Exact superpartners of $N=2$ supergravity solitons

P. C. Aichelburg and F. Embacher
Institut für Theoretische Physik, Universität Wien, Wien, Austria
(Received 28 July 1986)

By applying long-range $N=2$ supergauge transformations to the Majumdar-Papapetrou configuration, the set of all superpartners to the bosonic multisolitons is exhibited. The resulting configuration is parametrized by two additional complex Grassmann numbers. Apart from mass and electric (possibly magnetic) charge it carries a supercharge which is algebraically constrained due to the central charges in the supersymmetry algebra. The supercharge gives rise to an "intrinsic" angular momentum. Similar to the Kerr metric, the configuration is stationary with the quantum-mechanical magnetic moment.

I. INTRODUCTION

In ordinary field theory a classical solution of the field equations may be interpreted as a soliton, if it has the following properties. The configuration (i) is stationary, (ii) has finite total energy, (iii) is nonsingular, (iv) is classically and quantum-mechanically stable, and (v) shows a high, though nonmaximal symmetry (thus interpolating between different vacua of the theory).

The soliton concept has to be modified in general relativity because there are no everywhere-regular time-independent solutions of Einstein's equations that are asymptotically flat (except the flat space-time).

There exist however solutions with the properties (i), (ii), (iv), and (v) displaying singularities which are hidden behind event horizons. Outside of the horizons space-time is predictable. Classically one may consider the stationary black-hole solutions as solitons.

On the quantum level one knows, since the work of Hawking,[1] that black holes radiate thermally. In pure gravity the temperature of the radiation is proportional to the inverse mass of the black hole. It is reasonable to believe that the back reaction of the radiation extracts energy and will eventually lead to the evaporation of the hole. From this quantum instability one concludes that there are no solitons in pure Einstein gravity.

The situation changes if other fields are coupled to gravity and conserved charges prevent total evaporation. Within the Einstein-Maxwell theory a generic stationary black hole is completely characterized by its mass (M), angular momentum (a), and electric (e) and magnetic (q) charges. If the fundamental fields of the theory are uncharged (like the Maxwell field), then e and q are unchanged by the Hawking radiation. One expects the hole to reach a configuration with the smallest possible mass for given e and q (Refs. 2 and 3):

$$4\pi GM^2=e^2+q^2$$

(in units where $c=1$). These extreme Reissner-Nordström black holes (a has to vanish in the extreme case in order that the singularity is hidden by a horizon) have Hawking temperature zero and may be considered as gravitational solitons.[4]

In a series of papers the question was examined whether solitonlike configurations exist in supergravity theories. The symmetry between Bose and Fermi fields in these theories gives rise to a conserved spinorial charge. However, this supercharge expressed as a surface integral over the spin-$\frac{3}{2}$ field at spatial infinity may change under supergauge transformations which do not vanish asymptotically. One line of reasoning is to ask if a stationary black-hole solution with *nongauge* long-range fermionic fields exists. Within $N=1$ and $N=2$ supergravity this question is solved. The result is that, among the class of Kerr-Newman black holes, only the extreme Reissner-Nordström metric admits a supersymmetric generalization.[5–9] This result generalizes to the Majumdar-Papapetrou metric of several extreme black holes.[10,11] The most general exact single black-hole configuration carrying a nongauge supercharge was found.[12] Whether this classical field configuration may be considered as a soliton, i.e., whether it is in some sense quantum stable, is not clear.

The other line of investigations is concerned with the role of the purely bosonic solitons in the context of supergravity (for a review see Ref. 13). The extreme Reissner-Nordström black holes, as well as the Majumdar-Papapetrou multi-black-holes, are classical solutions of the $N=2$ supergravity field equations. Within the Einstein-Maxwell theory, they should acquire position parameters as "collective coordinates" associated with the translational coordinate freedom at spatial infinity. In the limit of slow motion, a collection of moving extreme black holes would then be described by the time dependence of their collective coordinates.[14] This philosophy, well known for flat space, has led to considerable success in the quantization of solitons.[15,16] In gravity the introduction of collective coordinates is subtle. Hajicek[17] generalizing the work of Regge and Teitelboim[18] has shown how to set up the dynamics of Einstein-Maxwell solitons.

In supergravity the question arises whether the additional local gauge symmetry leads to "supertranslated" partners (shortly called superpartners), thus introducing new fermionic variables (essentially the gauge parameters) as collective coordinates.

Gibbons[13,19] has considered this problem within $N=1$

and $N=2$ supergravity. By analyzing the infinitesimal supergauge translations, he concludes that only the extreme Reissner-Nordström or, more general, the Majumdar-Papapetrou configurations admit superpartners.

In this paper we construct the *exact* superpartners to the Majumdar-Papapetrou configurations. We obtain the finite supertranslation by iterating the gauge transformations performed on the initial background to *all* orders. Thereby the generated supercharges are constrained due to the extreme central charges of the background.

The resulting configuration is characterized by two complex Grassmann numbers. We discuss the solution by analyzing its asymptotic structure. While mass and electric charge remain unchanged, the configuration carries a supercharge which in turn gives rise to an "internal" angular momentum. Similar to the Kerr metric the configuration is stationary with the correct quantum-mechanical magnetic moment.

II. CONSTRUCTION OF THE EXACT SUPERPARTNER

The fundamental fields of ungauged $N=2$ supergravity[20] are the gravitational vierbien field $e^a=e^a{}_\mu dx^\mu$, the electromagnetic potential one-form $A=A_\mu dx^\mu$ and two Majorana spinor-valued one-forms ψ^j, combined to a complex (Dirac) field $\psi=\psi^1+i\psi^2=\psi_\mu dx^\mu$ (Rarita-Schwinger field). All fields are Grassmann valued, the bosonic fields (e,A) being even elements while ψ is odd (anticommuting).

The theory is invariant under local supergauge transformations generated by a complex spinor field $\epsilon=\epsilon^1+i\epsilon^2$ (ϵ^j Majorana). The infinitesimal form reads

$$\delta e^a=-\frac{ik}{2}(\bar\epsilon\gamma^a\psi-\bar\psi\gamma^a\epsilon)\ , \tag{2.1a}$$

$$\delta A=\frac{i}{2}(\bar\epsilon\psi-\bar\psi\epsilon)\ , \tag{2.1b}$$

$$\delta\psi=\frac{1}{k}\hat D\epsilon\ . \tag{2.1c}$$

[The explicit form of the field equations, notations and conventions may be found in Ref. 21, especially $k^2=4\pi G$, signature $(+\,-\,-\,-)$,

$$\hat D=D-\frac{k}{2}\hat F^{ab}\sigma_{ab}\gamma,\quad D=d+\tfrac{1}{2}\omega^{ab}\sigma_{ab},\quad \gamma=\gamma_a e^a\ .$$

We use the Weyl representation of the γ matrices which may be found in Ref. 22.]

The global conserved quantity associated with this symmetry is the (spinorial) supercharge[23,24]

$$\mathcal{S}=-\frac{i}{k}\oint_{S^2_\infty}\gamma_5\gamma\wedge\psi \tag{2.2}$$

which also acts as generator for the (asymptotic) global supersymmetry (SUSY) transformations (in the SUSY literature usually denoted by Q). Throughout the paper we consider only asymptotically flat configurations, and ψ has to fall off like $O(r^{-2})$ in order to render $\mathcal{S}$ finite.

The supercharge (2.2) is invariant under gauge transformations for which the gauge spinor ϵ tends to zero at spatial infinity. On the other hand, consider supergauge transformations for which ϵ is bounded away from zero at infinity. Then, the falloff conditions imposed on ψ require that

$$\lim_{r\to\infty}\epsilon\to\epsilon_\infty\ , \tag{2.3}$$

where ϵ_∞ is a *constant* (and also time-independent) four-spinor. This transformation induces a change in $\mathcal{S}$ which may be written as

$$\begin{aligned}\delta\mathcal{S}&=-\frac{i}{k^2}\oint_{S^2_\infty}\gamma_5\gamma\wedge\hat D\epsilon\\&=\left[-i\gamma^aP_a^{\rm ADM}+\frac{i}{k}e+\frac{i}{k}\gamma_5 q\right]\epsilon_\infty\ ,\end{aligned} \tag{2.4}$$

where

$$P_a^{\rm ADM}=\frac{1}{4k^2}\oint_{S^2_\infty}\omega^{bc}\wedge *e_{abc} \tag{2.5}$$

is the usual Arnowitt-Deser-Misner (ADM) four-momentum of the configuration, while e and q are the total electric and magnetic charge, respectively:

$$e=\oint_{S^2_\infty}*F\ , \tag{2.6a}$$

$$q=\oint_{S^2_\infty}F\ . \tag{2.6b}$$

Equation (2.4) expresses the global $N=2$ supersymmetry algebra with central charges e,q.

In what follows we concentrate on configurations which are related by gauge transformations of the form (2.3), i.e., we wish to identify all configurations which can be obtained by "short-range" gauge transformations ($\epsilon\to0$) from one another, thereby dividing the solutions into equivalence classes. Usually this is made explicit by imposing a gauge condition. Here we merely assume that this is in principle possible and return to this point later.

If the original configuration is extreme, i.e., if

$$k^2M^2=k^2P_a^{\rm ADM}P^a_{\rm ADM}=e^2+q^2\ , \tag{2.7}$$

the matrix on the right-hand side (RHS) of Eq. (2.4) is singular. As a consequence there are ϵ_∞ to the eigenvalue zero leaving $\mathcal{S}$ invariant. Also from (2.4) follows that the variations of $\mathcal{S}$ induced by gauge transformations are constrained by

$$\left[\gamma^aP_a^{\rm ADM}+\frac{1}{k}(e-\gamma_5 q)\right]\delta\mathcal{S}=0\ . \tag{2.8}$$

For $\psi=0$ the $N=2$ supergravity field equations reduce to the source-free Einstein-Maxwell system. Thus, any classical solution of the Einstein-Maxwell equations is automatically also a solution of the $N=2$ theory. The local gauge freedom (2.1) may now be used to generate configurations with nonzero Rarita-Schwinger fields from such a purely bosonic "background."

The gauge transformations (2.1) leave the field equations invariant only at the linearized level (i.e., to first order in ϵ). Finite transformations are obtained by iteration (or, if one prefers, exponentiation) of (2.1). Formally setting $\Phi=(e^a,A,\psi)$, one may write

$$\Phi \to e^{\delta}\Phi \equiv \Phi + \delta\Phi + \frac{1}{2!}\delta\delta\Phi + \cdots , \tag{2.9}$$

thereby the law (2.1) is simply repeated. As an example, we compute

$$\delta\delta e^a = -\frac{ik}{2}(\bar\epsilon\gamma^a\delta\psi - \delta\bar\psi\gamma^a\epsilon)$$
$$= -\frac{i}{2}[\bar\epsilon\gamma^a\hat D\epsilon - (\hat D\,\bar\epsilon)\gamma^a\epsilon] . \tag{2.10}$$

After the calculation, all fields (in this example they are contained in $\hat D$) are set equal to their original (background) values. These transformations leave the full nonlinear field equations invariant.

Since we shall be interested in the soliton structure of the theory, the backgrounds under consideration will be time independent. The identification made among configurations lying in the same equivalence class still permits gauge transformations of the type (2.3) acting between different classes. All equivalence classes obtained from the background by time-independent gauge transformations are called *superpartners*, leading to a degeneracy of the soliton sector. On the linearized level, i.e., to first order in ϵ, the supercharge $\mathscr{S}$ of the superpartners to a given background configuration is given by Eq. (2.4) with $\delta\mathscr{S}$ replaced by $\mathscr{S}$ (since $\psi = k^{-1}\hat D\epsilon$).

We now specify the background to the Majumdar-Papapetrou solutions[25,26] given by

$$e^0 = V^{-1}dt, \quad e^i = V\,dx^i, \quad i=1,2,3 , \tag{2.11}$$

$$A = -(kV)^{-1}dt , \tag{2.12}$$

where

$$\Delta V = 0, \quad \Delta = \sum_{i=1}^{3} \frac{\partial^2}{\partial x^{i_2}} . \tag{2.13}$$

For

$$V = 1 + \sum_{J=1}^{n} \frac{GM_J}{|\mathbf{x}-\mathbf{x}_J|}, \quad \mathbf{x}_J \neq \mathbf{x}_K \text{ for } J\neq K , \tag{2.14}$$

this configuration describes n static extreme charged black holes with mass parameters M_J and electric charges $e_J = kM_J$. Without loss of generality we restrict ourselves to the case of positive electric and zero magnetic charges. The sign of all electric charges may be reversed by changing the sign of A, while magnetic charge can be generated by a duality transformation.

Moreover

$$P_a^{\rm ADM} = (M,0,0,0), \quad M = \sum_{J=1}^{n} M_J , \tag{2.15}$$

and

$$e = \sum_{J=1}^{n} e_J . \tag{2.16}$$

The domain of outer communication for the metric (2.11) and (2.14) is

$$\mathbb{R} \times (\mathbb{R}^3 - \{\mathbf{x}_1, \ldots , \mathbf{x}_n\}) \tag{2.17}$$

on which the source-free Einstein-Maxwell equations are satisfied. The points $\mathbf{x}_J = 0$ are coordinate singularities, the location of the inner boundary of the extreme holes.[27]

To exhibit the parametrization of all superpartners we consider first linear gauge transformations

$$\psi = \frac{1}{k}\hat D\epsilon , \tag{2.18}$$

where ϵ goes to a constant spinor ϵ_∞ at infinity. According to (2.4), the associated supercharge is

$$\mathscr{S} = iM(\mathbb{1}-\gamma_0)\epsilon_\infty . \tag{2.19}$$

Splitting the gauge parameter ϵ into

$$\epsilon = \tfrac{1}{2}(\mathbb{1}-\gamma_0)\epsilon + \tfrac{1}{2}(\mathbb{1}+\gamma_0)\epsilon \equiv \epsilon_- + \epsilon_+ , \tag{2.20}$$

we see that the ϵ_+ generates no supercharge.

It is well known[3,28] that the Majumdar-Papapetrou metrics allow for supercovariantly constant spinor fields $\chi_{\rm sc}$, satisfying

$$\hat D\chi_{\rm sc} = 0 , \tag{2.21}$$

where

$$\chi_{\rm sc} = \tfrac{1}{2}V^{-1/2}(\mathbb{1}+\gamma_0)\epsilon_\infty . \tag{2.22}$$

Comparing (2.20) with (2.22) one notices that

$$\lim_{r\to\infty}(\epsilon_+ - \chi_{\rm sc}) = 0 . \tag{2.23}$$

But since $\chi_{\rm sc}$ does not generate any ψ field, ϵ_+ contains only short-range gauge freedom. All superpartners are generated by ϵ_- whose asymptotic form satisfies

$$(\mathbb{1}+\gamma_0)\epsilon_\infty = 0 , \tag{2.24}$$

and the associated supercharge is then given by

$$\mathscr{S} = 2iM\epsilon_\infty . \tag{2.25}$$

In the following we restrict ourselves to gauge spinors satisfying (2.24). In the Weyl representation of the γ matrices[22] this means that ϵ_∞ has the form

$$\epsilon_\infty = \begin{bmatrix} c \\ -c \end{bmatrix} , \tag{2.26}$$

where c is a complex two-spinor. Two gauge spinor fields which approach the same ϵ_∞ give rise to short-range equivalent ψ fields. Thus the set of gauge equivalence classes is parametrized by ϵ_∞, and hence by c.

A suitable way to construct one representative of each equivalence class explicitly is to impose the gauge condition

$$\gamma^\mu\psi_\mu = 0 \tag{2.27}$$

at the linearized level. This implies that ϵ satisfies the supercovariant Dirac equation

$$\gamma^\mu\hat D_\mu\epsilon = 0 . \tag{2.28}$$

As pointed out by Gibbons,[13] these conditions plus the requirement of ϵ being regular at the horizon and time independent project out all the short-range gauge transformations. Thus any time-independent regular solution of (2.28) is uniquely determined by its asymptotic value ϵ_∞.

The solutions are spanned by

$$\epsilon=V^{-1/2}\begin{bmatrix} c \\ -c \end{bmatrix}=-i\gamma_5\chi_{\rm sc} \tag{2.29}$$

and

$$\chi_{\rm sc}=V^{-1/2}\begin{bmatrix} c \\ c \end{bmatrix} . \tag{2.30}$$

Clearly, only the first class gives rise to superpartners.

At this point we wish to stress that for the construction of one representative of each equivalence class it is sufficient to choose *any* regular spinor field ϵ with the corresponding asymptotic values (2.26). On the classical level, imposing the gauge condition (2.27) is just for convenience.

Applying the same reasoning to the Schwarzschild black hole in $N=1$ supergravity, Gibbons has argued that, because there are no regular time-independent solutions to the Dirac equation, no superpartners exist. This means that regular static gauge configurations $\psi=k^{-1}D\epsilon$, which carry supercharge, are projected out by the gauge condition. However, unless a gauge condition of the type (2.27) is dictated by the quantum theory, classically these configurations are acceptable.

Now we turn to the construction of finite gauge transformations with the gauge parameter (2.29). Because c contains only two complex (four real) anticommuting Grassmann numbers, the series (2.9) stops after the fourth order.

Iterating the variations δ, one may make use of Eq. (2.21). The computation is tedious but straightforward (part of the calculations have been checked using the algebraic computer program REDUCE). At second order one introduces the quantity

$$\mathscr{C}_i=c^\dagger\sigma_i c \tag{2.31}$$

satisfying the identities

$$\mathscr{C}_i\mathscr{C}_j=-\delta_{ij}(c^\dagger c)(c^\dagger c) , \tag{2.32}$$

$$\mathscr{C}_i c=-(c^\dagger c)\sigma_i c , \tag{2.33}$$

which follow from the Grassmann nature of c. Using these relations in the third- and fourth-order calculations, one arrives at the final form of the solution. The exact superpartners of the Majumdar-Papapetrou configurations (satisfying the full nonlinear field equations) are given by the fields

$$e^0=[V^{-1}+\tfrac{1}{4}V^{-7}V_{,k}V_{,k}(c^\dagger c)(c^\dagger c)]dt + V^{-2}V_{,k}\epsilon_{kji}\mathscr{C}_j dx^i , \tag{2.34a}$$

$$e^i=V^{-4}V_{,k}\epsilon_{kij}\mathscr{C}_j dt + [V\delta_{ij}+\tfrac{1}{6}V^{-5}W_{ij}(c^\dagger c)(c^\dagger c)]dx^j , \tag{2.34b}$$

$$A=\frac{1}{k}[-V^{-1}+\tfrac{1}{4}V^{-7}V_{,k}V_{,k}(c^\dagger c)(c^\dagger c)]dt + \frac{1}{k}V^{-2}V_{,k}\epsilon_{kji}\mathscr{C}_j dx^i , \tag{2.34c}$$

$$\psi=\frac{1}{k}\left[V^{-3}V_{,k}\gamma_k+\frac{i}{2}V^{-6}V_{,k}V_{,k}(c^\dagger c)\right]dt\,\epsilon + \frac{1}{k}\left[V^{-1}V_{,k}\gamma_k\gamma_i-\frac{i}{3}V^{-4}W_{ik}\gamma_k(c^\dagger c)\right]dx^i\epsilon , \tag{2.34d}$$

where

$$W_{ij}=3VV_{,ij}-8V_{,i}V_{,j}+2\delta_{ij}V_{,k}V_{,k} \tag{2.35}$$

and V is still given by (2.14). The above given configurations (e^a,A,ψ), parametrized by the two complex parameters c, are exact solutions to the $N=2$ supergravity field equations and constitute the superpartners to the Majumdar-Papapetrou spacetimes.

For a discussion of the solution in the next section we also display the metric following from the vierbien (2.34a) and (2.34b):

$$g=\eta_{ab}e^a\otimes e^b=[V^{-2}+\tfrac{5}{2}V^{-8}V_{,k}V_{,k}(c^\dagger c)(c^\dagger c)]dt^2+4V^{-3}V_{,k}\epsilon_{kji}\mathscr{C}_j dx^i dt - V^2\delta_{ij}dx^i dx^j - V^{-4}(VV_{,ij}-\tfrac{11}{3}V_{,i}V_{,j}+\tfrac{5}{3}\delta_{ij}V_{,k}V_{,k})(c^\dagger c)(c^\dagger c)dx^i dx^j \tag{2.36}$$

and the determinant of $e^a{}_\mu$ is given by

$$\det(e^a{}_\mu)=\sqrt{-g}=V^2-\tfrac{25}{12}V^{-4}V_{,k}V_{,k}(c^\dagger c)(c^\dagger c) . \tag{2.37}$$

We close this section by noting that the gauge condition (2.27) does not carry over to the nonlinear level:

$$\gamma^\mu\psi_\mu=\frac{i}{k}\frac{25}{6}V^{-5}V_{,k}V_{,k}(c^\dagger c)\epsilon . \tag{2.38}$$

III. DISCUSSION OF THE SOLUTION

Let us first note that the superpartner configurations are regular at the horizons (which are located at $\mathbf{x}_J$) when transformed to a regular tetrad. Since the higher-order terms behave softer at $\mathbf{x}_J$ than the "body" and the first-order part of ψ, this is just a consequence of the regularity of the linearized field $\hat{D}\epsilon$.

The behavior at spatial infinity is governed by the lowest-order parts, the geometry being asymptotically flat.

Since the superpartner solutions may be used as candidates for solitons in $N=2$ supergravity, they have to be treated, in some approximation, as point particles described by a set of physical quantities. These quantities should clearly be extracted from the asymptotic behavior of the solution presented. Moreover, their transformation properties under asymptotic Lorentz transformations have to be known in order to describe moving solitons. For these reasons we study now the asymptotic (or "global") characterization of the superpartners.

A. ADM momentum

The ADM four-momentum $P_a^{\rm ADM}$ as given by Eq. (2.5) has still the form (2.15), the rest mass M being unchanged. This is because the Grassmann valued (or "soul") part of g_{ij} goes like r^{-3} at infinity. The same result is obtained when the Komar mass[29] is calculated from g_{00}. Here the soul part goes like r^{-4}.

B. Electric charge

The total charge is still given by the body of the electric field. The soul part behaves like r^{-5}, and thus does not contribute to the integral (2.6a). As a consequence the condition $e=kM$ is maintained. For a single black hole $(n=1)$, the potential is spherically symmetric and there are no higher multipole moments.

C. Supercharge

Only the linearized part of ψ contributes to the integral (2.2), $\mathcal{S}$ being given by (2.25). As noted in the foregoing section, it is not arbitrary but obeys the constraint (2.24) which in a Lorentz-invariant form reads

$$(\gamma^a P_a^{\rm ADM}+M)\mathcal{S}=0 \ . \tag{3.1}$$

$\mathcal{S}$ is conserved and behaves as a spinor under asymptotic Lorentz transformations.

D. Angular momentum

Inspection of (2.36) shows that the metric g is nonstatic but only stationary due to nondiagonal terms g_{0i}. Asymptotically

$$g_{0i}\sim 2GM\frac{x^k}{r^3}\epsilon_{ijk}\mathcal{C}_j \tag{3.2}$$

from which one reads off the angular momentum

$$S_i=M(c^\dagger\sigma_i c) \ . \tag{3.3}$$

Clearly, this coincides with the value of the ADM angular momentum.[24] Because this angular momentum is generated by a *static* spinor field it is more appropriate to speak about the intrinsic angular momentum or *spin* of the configuration. This interpretation is strengthened by the correct relation between spin and magnetic moment (see below).

The Lorentz-covariant formulation of (3.3) is easily found by noting

$$\bar\epsilon_\infty\sigma_{ij}\epsilon_\infty=i\epsilon_{ijk}\mathcal{C}_k \ . \tag{3.4}$$

The spin tensor is given by

$$S_{ab}=-iM\bar\epsilon_\infty\sigma_{ab}\epsilon_\infty=-\frac{i}{4M}\bar{\mathcal{S}}\sigma_{ab}\mathcal{S} \tag{3.5}$$

expressed in terms of the supercharge. It is related to (3.3) by

$$S_{ij}=\epsilon_{ijk}S_k, \quad S_{0i}=0 \ . \tag{3.6}$$

The last equation reads covariantly

$$P_a^{\rm ADM}S^{ab}=0 \tag{3.7}$$

and follows from (2.15) and (3.5). Any spin tensor which satisfies (3.7) is equivalent to a spin-four-vector

$$S^a=-\frac{1}{2M}\epsilon^{abcd}S_{bc}P_d^{\rm ADM} \ , \tag{3.8}$$

$$P_a^{\rm ADM}S^a=0 \ , \tag{3.9}$$

and one finds

$$S^a=\frac{i}{4M^2}P_b^{\rm ADM}(\bar{\mathcal{S}}\gamma_5\sigma^{ab}\mathcal{S})$$
$$=-\frac{i}{8M}(\bar{\mathcal{S}}\gamma_5\gamma^a\mathcal{S}) \ , \tag{3.10}$$

the last identity coming from (3.1). In the rest frame the spatial components S_i are just (3.3) and $S_0=0$. The inverse formula to recover S_{ab} is

$$S_{ab}=-\frac{1}{M}\epsilon_{abcd}P^c_{\rm ADM}S^d \ . \tag{3.11}$$

E. Magnetic moment

Since the vector potential behaves like

$$A^i\sim\epsilon_{ijk}e\mathcal{C}_j\frac{x^k}{4\pi r^3} \ , \tag{3.12}$$

the supergauge transformation has generated a magnetic field with magnetic dipole moment

$$\mu_i=e(c^\dagger\sigma_i c) \ . \tag{3.13}$$

Comparing this with (3.3), one finds

$$\mu_i=\frac{e}{M}S_i\equiv kS_i \ . \tag{3.14}$$

Note that this gives not the classical but rather the quantum-mechanical gyromagnetic ratio, just as for the Kerr metric.

Because of relation (3.14) the Lorentz-invariant notation of μ_i runs along the same line as for S_i: μ_i is incorporated in an antisymmetric tensor μ_{ab}, just as the magnetic field vector in F_{ab}. Then we set $\mu_{ab}=kS_{ab}$, and the relation $\mu_{0i}=0$ denotes the absence of an electric dipole moment.[30]

F. Magnetic charge

It is zero in the solution presented but may be generated by means of a finite duality rotation (which leaves the field equations invariant:[31]

$$e^a\to e^a \ ,$$
$$F\to F\cos\theta+\left[*\hat F-\frac{ik}{2}\bar\psi\wedge\gamma_5\psi\right]\sin\theta \ , \tag{3.15}$$
$$\psi\to\exp(-\tfrac{1}{2}\theta\gamma_5)\psi \ .$$

Electric and magnetic charge thereby transform as

$$e+iq\to e^{i\theta}(e+iq) \ , \tag{3.16}$$

the supercharge changes according to

$$\mathscr{S} \rightarrow \exp(\tfrac{1}{2}\theta\gamma_5)\mathscr{S} . \tag{3.17}$$

Note that by setting $\theta=\pi$, one may change the sign of the charges e_J.

Concluding the analysis of the asymptotic behavior, we summarize that, seen from infinity, our (positively charged) superpartner solution (for $n=1$) looks like a point particle (in flat space) with rest mass M (equal to its electric charge), four-momentum $p_a=P_a^{\mathrm{ADM}}$ and a spinorial quantity $\mathscr{S}$ which obeys the constraint (3.1). The spin tensor as well as the magnetic dipole moment are derived from these quantities. As the basic "equations of motion" of such a particle (soliton) one would take

$$\frac{d}{ds}p_a=0 , \tag{3.18}$$

$$\frac{d}{ds}\mathscr{S}=0 . \tag{3.19}$$

Far from having touched upon the interaction of such objects, we have exhibited the kinematics, i.e., the physical quantities by which solitons might be described. Since we have constructed time-independent multisoliton solutions, one expects that—in any reasonable theory of soliton dynamics—an equilibrium between a certain set of solitons is possible. In the limit $|\mathbf{x}_J-\mathbf{x}_K| \gg GM_L$ we may attach an almost asymptotically flat region around each hole and identify the individual supercharges (cf. Ref. 32):

$$\mathscr{S}_J=2iM_J \begin{bmatrix} c \\ -c \end{bmatrix}, \quad J=1,\ldots,n . \tag{3.20}$$

Thus, solitons equipped with supercharges as in (3.20) and which are at rest initially should remain in equilibrium. All questions concerning the interaction in other cases require to include the dynamics of $N=2$ supergravity.

ACKNOWLEDGMENTS

We thank R. Beig, H. Grosse, and H. Rumpf for helpful discussions. This work was supported in part by "Fonds zur Förderung der wissenschaftlichen Forschung in Österreich," Project No. P5766, and the "Einstein Memorial Foundation."

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