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Stech and Fritzsch hypotheses in the leptonic sector: Addendum to "Neutrino mass limits from the Fritzsch mass matrix"

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Following the idea suggested by Gronau, Johnson, and Schecter we further constrain the masses of the second- and third-generation neutrinos to a very small portion of the ν_2-ν_3 mass plane by combining the models of Fritzsch and Stech in the leptonic sector and demanding consistency with current experimental limits.

In a recent paper[1] the implications of the Fritzsch model[2] in the leptonic sector were examined. Using a similar analysis in this paper, the Fritzsch matrix will be combined with the ansatz of Stech,[3] resulting in more specific predictions for neutrino masses and mixing angles which are consistent with neutrino mass and oscillation experiments.

The lack of observation of neutrinoless double-β decay[4] may suggest that neutrinos are Dirac particles. Light Dirac neutrinos resulting from grand unified theories, which generally involve larger unification groups than SU(5), have recently been discussed in the literature.[5] In what follows we will limit ourselves to the case of Dirac neutrinos. If neutrinos are Dirac particles, then quarks and leptons may have similar horizontal symmetries in the electroweak Lagrangian. Since these symmetries act on quarks and leptons in an identical fashion, we expect the successful relationships found between the Kobayashi-Maskawa[6] (KM) angles and quark masses to be equally applicable in the leptonic sector. There have been extensive analyses[7] of the KM relationships for the quark sector. The most popular scenario in the literature[8] is the Fritzsch model,[2] which is in agreement with present experiment for the quark sector. Gronau, Johnson, and Schechter[9] have shown that the hypothesis of Stech does not conflict with that of Fritzsch, and hence the two models may be combined, resulting in the complete determination of the parameters in the KM matrix in terms of the quark masses. In this paper we will examine the implications of simultaneously imposing the Fritzsch and Stech hypotheses in the leptonic sector. As we will see, only a very tiny region of the ν_2-ν_3 mass plane will remain as allowed by the data.

In the Fritzsch model the fermion mass matrix takes the form

$$M_F=\begin{pmatrix} 0 & A_F e^{i\phi_A^F} & 0 \\ A_F e^{i\phi'_A} & 0 & B_F e^{i\phi_B^F} \\ 0 & B_F e^{i\phi_B'^F} & C_F e^{i\phi_C^F} \end{pmatrix} , \tag{1}$$

where $F=\nu,(l)$ for the neutral- (charged-) lepton mass matrix. The weak-charged-current mixing matrix is given by

$$U=V_\nu^T\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{i\tau} \end{pmatrix} V_l . \tag{2}$$

The matrices $V_{\nu(l)}$ as well as the coefficients $A_{\nu(l)}$, $B_{\nu(l)}$, and $C_{\nu(l)}$ are obtained in terms of the fermion masses in the work of Georgi and Nanopoulos.[10] σ and τ are two phase parameters which are related to the phases in M_F by

$$\begin{aligned} &\sigma=\phi_A^\nu+\phi_C^\nu-\phi_B^\nu-\phi_B'^\nu-(\phi_A^l+\phi_C^l-\phi_B^l-\phi_B'^l) , \\ &\tau=\phi_A^\nu-\phi_B'^\nu-(\phi_A^l-\phi_B'^l) . \end{aligned} \tag{3}$$

For the quark sector, the Stech model is defined by the following hypothesis:

$$\begin{aligned} &M_u=M_u^\dagger=M_u^T , \\ &M_d=M_d^\dagger=aM_u+A , \end{aligned} \tag{4}$$

where a is a constant, A is an antisymmetric matrix, and $M_{u(d)}$ is the up- (down-) quark mass matrix. Taking this directly over to the lepton sector we have instead

$$M_\nu=M_\nu^\dagger=M_\nu^T , \tag{5a}$$

$$M_l=M_l^\dagger=aM_l+A . \tag{5b}$$

If M_ν and M_l are also of the Fritzsch form (1), Eq. (5a) implies

$$\phi_A^\nu=\phi_A'^\nu=\phi_B^\nu=\phi_B'^\nu=\phi_C^\nu=0 . \tag{6}$$

From Eq. (5b) we can see that the antisymmetric matrix A must be imaginary and also of the Fritzsch form, implying

$$M_l=aM_\nu+\begin{pmatrix} 0 & ia & 0 \\ -ia & 0 & ib \\ 0 & -ib & 0 \end{pmatrix} . \tag{7}$$

From this we find the relations

$$\begin{aligned} &\phi_A'^l=-\phi_A^l , \\ &\phi_B'^l=-\phi_B^l , \\ &\phi_C^l=0 . \end{aligned} \tag{8}$$

Examining the elements of M_l, we see the equalities

$$A_l e^{i\phi_A^l} = aA_\nu + ia \ ,$$
$$B_l e^{i\phi_B^l} = aB_\nu + ib \ , \qquad (9)$$

from which it immediately follows that

$$\phi_A^l = \arctan\left[\left(\frac{A_l}{aA_\nu}\right)^2 - 1\right]^{1/2} ,$$
$$\phi_B^l = \arctan\left[\left(\frac{B_l}{aB_\nu}\right)^2 - 1\right]^{1/2} . \qquad (10)$$

Taking the trace of Eq. (7) gives the additional relation

$$a = \frac{m_\tau - m_\mu + m_e}{m_{\nu_3} - m_{\nu_2} + m_{\nu_1}} \ . \qquad (11)$$

Since $A_{\nu(l)}$ and $B_{\nu(l)}$ are known in terms of lepton masses, the phases ϕ_A^l, ϕ_B^l are completely determined. σ and τ, the *CP* parameters in the weak-charged-current mixing matrix, then become

$$\sigma = -\phi_A^l \ ,$$
$$\tau = \sigma - \phi_B^l \ . \qquad (12)$$

Hence, all the parameters in the KM matrix U are determined by the lepton masses.

Lubimov *et al.*[11] have examined the end-point spectrum of tritium β decay and report a finite result for the electron neutrino mass of 33 ± 1.1 eV. Combining these results with the improved upper mass bounds for neutrinos from π and τ decays,[12] yields the following constraints: $M_{\nu_1} = 33 \pm 1.1$ eV, 33 eV $< M_{\nu_2} <$ 250 keV, $M_{\nu_2} < M_{\nu_3} <$ 70 MeV. As in our previous work, the lower mass limits of ν_2 and ν_3 result from the assumption that massive neutrinos obey the usual hierarchal structure. Exploring the above mass region, and implementing the Fritzsch and Stech hypotheses, we have calculated the probability of neutrino oscillations. The standard result[13] for the probability of $\nu_\alpha \rightarrow \nu_\beta$ for large

$$\Delta M^2_{\alpha\beta} \equiv M_{\nu_\alpha}{}^2 - M_{\nu_\beta}{}^2 \ (\Delta M^2_{\alpha\beta} \gtrsim 10 \text{ eV}^2)$$

is

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{i=1}^{3} |U_{\alpha i} U_{\beta i}|^2 \ . \qquad (13)$$

A comparison of our calculations with the data[14] results in a very small allowed region in the M_{ν_2}-M_{ν_3} plane. This region is the doubly cross-hatched area shown in Fig. 1. The oscillation probabilities are the same as those found by us earlier.[1] As in our previous work[1] we ask how an improvement in the present data will modify our results. To answer this question we considered a twofold improvement in the present limits on neutrino oscillations, the most sensitive process being $\nu_\mu \leftrightarrow \nu_e$ oscillations. This shrinks the allowed region down to the small solid area shown in Fig. 1 and leads to an almost unique determination of the masses of ν_2 and ν_3. Sample values of $|U_{ij}|^2$ are given in Table I for the representative values of $M_{\nu_2} = 7$ keV, $M_{\nu_3} = 120$ keV, $\sigma = 14°$, and $\tau = 21°$. These mixing angles are clearly too small for observation of these neutrinos in π- and β-decay experiments.

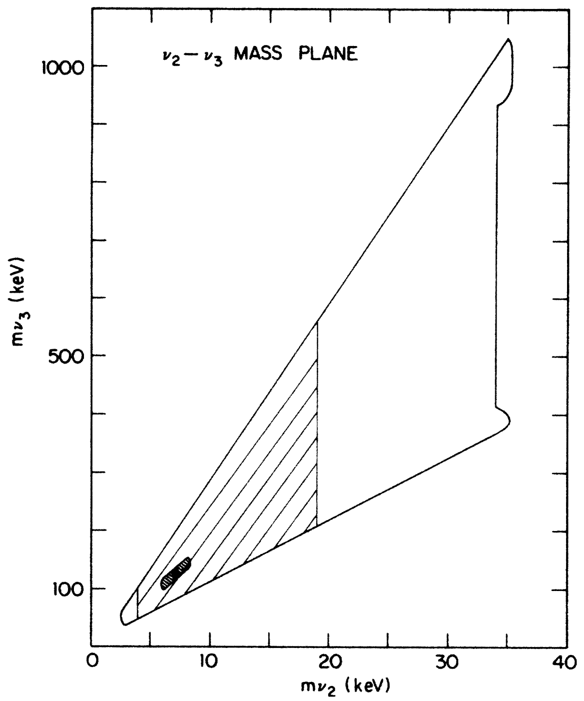

FIG. 1. A plot of the ν_2-ν_3 mass plane. The large unshaded area is the region where the Fritzsch model is consistent with present neutrino-oscillation experiments. The singly cross-hatched region corresponds to an improvement by a factor of 2 in $\nu_\mu \leftrightarrow \nu_e$ oscillation experimental limits. The doubly cross-hatched area is the region where the combined Fritzsch and Stech models are consistent with present oscillation data. The small solid region is where the Fritzsch and Stech models are combined with a twofold improvement in $\nu_\mu \leftrightarrow \nu_e$ oscillation experiments.

An interesting outcome of these calculations is that the neutral- and charged-lepton intergenerational mass ratios are almost equal:

$$\frac{M_{\nu_2}}{M_{\nu_1}} \simeq \frac{M_\mu}{M_e} \ , \quad \frac{M_{\nu_3}}{M_{\nu_2}} \simeq \frac{M_\tau}{M_\mu} \ . \qquad (14)$$

TABLE I. The calculated values of $|U_{ij}|^2$ for $M_{\nu_2} = 7$ keV, $M_{\nu_3} = 120$ keV, $\sigma = 14°$, $\tau = 21°$.

	Predicted value
$\|U_{11}\|^2$	9.99×10^{-1}
$\|U_{12}\|^2 = \|U_{21}\|^2$	2.92×10^{-4}
$\|U_{13}\|^2 = \|U_{31}\|^2$	5.70×10^{-6}
$\|U_{22}\|^2$	9.99×10^{-1}
$\|U_{23}\|^2 = \|U_{32}\|^2$	8.71×10^{-4}
$\|U_{33}\|^2$	9.99×10^{-1}

Equality yields $M_{\nu_2}=6.8$ keV and $M_{\nu_3}=115.2$ keV, which lie in the small solid area of Fig. 1. These relations should not be surprising, as they follow immediately from the small size of mixing angles, since in the limit that (14) becomes exact, all mixing angles will tend to zero.[15]

Very recently[16] a new tritium β-decay experiment has been performed at SIN which conflicts with Lubimov *et al.* They report a limit on the electron neutrino mass of $M_{\nu_e} < 9$ eV. How would our results change if these new results are verified? As an example, we have repeated our analysis assuming $M_{\nu_e}=5$ eV. We find that, as expected, the allowed masses for ν_2 and ν_3 are given quite accurately by Eq. (14) with $M_{\nu_1}=5$ eV. This is because all the values of the mixing angles remain small due to the bounds from neutrino-oscillation experiments. In fact, we find that for any value of $M_{\nu_e} \leq 30$ eV the allowed values for the masses of ν_2 and ν_3 are simply obtainable via Eq. (14).

In conclusion, we see that the combination of the Fritzsch and Stech hypotheses are remarkably restrictive in the leptonic sector.

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