causally connected region.[49]

Applying this analysis to our case, we remark that, as soon as $\hat{c}\neq\hat{c}_0$, $m^2\sim\mu^4/M_{\rm Pl}{}^2$. Since $\lambda\sim\mu^4/M_{\rm Pl}{}^4$ and $H\sim\mu^2/M_{\rm Pl}{}^2$, $\lambda^{1/2}H^2\ll m^2\sim H^2$ which, barring some fine-tuning of the parameters, corresponds to the troublesome region (ii). To see this more quantitatively, let us make an expansion around $\hat{c}_0$: $\hat{c}=\hat{c}_0+\epsilon, \alpha_1=A-\delta\alpha$.

Since the relation between $\hat{c}$ and α_1 reads

$$\hat{c}=(1+\alpha_1{}^2)e^{-\alpha_1/2} \tag{2.39}$$

we obtain

$$\delta\alpha=(2e^{A/2}/\sqrt{3})^{1/2}\epsilon^{1/2}\ . \tag{2.40}$$

The two relevant parameters are therefore

$$\begin{aligned} m^2&=\frac{V''(\alpha_1'')}{M_{\rm Pl}{}^2}=-\frac{V'''(A)}{M_{\rm Pl}{}^2}\delta\alpha \\ &=\frac{\mu^4}{M_{\rm Pl}{}^2}\alpha_0\frac{3^{1/4}}{8}A^{-5/2}e^{A/4}\epsilon^{1/2}\ , \\ H^2&=\frac{V(\alpha_1'')}{3M_{\rm Pl}{}^2}\sim\frac{V(A)}{3M_{\rm Pl}{}^2}=\frac{\mu^4}{M_{\rm Pl}{}^2}\alpha_0\frac{1}{24}\ , \end{aligned} \tag{2.41}$$

and a scenario in the manner of Hawking and Moss is possible [case (iii)] if $m^2<2H^2$, that is,

$$\epsilon<\frac{4\sqrt{3}}{27}A^5e^{-A/2}\sim 3\times10^{-4}\ . \tag{2.42}$$

Therefore the only viable scenario we have found, based on the properties of the potential V given by Eq. (2.30), is for the range of parameters

$$\hat{c}=0.9374+\delta\hat{c},\ \ \delta\hat{c}<3\times10^{-4}\ . \tag{2.43}$$

We have noted earlier that $\hat{c}$ is a calculable quantity, once one knows the underlying Kaluza-Klein compact manifold. As long as such a manifold with the value of $\hat{c}$ given in (2.43) has not been exhibited (this manifold should also yield the complete low-energy phenomenology), we do not think it wise to pursue the analysis of the corresponding scenario.

Another reason not to do so is the danger that the field α might overshoot, in which case, instead of oscillating around the ground state α_0, it would reach the region beyond the maximum α_2 at the right of α_0 and escape to infinity. The reason is that, for $\hat{c}=\hat{c}_0$,

$$V(\alpha_1)\sim 0.35\mu^4>V(\alpha_2)\sim 0.24\mu^4 \tag{2.44}$$

and for any reasonable value of μ, the "friction" due to the Universe expansion might not be large enough to prevent the field from reaching α_2.

Instead of taking $\alpha=\alpha_1$ as an initial condition which led us to the unrealistic choice of parameter (2.43) and potential trouble with overshooting, one could try to start with $\alpha=\alpha_2$, the maximum which separates the ground state α_0 from $\alpha\to\infty$. The problem here is that any initial configuration $\alpha\sim\alpha_2$ would lead to the superposition of two wave packets: one drawing toward the ground state and the other one to infinity. After some time, the only relevant one, as far as the evolution of the Universe is concerned, will be the second one. Let us point out here that this would be a serious drawback to any chaotic scenario of inflation.[50] Chaotic scenarios evade the determination of initial conditions by letting the different causally connected patches which form the Universe choose them at random. The problem is that for those regions that start with $\alpha>\alpha_2$, there will be no end to inflation and they will soon dominate the Universe.

Therefore, although the flat direction associated with the dilation field in superstring theories seems to provide the right ingredient for an inflationary scenario, the four-dimensional theory that is believed to emerge from it does not satisfy this expectation. Besides problems with determining the right initial conditions (which are shared by almost any supergravity model), the potential $V(S)$, where S is related to the dilaton through Eq. (1.6), does not seem to give rise to an inflationary evolution, at least for any reasonable range of parameters.

We finally comment on the possible role of the other structure-related fields mentioned at the end of Sec. I: $T^{(k)}, k=2,\ldots,b_{1,1}$ and $C^{(\alpha)}\alpha=1,\ldots,b_{2,1}$.

The $T^{(k)}$ fields, defined in (1.7) have properties very similar to the T field described above. In particular they do not appear in the superpotential (because of a Peccei-Quinn symmetry associated with $\beta^{(k)}$) and they remain massless after supersymmetry breaking. A determination of their potential would thus require a detailed knowledge of the one-loop corrections. As in the case of T we will not pursue their analysis further.

On the other hand, some of the properties of $C^{(\alpha)}$, the fields related to the description of the complex structure of the compact manifold K, make them plausible candidates for the inflaton field. First of all, they acquire a mass at the scale of supersymmetry breaking. The reason[28] is that a given VEV for H [i.e., a value of c in (2.4)] represents a given integral cohomology class for a particular choice of complex structure. The complex structure is therefore fixed once supersymmetry is broken (through nonzero $\langle H \rangle$), which amounts to the $C^{(\alpha)}$ acquiring a mass. This means in particular that the scale that determines the nontrivial structure in the potential of the $C^{(\alpha)}$ is given in terms of $m_{3/2}$. As in the case of S discussed above, this should allow for the hierarchy of scales needed for inflation [cf. (1.3)]. We might even have more freedom than we had with S because the $C^{(\alpha)}$ fields could be present in the superpotential: there is no Peccei-Quinn-type symmetry that prevents it (actually, some of the F terms depend on the choice of complex structure). On the other hand, renormalizable terms in the superpotential should not couple the $C^{(\alpha)}$ to the matter fields. As discussed in Ref. 51, such a coupling—necessarily of the form $C^{(\alpha)}\mathbf{27}\overline{\mathbf{27}}$—would mean that the number of massless fields depends on whether the $C^{(\alpha)}$ have a nonzero VEV or not. But this number is a topological invariant that cannot depend on shifts in the complex structure of the compact manifold. The absence of such couplings means that the $C^{(\alpha)}$ are in a hidden sector, i.e., a sector that interacts with ordinary matter only through gravitational interactions. As stressed earlier, this is a welcome feature for an inflaton field.

On the basis of these remarks, we could build a model where one of the $C^{(\alpha)}$ fields would play the role of inflaton. We will refrain from doing so because the freedom that we have is only a reflection of our poor knowledge of the detailed couplings of these fields. More insight is necessary—in particular through the construction of an explicit example where the general remarks that we have made would receive an illustration—before one can make an attempt at using the $C^{(\alpha)}$ in an inflationary scenario.

III. MATTER FIELDS

We now turn to the fields in the matter multiplets transforming as parts of **27** and $\overline{\mathbf{27}}$ representations of E_6. Let us consider first one **27**. Under SO(10), it decomposes as

$$\Phi \equiv \mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1} \ .$$

We will require the inflaton to be a gauge singlet of the strong and electroweak interactions $SU(3)\times SU(2)\times U(1)_Y$. This leaves us with only two candidates: the SO(10) singlet N and the SU(5) singlet in the 16 of SO(10),N'. Under the minimal subgroup of rank 6 $SU(3)\times SU(2)\times U(1)_Y\times U(1)_{Y'}\times U(1)_{Y''}$, these two fields have the charges [we adopt here the normalizations of Ref. 52 which ensures that all U(1) couplings g_Y are equal at the grand unification scale, and to a good approximation, at all scales]

$$N:\ Y=0,\ \ Y'=-\tfrac{5}{3},\ \ Y''=-(\tfrac{5}{3})^{1/2} \ ,$$

$$N':\ Y=0,\ \ Y'=\tfrac{5}{3},\ \ Y''=(\tfrac{5}{3})^{1/2} \ .$$

Putting to zero all the fields which are not singlets under $SU(3)\times SU(2)\times U(1)_Y$, we readily see that N and N' correspond to flat directions of the F terms in the potential, since there is no term in the superpotential ($\sim\Phi^3$) involving only N and N' (such a term would have $Y'=-5$). On the other hand, the D terms do not cancel and the potential is therefore minimized for $N=N'=0$. Since the gauge symmetry is always larger than $SU(3)\times SU(2)\times U(1)_Y$ by one or two units of rank,[53] this implies that the extra gauge group is only broken at energies comparable to the weak breaking scale M_W (since N,N' are the only fields that could break it, and they cannot do it alone). [This in turn implies a strong mixing between the Z gauge boson and extra neutral gauge boson(s) which in most cases, proves to be ruled out by experiment.[52]] For similar reasons, neither N nor N' can be used for inflation.

To evade these conclusions, one has simply to remark[25] that the presence of an $\overline{N}$ field (or $\overline{N}'$), mirror image of the N (or N') in the $\overline{\Phi}=\overline{\mathbf{27}}$ would allow the possibility of canceling the D term. We will suppose for the moment that the only $SU(3)\times SU(2)\times U(1)_Y$ singlet of $\overline{\mathbf{27}}$ present in the theory is a $\overline{N}$ field (the same conclusions would be reached with $\overline{N}'$):

$$\overline{N}:\ Y=0,\ \ Y'=\tfrac{5}{3},\ \ Y''=(\tfrac{5}{3})^{1/2} \ .$$

Clearly, $N=\overline{N}$ is a flat direction of the potential (none of the terms in the superpotential originating from Φ^3 and $\overline{\Phi}^3$ involve N and $\overline{N}$ only). Therefore, once again a flat direction appears naturally in a sector of the theory, which could account for the development of an inflationary era. Of course, this direction cannot remain and some sort of structure has to develop, since $N=\overline{N}$ has to break the extra symmetry. The main question is at what scale does this occur?

It has been recognized[25] that nonrenormalizable terms in the effective low-energy superpotential g will lift the degeneracy:

$$g(\Phi)=\Phi^3+\overline{\Phi}^3+\sum_{n=n_0}\frac{\lambda_n}{M_{\rm Pl}{}^{2n-3}}(\Phi\overline{\Phi})^n \ , \tag{3.1}$$

where $n_0\geq 2$. [Note that as far as $N,\overline{N}$ are concerned, (3.1) includes the most general nonrenormalizable terms neutral under the U(1) charges.] The corresponding low-energy potential for N and $\overline{N}$ reads, from (2.5)

$$V(N,\overline{N})=m^2(|N|^2+|\overline{N}|^2)+\frac{n_0{}^2\lambda_{n_0}{}^2}{M_{\rm Pl}{}^{4n_0-6}}(|N\overline{N}|)^{2n_0-2}(|N|^2+|\overline{N}|^2)+D\ \text{terms} \ , \tag{3.2}$$

where we have kept only the term of lowest dimension and have added a mass term that is assumed to arise from supersymmetry breaking (as discussed in the last section).

[We rescale the fields so that their kinetic energy has a canonical form in the low-energy limit. Similarly, in order that the F terms in the low-energy potential be expressed in terms of the superpotential g in the standard way ($|g^i|^2$), g differs from $\tilde{W}$ in (2.4) by the rescaling $g=u_0{}^{3/4}(2b_0\alpha_0/3)^{-1/2}\tilde{W}$.] In the direction $N=\overline{N}$ (real), the D terms cancel and

$$V(N=\overline{N})=2m^2N^2+\frac{2n_0{}^2\lambda_{n_0}{}^2}{M_{\rm Pl}{}^{4n_0-6}}N^{4n_0-2} . \tag{3.3}$$

The ground-state value and the inflation scale are given by

$$\begin{aligned}\sigma &\equiv \langle N\rangle \\ &=\langle \overline{N}\rangle \\ &\sim M_{\rm Pl}[(|m^2|)^{1/2}/M_{\rm Pl}]^{1/(2n_0-2)}\lambda_{n_0}{}^{-1/(2n_0-2)} \\ &\sim M_{\rm Pl}(M_W/M_{\rm Pl})^{1/(2n_0-2)} , \\ V_0 &\equiv \mu_0{}^4 \\ &\equiv V(0)-V(\sigma) \\ &\sim M_{\rm Pl}{}^4[(|m^2|)^{1/2}/M_{\rm Pl}]^{(2n_0-1)/(n_0-1)}\lambda_{n_0}{}^{-1/(n_0-1)} \\ &\sim M_{\rm Pl}{}^4(M_W/M_{\rm Pl})^{(2n_0-1)/(n_0-1)} ,\end{aligned} \tag{3.4}$$

where we have used the fact that $-m^2$ should be of the order of the weak-interaction breaking scale M_W [since the same supersymmetry-breaking mass term is expected to be present for the Higgs doublets of $SU(2)\times U(1)$].

Let us briefly discuss the value of n_0 that we should expect. It was first considered[25] that a quartic coupling $(\Phi\overline{\Phi})^2$ is always present when one integrates out the superheavy fields corresponding to the internal degrees of freedom: if this is true, $n_0=2$ and the intermediate scale is $\langle N\rangle\sim\sqrt{M_W/M_{\rm Pl}}\sim 10^{10}$ GeV. However, it has been recently argued by del Aguila *et al.*[54] that in a specific model, $n_0=3$. This model was first discussed by Witten[53] and is based on the gauge group $SU(3)\times SU(2)_L \times SU(2)_R\times U(1)_F\times U(1)_{Y'}$. Its representation content is four generations (complete 27) plus parts of $27+\overline{27}$ which include the singlets of SO(10) (we will denote them by N_0 and $\overline{N}_0$). The symmetry of the Calabi-Yau manifold yields a discrete symmetry $Z_2\times Z_5$, where the Z_5 symmetry is associated with a generation index. We will therefore denote the six N fields by $N_{\pm1},N_{\pm2},N_0,\overline{N}_0$. As before, if we do not include the nonrenormalizable terms, $\overline{N}_0{}^2=\sum_{i=-2}^{2}N_i{}^2$ is a flat direction of the potential (it cancels the D terms). The new feature[54] is that quasisymmetries (see Ref. 53 for a discussion of these quasisymmetries) impose that $N_{\pm1}$ are not present in the $n=2$ term of the superpotential

$$g=\frac{\lambda_2}{M_{\rm Pl}}\overline{N}_0{}^2N_0{}^2+\frac{\lambda_2'}{M_{\rm Pl}}\overline{N}_0{}^2N_2N_{-2}+O(M_{\rm Pl}{}^{-3}) . \tag{3.5}$$

(However, one should be aware of the fact that these quasisymmetries hold only at the tree level[53] and are therefore not necessarily valid for the nonrenormalizable terms we are considering. As a matter of fact, these terms violate an R symmetry which is another of these quasisymmetries.[53]) Therefore $\overline{N}_0{}^2=N_1{}^2+N_{-1}{}^2$ is a flat direction of the potential up to terms of order $M_{\rm Pl}{}^{-3}$ and $n_0\geq 3$. It is easy to check that actually $n_0=3$ which yields an intermediate scale $\langle N\rangle\sim(M_WM_{\rm Pl}{}^3)^{1/4}\sim 10^{15}$ GeV. This provides an elegant way to solve the problem of baryon-number violation in these theories.[54]

The question of the origin of nonrenormalizable terms in the superpotential has been recently discussed by Witten.[51] It is rather simple to realize that higher orders in the σ-model perturbation theory do not renormalize the superpotential. Indeed, the σ-model coupling γ is given by[38] $\gamma^{-2}={\rm Re}T$; the superpotential being an analytic function, any dependence in that coupling would imply a dependence in ${\rm Im}\,T$. This is forbidden by the Peccei-Quinn symmetry discussed earlier, which is respected to all finite orders of σ-model perturbation theory.[55] Furthermore, Witten argued that, at least in some cases, even when integrating our massive Kaluza-Klein states, the superpotential remains unchanged. In such instances, the flat directions are not lifted as long as one stays away from the region close to the compactification scale, or $M_{\rm Pl}$. (For example, when N or $\overline{N}$ reach scales of order $M_{\rm Pl}$, their kinetic energy starts playing an important role because of its noncanonical form: for the rescaled fields that we are considering here, the kinetic term in the Lagrangian has a nontrivial factor $[1-(|N|^2+|\overline{N}|^2)/(3M_{\rm Pl}{}^2)]^{-1}$ which can be approximated to 1 only for $N,\overline{N}\ll M_{\rm Pl}$.) It has been noted recently[56] however that world-sheet nonperturbative effects (instantons) will, in most cases, life the degeneracy. We will return to this question below. For the moment, we will let n_0 in (3.1) have arbitrarily large values.

When studying the potential V [Eq. (3.3)], one has to remember that the mass is a running mass that, through quantum corrections, depends itself on the value of the field N. The renormalization-group equations (RGE) describing the evolution of the scalar masses form a system of coupled differential equations (see, for example, Ref. 52). For simplicity, we will assume here that m satisfies the oversimplified renormalization-group equation

$$\mu\frac{dm^2}{d\mu}=\tilde{\lambda}^2(m^2+m_0{}^2) , \tag{3.6}$$

where $\tilde{\lambda}$ is a typical Yukawa coupling, taken to be constant (once again a crude approximation) and μ is the running scale (not to be confused with the scale of the potential in Sec. II). We retain here only one feature of the solutions of the coupled equations: when the scale μ goes to zero, the scalar masses approach a fixed point.[57,52] In particular, the fixed-point value for the mass squared of N is negative (it would be difficult to induce gauge symmetry breaking otherwise); we denote it by $(-m_0{}^2)$. The solution of Eq. (3.6) therefore reads

$$m^2=-m_0{}^2\left[1+\xi\left[\frac{\mu}{N}\right]^{\tilde{\lambda}^2}\right] , \tag{3.7}$$

where ξ is a constant, and we have used the fact that m^2 scales as $m_0{}^2$ throughout its evolution (to put it differently, there is basically one mass parameter, the mass at the compactification scale—or rather Λ_c—that determines in particular the mass at the fixed point). In Eq. (3.7), N appears because, at least in the region $N \gg m_0$ (and therefore in particular for $N \sim \sigma$), it is the only relevant dimensionful parameter in the problem.[58] The complete N dependence of the potential V is obtained from Eqs. (3.3) and (3.7). Using the condition $(dV/dN)(\sigma)=0$ to express μ in terms of σ, we obtain

$$V=2m_0{}^2N^2\left[-1+\frac{2}{2-\tilde{\lambda}^2}\left[\frac{\sigma}{N}\right]^{\tilde{\lambda}^2}\right]+2n_0{}^2\lambda_{n_0}{}^2N^2M_{\rm Pl}{}^2\left[\left[\frac{N}{M_{\rm Pl}}\right]^{4n_0-4}-\frac{4n_0-2}{2-\tilde{\lambda}^2}\left[\frac{\sigma}{N}\right]^{\tilde{\lambda}^2}\left[\frac{\sigma}{M_{\rm Pl}}\right]^{4n_0-4}\right]. \tag{3.8}$$

Going back to our discussion of inflation, there is no problem here with determining the initial conditions: at high temperature, the symmetry that is broken by N or $\overline{N}$ is restored and the ground state is $N=\overline{N}=0$. As the Universe cools down, the temperature reaches $T_i \sim \mu_0$, where the energy stored in the vacuum V_0 dominates the energy density of the Universe, leading to an exponential expansion. We need therefore to know the precise shape of the potential V near $N=0$, for which value unfortunately Eq. (3.8) breaks down. The reason is that the parametrization for m given in (3.7) is not valid near $N=0$ because it generates spurious large logarithms $\ln\mu/N$. One can think first of replacing the scale N by m_0, but the important point is that renormalizing the mass at any large scale μ will generate large logarithms. However, in the region where N is very close to zero, one can safely assume that m has reached its fixed-point value $-m_0$, and

$$V(N\sim 0)=-2m_0{}^2N^2+2n_0{}^2\lambda_{n_0}{}^2M_{\rm Pl}{}^4\left[\frac{N}{M_{\rm Pl}}\right]^{4n_0-2}. \tag{3.9}$$

The temperature corrections are once again given by Eq. (2.20) and from the results of Ref. 35 one obtains, using the same field normalization as for the zero-temperature potential,

$$\Delta V_T(N\sim 0)=\frac{1}{8}T^2\left[\frac{80}{9}g_Y{}^2N^2\frac{1}{\left[1-\frac{2}{3}\left[\frac{N}{M_{\rm Pl}}\right]^2\right]}+\left[\alpha_0{}^2+\frac{4}{3}\right]\frac{m_{3/2}{}^2}{\left[1-\frac{2}{3}\left[\frac{N}{M_{\rm Pl}}\right]^2\right]^3}\right.$$
$$\left.-\frac{8}{3}m_{3/2}M_{\rm Pl}\lambda_{n_0}\left[\frac{N}{M_{\rm Pl}}\right]^{2n_0}+2n_0{}^2[(n_0-1)^2+n_0{}^2]\lambda_{n_0}{}^2\left[\frac{N}{M_{\rm Pl}}\right]^{4(n_0-1)}M_{\rm Pl}{}^2\right], \tag{3.10}$$

where g_Y is the coupling constant associated with the U(1) gauge symmetry broken by N. It is clear from (3.9) and (3.10) that the behavior of the temperature-dependent potential is different from the Coleman-Weinberg[58] case (see, for example, the discussion of Ref. 59). For temperatures larger than $T_c \sim (\frac{9}{5})^{1/2}(m_0/g_Y)$, the only minimum is $N=0$, but as soon as $T<T_c$, $N=0$ becomes a local maximum and a nonzero minimum starts developing which will eventually reach the value σ. There is therefore no barrier as there was in the Coleman-Weinberg case (see Fig. 4).

In order to determine the number N_e of e-foldings undergone by the cosmic scale factor, we compute the value of the entropy of the Universe S_i at T_c and S_f after reheating:[59]

$$S_i=\frac{4\pi^2}{90}n_iT_c{}^3,\quad S_f=\frac{4\pi^2}{90}n_fT_{\rm rh}{}^3, \tag{3.11}$$

where n_i,n_f are the effective numbers of degrees of freedom at T_c and $T_{\rm rh}$. The reheating temperature $T_{\rm rh}$ is given by

$$V_0=\mu_0{}^4=\frac{\pi^2}{30}n_fT_{\rm rh}{}^4. \tag{3.12}$$

Hence, the factor of entropy increase is

$$\frac{S_f}{S_i}=\frac{n_f{}^{1/4}}{n_i}\left[\frac{30}{\pi^2}\right]^{3/4}\left[\frac{\mu_0}{T_c}\right]^3 =\frac{n_f{}^{1/4}}{n_i}g_Y{}^3\left[\frac{250}{27\pi^2}\right]^{3/4}\left[\frac{\mu_0}{m_0}\right]^3, \tag{3.13}$$

and using (3.4) with $|m^2|=m_0{}^2\approx M_W{}^2$,

$$\frac{S_f}{S_i}=e^{3N_e}\approx\frac{n_f{}^{1/4}}{n_i}g_Y{}^3\left[\frac{M_{\rm Pl}}{m_0}\right]^{3(2n_0-3)/4(n_0-1)}. \tag{3.14}$$

This yields an entropy release of a $10^8 (N_e \sim 6)$ for $n_0=2$, and $10^{20} (N_e\sim 16)$ for $n_0 \gg 1$. Therefore, although a substantial amount of entropy is released during the period $T_c<T<\mu_0$ when, due to thermal effects, the minimum is located at the origin but the vacuum energy dominates the energy density, this is definitely not enough to satisfy the constraint (1.1). Once again, although the model provides a natural flat direction in the potential of some scalar fields, our analysis shows that this does not provide a framework for an inflationary scenario.

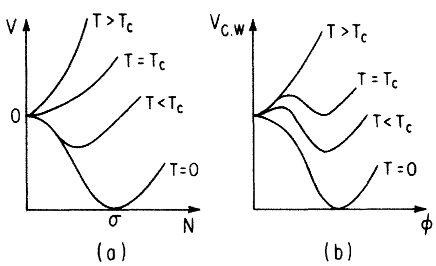

FIG. 4. Comparison of the temperature dependence of the potential $V(N)$ of Eqs. (3.9) and (3.10) and of a Coleman-Weinberg potential.

We mentioned earlier that nonperturbative effects on the world sheet[56] seem, in most cases, to lift the degeneracy associated with flat directions. More precisely, world-sheet instantons generate corrections to the superpotential of the four-dimensional theory. Of particular interest to us is the term $N^2\bar{N}^2/M_{\rm Pl}$ which appears with a coupling of order $e^{-1/\gamma^2}\sim e^{-{\rm Re}T}$, typical of world-sheet instanton contribution (γ, the σ-model coupling constant, has been defined earlier). In the notation of (3.1), $n_0=2$ and λ_2 is of order $e^{-{\rm Re}T}$. We obtain, from (3.4),

$$\sigma/M_{\rm Pl}\sim e^{{\rm Re}T/2}(M_W/M_{\rm Pl})^{1/2}\ ,$$
$$\mu_0/M_{\rm Pl}\sim e^{{\rm Re}T/4}(M_W/M_{\rm Pl})^{3/4}\ . \quad (3.15)$$

For $e^{-{\rm Re}T}$ small, the intermediate scale σ is thus larger than previously expected.[25] A small value for $e^{-{\rm Re}T}$ is also desirable to generate hierarchies between Yukawa couplings. Moreover, it gives a larger inflation scale μ_0 which in turn yields a longer inflationary epoch ($T_c<T<\mu_0$). As one can check from (3.13) and (3.14), the number of e-foldings N_e receives an additional contribution $({\rm Re}T)/4>0$. But this is presumably not enough to obtain the amount of entropy release S_f/S_i that is needed ($N_e\sim 6+{\rm Re}T/4$ is still much smaller than 65).

IV. CONCLUSIONS

Because the scale of compactification is presumably close to the Planck scale, there are good reasons to believe that the compactified versions of superstring theories are subject to the same cosmological problems as all other four-dimensional models: production of primordial gravitons, monopoles, gravitinos that makes these species too abundant at the present time. This suggests that, if the Universe went through an inflationary era, it must have occurred after compactification.

This has prompted us to study in this paper whether the cosmological scenario that has been devised for inflation in locally supersymmetric theories[2–8] can be implemented in superstring models. The flat directions of the scalar potential, that play an important role in these models, seem to provide the right starting point for developing such a scenario. We have studied in detail two of these flat directions.

The first one is associated with the dilaton field of superstring theories (to be precise, its four-dimensional version S). The corresponding degeneracy is lifted by supersymmetry breaking and the nontrivial scalar potential that is generated scales as $\mu_0{}^4$, where μ_0 is of order $\sqrt{m_{3/2}M_{\rm Pl}}$. Besides the flatness of its potential, some other properties of S make it a good candidate for the inflaton field. It is in a hidden sector (its interactions with ordinary matter are only gravitational); its ground state is supersymmetry breaking (choosing the inflaton field in the supersymmetry-conserving sector often leads to the "entropy crisis" problem[60]); μ_0 has the right scale ($\mu_0/M_{\rm Pl}\sim 10^{-3}$) for values of $m_{3/2}$ that might be realistic and that take care of the gravitino problems ($m_{3/2}\sim 10^{13}$ GeV). The first difficulty that we encountered is the determination of the initial conditions. The full temperature corrections to the potential stabilize the field at high temperature near its ground state, where no vacuum energy is stored. It is only by restricting the temperature corrections to the fluctuations of the S and T fields that one obtains a nontrivial behavior: a potential energy of order $\mu_0{}^4$ at the minimum of the temperature-dependent potential.

However, whatever the initial conditions are, the evolution of the S field in the potential gives rise to an inflationary scenario only for an intolerably narrow range of parameters [Eq. (2.43)]:

$$\hat{c}=0.9374+\delta\hat{c},\quad \delta\hat{c}<3\times 10^{-4}\ , \quad (4.1)$$

where $\hat{c}$ is given in terms of the gauge coupling constant $\alpha_{\rm GUT}$ at the compactification scale by [see Eqs. (2.10) and (2.15)]

$$\hat{c}=\left[1+\frac{3}{4\pi b_0\alpha_{\rm GUT}}\right]e^{-3/(8\pi b_0\alpha_{\rm GUT})}\ . \quad (4.2)$$

The second flat direction of potential that we have studied occurs in the nonsinglet sector of the theory and is associated with the possible presence of an intermediate scale of gauge-symmetry breaking between M_W and $M_{\rm Pl}$. The properties of the corresponding field N (or N') are somewhat less attractive: it is not a gauge singlet (although it is a singlet under the low-energy gauge symmetry) and it is not weakly coupled to ordinary matter (a coupling to matter is necessary in order to drive its mass-squared negative through renormalization-group equations, and induce a nonzero VEV). We found that, for some time during the evolution of the Universe, the temperature is large enough to keep the field N at the origin (gauge symmetry restored) but small enough so that the nonzero potential energy at the origin dominates over the thermal energy. This leads to an inflationary evolution but the amount of entropy released is not large enough to solve the flatness and horizon problems.

In view of the negative conclusions that we have reached, one might wonder what modifications of the models considered should be made that would lead to a successful inflationary scenario.

First of all, since there is not at present a compelling correspondence between superstring theories and the models that are believed to arise from their compactification, there is some uncertainty in the couplings. This is crucial here because the occurrence of inflation depends on the precise shape of the scalar potential. Let us note however that, in the case of the S field, the couplings are fairly independent of the precise compactification scheme used, at least for those schemes that lead to a positive-definitive scalar potential [no-scale models,[30] where the couplings of S are fixed by a SU(1,1)/U(1) symmetry]. The same would not hold, for example, for the T field whose couplings strongly depend on the compactification scheme. However it is the breaking of supersymmetry that induces a nontrivial structure in the potential of S and any supersymmetry-breaking mechanism other than gaugino condensation in the hidden sector would yield a different shape for this potential.

Another possibility is that the cosmological evolution is more complicated than the one we assumed. For example, it is possible (although improbable, as we stressed in Sec. II) that inflation occurs while the field H is tunneling from a zero value to the value $c \neq 0$ that corresponds to a minimum of the total energy (once gauginos have condensed).

We have also noted at the end of Sec. II that the general properties of the fields $C^{(\alpha)}$ related to the complex structure of the complex manifold make them plausible candidates for the inflaton. But a more detailed knowledge of their couplings is needed in order to study further this possibility.

Finally, inflation could occur before compactification or, more probably, in connection with compactification. But, in this case, one would be taking somewhat of a step backward and one would have to explain why there are no graviton, gravitino, monopole, domain wall (in particular those associated with compactification itself), etc., problems in the four-dimensional theory. Of course, the fact that one would rather have an inflationary epoch following compactification does not preclude the possibility of other departures from the standard big-bang theory associated with superstrings and/or Kaluza-Klein compactification.

ACKNOWLEDGMENTS

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contracts Nos. DE-AC02-82ER 40073 and DE-AC03-76SF 00098 and in part by the National Science Foundation under Grant No. PHY-85-15857.

---

*Present address: Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720.

[1]A. Guth, Phys. Rev. D **23**, 347 (1981); A. D. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, *ibid.* **140B**, 44 (1984).

[2]J. Ellis, D. V. Nanopoulos, K. A. Olive, and K. Tamvakis, Phys. Lett. **118B**, 335 (1982); Nucl. Phys. **B221**, 524 (1983); D. V. Nanopoulos, K. A. Olive, M. Srednicki, and K. Tamvakis, Phys. Lett. **123B**, 41 (1983); D. V. Nanopoulos, K. A. Olive, and M. Srednicki, *ibid.* **127B**, 30 (1983).

[3]A. D. Linde, Phys. Lett. **132B**, 317 (1983).

[4]B. A. Ovrut and P. J. Steinhardt, Phys. Lett. **133B**, 161 (1983); Phys. Rev. Lett. **53**, 732 (1984); Phys. Rev. D **30**, 2061 (1984); Phys. Lett. **147B**, 263 (1984).

[5]R. Holman, P. Ramond, and G. G. Ross, Phys. Lett. **137B**, 343 (1984).

[6]M. Dine, W. Fischler, and D. Nemeschansky, Phys. Lett. **136B**, 169 (1984).

[7]P. Binétruy and S. Mahajan, Nucl. Phys. **B263**, 413 (1986).

[8]G. B. Gelmini, C. Kounnas, and D. V. Nanopoulos, Nucl. Phys. **B250**, 177 (1985); C. Kounnas and M. Quirós, Phys. Lett. **151B**, 189 (1985); K. Enqvist and D. V. Nanopoulos, *ibid.* **142B**, 349 (1984); Nucl. Phys. **B252**, 508 (1985); K. Enqvist *et al.*, *ibid.* **B262**, 538 (1985); J. Ellis *et al.*, Phys. Lett. **152B**, 175 (1985); K. Enqvist *et al.*, *ibid.* **152B**, 181 (1985).

[9]A. M. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. **49**, 970 (1982); R. Barbieri, S. Ferrara, and C. Savoy, Phys. Lett. **119B**, 343 (1982); E. Cremmer, P. Fayet, and L. Girardello, *ibid.* **122B**, 41 (1983).

[10]P. J. Steinhardt and M. S. Turner, Phys. Rev. D **29**, 2162 (1984).

[11]A. D. Linde, Rep. Prog. Phys. **47**, 925 (1984).

[12]P. Binétruy, in *Proceedings of the Sixth Workshop on Grand Unification*, Minneapolis, 1985, edited by S. Rudaz and T. Walsh (World Scientific, Singapore, 1986).

[13]P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985).

[14]J. H. Schwarz, Phys. Rep. **89**, 223 (1982); M. Green, Surv. High Energy Phys. **3**, 127 (1984).

[15]M. B. Green and J. H. Schwarz, Nucl. Phys. **B218**, 43 (1983); Phys. Lett. **149B**, 117 (1984).

[16]D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985); Nucl. Phys. **B256**, 253 (1985); **B267**, 75 (1986).

[17]J. Polchinski, Commun. Math. Phys. **104**, 37 (1986).

[18]W. Fischler and L. Susskind, Phys. Lett. **171B**, 383 (1986); **173B**, 262 (1986).

[19]E. Witten, Nucl. Phys. **B188**, 513 (1981).

[20]I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. **B256**, 557 (1985).

[21]S. W. Hawking, Phys. Lett. **115B**, 295 (1982); A. A. Starobinsky, *ibid.* **117B**, 175 (1982); A. Guth and S. Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).

[22]V. A. Rubakov, M. V. Sazhin, and A. V. Veryaskin, Phys. Lett. **115B**, 189 (1981).

[23]E. Witten, Phys. Lett. **153B**, 243 (1985).

[24]A. Vilenkin and Q. Shafi, Phys. Rev. Lett. **51**, 1716 (1983); J. Silk and A. Vilenkin, *ibid.* **53**, 1700 (1984); N. Turok, Nucl. Phys. **B242**, 520 (1984).

[25]M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, Nucl. Phys. **B259**, 549 (1985).

[26]E. Witten, Phys. Lett. **155B**, 151 (1985).

[27]J. P. Derendinger, L. E. Ibáñez, and H. P. Nilles, Nucl. Phys. **B267**, 365 (1986).

[28]R. Rohm and E. Witten, Ann. Phys. (N.Y.) **170**, 454 (1986).

[29]E. Cremmer *et al.*, Phys. Lett. **79B**, 231 (1978); Nucl. Phys. **B147**, 105 (1979); Phys. Lett. **116B**, 231 (1982); Nucl. Phys. **B212**, 413 (1983); E. Witten and J. Bagger, Phys. Lett. **115B**,

202 (1982); J. Bagger, Nucl. Phys. **B211**, 302 (1983).
[30]E. Cremmer, S. Ferrara, C. Kounnas, and D. V. Nanopoulos, Phys. Lett. **133B**, 61 (1983); J. Ellis, C. Kounnas, and D. V. Nanopoulos, Nucl. Phys. **B241**, 406 (1984); **B247**, 373 (1983); Phys. Lett. **143B**, 410 (1984); J. Ellis, K. Enqvist, and D. V. Nanopoulos, *ibid.* **147B**, 99 (1984).
[31]J. P. Derendinger, L. E. Ibáñez, and H. P. Nilles, Phys. Lett. **155B**, 65 (1985).
[32]H. P. Nilles, Phys. Lett. **115B**, 193 (1982); S. Ferrara, L. Girardello, and H. P. Nilles, *ibid.* **125B**, 457 (1983).
[33]M. Dine, R. Rohm, N. Seiberg, and E. Witten, Phys. Lett. **156B**, 55 (1985).
[34]J. D. Breit, B. A. Ovrut, and G. Segrè, Phys. Lett. **162B**, 303 (1985).
[35]P. Binétruy and M. K. Gaillard, Phys. Lett. **168B**, 347 (1986).
[36]M. Quirós, Phys. Lett. **173B**, 265 (1986).
[37]Y. J. Ahn and J. D. Breit, Nucl. Phys. **B273**, 75 (1986).
[38]M. Dine and N. Seiberg, Phys. Rev. Lett. **55**, 366 (1985); Phys. Lett. **162B**, 299 (1985).
[39]V. Kaplunovsky, Phys. Rev. Lett. **55**, 1036 (1985).
[40]M. K. Gaillard *et al.*, Phys. Lett. **122B**, 355 (1983).
[41]P. Binétruy, S. Dawson, and I. Hinchliffe, Report No. LBL-21606 and EFI-86-28, 1986 (unpublished).
[42]J. Ellis, K. Enqvist, D. V. Nanopoulos, and M. Quirós, Report No. CERN TH-4325, 1985 (unpublished).
[43]A. H. Guth and E. J. Weinberg, Phys. Rev. D **23**, 876 (1981); Nucl. Phys. **B212**, 321 (1983).
[44]K. A. Olive and M. Srednicki, Phys. Lett. **148B**, 437 (1984).
[45]P. Binétruy and M. K. Gaillard, Nucl. Phys. **B254**, 388 (1985); Phys. Rev. D **32**, 931 (1985).
[46]A. Vilenkin and L. H. Ford, Phys. Rev. D **26**, 1231 (1982); A. D. Linde, Phys. Lett. **116B**, 335 (1982); S. Hawking and I. Moss, Nucl. Phys. **B224**, 180 (1983).
[47]K. Enqvist, E. Papantonopoulos, and K. Tamvakis, Phys. Lett. **165B**, 299 (1985).
[48]S. Coleman, Nucl. Phys. **B262**, 263 (1985).
[49]S. Hawking and I. Moss, Phys. Lett. **110B**, 35 (1982).
[50]A. D. Linde, Phys. Lett. **129B**, 177 (1983); **132B**, 317 (1983).
[51]E. Witten, Nucl. Phys. **B268**, 79 (1986).
[52]P. Binétruy, S. Dawson, I. Hinchliffe, and M. Sher, Nucl. Phys. **B273**, 501 (1986).
[53]E. Witten, Nucl. Phys. **B258**, 75 (1985).
[54]F. del Aguila, G. Blair, M. Daniel, and G. G. Ross, Nucl. Phys. **B272**, 413 (1986).
[55]X. G. Wen and E. Witten, Phys. Lett. **166B**, 397 (1986).
[56]M. Dine, N. Seiberg, X. G. Wen, and E. Witten, Princeton report, 1986 (unpublished).
[57]M. Mangano, Z. Phys. C **28**, 613 (1985).
[58]S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[59]E. Witten, Nucl. Phys. **B177**, 477 (1981).
[60]G. D. Coughlan *et al.*, Phys. Lett. **131B**, 59 (1983).