Quantization of a gauge theory with independent metric and connection fields

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We carry out the Hamiltonian quantization of a member of a class of gauge theories in which the internal metric becomes an independent degree of freedom and the gauge group is generalized to SL(N,C). The Hamiltonian quantization is carried out both in a gauge where only the SU(N) symmetry is manifest and in a gauge-invariant way. These theories have been proposed by Cahill, and they also arise as the nongravitational sector of a unified theory of gravitational and gauge fields which has been proposed by one of us (J.D.). The physical degrees of freedom are identified and a relativistically invariant functional generator is constructed. The resulting quantum theory is stable, but not perturbatively renormalizable.

I. INTRODUCTION

Given the successful description of the strong, electromagnetic, and weak interactions in terms of minimally coupled Yang-Mills theories, is seems that this is a good time to reconsider the question of the relationship between gravity and the other forces in nature, raised by Einstein in this work on unified field theories. Indeed, as both Yang-Mills theory and general relativity use a differential-geometric description of fields on spacetime, it seems most natural to look for a geometrical construction which could provide a unified description of all the known interactions in nature. Such a construction has been proposed by one of us (J.D.) and leads to a theory which is a simultaneous generalization of both Einstein and Yang-Mills theories.[1]

The purpose of this paper is to examine the dynamics and quantization of the resulting theory. The basic ideas which motivated the construction of this theory are the following: In order to construct a theory that unifies general relativity and Yang-Mills theory we must first understand what is common and what is different about the geometrical formulation in each case. What is common to both theories is that they can be formulated in terms of the fundamental objects of differential geometry. These are the *connections,* which define a notion of parallel transport to move objects between points in spacetime, and *metrics,* which allow us to quantitatively compare objects at the same point. In each case a local symmetry group acts at each point of spacetime, the Lorentz group SO(3,1) in general relativity, and a compact semisimple Lie group like SU(N) in Yang-Mills theory. What is different for each theory is, first of all, the space on which the metric and connection act: in general relativity the metric and connection act on vectors and tensors defined on spacetime itself whereas in Yang-Mills theory they act on objects in an internal vector space set up over each point of spacetime. We will have nothing to say here about bridging this difference between the two theories. Another way in which they differ is in the dynamical roles assigned to the metric and the connection in each theory. In general relativity the metric is the dynamical field and the connection is constrained by the conditions

$$\nabla_\lambda g_{\mu\nu}=0 \text{ and } \theta^\alpha_{\mu\nu}=0 \qquad (1.1)$$

to be a function of the metric. Here θ is the torsion. In Yang-Mills theory, on the other hand, the metric on the internal space is fixed to be δ_{ab} and the connection is the dynamical variable, and is again restricted by a metric compatibility condition:

$$D_\lambda\delta_{ab}=\partial_\lambda\delta_{ab}-\omega_{\lambda ab}-\omega^\dagger_{\lambda ab}=0 . \qquad (1.2)$$

Because the metric is constant, this means only that the connection is restricted by the condition $\omega_\lambda=-\omega^\dagger_\lambda$ to be in the U(N) subgroup generated by the anti-Hermitian transformations which preserve δ_{ab}.

The basic idea is then that, in order to unify gravity with the other interactions, it may be necessary to generalize both general relativity and Yang-Mills theory to theories in which both metrics and both connections are independent dynamical fields. In accordance with the Einstein philosophy that all aspects of the geometry should be determined dynamically, we assume no *a priori* constraints relating each metric to the corresponding connection. This means that each of the connections must be taken to gauge the whole invariance group of the space on which it acts. The spacetime connection must then gauge GL($4,R$), and the internal connection SL(N,C). The constancy of the Yang-Mills metric and the conditions (1.1) and (1.2) will be recovered in the low-energy limit, where the extra degrees of freedom introduced are frozen out, and will no longer be *a priori* conditions. At high energies the non-metric-compatible parts of both the spacetime and the internal connections will be dynamical degrees of freedom.

Based on these ideas, one of us (J.D.) found a simple expression for an action involving both metrics and both connections which gives a unified theory with the characteristics above.[2] This action is formulated in terms of the bundle of general linear frames over the product of spacetime and the internal space and is described in Ref. 2. Here we will only exhibit the form of the action, which is similar to a harmonic map or chiral Lagrangian on the natural metric G on the tangent space of the aforemen-

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tioned bundle:

$$I_u = \frac{1}{32\pi G_N} \int d^4x \sqrt{-g}\, g^{\mu\nu} \mathrm{Tr}[(L_{B_\mu} G)(L_{B_\nu} G^{-1})] \ . \tag{1.3}$$

Here $g_{\mu\nu}$ is the metric on spacetime and, for those familiar with fiber-bundle notation, we mention that L_{B_μ} is the Lie derivative in the bundle with respect to the basic vector field B_μ, the trace is taken in the tangent space of the bundle and the integral is over any cross section of the bundle. In addition we note that the theory depends on the single dimensional parameter G_{Newton}. When Eq. (1.3) is written out in terms of component fields it is found to contain a purely gravitational part and an internal part

$$I_u = I_g + I_i \ . \tag{1.4}$$

In terms of the spacetime connection $\Gamma^\alpha_{\mu\nu}$ and the spacetime metric $g_{\mu\nu}$ the gravitational part is

$$L_g = \frac{m^2}{2} g^{\mu\nu} R(\Gamma)_{\mu\nu} + \frac{1}{\alpha^2} g^{\mu\lambda} g^{\nu\sigma} R_{\mu\nu\alpha}{}^{\beta} R_{\lambda\sigma\beta}{}^{\alpha} + \frac{m^2}{2} g^{\mu\nu} (\nabla_\mu g_{\alpha\beta} - 2\theta_{\mu(\alpha\beta)})(\nabla_\nu g^{\alpha\beta} - 2\theta_\nu^{(\alpha\beta)}) \ . \tag{1.5}$$

The internal part is a function of the internal metric q, which is a Hermitian matrix of scalar fields, and the internal $\mathrm{SL}(N,C)$ connection ω_μ, a set of traceless matrices forming a four-vector:

$$L_i = -\frac{1}{4g^2} \mathrm{Tr}(q^{-1} F^\dagger_{\mu\nu} q F^{\mu\nu}) + \frac{m^2}{8} \mathrm{Tr}[(q^{-1} \nabla_\mu q)(q^{-1} \nabla^\mu q)] \ , \tag{1.6}$$

where $F_{\mu\nu}$ is the $\mathrm{SL}(N,C)$ field strength formed from ω_μ in the usual way,

$$F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu + [\omega_\mu, \omega_\nu] \ , \tag{1.7}$$

and the covariant derivative of q is given by

$$\nabla_\lambda q = \partial_\lambda q - \omega^\dagger_\lambda q - q\omega_\lambda \ . \tag{1.8}$$

Note that the internal part and the gravitational part are very similar in form, the major difference being the term linear in the Ricci curvature in the spacetime part, which arises naturally from the Lagrangian in (1.3). A similar term does not arise in the internal part because the corresponding contraction cannot be made with $F_{\mu\nu}$. The same dimensional constant $M^2 = 16\pi G_{\mathrm{Newton}}{}^{-1}$ occurs in both parts and sets the scale for the low-energy limit of the theory. The theory has, in addition, two dimensionless constants, one associated with the internal connection (g) and one associated with the spacetime connection (α). In addition, note that the internal part turns out to be a gauge theory of $\mathrm{SL}(N,C)$ rather than $\mathrm{GL}(N,C)$, because expression (1.6) is actually independent of the determinant of q, as we will show, and the Abelian factor in ω_λ corresponding to $\mathrm{GL}(N,C)/\mathrm{SL}(N,C)$ decouples from the other degrees of freedom. Thus, we may also impose on q the condition

$$\det q = 1 \ , \tag{1.9}$$

This is all we will say here about the unified theory and its derivation. For details the reader may consult Ref. 2. We will also have little more to say here about the gravitational part of the theory. It is more complex than other gravitational theories that have been considered because it involves the full spacetime connection dynamically, including nonmetric and torsion components, and because it involves a much larger local invariance group and set of fields than is usually considered in gravitational theories. If we constrain the spacetime connection in various ways then this theory reduces to others that have been studied before. For example, if the connection is constrained to be metric compatible the theory is one of the theories with propagating torsion that have been studied by a number of authors.[3] If all of the components of nonmetricity are constrained except for the Abelian component corresponding to local scale transformations then the theory becomes the conformally metric theory which has also been studied before.[4] Of course, if the connection is constrained to be both metric compatible and torsion-free then it is one of the higher derivative extensions of general relativity that have recently been studied a good deal.

None of these special cases have led to a satisfactory quantum theory of gravitation; it has not been possible to invent a theory which has a stable ground state and which is at the same time perturbatively renormalizable, unitary, and causal.[5] There is, however, some basis for the conjecture that in this theory with a completely free $\mathrm{GL}(4,R)$ connection it will be possible to satisfy all three of these conditions.[6] This is based on the possibility of using the $\mathrm{GL}(4,R)$ invariance to choose a gauge in which all of the dynamics are carried by the connection fields, in terms of which the theory could scale at high energy without introducing the pathologies that arise from higher derivatives. Unfortunately, the complexity of the theory makes the examination of this conjecture difficult.

Thus, before trying to understand the dynamics of the gravitational part of the theory it seems prudent to first understand the dynamics of the internal part. Considered by itself this theory, given by Eqs. (1.6)–(1.9) gives an interesting generalization of Yang-Mills theory, as well as a simpler example than the gravitational part of a theory in which both metric and connection are dynamical fields. This theory has been proposed first by Cahill;[1] however, its dynamics and quantization have never, to our knowledge, been studied in any detail. Consequently the body of this paper is devoted to establishing the correct Hamiltonian quantization of this theory. For systems with constraints this is a necessary prelude to the construction of a relativistic functional integral from which a covariant perturbation theory may be defined.

The paper is organized as follows. In Sec. II we describe the classical theory. In Sec. III we work out the Hamiltonian analysis of the theory in the gauge $q = I$, and try to construct the path integral, but meet a difficulty due to the presence of second-class constraints. In Sec. IV we extend the Hamiltonian analysis to the arbitrary gauge case and construct the relativistically invariant path integral form for the functional generator. Section V contains our conclusions.

II. THE CLASSICAL THEORY IN FLAT SPACE

We begin by reviewing the basic properties of the classical theory which follows from the Lagrangian (1.6). Under a local $SL(N,C)$ gauge transformation parametrized by the matrix λ the internal metric q and connection ω_μ transform as

$$q \rightarrow \lambda^\dagger q \lambda , \tag{2.1}$$

$$\omega_\mu \rightarrow \lambda^{-1}\omega_\mu \lambda + \lambda^{-1}\partial_\mu \lambda , \tag{2.2}$$

where $\lambda^\dagger$ is the Hermitian conjugate of λ in the usual sense. The covariant derivative ∇_μ transforms homogeneously under these transformations.

Given q at a point the generators of $SL(N,C)$ (traceless $N \times N$ complex matrices), denoted by τ, can be decomposed into two pieces, which are, respectively, "anti-Hermitian" and "Hermitian" *with respect to q:*

$$\tau = i\alpha + \beta , \tag{2.3}$$

$$i\alpha = \tfrac{1}{2}(\tau - q^{-1}\tau^\dagger q) , \tag{2.4}$$

$$\beta = \tfrac{1}{2}(\tau + q^{-1}\tau^\dagger q) . \tag{2.5}$$

We call $(i\alpha)$ and β anti-Hermitian and Hermitian with respect to q because they satisfy

$$(i\alpha) = -q^{-1}(i\alpha)^\dagger q \quad \text{and} \quad \beta = q^{-1}\beta^\dagger q . \tag{2.6}$$

The anti-Hermitian piece $i\alpha$ generates the $SU(N)$ subgroup of $SL(N,C)$ which preserves q, while under an infinitesimal β transformation q transforms as

$$\delta q = q\beta + \beta^\dagger q . \tag{2.7}$$

The connection ω_μ may be split into anti-Hermitian and Hermitian parts with respect to q:

$$\omega_\mu = iA_\mu + B_\mu , \tag{2.8a}$$

$$A_\mu = \tfrac{1}{2}(\omega_\mu - q^{-1}\omega_\mu^\dagger q) , \tag{2.8b}$$

$$B_\mu = \tfrac{1}{2}(\omega_\mu + q^{-1}\omega_\mu^\dagger q) . \tag{2.8c}$$

However, this separation is not invariant under gauge transformations, which mix the two parts. The infinitesimal transformation properties of ω_μ are

$$\delta\omega_\mu = [\omega_\mu, \tau] + \partial_\mu \tau , \tag{2.9}$$

and under separate α and β transformations,

$$\delta_\alpha A_\mu = [A_\mu, \alpha] + \partial_\mu \alpha + \tfrac{1}{2} q^{-1}[(\partial_\mu q)\alpha + \alpha^\dagger \partial_\mu q] , \tag{2.10a}$$

$$\delta_\alpha B_\mu = [B_\mu, \alpha] - \tfrac{1}{2} q^{-1}[(\partial_\mu q)\alpha + \alpha^\dagger \partial_\mu q] , \tag{2.10b}$$

$$\delta_\beta A_\mu = [B_\mu, \beta] - \tfrac{1}{2} q^{-1}[(\partial_\mu q)\beta + \beta^\dagger \partial_\mu q] , \tag{2.10c}$$

$$\delta_\beta B_\mu = [A_\mu, \beta] + \partial_\mu \beta + \tfrac{1}{2} q^{-1}[(\partial_\mu q)\beta + \beta^\dagger \partial_\mu q] . \tag{2.10d}$$

The fields A_μ are perturbatively equivalent to the ordinary Yang-Mills gauge fields for $SU(N)$, and correspond to the part of the connection that is metric compatible with respect to q. The fields B_μ gauge the additional $SL(N,C)/SU(N)$ part of the gauge group and correspond to the non-metric-compatible part of the connection. Indeed the covariant derivative of q may be written as

$$\nabla_\mu q = \partial_\mu q - 2qB_\mu , \tag{2.11}$$

involving only the B_μ fields. The Lagrangian (1.6) is easily shown to be invariant under $SL(N,C)$. Taking variations with respect to ω_μ, $\omega_\mu^\dagger$, and q the following classical field equations are found:

$$\nabla_\mu(q^{-1}F^{\dagger\mu\nu}q) - \frac{g^2m^2}{2}(q^{-1}\nabla^\nu q) = 0 , \tag{2.12}$$

$$\nabla_\mu^\dagger(qF^{\mu\nu}q^{-1}) - \frac{g^2m^2}{2}(\nabla^\nu q)q^{-1} = 0 , \tag{2.13}$$

$$[(q^{-1}F_{\mu\nu}^\dagger q), F^{\mu\nu}] - g^2m^2\nabla_\nu(q^{-1}\nabla^\nu q) = 0 , \tag{2.14}$$

where $\nabla_\mu^\dagger$ is defined by $\nabla_\mu^\dagger X^\dagger = (\nabla_\mu X)^\dagger$. It is important to note that these equations are redundant in that the q field equation actually follows from the $SL(N,C)$ covariant divergence of the field equation ω_μ. One may check this directly using the fact that for any mixed second-rank internal tensor E one has

$$(\nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu)E = [F_{\mu\nu}, E] , \tag{2.15}$$

or one may derive identities coming from the variation of the Lagrangian with respect to an arbitrary gauge transformation λ:

$$\int d^4x\, \delta L = \int d^4x \,\mathrm{Tr}\left[\frac{\delta L}{\delta q}\delta q^T + \frac{\delta L}{\delta \omega_\mu}\delta\omega_\mu^T + \frac{\delta L}{\delta \omega_\mu^+}\delta\omega_\mu^*\right]$$

$$= \int d^4x \,\mathrm{Tr}\left[\mathrm{Tr}\left[\frac{\delta L}{\delta q}\frac{\delta q^T}{\delta\lambda} + \frac{\delta L}{\delta \omega_\mu}\frac{\delta\omega_\mu^T}{\delta\lambda} + \frac{\delta L}{\delta \omega_\mu^+}\frac{\delta\omega_\mu^*}{\delta\lambda}\right]\delta\lambda^T\right] = 0 , \tag{2.16}$$

where the first trace acts on q's and the second on λ's. Restricting λ alternatively to be anti-Hermitian or Hermitian with respect to q one finds

$$\nabla_\mu\left[\frac{\delta L}{\delta\omega_\mu^T}\right] - q^{-1}\left[\nabla_\mu^\dagger \frac{\delta L}{\delta\omega_\mu^*}\right]q = 0 , \tag{2.17a}$$

$$2\frac{\delta L}{\delta q^T}q - \left[\nabla_\mu\left[\frac{\delta L}{\delta\omega_\mu^T}\right] + q^{-1}\left[\nabla_\mu^\dagger \frac{\delta L}{\delta\omega_\mu^*}\right]q\right] = 0 . \tag{2.17b}$$

These equations are an extension of the familiar identity in Yang-Mills theory by which the covariant divergence of the field equations vanishes.

Thus q is a redundant variable in the sense that its field equation is automatically satisfied whenever the field equations for the connection ω_μ are satisfied. Moreover because we have set $\det q = 1$, q has no singularities and we can always use a gauge transformation a to set $q = I$ everywhere:

$$q \to q' = a^\dagger q a = I \ . \tag{2.18}$$

Given that q is a Hermitian matrix with unit determinant this condition is exactly sufficient to fix the $SL(N,C)/SU(N)$ part of the gauge freedom, so that it fixes completely the Hermitian part of the gauge invariance. Thus with q fixed one is left with only the anti-Hermitian part of the gauge transformations, which generates $SU(N)$.

With q set equal to I many of the previous equations simplify. For example the Lagrangian (1.6) becomes in this gauge

$$L = -\frac{1}{4g2}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu}) + \frac{m^2}{2}\mathrm{Tr}(B_\mu B^\mu) \ , \tag{2.19}$$

where we have defined

$$G_{\mu\nu} = f_{\mu\nu} + [B_\mu, B_\nu] \ , \tag{2.20}$$

$$W_{\mu\nu} = D_\mu B_\nu - D_\nu B_\mu \ , \tag{2.21}$$

and $f_{\mu\nu}$ and D_μ are the ordinary Yang-Mills quantities for the gauge fields A_μ:

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \ , \tag{2.22a}$$

$$D_\mu B_\nu = \partial_\mu B_\nu + [A_\mu, B_\nu] \ . \tag{2.22b}$$

In this gauge the particle content of the theory is manifest. The theory consists of a $SU(N)$ Yang-Mills field coupled to a multiplet of massive spin-one vector bosons in the adjoint representation, with the couplings chosen such that the theory can be extended by addition of the variable q to be invariant under the larger group $SL(N,C)$. What has happened is that the Hermitian metric field q has combined with the Hermitian part of the gauge fields B_μ to make massive vector fields, while the anti-Hermitian fields A_μ remain massless. (At the quantum level it is possible that the fields A_μ will acquire masses from vacuum expectation values involving the fields B_μ. This will be discussed in another paper.)

This is somewhat analogous to the Higgs mechanism in ordinary gauge theories. However, there are three important differences. First, there are no scalar particles left over after the longitudinal parts of the massive vector fields have been taken into account. Second, there is no way in which one could form mass or self-interaction terms directly for the q field. This is forbidden by the $SL(N,C)$ symmetry; the only invariant that one could form out of q is $\det q$ and this is one by construction. Third, although one may say that in the mass spectrum of the theory $SL(N,C)$ has broken down to $SU(N)$, this is not spontaneous symmetry breaking, in that nowhere have we needed to assume that the vacuum, or ground state, is not invariant under the whole $SL(N,C)$ symmetry. All that we have done is to exhibit a gauge in which the particle content of the theory is manifest.

In the $q = I$ gauge, the field equations for A_μ and B_μ are

$$D_\mu G^{\mu\nu} + i[B_\mu, W^{\mu\nu}] = 0 \ , \tag{2.23}$$

$$D_\mu W^{\mu\nu} - i[B_\mu, G^{\mu\nu}] + g^2 m^2 B^\nu = 0 \ . \tag{2.24}$$

The q field equation in turn becomes

$$D_\mu B^\mu - \frac{i}{g^2 m^2}[G_{\mu\nu}, W^{\mu\nu}] = 0 \ . \tag{2.25}$$

Note the important fact that the q equation may be considered to be an evolution equation for B_0.

We close this section with a brief description of how one couples the $SL(N,C)$ theory to matter. We may form the usual minimal coupling of the $SL(2,C)$-invariant theory to multiplets of spinors and scalars by

$$L_m = \tfrac{1}{2}(i\bar{\psi}q\gamma_\mu \nabla^\mu \psi + \mathrm{H.c.}) + \tfrac{1}{2}(\nabla_\mu \phi)^\dagger q (\nabla^\mu \phi) \ . \tag{2.26}$$

Choosing the gauge $q = I$ one can see that the new nonmetric fields B_μ couple to scalar fields but not to spinor fields. This is unfortunate as it makes it difficult to use the massive B_μ fields to model the weak interactions. One can remedy this problem somewhat by introducing nonminimal couplings of the fields B_μ to the spinors. This may be done in two different ways by adding Hermitian terms to the Lagrangian:

$$L_{n-m} = \frac{i}{2}(i\bar{\psi}q\gamma_\mu \nabla^\mu \psi - \mathrm{H.c.}) + c\bar{\psi}(\gamma_\mu \nabla^\mu q)\psi \ . \tag{2.27}$$

When we go to the $q = I$ gauge both of these terms reduce to

$$L_{n-m} = -(1+2C)\bar{\psi}\gamma_\mu B^\mu \psi \ . \tag{2.28}$$

It might be interesting to see if one could attempt to model the weak interactions with terms of this type. One difficulty that may arise is that they break a discrete symmetry of the original Lagrangian (2.19), which is invariant under the operation

$$B_\mu \to -B_\mu, \ \ A_\mu \to A_\mu \ . \tag{2.29}$$

We call this operation B parity. Thus, once one introduces direct couplings of the fields B_μ to spinors one will no longer have conservation of B parity and it may be necessary to introduce into the Lagrangian additional dimension-four counterterms that would otherwise be excluded. Conversely we can see from the fact that the nonminimal couplings to spinors violate B parity that such couplings will not arise in the effective action generated from (2.19) to any order in perturbation theory.

III. HAMILTONIAN QUANTIZATION OF THE SL(2,C) THEORY

We proceed in this section to the construction of the Hamiltonian quantization of the theory. For simplicity we will begin by working in the gauge $q = I$. We expect that we would get the same answer if we quantized the theory with q as a dynamical variable and only at the end made the gauge choice $q = I$. This is, however, a delicate point, as we will see later.

In addition, to simplify the notation we will work with the simplest case, in which the group is chosen to be $SL(2,C)$. This will allow us to use three-vector notation for the internal Lie-algebra indices. $SL(2,C)$ is familiar to us as the covering group of the Lorentz group $SO(3,1)$ and has the Lie algebra given by

$$c_a=\sigma_a/2,\ \ d_a=i\sigma_a/2,\ \ a=1,2,3\ , \tag{3.1a}$$

$$[c_a,c_b]=i\epsilon_{abc}c^c\ , \tag{3.1b}$$

$$[c_a,d_b]=i\epsilon_{abc}d^c\ , \tag{3.1c}$$

$$[d_a,d_b]=-i\epsilon_{abc}c^c\ . \tag{3.1d}$$

We will see that this is related to the constraint algebra we find. We proceed to write the theory in component form, expanding in terms of the SU(2) generators c_a. We use the three-vector notations

$$A_aB^a=A\cdot B,\ \ \epsilon_{abc}A^bB^c=(A\times B)_a\ , \tag{3.2}$$

and so we get for all relevant quantities, up to an irrelevant overall factor in the Lagrangian:

$$L=-\frac{1}{4g^2}(G_{\mu\nu}\cdot G^{\mu\nu}+W_{\mu\nu}\cdot W^{\mu\nu})+\frac{m^2}{2}B_\mu\cdot B^\mu\ , \tag{3.3}$$

$$G_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-A_\mu\times A_\nu+B_\mu\times B_\nu\ , \tag{3.4}$$

$$W_{\mu\nu}=D_\mu B_\nu-D_\nu B_\mu\ , \tag{3.5}$$

$$D_\mu B_\nu=\partial_\mu B_\nu-A_\mu\times B_\nu\ . \tag{3.6}$$

A. Classical Hamiltonian analysis

We begin the construction of the canonical theory by finding the canonical momenta of A_i and B_i:

$$\pi_a^0\equiv\frac{\partial L}{\partial\dot A_0^a}=0,\ \ P_a^0\equiv\frac{\partial L}{\partial\dot B_0^a}=0\ , \tag{3.7}$$

$$\pi_a^i\equiv\frac{\partial L}{\partial\dot A_i^a}=-\frac{1}{g^2}G_a^{0i}\ , \tag{3.8}$$

$$P_a^i\equiv\frac{\partial L}{\partial\dot B_i^a}=-\frac{1}{g^2}W_a^{0i}\ . \tag{3.9}$$

As in ordinary Yang-Mills theory the Lagrangian (3.3) is not a function of A_0^a and B_0^a, and so the momenta conjugate to them vanish and we have two sets of constraints. The canonical Hamiltonian is then

$$H_c(x)\equiv\pi_a^i\dot A_i^a+P_a^i\dot B_i^a-L\ . \tag{3.10}$$

Inverting the expressions for the momenta to solve in terms of the velocities we find

$$\partial_0A_i^a=-g^2\pi_i^a+\partial_iA_0^a+(A_0\times A_i)^a-(B_0\times B_i)^a\ , \tag{3.11}$$

$$\partial_0B_i^a=-g^2P_i^a+\partial_iB_0^a+(A_0\times B_i)^a-(A_i\times B_0)^a\ . \tag{3.12}$$

After some integrations by parts we get for the Hamiltonian in terms of canonical variables:

$$\begin{aligned}H_c(x)=&-\frac{g^2}{2}(\pi_i\cdot\pi^i+P_i\cdot P^i)-\frac{m^2}{2}B_i\cdot B^i\\&+\frac{1}{4g^2}(G_{ij}\cdot G^{ij}+W_{ij}\cdot W^{ij})\\&-A_0\cdot(D_i\pi^i-B_i\times P^i)\\&-B_0\cdot\left[D_iP^i+B_i\times\pi^i+\frac{m^2}{2}B_0\right]\ .\end{aligned} \tag{3.13}$$

Note that because of our convention for the metric

$$\eta_{\mu\nu}=\text{diag}(+,-,-,-)\Longrightarrow B_i\cdot B^i=-\sum_i(B_i^a)^2\ . \tag{3.14}$$

In order to ensure that a constrained Hamiltonian system such as this defines a consistent theory we must make sure that the constraints once fixed at an initial time are preserved in the evolution defined by the Hamiltonian. If they are not, we must modify the Hamiltonian in such a way that the initial constraints are preserved or, that failing, add additional constraints in order to guarantee the preservation of the original ones. In order to be consistent with the classical equations of motion the Hamiltonian can only be modified by adding terms proportional to the initial constraints, times arbitrary functions of the fields. If this does not suffice to guarantee the preservation of the constraints and we have to add additional constraints we must then make sure that there additional constraints are also preserved under evolution defined by the Hamiltonian, and the procedure is thus, repeated. The procedure continues until a system of a Hamiltonian and constraints is found such that all of the constraints are preserved by the evolution generated by the Hamiltonian. (For an exposition of the Hamiltonian quantization of a constrained system the best reference we have found is the book by Dirac.[7] There is also the excellent review by Hanson, Regge, and Teitelboim.[8])

We begin by seeing if the initial constraints $\pi_0^a=0$ and $p_0^a=0$ are preserved. Taking the first case,

$$[H_c,\pi_0^a(x)]=-C^a(x)=(D_i\pi^i-B_i\times P^i)^a(x)\approx 0\ , \tag{3.15}$$

where $H_c=\int dx^3H_c(x)$. This defines the constraint C^a, which is just the usual constraint arising from gauge invariance in Yang-Mills theory, and generates the time-independent SU(2) transformation laws of the various fields, as the reader may verify. An additional set of constraints arise from the second case:

$$\begin{aligned}[H_c,P_0^a(x)]&=-D^{*a}(x)\\&=(D_iP^i+B_i\times\pi^i+m^2B_0)^a(x)\approx 0\ .\end{aligned} \tag{3.16}$$

These new constraints D^{*a} correspond to the SL(2,C)/SU(2) gauge symmetry of the original theory. However, as we have broken this gauge invariance by setting $q=I$ this constraint does not function like the usual gauge constraints.

We may now ask if the constraints we have found, π_0^a, P_0^a, C^a, and D^{*a}, generate a closed algebra under the Poisson-brackets operation. We find

$$[C_a(x),C_b(x)]=-\delta^3(x-y)\epsilon_{abc}C^c(x)\ , \tag{3.17}$$

which is just the Lie algebra of SU(2), up to a sign convention. However, the Poisson brackets of C^a with D^{*a} does not close. Instead

$$[C_a(x),D_b^*(y)]=-\delta^3(x-y)\epsilon_{abc}[D^{*c}(x)-m^2B_0^c(x)]\ . \tag{3.18}$$

This is not a constraint, and so C^a and D^{*a} are not first-class constraints. Luckily this problem is easy to fix. We can add to C^a a linear combination of other constraints to make it first class. We define a new SU(2) constraint by

$$C^{*a} \equiv C^a - (B_0 \times P_0)^a \, . \tag{3.19}$$

The new constraint also generates the SU(2) algebra

$$[C^{*a}(x), C^{*b}(x)] = -\delta^3(x-y)\epsilon^{abc} C_c^*(x) \, , \tag{3.20}$$

and the Poisson brackets with D^{*a} now closes

$$[C_a^*(x), D_b^*(y)] = -\delta^3(x-y)\epsilon_{abc} D^{*c}(x) \, , \tag{3.21}$$

which shows that D^{*a} transforms properly under the SU(2) transformations generated by C^{*a}. In addition the Poisson brackets of the D^{*a}'s with themselves also closes:

$$[D^{*a}(x), D^{*b}(y)] = \delta^3(x-y)\epsilon^{abc}[C_c^*(x) + (B_0 \times P_0)_c(x)] \, . \tag{3.22}$$

However, because of the extra term containing P_0 the constraints C^{*a} and D^{*a} do not form a representation of the Lie algebra of SL(2,C). The complete algebra of all the constraints is now

$$[\pi_a^0(x), \pi_b^0(y)] = 0 \, , \tag{3.23a}$$

$$[P_a^0(x), P_b^0(y)] = 0 \, , \tag{3.23b}$$

$$[\pi_a^0(x), C_b^*(y)] = 0 \, , \tag{3.23c}$$

$$[\pi_a^0(x), D_b^*(y)] = 0 \, , \tag{3.23d}$$

$$[\pi_a^0(x), P_b^0(y)] = 0 \, , \tag{3.23e}$$

$$[C_a^*(x), C_b^*(y)] = -\delta^3(x-y)\epsilon_{abc} C^{*c}(x) \approx 0 \, , \tag{3.24a}$$

$$[C_a^*(x), D_b^*(y)] = -\delta^3(x-y)\epsilon_{abc} D^{*c}(x) \approx 0 \, , \tag{3.24b}$$

$$[D_a^*(x), D_b^*(y)] = \delta^3(x-y)\epsilon_{abc}[C^{*c}(x) + (B_0 \times P_0)^c(x)] \approx 0, \tag{3.24c}$$

$$[P_a^0(x), C_b^*(y)] = -\delta^3(x-y)\epsilon_{abc} P_0^c(x) \approx 0 \, , \tag{3.24d}$$

$$[P_a^0(x), D_b^*(y)] = -m^2 \delta_{ab} \delta^3(x-y) \not\approx 0 \, . \tag{3.25}$$

We see from the fact that this last commutator does not vanish on the constraint surface that P_0^a and D^{*a} are second-class constraints. The other constraints π_0^a and C^{*a} are first class as in Yang-Mills theory. The reader may check that no further linear combination of the second-class constraints is first class. The presence of these second-class constraints is due to the gauge choice $q = I$, as we shall see, and it makes the analysis of the theory in the present form awkward.

We must now check to see if the new constraints C^{*a} and D^{*a} are preserved by the Hamiltonian. Calculating the brackets with H_c we find that, as in Yang-Mills theory,

$$[C^{*a}(x), H_c] = (A_0 \times C^* + B_0 \times D^*)^a(x) \approx 0 \, . \tag{3.26}$$

However, the bracket of H_c with D^a does not vanish on the constraint surface. Instead

$$[D^{*a}(x), H_c] = (A_0 \times D^* - B_0 \times C)^a(x) + m^2 F^a \, , \tag{3.27}$$

$$F^a = \left[D_i B^i - A_0 \times B_0 + \frac{2g^2}{m^2} \pi_i \times p^i + \frac{1}{g^2 m^2} G_{ij} \times W^{ij} \right]^a (x) \, . \tag{3.28}$$

In order to solve this problem we must add a term proportional to P_0 to the Hamiltonian

$$H_t \equiv H_c - \int d^3x \, F^a(x) P_a^0(x) \, . \tag{3.29}$$

This makes the new brackets vanish on the constraint surface:

$$[C^{*a}(x), H_t] = (A_0 \times C^* + B_0 \times D^* + P_0 \times F)^a(x) \approx 0 \, , \tag{3.30}$$

$$[D^{*a}(x), H_t] = (A_0 \times D^* - B_0 \times C^*)^a(x) - \int d^3x \, [D^{*a}(x), F^b(y)] P_b^0(y) \approx 0 \, . \tag{3.31}$$

The expression F^a relates to the time evolution of B_0 in the following way: we have

$$[B_0^a(x), H_t] = \frac{\partial B_0^a(x)}{\partial t} = -F^a(x) \, . \tag{3.32}$$

If we plug in the momenta in terms of the velocities back in this equation we get

$$D_\mu B^\mu - \frac{i}{g^2 m^2}[G_{\mu\nu}, W^{\mu\nu}] = 0 \, . \tag{3.33}$$

This is just the field equation for q, in the $q = I$ gauge. We see then that our gauge choice implies, in the present formalism, a definite time evolution for B_0, which would be normally undetermined.

Note that in the presence of the constraints we may write the Hamiltonian as

$$H_f(x) = \frac{g^2}{2}[(\pi_a^i)^2 + (P_a^i)^2] + \frac{m^2}{2}[(B_i^a)^2 + (B_0^a)^2] + \frac{1}{4g^2}[(G_{ij}^a)^2 + (W_{ij}^a)^2] \geq 0 \, . \tag{3.34}$$

Thus we see that, while not manifestly positive, the Hamiltonian is positive in the presence of the constraints. A problem that could arise from the mass term due to the indefiniteness of the Minkowski metric in $B_\mu B^\mu$ is resolved because the constraint D^{*a} serves to switch the sign of the term $m^2 B_0^2$ in the Hamiltonian.

Note also that, as in Yang-Mills theory, the field A_0 in the presence of the constraints is absent from the Hamiltonian and the constraints, and may be used as a Lagrange multiplier for the constraints $C^{*a}(x)$. This is not the case for B_0, which is present in both the Hamiltonian and the constraints and has its own time evolution, given by H_t.

B. Canonical quantization and the functional integral

Thus we have found that the total Hamiltonian H_t together with the constraints C^{*a}, D^{*a}, π_0^a, and P_0^a define a consistent generalized Hamiltonian system. While this is enough to show that the theory can be formulated consistently at the classical level, we need to do some more work to construct the quantum theory. First we need to rework the canonical analysis performed so far but in terms of commutators instead of Poisson brackets, and verify that we do not run into ordering problems. This was done, and it was found that the antisymmetry of the structure constants of the Lie group is enough to guarantee the absence of ordering problems, because it precludes the appearance of products of two fields with the same internal index, which are the ones which might not commute. Second, we must identify the physical variables satisfying canonical commutation relations and then construct the physical Hamiltonian which will give us the unconstrained evolution of the canonical variables on the physical subspace of phase space, that is, the constraint surface.

We do this in several stages. First, we solve all of the constraints to express the theory in terms of unconstrained variables. Second, for each first-class constraint associated with gauge invariance we add a subsidiary or gauge condition. Third we may have to introduce Dirac brackets in terms of which the commutation relations of all quantities with the second-class constraints will be zero, closing the constraint algebra. Fourth, we construct the path-integral representation of the functional generator of the Green's functions of the theory, in order to have a perturbative expansion and a set of Feynman rules for the theory.

Let us have a look at our constraint system. The solutions for π_0^a and P_0^a are obvious, but we still have a coupled system of equations:

$$(\partial_i \pi^i - A_i \times \pi^i - B_i \times P^i)^a(x) = 0 \;, \tag{3.35}$$

$$(\partial_i P^i - A_i \times P^i + B_i \times \pi^i)^a(x) = -m^2 B_0^a(x) \;. \tag{3.36}$$

We may choose now to solve the second equation for B_0^a and substitute in H_t. This is convenient because, besides eliminating the variable B_0^a from the system, it decouples the constraint system, and one is left with only one equation. At this point we have

$$H_t = H_t[A_i^a, B_i^a, \pi_a^i, P_a^i] \;, \tag{3.37}$$

$$(\partial_i \pi^i - A_i \times \pi^i - B_i \times P^i)^a(x) = 0 \;, \tag{3.38}$$

where the square brackets in (3.37) are used to indicate that H is a functional of the quantities in the bracket. To proceed further it is convenient to decompose the remaining fields and momenta into transverse and longitudinal parts in the usual way

$$V_i^a = V_i^{aT} + V_i^{aL} = \lambda_i^{Tj} V_j^a + \lambda_i^{Lj} V_j^a \;, \tag{3.39}$$

where

$$\lambda_i^{Lj} = \partial_i (\partial_k \partial^k)^{-1} \partial^j, \quad \lambda_i^{Tj} = \delta_i^j - \lambda_i^{Lj} \tag{3.40}$$

and

$$\partial^i V_i^{aT} = 0 \quad \text{and} \quad \partial_{[i} V_{j]}^{aL} = 0 \;, \tag{3.41}$$

where V_i^a stands for any of our three-vectors. Now the constraints C_a can be solved for the variables π_a^{iL}, giving some solutions $\bar{\pi}_a^{iL}$,

$$C_a = \partial_i \pi_a^{iL} - (A_i \times \pi^i)_a - (B_i \times P^i)_a = 0 \;, \tag{3.42}$$

$$C_a(\bar{\pi}_b^{iL}) = 0 \;. \tag{3.43}$$

We now need to choose a subsidiary condition to eliminate the gauge freedom represented by the first-class constraints. The constraints $\pi_a^0 = 0$ are obviously related to the conditions $A_0^a = 0$, which are not necessary to impose explicitly, as A_0^a has vanished from sight already. For C^a it is convenient to choose $A_a^{iL} = 0$, or

$$\chi^a \equiv \partial_i A^{ia}(x) = 0 \;. \tag{3.44}$$

As $\det[\chi^a, C^b] \neq 0$, this condition suffices to fix the gauge. We are at this stage left only with the unconstrained variables on the constraint surface:

$$H = H_t[A^T, \pi^T, B^T, P^T, B^L, P^L] \;. \tag{3.45}$$

These are the degrees of freedom of a set of massless (A) and massive (B) vector particles, as expected. These coordinate fields satisfy the usual canonical Poisson-brackets relations:

$$[A_i^{aT}(x), \pi_b^{jT}(y)] = \lambda_i^{Tj} \delta_b^a \delta^3(x-y) \;, \tag{3.46}$$

$$[B_i^{aT}(x), P_b^{jT}(y)] = \lambda_i^{Tj} \delta_b^a \delta^3(x-y) \;, \tag{3.47}$$

$$[B_i^{aL}(x), P_b^{jL}(y)] = \lambda_i^{Lj} \delta_b^a \delta^3(x-y) \;. \tag{3.48}$$

The next step would now be to introduce the Dirac brackets for these quantities, but in our case this is not really necessary. This is a further advantage of our choice of field variables, for, as they are just some of the original canonical variables in their original roles, their Dirac brackets coincide with their Poisson brackets, as can be seen from the definition

$$[A(x),B(y)]_D = [A(x),B(y)] + \frac{1}{2m^4}\int d^3z\, \epsilon^{abc} C_c(z) \{ [A(x),P_a^0(z)][B(y),P_b^0(z)] - [A(x),P_b^0(z)][B(y),P_a^0(z)] \}$$
$$- \frac{1}{m^2}\int d^3z \{ [A(x),P_a^0(z)][D^{*a}(z),B(y)] - [A(x),D^{*a}(z)][P_a^0(z),B(y)] \} \;. \tag{3.49}$$

We may therefore directly identify the Poisson brackets of the remaining fields with the quantum commutators. We are therefore at this point ready to define the quantum theory, with the use of the Hamiltonian (3.45) and the canonical brackets (3.46), (3.47), and (3.48). Equivalently we may define the functional generator as a Hamiltonian path integral over the constraint surface:

$$Z = \int d[A_i^{aT}, \pi_a^{iT}, B_i^a, P_a^i] \times \exp\left[i \int d^4x [\pi_a^{iT} \dot{A}_i^{aT} + P_a^i \dot{B}_i^a - H(x)] \right] \;, \tag{3.50}$$

where we denote $[dA\, dB\, dC\, dD]$ by $d[A,B,C,D]$ and H is given by (3.45). Unfortunately this Hamiltonian is a highly nonlinear function of the physical variables, as a result of the substitution of the solutions for the constraints into it. In particular, the integration over the momenta is nontrivial and the result will not be manifestly relativistically invariant.

We proceed then to reintroduce the missing variables and appropriate δ functions in order to express our theory in terms of an equivalent functional integral over the whole phase space, in the hope that the result will be relativistically invariant.

We begin by reintroducing the longitudinal components of A and π. We had the conditions

$$A^a_{iL}=0,\quad \pi^i_{aL}=\bar{\pi}^i_{aL},\quad C^a(\bar{\pi}^i_{bL})=0\;; \tag{3.51}$$

and we may therefore write for Z,

$$Z=\int d[A^a_i,\pi^i_a,B^a_i,P^i_a]\delta[A^a_{iL}]\delta[\pi^i_{aL}-\bar{\pi}^i_{aL}]\times\exp\left[i\int d^4x[\pi^i_a\dot{A}^a_i+P^i_a\dot{B}^a_i-H(x)]\right], \tag{3.52}$$

where H is now a function of the variables

$$H=H[A^a_i,B^a_i,\pi^i_a,P^i_a]\;. \tag{3.53}$$

We may change variables in the δ functions to C^a and the gauge condition $\chi_a=\partial_i A^i_a=0$, in terms of the field A^i_a:

$$Z=\int d[A^a_i,\pi^i_a,B^a_i,P^i_a]\delta[\chi^a]\delta[C^a]\det\left|\left[\frac{\delta\chi_a}{\delta A^i_{cL}},\frac{\delta C_b}{\delta\pi^c_{iL}}\right]\right|\exp\left[i\int d^4x[\pi^i_a\dot{A}^a_i+P^i_a\dot{B}^a_i-H(x)]\right]. \tag{3.54}$$

We must now reintroduce B^a_0 in order to get back the original version of H

$$Z=\int d[A^a_i,\pi^i_a,B^a_\mu,P^i_a]\delta[\chi^a]\delta[C^a]\det|[\chi_a,C_b]|\,\delta[B^0_a-\bar{B}^0_a]\exp\left[i\int d^4x[\pi^i_a\dot{A}^a_i+P^i_a\dot{B}^a_i-H(x)]\right], \tag{3.55}$$

where $\bar{B}^0_a$ is the solution of the constraint equation $D^{*a}=0$ and we now have for the functional dependence of H on the fields

$$H=H[A^a_i,B^a_\mu,\pi^i_a,P^i_a]\;. \tag{3.56}$$

We may now exponentiate $\delta[C^a]$ by means of the reintroduction of A^a_0, which is a free function at our disposal, and we may also change variables in the last δ functional to D^{*a}

$$Z=\int d[A^a_\mu,B^a_\mu,\pi^i_a,P^i_a]\delta[\chi_a]\delta[D^*_a]\det\left|\left[\frac{\delta D^{*a}}{\delta B^b_0}\right]\right|\det|[\chi^a,C^b]|\exp\left[i\int d^4x[\pi^i_a\dot{A}^a_i+P^i_a\dot{B}^a_i-H(x)+A^a_0C_a]\right], \tag{3.57}$$

where H is now just H_t in the constraint surface (CS)

$$H=H[A^a_\mu,B^a_\mu,\pi^i_a,P^i_a]=H_t\,|_{\rm CS}\;. \tag{3.58}$$

At this point we almost have H_t in the argument of the exponential. Let us recall that

$$H_t(x)=H(x)-A^a_0C^*_a-B^a_0D^*_a\;, \tag{3.59}$$

where $C^*_a=C_a$ for $P^0_a=0$, so we see that the term proportional to D^*_a is missing, due to the fact that we cannot use B^a_0 to exponentiate the δ functional in D^*_a, because B^a_0 is not a free parameter. In order to do this exponentiation it is necessary to introduce a new set of fields λ_a, but we would be introducing the time components of some four-vector without introducing the space components, and it is difficult to see how the result could be relativistically invariant. Also, one would still have extra terms in the exponent, of the form

$$(\lambda_a-B^0_a)D^{*a}\;, \tag{3.60}$$

in addition to the original Hamiltonian H_t, which would change the integration over momenta.

This is a strange situation, as the relativistically invariant gauge condition

$$q=I \tag{3.61}$$

seems to give rise to a nonrelativistically invariant result in the end.

At this point it is possible, however, as was shown by Popovic,[9] using methods developed previously by Senjanovic, and also by Fradkin and Vilkovisky,[10] to perform a sequence of formal substitutions and other operations on the path integral that render it in a relativistically invariant form. The result of Ref. 9 is what one would get from the direct application of the Faddeev-Popov ansatz to the A^a_μ field only, and there are no extra fields, as the λ_a parameters are integrated out. This is a surprising result, in a theory with second-class constraints, in particular because the content of Ref. 9 implies the remarkable result

$$\int d[A^a_\mu,B^a_\mu,\pi^i_a,P^i_a]\delta[\chi^a]\det|[\chi^a,C^b]|\,\delta[D^*_a]\exp\left[i\int d^4x[\pi^i_a\dot{A}^a_i+P^i_a\dot{B}^a_i-H_t(x)]\right]$$
$$=\alpha\int d[A^a_\mu,B^a_\mu,\pi^i_a,P^i_a]\delta[\chi^a]\det|[\chi^a,C^b]|\exp\left[i\int d^4x[\pi^i_a\dot{A}^a_i+P^i_a\dot{B}^a_i-H_t(x)]\right], \tag{3.62}$$

which means that the second-class constraint $D^{*a}(x)$ may be lifted, in this case. This is in sharp contrast to the original Dirac treatment,[7] in which the second-class constraints must be imposed as strong relations between the quantum operators. This situation is similar to the one encountered in the case of the massive Yang-Mills field,[10] in which the breakdown of gauge invariance introduces second-class constraints. In our case, however, there is no such breakdown, and in order to clarify the role that the second-class constraints and the variable q play in the theory, we, in what follows, repeat the whole analysis of the theory without the $q=I$ gauge condition imposed beforehand.

IV. GAUGE-INDEPENDENT ANALYSIS OF THE SL(2,C) THEORY

We begin with the general Lagrangian (1.6) and write both q and ω_μ in terms of component fields and the SU(2) generators $\theta_a=\theta_a^\dagger$. For convenience we decompose q in terms of its determinant explicitly, so that we have the expansions

$$q=\sqrt{\lambda}(\phi_0+2\phi_a\theta^a)\ ,\qquad q^{-1}=\frac{1}{\sqrt{\lambda}}(\phi_0-2\phi_a\theta^a)\ , \tag{4.1}$$

$$\lambda=\det q,\ \ \phi_0=(1+\phi_a\phi^a)^{1/2}\ , \tag{4.2}$$

$$\omega_\mu=(iA_\mu^a+B_\mu^a)\theta_a\ . \tag{4.3}$$

Note that because in (3.1) $c_a=\theta_a$, $d_a=i\theta_a$, this last expansion is just an expansion in the generators of SL(2,C). Next we compute $\nabla_\mu q$ and $F_{\mu\nu}$ in component form, and get

$$\nabla_\mu q=\sqrt{\lambda}\left\{\phi_0\partial_\mu\ln\sqrt{\lambda}+\frac{1}{\phi_0}\phi\cdot\partial_\mu\phi-\phi\cdot B_\mu+2\theta_a[(D_\mu\phi)^a+\phi^a\partial_\mu\ln\sqrt{\lambda}-\phi_0B_\mu^a]\right\}, \tag{4.4}$$

$$F_{\mu\nu}=iG_{\mu\nu}+W_{\mu\nu}=(iG_{\mu\nu}^a+W_{\mu\nu}^a)\theta_a\ . \tag{4.5}$$

Here we are using the vector notations defined in (3.2), and $G_{\mu\nu}^a, W_{\mu\nu}^a$ are given by the expressions (3.4), (3.5), and (3.6) in terms of the potentials. Note, however, that our fields are now decomposed into Hermitian and anti-Hermitian parts in the usual sense, not with respect to q as in Sec. II.

Our next task is to express the Lagrangian in terms of the component fields. Apart from an unimportant overall factor of 2 we get

$$\begin{aligned}L=&-\frac{1}{4g^2}(G_{\mu\nu}\cdot G^{\mu\nu}+W_{\mu\nu}\cdot W^{\mu\nu})+\frac{m^2}{2}B_\mu\cdot B^\mu+\frac{m^2}{2}D_\mu\phi\cdot D^\mu\phi+\frac{m^2}{2}(B_\mu\times\phi)\cdot(B^\mu\times\phi)\\&-\frac{1}{2g^2}(\phi\times G_{\mu\nu})\cdot(\phi\times G^{\mu\nu})-\frac{1}{2g^2}(\phi\times W_{\mu\nu})\cdot(\phi\times W^{\mu\nu})-\frac{m^2}{2}\frac{1}{\phi_0^2}(\phi\cdot\partial_\mu\phi)(\phi\cdot\partial^\mu\phi)+\frac{1}{g^2}\phi_0\phi\cdot(G_{\mu\nu}\times W^{\mu\nu})\\&-\frac{m^2}{\phi_0}[B_\mu\cdot D^\mu\phi+(\phi\times B_\mu)\cdot(\phi\times D^\mu\phi)]+\frac{m^2}{2}(\partial_\mu\ln\sqrt{\lambda})(\partial^\mu\ln\sqrt{\lambda})\ .\end{aligned} \tag{4.6}$$

Note that, as stated earlier, λ decouples from the other degrees of freedom, and may be dropped from the theory. We are now ready to begin the canonical analysis. We have the pairs of canonically conjugate variables:

$$(A_\mu^a,\pi_a^\mu),\ \ (B_\mu^a,P_a^\mu),\ \ (\phi^a,Q_a)\ . \tag{4.7}$$

We calculate then directly the momenta. In order to write the results in a concise way, as well as to facilitate the inversion for the velocities in terms of the momenta, we use a new matrix notation. We get

$$-\frac{g^2}{2}\pi_a^\mu=\frac{1+2\phi^2}{2}\Delta_a^{(2)b}G_b^{0\mu}+\phi_0\Sigma_a^bW_b^{0\mu}\ , \tag{4.8}$$

$$-\frac{g^2}{2}P_a^\mu=\frac{1+2\phi^2}{2}\Delta_a^{(2)b}W_b^{0\mu}-\phi_0\Sigma_a^bG_b^{0\mu}\ , \tag{4.9}$$

$$\frac{1}{m^2}Q_a=\Delta_a^{(1)b}\partial_0\phi_b+\Sigma_a^bA_b^0-\phi_0\Delta_a^{(1)b}B_b^0\ , \tag{4.10}$$

where $\phi^2=\phi_a\phi^a$ and

$$\Delta_a^{(n)b}=\delta_a^b-\frac{n\phi_a\phi^b}{1+n\phi^2}\ , \tag{4.11}$$

$$[\Delta_a^{(n)b}]^{-1}=\delta_a^b+n\phi_a\phi^b\ , \tag{4.12}$$

$$\Sigma_a{}^b=\phi_c\epsilon_a{}^{cb}=-\Sigma^b{}_a\ . \tag{4.13}$$

Note that in the $\phi_a=0$ (or $q=I$) gauge π_a^μ and P_a^μ reduce to the previous results, and Q_a becomes $-m^2B_a^0$. In our previous analysis Q_a was missing, and because $Q_a=0$ is not a relativistically invariant statement, we can see now where the apparent noninvariance of our result came from. In order to invert these relations we first establish the following properties of $\Delta^{(n)}$ and Σ:

$$\Delta^{(n)}\Sigma=\Sigma\Delta^{(n)}=[\Delta^{(n)}]^{-1}\Sigma=\Sigma[\Delta^{(n)}]^{-1}=\Sigma\ , \tag{4.14}$$

$$\Sigma\Sigma=-\phi^2\Delta^{(0)},\ \ \Delta^{(0)}\Delta^{(0)}=\Delta^{(0)}\ . \tag{4.15}$$

Note that $\Delta^{(0)}$ is a projector and has no inverse. We then get, for the velocities,

$$G^{0i}=-g^2(1+2\phi^2)\Delta^{(2)}\pi^i+2g^2\phi_0\Sigma P^i\ , \tag{4.16}$$

$$W^{0i}=-g^2(1+2\phi^2)\Delta^{(2)}P^i-2g^2\phi_0\Sigma\pi^i\ , \tag{4.17}$$

$$\partial_0\phi=\frac{1}{m^2}[\Delta^{(1)}]^{-1}Q-\Sigma A_0+\phi_0B_0\ . \tag{4.18}$$

From these one easily gets $\dot{A}^i$ and $\dot{B}^i$. Note that we have two primary constraints as before: namely,

$$\pi_a^0 = 0 = P_a^0 . \tag{4.19}$$

We are now in a position to construct the canonical Hamiltonian, given by

$$H_c(x) = \dot{A}_i^a \pi_a^i + \dot{B}_i^a P_a^i + \phi^a Q_a - L(\text{momenta}) . \tag{4.20}$$

After some calculation with our matrices and integration by parts we get, dropping total space derivatives,

$$\begin{aligned} H_c(A_\mu^a, B_\mu^a, \phi^a, \pi_a^i P_a^i, Q_a) = H_c(x) = & -g^2(\phi^2 + \tfrac{1}{2})(\pi_i \Delta^{(2)} \pi^i + P_i \Delta^{(2)} P^i) + 2g^2 \phi_0 \pi_i \Sigma P^i \\ & + \frac{1}{2g^2}(\phi^2 + \tfrac{1}{2})(G_{ij}\Delta^{(2)} G^{ij} + W_{ij}\Delta^{(2)} W^{ij}) - \frac{1}{g^2}\phi_0 W_{ij} \Sigma G^{ij} \\ & - \frac{m^2}{2}(\phi^2 + 1) B_i \Delta^{(1)} B^i - \frac{m^2}{2}\phi^2 A_i \Delta^{(0)} A^i + m^2 \phi_0 B_i \Sigma A^i \\ & - \frac{m^2}{2}\partial_i \phi \Delta^{(1)} \partial^i \phi + m^2 A_i \Sigma \partial^i \phi + m^2 \phi_0 B_i \Delta^{(1)} \partial^i \phi \\ & + \frac{1}{2m^2} Q[\Delta^{(1)}]^{-1} Q - A_0 \cdot \bar{C} - B_0 \cdot \bar{D} , \end{aligned} \tag{4.21}$$

where $\pi_i \Delta^{(2)} \pi^i$, for example, stands for $\pi_i^a \Delta_a^b \pi_b^i$, and in the last two terms we have

$$\bar{C}^a = C^a + (Q \times \phi)^a , \tag{4.22a}$$

$$C^a = (D_i \pi^i - B_i \times P^i)^a , \tag{4.22b}$$

$$\bar{D}^a = D^a - \phi_0 Q^a , \tag{4.23a}$$

$$D^a = (D_i P^i + B_i \times \pi^i)^a . \tag{4.23b}$$

Imposing now the consistency of the primary constraints with the time evolution generated by H_c we get two secondary constraints:

$$[\pi_0^a(x), H_c] = \bar{C}^a(x) = 0 , \tag{4.24}$$

$$[P_0^a(x), H_c] = \bar{D}^a(x) = 0 , \tag{4.25}$$

where $H_c = \int d^3(x) H_c(x)$. Using our previous knowledge about C^a and D^a it is not difficult to compute the whole constraint algebra. The nonvanishing brackets are

$$[\bar{C}^a(x), \bar{C}^b(y)] = -\delta^3(x-y)\epsilon^{abc}\bar{C}_c(x) , \tag{4.26a}$$

$$[\bar{C}^a(x), \bar{D}^b(y)] = -\delta^3(x-y)\epsilon^{abc}\bar{D}_c(x) , \tag{4.26b}$$

$$[\bar{D}^a(x), \bar{D}^b(y)] = +\delta^3(x-y)\epsilon^{abc}\bar{C}_c(x) . \tag{4.26c}$$

We see that this constraint algebra closes. Note that it reproduces the Lie algebra of SL(2,C), Eq. (3.1). We still have to impose the consistency of the secondary constraints with respect to H_c. After a rather long calculation we get the brackets

$$[\bar{C}^a(x), H_c] = (A_0 \times \bar{C} + B_0 \times \bar{D})^a(x) \approx 0 , \tag{4.27}$$

$$[\bar{D}^a(x), H_c] = (A_0 \times \bar{D} - B_0 \times \bar{C})^a(x) \approx 0 . \tag{4.28}$$

The secondary constraints are therefore also consistent, and so *all* the constraints are first class. This tells us that the existence of second class constraints in our previous analysis is purely an artifact of the gauge choice we made beforehand.

Next we examine the positivity of the Hamiltonian. For this it is convenient to introduce some further, 6×6 matrix notation and write $H_c(x)$ in the form

$$\begin{aligned} H_c(x) = & \frac{1}{2m^2}[Q^2 + (\phi \cdot Q)^2] - A_0 \cdot \bar{C} - B_0 \cdot \bar{D} + g^2 \begin{bmatrix} \pi_i, P_i \end{bmatrix} \begin{bmatrix} M + \frac{I}{2} \end{bmatrix} \begin{bmatrix} \pi_i \\ P_i \end{bmatrix} + \frac{1}{2g^2} \begin{bmatrix} W_{ij}, G_{ij} \end{bmatrix} \begin{bmatrix} M + \frac{I}{2} \end{bmatrix} \begin{bmatrix} W_{ij} \\ G_{ij} \end{bmatrix} \\ & + \frac{m^2}{2} \begin{bmatrix} A_i, \frac{\partial_i \phi}{\phi_0} - B_i \end{bmatrix} \begin{bmatrix} M + \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \end{bmatrix} \begin{bmatrix} A_i \\ \frac{\partial_i \phi}{\phi_0} - B_i \end{bmatrix} , \end{aligned} \tag{4.29}$$

where

$$M = \begin{bmatrix} (\phi^2 \Delta^{(0)}) & (-\phi_0 \Sigma) \\ (\phi_0 \Sigma) & (\phi^2 \Delta^{(0)}) \end{bmatrix} . \tag{4.30}$$

is a 6×6 real symmetric matrix, and

$$\begin{bmatrix} \pi_i, P_i \end{bmatrix} ,$$

for example, stands for a six-vector. Note that we lowered all spatial indices and changed the signs accordingly. We see that H_c will be positive in the presence of the constraints if and only if all the eigenvalues of the matrices

$$M+\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{4.31a}$$

and

$$M+\begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \tag{4.31b}$$

are positive. We must therefore write out the secular equation for each matrix, find the roots and see if they are all positive. Note that the matrices depend only on the three parameters ϕ_a $(a=1,2,3)$. This calculation was done on the computer with the help of the symbolic manipulation program (SMP) package:

$$\det\left[M+\frac{I}{2}-I\lambda\right]=(\lambda-\tfrac{1}{2})^2\left[\lambda-\frac{[\phi_0+(\phi^2)^{1/2}]^2}{2}\right]^2\left[\lambda-\frac{[\phi_0-(\phi^2)^{1/2}]^2}{2}\right]^2 , \tag{4.32}$$

$$\det\left[M+\begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}-\lambda I\right]=\lambda^3(\lambda-1)[\lambda-(\phi^2+\phi_0{}^2)]^2 . \tag{4.33}$$

Note that the roots are all SU(2) invariant, as one should expect. An independent proof of the positivity of the Hamiltonian can be achieved by remarking that as $\overline{C}^a$ and $\overline{D}^a$ form a representation of the SL(2,C) Lie algebra (just integrate the constraints over $\int d^3x$) and are consistent with H_c, one can rotate the vector (ϕ_0,ϕ_a) to $(\phi_0,0)$ by means of a gauge transformation, and return to our old form of H_c, which we know to be positive.

The final step is to construct the path integral for the functional generator. We shall see that it is possible to choose the gauge in such a way that the resulting perturbation expansion is relativistically invariant. The Hamiltonian in the presence of the constraints is

$$H=H_c[A_i^a,\pi_a^i,B_i^a,P_a^i,\phi^a,Q_a] , \tag{4.34}$$

and we have the four first-class constraints π_0^a, P_0^a, $\overline{C}^a$, and $\overline{D}^a$. The first two are trivially solved and relate to the fact that now both A_0^a and B_0^a have vanished from sight. We must impose two additional subsidiary conditions, and we choose

$$A_{iL}^a=0,\ \ \phi^a=0 , \tag{4.35}$$

so that we are back at the $q=I$ gauge. We then use the constraints to solve for the corresponding momenta, π_{iL}^a and Q_a. This choice of gauge is convenient because the constraint system decouples into

$$C^a(\overline{\pi}_{iL}^b)=0 \tag{4.36a}$$

and

$$Q_a=D_a , \tag{4.36b}$$

so that we may solve explicitly for Q_a. We have, for the Hamiltonian at this point,

$$H=H[A_{iT}^a,\pi_{aT}^i,B_i^a,P_a^i] , \tag{4.37}$$

and therefore our physical variables are the same as before, and we have, for the path integral,

$$Z=\int d[A_{iT}^a,\pi_{aT}^i,B_i^a,P_a^i]\times\exp\left[i\int d^4x[\dot{A}_{iT}^a\pi_{aT}^i+\dot{B}_i^aP_a^i-H(x)]\right] . \tag{4.38}$$

We may now extend the integration over the whole phase space as before. After reintroduction of the longitudinal components we get

$$Z=\int d[A_i^a,\pi_a^i,B_i^a,P_a^i]\delta[\chi^a]\delta[C^a]\det|[\chi^a,C^b]|\times\exp\left[i\int d^4x[\dot{A}_i^a\pi_a^i+\dot{B}_i^ap_a^i-H(x)]\right] . \tag{4.39}$$

We next reintroduce the momenta Q_a and the constraint $\overline{D}^a$, and get

$$Z=\int d[A_i^a,\pi_a^i,B_i^a,P_a^i,Q^d]\delta[\chi^a]\delta[C^a]\delta[\overline{D}^a]\times\det|[\chi^a,C^b]|\det\left[\frac{\delta\overline{D}^a}{\delta Q^b}\right]\times\exp\left[i\int d^4x[\dot{A}_i^a\pi_a^i+\dot{B}_i^aP_a^i-H(x)]\right] . \tag{4.40}$$

We may now use both A_0^a and B_0^a to exponentiate the constraints, and so we get

$$z=\int d[A_\mu^a,\pi_a^i,B_\mu^a,P_a^i,Q^a]\delta[\chi^a]\det|[\chi^a,C^b]|\det|[\phi^a,\overline{D}^b]|\exp\left[i\int d^4x[\dot{A}_i^a\pi_a^i+\dot{B}_i^aP_a^i-H(x)+A_0\cdot C+B_0\cdot\overline{D}]\right] , \tag{4.41}$$

where now we substitute back in $H(x)$ all of the variables that were reintroduced. The variable Q_a may now be integrated out by the completion of a square, because

$$[\phi^a,\overline{D}_b]=\frac{\delta\overline{D}^a}{\delta Q^b}=1 \tag{4.42}$$

is independent of Q^a, and so we get back the complete $H_c(x)$ in the $q=I$ gauge:

$$Z=\int d[A_\mu^a,\pi_a^i,B_\mu^a,P_a^i]\delta[\chi^a]\det|[\chi^a,C^b]|\times\exp\left[i\int d^4x[\dot{A}_i^a\pi_a^i+\dot{B}_i^aP_a^i-H_c(x)]\right] . \tag{4.43}$$

The integration over momenta now gives us back L:

$$Z=\int d[A^a_\mu,B^a_\mu]\delta[\chi^a]\det|[\chi^a,C^b]|\exp\left[i\int d^4x\,L\right]. \tag{4.44}$$

This result coincides with the one found by Popovic.[9] The relation (4.43) tells us that the $(q=I)$ gauge happens to be a ghost-free gauge for the SL(2,C)/SU(2) part of the gauge group, and this is why we are reduced to the Faddeev-Popov ansatz. One can now easily change from the transverse gauge $A_L=0$ to a covariant gauge,[11] and derive a covariant perturbation expansion from (4.45). We see that one can in fact lift the constraint $\bar{D}$. In doing so we are extending the integration to the orbits of the generators of SL(2,C)/SU(2), but this is immaterial since the weight (volume) of the orbits is a constant, independent of the fields. The reason why this was not clear before was the absence from our previous analysis of the momenta Q_a conjugate to ϕ^a.

V. CONCLUSIONS

We have shown that the SL(N,C) metric connection theories proposed by Cahill, Kim and Zee,[1] and Dell[2] can be consistently quantized in the canonical formalism. In spite of the fact that the group is noncompact a consistent constrained Hamiltonian quantum theory can be constructed in which the constraints are first class and consistent quantum mechanically and the Hamiltonian is bounded from below. We have also shown that a relativistically invariant functional integral may be defined, given an arbitrary fixing of the SL(N,C) local gauge freedom. From this we can in principle define a relativistically invariant perturbation theory in any gauge, and show that the resulting S-matrix elements are gauge invariant. Furthermore, by constructing the full Hamiltonian theory without any prior gauge fixing, we were able to show that the $q=I$ gauge is indeed a ghost-free gauge with respect to the SL(2,C)/SU(2) part of the gauge group, and that the form of the functional constructed by Popovic is in fact correct.

The resulting perturbation theory is, however, not renormalizable. This is easily seen from the fact that the propagator for the B^μ field will take the form, in $q=I$ gauge,

$$D_{\mu\nu}(P)=\frac{-i(\eta_{\mu\nu}-P_\mu P_\nu/P^2)}{P^2-m^2}+\frac{-iP_\mu P_\nu/P^2}{m^2}. \tag{5.1}$$

Thus we see that the longitudinal mode is not damped, and this will lead to the usual problems involving massive vector fields.

One may of course attempt to solve the problem of nonrenormalizability by adding to the theory additional terms as was done in Ref. 12 by one of us (L.S.). There are, to begin with, a number of terms which one can add to the theory without compromising the stability of the Hamiltonian. These are (assuming that we do not want to break B parity)

$$\frac{1}{8g'^2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}+F^{+}_{\mu\nu}F^{+\mu\nu}) -\frac{\lambda_1}{32}\{\mathrm{Tr}[(q^{-1}\nabla_\mu q)(q^{-1}\nabla^\mu q)]\}^2 -\frac{\lambda_2}{32}\{\mathrm{Tr}[(q^{-1}\nabla_\mu q)(q^{-1}\nabla_\nu q)]\}^2 \tag{5.2}$$

and they lead, in the $q=I$ gauge, to the terms

$$-\frac{1}{4g'^2}(G_{\mu\nu}\cdot G^{\mu\nu}-W_{\mu\nu}\cdot W^{\mu\nu})-\frac{(\lambda_1-8h^{-1})}{4}(B_\mu\cdot B^\mu)^2 -\frac{\lambda_2}{4}(B_\mu\cdot B_\nu)(B^\mu\cdot B^\nu). \tag{5.3}$$

These terms do not affect the analysis of the constraints, either in a general gauge or when $q=I$, and, for the proper signs of the coefficients, will lead to a positive-definite Hamiltonian. However, while they will absorb a number of divergences, they are not enough to lead to a renormalizable theory. This is because the longitudinal part of the B^μ propagator has not been modified. In order to do this we must add an additional dimension-four term

$$\frac{1}{8h}\mathrm{Tr}[(\nabla^2 q^{-1})(\nabla^2 q)], \tag{5.4}$$

which becomes, when $q=I$,

$$-\frac{1}{2h}\mathrm{Tr}(D_\mu B^\mu D_\nu B^\nu). \tag{5.5}$$

When this term is also added, the theory becomes renormalizable, as was shown in Ref. 12. However, in this case the Hamiltonian is no longer bounded from below. In Ref. 12 an ansatz for a perturbatively stable vacuum state was found which led to a unitary perturbation theory. While this is interesting, it does not change the fact that the Hamiltonian is not bounded from below, so that nonperturbatively, there is an instability.

Thus, the situation for the metric-connection gauge theories is exactly the same as the one that holds for the metric theories of gravity. We have a choice between a theory which is stable, but nonrenormalizable and a theory which is renormalizable, but unstable.

Is there then anything further to be done with these theories? It would perhaps be easier to dismiss theories which lead to this dilemma, were it not for the fact that the dilemma is shared by all known local quantum field theories which involve gravity. Therefore, if gravity is described by a local quantum field theory, there must be some way out of the dilemma. In this regard we may note that while one horn of the dilemma, that associated with stability, is apparently a desease of the full nonperturbative theory (in our case is need not show up at all in the perturbation theory), the other horn, nonrenormalizability, is known only to be a desease of the perturbation theory. Thus, it may still be useful to look for nonperturbative mechanisms to cure the problem of renormalizability.

As a final comment, we may note that there is one possibility concerning these metric-connection gauge theories which has not yet been studied sufficiently to permit definite conclusions. This is the case when we set the parameter m to zero. In this case the theory has genuine second-class constraints in any gauge, these being just the q field equations when $m=0$. These constraints are purely quadratic, for example, in the $q=I$ gauge they are of the form

$$E_a = 2g^2(\pi_i \times P^i)_a + \frac{1}{g^2}(G_{ij} \times W^{ij})_a = 0 \ . \tag{5.6}$$

It is thus not straightforward to work out the consequences of these constraints. However, as they are second class in any gauge on can say that they decrease the number of degrees of freedom of the theory. Thus, the limit $m \to 0$ of the theory is not continuous, and it is not true that the properties of the quantum theory for m strictly zero will be the same as the theory we have analyzed in this paper, when we take the limit $m \to 0$. Thus, one may learn from an analysis of this case how purely second-class constraints, which do not follow from any gauge-fixing conditions, may affect perturbation theory.

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