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Does strange matter evaporate in the early Universe?

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Evaporation of nuggets of strange quark matter in the early Universe is investigated. A model of nugget evaporation is developed, and is shown to permit the survival of nuggets with initial baryon number $A \gtrsim 10^{46}$, when reabsorption of emitted hadrons is neglected. Reabsorption is shown to be potentially very important, making the survival of nuggets with a much lower initial baryon number possible. The evaporation rate of a nugget depends mainly on the effective binding energy of hadrons in a thin surface layer. This binding energy is shown to be several hundred MeV, determined by an equilibrium between emission of nucleons and kaons. A number of simplifying assumptions are explicitly discussed in order to stress the complicated physics involved. It is concluded that strange nuggets remain a possible candidate for the dark matter of the Universe.

I. INTRODUCTION

Quark matter consisting of roughly equal numbers of up, down, and strange quarks may have a lower energy per baryon than ordinary nuclei, and may thus be absolutely stable at zero temperature.[1,2] Witten[1] described a scenario for the production of this so-called strange matter in the early Universe, in connection with the QCD (quark-hadron) phase transition at a temperature T_c of order 100–200 MeV. It is not yet clear that strange matter nuggets with nuclear density are created,[3–5] but the possibility seems worth exploring.

If strange nuggets are created and survive they may serve as candidates for the dark matter of the Universe. Stable lumps can have sizes from a few fermis (baryon number $A \gtrsim 10^2$) up to ~ 10 km, the size of neutron stars ($A \lesssim 10^{57}$). If the lumps have a size exceeding ~ 1 cm, they would be unobservable on Earth.[6] Neutron absorption in nuggets will reduce the cosmological production of ^{4}He. This places a lower bound[7] on the acceptable size of nuggets at the time of nucleosynthesis (provided that nuggets and nucleons are homogeneously distributed) of order $r > 10^{-6}$ cmΩ_Q ($A > 10^{20}\Omega_Q{}^3$), where Ω_Q is the density of nuggets relative to the critical density of the Universe. This still leaves plenty of parameter space for cosmologically produced nuggets.

Alcock and Farhi[8] have recently suggested that evaporation of neutrons and protons at temperatures of order 10–50 MeV destroys nuggets with a baryon number less than $A \sim 10^{52-55}$. Since the mean baryon number within the particle horizon is $\sim 10^{52}(10\ \text{MeV}/T)^3$ this means that only lumps involving large perturbations in baryon number on the horizon scale should survive this evaporation. Such an evaporating scenario would still have interesting consequences in relation to big-bang nucleosynthesis and galaxy formation, but would not leave any strange nuggets.

The present paper analyzes some effects that may hinder rapid evaporation of cosmologically produced strange quark matter. *Nuggets with $A \gtrsim 10^{46}$ do not evaporate significantly.* This follows from detailed studies of how the chemical potentials of quarks in the hadron-emitting surface layer evolve. Not only neutrons and protons, but also Λ, Σ, Ξ, Ω, and in particular kaons are emitted, and the chemical potentials adjust in a way such that the effective binding energy of hadrons amounts to about 350 MeV, compared to 20 MeV for the scenario studied by Alcock and Farhi.[8]

Absorption of the cosmological background of neutrons and protons effectively stops the net evaporation of nuggets when the temperature drops below 15–20 MeV. *Reabsorption of emitted hadrons* will also contribute to the survival of nuggets. To calculate how, requires a detailed description of the transport of hadrons away from the nugget. Such a description is not attempted in this paper, but it is shown that time is insufficient to secure pressure equilibrium near the surface. As an extreme alternative we describe a scenario where nucleons are removed by diffusion. *In this case virtually all nuggets survive.*

An evaporating nugget is a very complex physical system, and we have had to make a number of simplifying assumptions. These are explicitly discussed in the text. A major reason for the complexity is, that the system in many respects deviates from a nugget in equilibrium. The surface layer is not in β equilibrium, and the speed of radius shrinkage is sufficiently fast to hinder the development of diffusion and convection in the way anticipated for a static system.

The rest of the paper is organized in the following way. In Sec. II we derive the emission rates of hadrons from quark nuggets, and calculate the rate of absorption of cosmological nucleons. Finally we discuss the importance of reabsorption of emitted hadrons. Section III describes a simple model of nugget evaporation and the physical processes that are important in the hadron-emitting surface layer. Section IV gives numerical results for nugget evaporation, and finally Sec. V contains a discussion of our results and compares them with other results from the literature.

II. ABSORPTION AND EMISSION OF HADRONS

At zero temperature strange matter is essentially a bag of degenerate up, down, and strange quarks, with a small number of electrons to ensure neutrality.[1,2] The system is kept together by a confining bag pressure; gravity plays a minor role except in very large, neutron-star-like systems. For strange matter to be stable at zero temperature, the energy per baryon must certainly be less than the mass of a nucleon, 938 MeV. With a baryon number A, a nugget consists of $3A$ quarks, and these have a number density n_0 of roughly 10^{39} cm^{-3}.

At finite temperatures thermally produced antiquarks and positrons will permeate a nugget. Because of the s-quark mass, very few $\bar{s}$ are produced at the temperatures of interest ($T < 100$ MeV).

Below T_c, the temperature of the QCD phase transition, quarks are confined; they can only escape the bag by joining and creating a hadron. Only baryons and mesons formed in a thin shell near the surface of a nugget have a chance of escaping. The strong interactions limit the mean free path of a hadron to approximately 1 F inside a nugget, so the actively emitting surface layer has a thickness of this order.

A. Emission rates

Alcock and Farhi[8] calculated the emission rate of neutrons dN_n/dt from considerations of equilibrium between a gas of nuggets and a gas of neutrons. They found the expression

$$\frac{dN_n}{dt}=\frac{2}{\pi}m_n T^2 e^{-I_n/T}R^2 . \tag{1}$$

Here, m_n denotes the mass of a neutron, R is the radius of a nugget, T is the temperature within a nugget, and I_n is the binding energy of a neutron, which they estimated to be 20 MeV.

By reducing the number of u and d quarks, emission of nucleons increases the chemical potential of s quarks, μ_s, relative to that of u and d quarks, μ_u and μ_d, in the surface region. This has several effects, which will be discussed in more detail later. First of all, it becomes harder to emit nucleons, since there are fewer u and d quarks available. In other words, the binding energy $I_n=m_n-\mu_u-2\mu_d$ increases. This is to some extent compensated by decay of s quarks, but as will be shown below, this process is not fast enough to fix μ_u and μ_d. Nor is quark diffusion or convection. The increase in μ_s makes it energetically favorable to emit strange baryons, Λ^0, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, and Ω^- in addition to n and p, thereby reducing μ_s. Even more important, due to the existence of thermal $\bar{u}$ and $\bar{d}$, also kaons ($\bar{K}^0$ and K^-) are emitted.

The rates for each of these emission processes are similar to Eq. (1) and may be derived from the following argument, which is slightly more intuitive than the derivation in Ref. 8. If the hadrons h of mass m_h, immediately outside the surface of a nugget are thermalized and nondegenerate, they have a number density

$$n_h=g_h\left[\frac{m_h T}{2\pi}\right]^{3/2} e^{(\mu_h-m_h)/T} , \tag{2}$$

where μ_h denotes the chemical potential, and g_h is the number of spin degrees of freedom. The thermal velocity perpendicular to the surface is $v_h=(T/2\pi m_h)^{1/2}$, and the rate of escape from the nugget is the flux of hadrons ($n_h v_h$) times the surface area of the nugget, $4\pi R^2$:

$$\frac{dN_h}{dt}=n_h v_h 4\pi R^2=\frac{g_h}{\pi}m_h T^2 e^{(\mu_h-m_h)/T}R^2 . \tag{3}$$

For neutrons this is the rate derived by Alcock and Farhi [Eq. (1)].

The chemical potential μ_h is given by the sum of the chemical potentials of the quarks producing the hadron (assuming hadron formation to be an equilibrium process): $\mu_h=\sum_i \mu_i$, e.g., $\mu_n=\mu_u+2\mu_d$. Table I lists the masses, compositions, and effective binding energies of the relevant hadrons. Figure 1 illustrates the relative importance of the different emission channels as a function of μ_u (for $\mu_u=\mu_d$; this assumption will be motivated later), using (3) on the form

$$\frac{dN_h}{dT}=-\frac{2}{\pi}\left[\frac{9}{4\pi n_0}\right]^{2/3} g_h m_h T^{-1}e^{(\mu_h-m_h)/T}A^{2/3}=-2.24\times10^{19}g_h\left[\frac{m_h}{m_n}\right][T\ (\text{MeV})]^{-1}e^{-I_h/T}A^{2/3}\ \text{MeV}^{-1} . \tag{4}$$

Equation (4) follows from Eq. (3) using $A=4\pi/3(n_0/3)R^3$, and the temperature-time relationship $t_{\text{sec}}\sim[T\ (\text{MeV})]^{-2}$, implying that $dt\sim-2T^{-3}\,dT$. Here and in what follows (unless stated otherwise) we assume the temperature within the nugget is equal to the temperature of the Universe. This is an overestimate of T, since evaporation of hadrons lowers the temperature, so we will overestimate the rate of emission.

We neglect Coulomb barriers. The errors in the net emission introduced by this assumption are of order a factor 2 (see Secs. III and IV).

B. Absorption rates

Quark nuggets will absorb nucleons at a rate that turns out to exceed the rate of emission for temperatures below 15–20 MeV or maybe even sooner. Two neutron and proton sources contribute to this. First, there is the fairly homogeneous background of nucleons created during the QCD phase transition (this is estimated to amount to somewhere between 1% and 20% of the total net baryon number[1]). And in addition to this, there are the nucleons emitted by other nuggets and, in particular, by the nugget

TABLE I. The quark composition, rest mass (m_h), binding energy (I_h), contribution to the emission rates in Eq. (4) [$g_h(m_h/m_n)$], effective number of u quarks (C_h), and baryon number (B_h) of the hadrons that can be emitted from strange nuggets.

Particle	Quarks	Mass (MeV)	I_h	$g_h\left[\frac{m_h}{m_n}\right]$	C_h	B_h
p	uud	938.3	$m_p-2\mu_u-\mu_d$	2.00	$\frac{3}{2}$	1
n	udd	939.6	$m_n-\mu_u-2\mu_d$	2.00	$\frac{3}{2}$	1
Λ^0	uds	1115.6	$m_\Lambda-\mu_u-\mu_d-\mu_s$	2.37	1	1
Σ^+	uus	1189.4	$m_{\Sigma^+}-2\mu_u-\mu_s$	2.53	1	1
Σ^0	uds	1192.5	$m_{\Sigma^0}-\mu_u-\mu_d-\mu_s$	2.54	1	1
Σ^-	dds	1197.3	$m_{\Sigma^-}-2\mu_d-\mu_s$	2.55	1	1
Ξ^0	uss	1314.9	$m_{\Xi^0}-\mu_u-2\mu_s$	2.80	$\frac{1}{2}$	1
Ξ^-	dss	1321.3	$m_{\Xi^-}-\mu_d-2\mu_s$	2.81	$\frac{1}{2}$	1
Ω^-	sss	1672.5	$m_\Omega-3\mu_s$	3.56	0	1
$\bar{K}^0$	$s\bar{d}$	497.7	$m_{\bar{K}^0}-\mu_s+\mu_d$	0.53	$-\frac{1}{2}$	0
K^-	$s\bar{u}$	493.7	$m_{K^-}-\mu_s+\mu_u$	0.53	$-\frac{1}{2}$	0

under study.

The absorption rate of nucleons per nugget is given as $\lambda_{Qn}=n_n v_n 4\pi R^2$. Here $4\pi R^2$ is the surface area of the nugget. For thermal nucleons the velocity perpendicular to the surface is $v_n=(T/2\pi m_n)^{1/2}$, and the local density of nucleons is $n_n=\rho_n/m_n=\Delta\Omega_n\rho_{\mathrm{crit},0}(T/T_0)^3/m_n$, where T_0 is the present photon temperature ($T_0\sim 3$ K), Ω_n is the contribution of nucleons to the Universal density in units of the critical density, and Δ is the relative local overdensity ($\Delta=1$ if all nucleons are homogeneously distributed). The present critical density is $\rho_{\mathrm{crit},0}=1.88\times 10^{-29}h^2$ g cm^{-3} with $100h$ km s^{-1} Mpc^{-1} being the present value of the Hubble parameter.

Collecting terms, the absorption rate of thermalized nucleons per nugget is calculated to be

$$\lambda_{Qn}(T)=3.06\times 10^7[T\ (\mathrm{MeV})]^{7/2}\Delta\Omega_n h^2 \times\left[\frac{3K}{T_0}\right]^3 A^{2/3}\ \mathrm{s}^{-1}\ . \tag{5}$$

Put in a form directly comparable to (4), this amounts to

$$\left.\frac{dN}{dT}\right|_{\mathrm{abs}}=-6.12\times 10^7[T\ (\mathrm{MeV})]^{1/2}\Delta\Omega_n h^2 \times\left[\frac{3K}{T_0}\right]^3 A^{2/3}\ \mathrm{MeV}^{-1}\ . \tag{6}$$

[Note the sign convention in Eqs. (4) and (6). dT is negative whereas dN is chosen to be positive.]

Figure 1 shows that a single emission channel dominates for most values of μ_u. If the binding energy for this channel is I, it follows that absorption of nucleons dominate over emission at temperature T if I exceeds the value $I_{\mathrm{eq}}=T\ln(7.3\times 10^{11}a^{-1}T^{-3/2})$, where $a=\Delta\Omega_n h^2(3K/T_0)^3(2/g_h)(m_n/m_h)$. Figure 2 shows $I_{\mathrm{eq}}(T)$ for different values of a. The effective binding energy for the relevant emission channels will typically exceed 350 MeV. Therefore the emission will be balanced by absorption of a homogeneous background of cosmological nucleons ($a\sim 1$) when the temperature drops below 15 MeV. If the local overdensity of nucleons surrounding a nugget gives $a\sim 100$, evaporation will stop at $T\sim 20$ MeV. And if $a\gtrsim 10^6$, essentially no evaporation is going to take place.

C. Estimating reabsorption

The importance of (re)absorption is thus parametrized by the value of a, which in turn depends on the local density of nucleons, n_n, around the nugget. It is hard to imagine nucleons anticorrelating with nuggets, so a lower bound on a is 0.1, corresponding to the contributions from the homogeneous cosmological background of nucleons. The actual value of n_n depends on the efficiency of transport of emitted hadrons from the nugget. Alcock and Farhi[8] calculated n_n from an assumption of pressure equilibrium, using the relation

$$n_n T_s+\frac{11\pi^2}{180}T_s{}^4=\frac{11\pi^2}{180}T_u{}^4\ . \tag{7}$$

Here T_s denotes the temperature near the surface, T_u is the temperature of the Universe far from the nugget, and the terms $(11\pi^2/180)T^4$ are the contributions to the pressure from electrons, positrons, and photons. The density of nucleons far from the nugget is here assumed to be zero. (The authors do not incorporate the cosmological background of nucleons in their calculations.) From Eq. (7) the local density of nucleons becomes

$$n_n=8\times 10^{31}\frac{T_u{}^4-T_s{}^4}{T_s}\ \mathrm{cm}^{-3} \tag{8}$$

(temperatures in MeV) a value that is zero when $T_u=T_s$, and quite small for "reasonable" temperatures. Thus reabsorption, assuming pressure equilibrium, has a minor effect on the net evaporation.

However, a simple estimate shows that Eq. (8) cannot be correct and that pressure equilibrium is therefore not obtained. If the net evaporation amounts to that given by Eq. (3), and the evaporated matter is removed uniformly from the surface with speed v, then the density near the surface will be of order

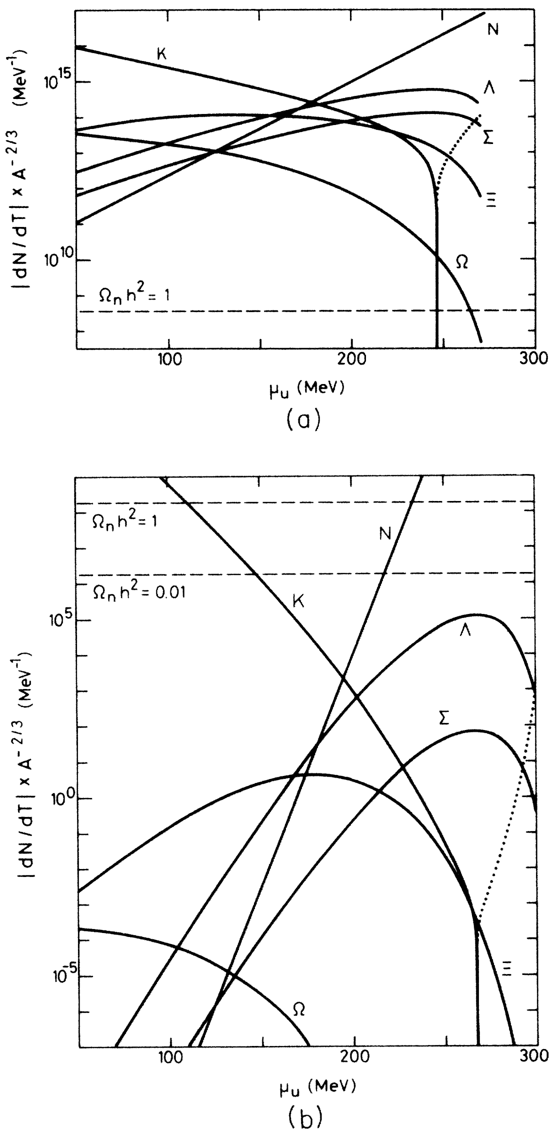

FIG. 1. (a) The evaporation rate of hadrons vs the chemical potential of the u quarks. The temperature T is 50 MeV, $\mu_d = \mu_u$, μ_s is found from the assumption of constant quark number density $n_0 = 7.6 \times 10^{38}$ cm^{-3}. The figure gives the normalized evaporation rates $|dN_h/dT| \times A^{-2/3}$ [dN_h/dT given by Eq. (4)] including the contribution from the corresponding antiparticles. The curves (solid lines) are labeled by the name of the hadron. Hadron parameters are given in Table I. The evaporation rate for kaons changes sign as indicated by the dotted line. For comparison the absorption rate for cosmological nucleons [Eq. (6)] is given for $\Delta\Omega_n h^2 = 1$ (dashed line). (b) As (a), but for a temperature T of 10 MeV. Note that the absorption rate of cosmological nucleons, given for two different values of $\Delta\Omega_n h^2$, dominates over particle emission for $\mu_u \sim 200$ MeV.

$$n_n \approx |dN_h/dt| / (4\pi R^2 v)$$

$$\approx 6 \times 10^{33} [T\ (\text{MeV})]^2 e^{-I/T} \left[\frac{c}{v} \right] \text{cm}^{-3} . \qquad (9)$$

If v equals the thermal velocity and $T_s = T_u \equiv T$, reabsorption with n_n given by (9), just balances emission (nuggets do not evaporate), whereas reabsorption, with n_n given by (8), would be zero. Even if $v = c$, the local density as given by (9) significantly exceeds the density calculated from (8).

A detailed calculation of reabsorption requires among other things an investigation of heating versus cooling, diffusion and convective motions, etc. We have not attempted to do this, but to illustrate, the potential importance of reabsorption, we have considered the extreme case where nucleons are removed by diffusion.

An emitted nucleon spends half the time as a neutron, and the other half as a proton. Proton diffusion is very slow due to the electric charge. Neutron diffusion is determined by electromagnetic scattering on electrons and photons. When the Universe reaches an age t a nucleon will at most have diffused a distance $L \sim (vt/n_\gamma \sigma)^{1/2}$, where $v = (8T_s/\pi m_n)^{1/2} c$ is the thermal speed of the nucleon at temperature T_s, $n_\gamma = 3 \times 10^{31}$ cm$^{-3} T_s^{\,3}$ is the density of photons, and $\sigma \sim 2$–30 mb is the relevant cross section.[3] With $t \sim T_u^{\,-2}$ we thus have

$$L \approx 230 \text{ cm} T_s^{\,-5/4} T_u^{\,-1} \sigma_{\text{mb}}^{\,-1/2} . \qquad (10)$$

The removal of nucleons thus takes place with a speed $v_{\text{diff}} = dL/dt$. If $T_s = T_u$ the diffusion speed is

$$v_{\text{diff}} \approx 260 \text{ cm s}^{-1} \sigma_{\text{mb}}^{\,-1/2} T^{-1/4} . \qquad (11)$$

This is many orders of magnitude smaller than the thermal speed. Thus according to Eq. (9) removal of nucleons is extremely inefficient, and nugget evaporation will proceed at a much slower rate than expected from Eq. (4).

Equation (11) underestimates v_{diff} due to the assumption that $T_s = T_u$. Furthermore the Hubble expansion of the nucleon gas was not incorporated. However the argument demonstrates that if the removal of nucleons is

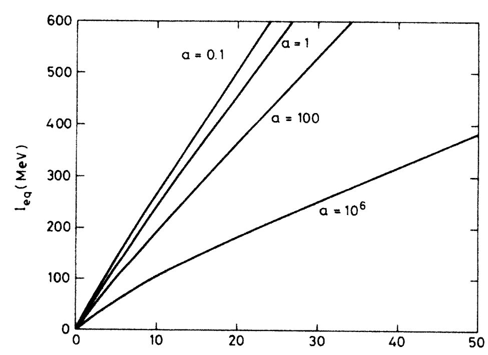

FIG. 2. The effective binding energy I_{eq} for which hadron emission is balanced by absorption of nucleons from the surroundings. The balance depends on the local density of nucleons, parametrized by the quantity $a = \Delta\Omega_n h^2 (3K/T_0)^3 (2/g_h)(m_n/m_h)$ (see text for further discussion).

governed by diffusion, reabsorption may well prevent evaporation for a large range of A.

We will now go on to show that nuggets with an interesting range of A values may survive, even when reabsorption is assumed to be negligible (corresponding to the pressure equilibrium assumption). In this case the A limits obtained in Ref. 8 are reduced by 6–9 orders of magnitude, bringing us into the regime of nuggets with a baryon number smaller than the mean baryon number within the horizon at the relevant temperatures. The physics of the evaporation process is quite different from what was assumed in Ref. 8, and therefore of interest in its own right.

III. A MODEL OF NUGGET EVAPORATION

In order to calculate the evaporation of strange nuggets, we have made a simple model, dividing a nugget into two regions: an inner part (with radius $R-D$, where R denotes the total radius of a nugget) in which u, d, and s quarks are homogeneously distributed, each with a number density $n_0/3$; and an outer part (of thickness D) in which the chemical potentials adjust according to the rates of emission, absorption, and quark conversion.

The central part is assumed to be inactive, in the sense that the chemical potentials are constant, and there is no exchange of particles with the surface layer. In the very early Universe ($T>m_s$) u, d, and s quarks existed in equal numbers, and its therefore reasonable to assume that nuggets were formed with $\mu_u \approx \mu_d \approx \mu_s$. This relation will not change significantly for the temperatures and times of interest here, and we will assume that $n_u = n_d = n_s = n_0/3$ throughout the central part with radius $R-D$.

The active layer is the outer shell which takes part in hadron exchange with the surroundings. When hadrons are emitted (absorbed) from a nugget, the radius shrinks (grows), keeping the pressure in the bag constant. For one flavor of massless quarks the constant pressure corresponds to a constant number density of quarks. This is not correct for a mixture of massless u and d quarks and massive s quarks, but for low s-quark masses ($m_s \sim 100$–150 MeV according to some recent estimates[9]) and the rather high values of μ_s ($\mu_s \sim 300$–350 MeV in the numerical calculations), the assumption of constant n_0 in the surface layer is a good approximation, and will be used throughout this paper. Thus $dA/dR = 4\pi(n_0/3)R^2$.

Neglecting Coulomb barriers (this can give errors of order a factor 2 in estimates of the net evaporation rate) u and d quarks are evaporated in equal numbers from a nugget, whereas the emission of s quarks differs significantly (cf. Table I). Since $\mu_u \approx \mu_d$ initially, this continues to be the case, whereas $\mu_s \neq \mu_u$ in the surface layer. We assume $\mu_u = \mu_d$ in the following.

S-quark conversion in the surface layer plays a role in reducing μ_s relative to μ_u and μ_d, but it is not sufficiently fast to keep $\mu_u \approx \mu_s$. The weak interactions of primary importance for s-quark conversion are $s+u \to u+d$, $s \to u+e^- + \bar{\nu}_e$, and $s+e^+ \to u+\bar{\nu}_e$.

Detailed calculations of the s-quark converting reactions are presented elsewhere,[10] taking account of Pauli-blocking and finite-temperature effects. Here it suffices to note that Γ (the net decay rate per s quark) for $m_s = 100$ MeV for the relevant chemical potentials drops from approximately 5×10^7 s^{-1} at $T=50$ MeV to 1×10^7 s^{-1} at $T=10$ MeV, whereas the values for $m_s=200$ MeV are less than a factor of 2 higher. This is 3–4 orders of magnitude *smaller* than the simple estimate[8] $\Gamma \simeq G_F{}^2 \mu_s{}^5 \sin^2\theta_C$, where G_F is the weak-interaction strength and θ_C the Cabibbo angle ($\sin^2\theta_C \approx 0.05$). The major contribution to Γ comes from $s+u \to u+d$.

If s-quark conversion leads to a surplus of either u or d quarks relative to the other flavor, reactions such as $d \to u+e^- + \bar{\nu}_e$ and $u+e^- \to d+\nu_e$ will quickly lead to $n_u = n_d$. These reactions are much faster than the corresponding reactions involving s quarks, since the rates do not contain the term $\sin^2\theta_C \approx 0.05$.

The thickness of the active surface layer D is at least 1 or 2 F, the distance which triquarks ("baryons") can move before they are disintegrated by strong interactions. Effectively this layer may be thicker, since diffusion and convection of quarks will tend to broaden the region that is influenced by the evaporation. However, as will be evident from the following, neither of these effects are important until $T \lesssim 15$ MeV.

A. Diffusion

The mean free path λ of a quark in degenerate quark matter is quite small at the temperature of interest here. At zero temperature, the mean free path is infinite due to Pauli-blocking, but at a finite temperature T the effective mean free path is $\lambda = [n_0\sigma_0(T/\mu)^2]^{-1}$, where $\mu \sim 300$ MeV is a characteristic Fermi energy, and $\sigma_0 \approx 13$ mb is the strong-interaction cross section.[11] Thus the mean free path is

$$\lambda = 7\times10^{-9}\ \text{cm} \left[\frac{\mu}{300\ \text{MeV}}\right]^2 \left[\frac{1\ \text{MeV}}{T}\right]^2 \left[\frac{10^{39}\ \text{cm}^{-3}}{n_0}\right] \times \left[\frac{13\ \text{mb}}{\sigma_0}\right]. \tag{12}$$

With speed αc ($\alpha=1$ for massless u and d quarks) the time between two scatterings is $\tau = \lambda(\alpha c)^{-1} \sim T^{-2}$. The age of the Universe at temperature T is $t_{\text{sec}} \approx [T\ (\text{MeV})]^{-2}$. This means that most of the time available for diffusion from temperature $T_{\max}$ to temperature T is at temperatures close to T. This leads to an estimated diffusion length $L = \lambda(t/\tau)^{1/2} \approx 15$ cm$[T\ (\text{MeV})]^{-2}$. Nuggets with *constant* $R>L$ ($A \gtrsim 3\times10^{42}[T\ (\text{MeV})]^{-6}$) would have "shell diffusion," whereas those with $R<L$ would be fully diffusive.

Our numerical calculations have shown that diffusion in general is a very slow process relative to the change of radius due to evaporation, when the temperature exceeds 15–20 MeV. This means that the actively emitting surface layer in this regime, where most of the total evaporation takes place, has a thickness $D_0 \approx 1$ F.

When $T \lesssim 20$ MeV a diffusion layer has time to build up since the diffusion speed, dL/dt, exceeds the speed of radius shrinkage, $-dR/dt$. We have in our models chosen the thickness of the active layer to be given by the expression

$$D(t+dt)=\max\left[D_0, D(t)+\left[\frac{dL}{dt}+\frac{dR}{dt}\right]dt\right] . \quad (13)$$

Different values were chosen for D_0 (1–10 F), with less than 5% changes in the resulting evaporation rates.

B. Convection

Building up a surplus of s quarks in a thin surface layer results in an increase in energy density of order 10% for a typical calculation. In a static system (e.g., a quark star) this would lead to convective instability, since a surface volume element would feel a net downward acceleration $\alpha \sim 0.1\, g$, where $g=GM/R^2=1.1\times 10^{-5}A^{1/3}$ cm s^{-2} is the gravitational acceleration on the surface of a nugget.

In a nonstatic system such as an evaporating nugget, convection may not have time to develop. Convection is likely to become important when $\alpha dt_D > |dR/dt|$, where αdt_D is the speed a convective element can reach in the time $dt_D = D/|dR/dt|$ it takes to evaporate the s-quark-enriched surface layer of thickness D (~ 1 F). The speed of nugget shrinkage, $|dR/dt|$, can be found from Eq. (4).

When nugget evaporation is governed by neutron and proton emission, it follows from Eq. (4) that convection can be important ($\alpha dt_D > |dR/dt|$) if

$$AD\ (\mathrm{F})^3 \gtrsim 5\times 10^{93}[T\ (\mathrm{MeV})]^{12}e^{-6I/T}\equiv K(I,T) , \quad (14)$$

where I is the nucleon binding energy in the surface layer.

Figure 3 shows $K_I(T)$ for different values of I. It is seen that for the nuggets of interest here $[AD\ (\mathrm{F})^3 \lesssim 10^{55}]$, convection is not important until the temperature has dropped to well below 20 MeV. For $I=20$ MeV as used by Alcock and Farhi, convection sets in at $T<1.5$ MeV. Thus convection cannot be the mechanism needed to supply u and d quarks to the surface in order to keep I as low as 20 MeV.

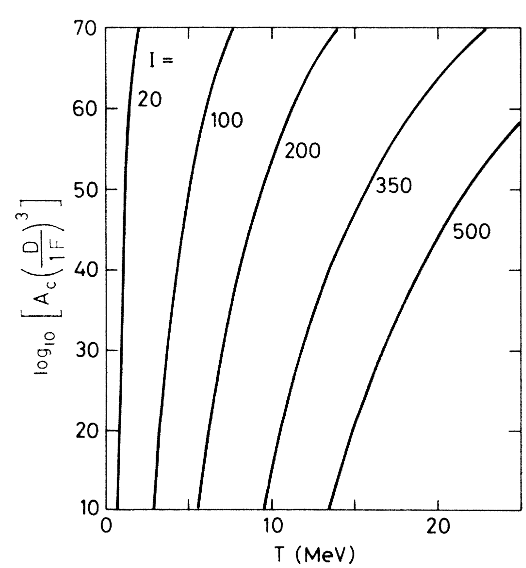

FIG. 3. The value of D (F)^3A for which convection sets in, as a function of temperature T and binding energy I in MeV of the dominating hadron emission channel [Eq. (14)].

As will be described in the following our model gives $I \sim 350$ MeV. In this case convection is unimportant for the nuggets of interest until T drops to 15–17 MeV.

Our model does not include convection. According to the discussion above this simplification is justified as long as T exceeds 15–17 MeV. When T drops to this value convective instability may set in, and also diffusion starts to widen the surface layer. At the same time, however, absorption will start dominating over emission, according to Fig. 2. The detailed balance between these different effects cannot be studied with our model. As discussed later, we expect that the net evaporation below $T=15$ MeV is negligible.

IV. NUMERICAL CALCULATIONS

A. Method

The total number of u and d quarks in a nugget, N_u and N_d, can be found from the expression

$$N_u=N_d=\frac{4\pi}{3}\frac{n_0}{3}(R-D)^3+n_u\frac{4\pi}{3}[R^3-(R-D)^3] , \quad D\le R , \quad (15)$$

where n_u is the number density of u quarks in the surface layer. (The chemical composition is assumed to be independent of position within the surface layer.) The change in N_u as a function of temperature can be found as dN_u/dT. Another expression for dN_u/dT is the net evaporation rate of u quarks, minus the rate of s-quark conversion, i.e.,

$$\frac{dN_u}{dT}=\left.\frac{dN_u}{dT}\right|_{\mathrm{evap}}-\Gamma n_s T^{-3}\frac{4\pi}{3}[R^3-(R-D)^3] . \quad (16)$$

In the second term a factor of 2 from the t-T relationship cancels a factor $\frac{1}{2}$ from the assumption that half of all converted s quarks eventually become u quarks. The term $dN_u/dT\,|_{\mathrm{evap}}$ is the net evaporation

$$\left.\frac{dN_u}{dT}\right|_{\mathrm{evap}}=\sum_h\left[\frac{dN_{\bar h}}{dT}-\frac{dN_h}{dT}\right]C_h+\frac{3}{2}\left.\frac{dN}{dT}\right|_{\mathrm{abs}} , \quad (17)$$

where $dN/dT\,|_{\mathrm{abs}}$ is the absorption of cosmological nucleons as given by Eq. (6) (in the mean each absorbed nucleon increases N_u by $\frac{3}{2}$). $dN_{\bar h}/dT$ is the emission of antihadrons, which can be calculated from Eq. (4) with μ_h replaced by $\mu_{\bar h}=-\mu_h$ (emission of an antihadron corresponds to absorption of one hadron, as far as the net change in N_u is concerned). The factor C_h is the number of u quarks lost from emission of one hadron, calculated as half the total number of u and d quarks (see Table I). The sum in Eq. (17) is over all hadrons in Table I.

Equating dN_u/dT from Eqs. (15) and (16) one gets the following relation (for $D=R$; for $D<R$ the relation is similar, but more cumbersome):

$$\frac{4\pi}{3}\frac{dn_u}{dT}R^3+4\pi n_u R^2\frac{dR}{dT}=\left.\frac{dN_u}{dT}\right|_{\rm evap}-\Gamma n_s T^{-3}\frac{4\pi}{3}R^3 . \tag{18}$$

Using $A=(4\pi/3)(n_0/3)R^3$ and $dA/dR=4\pi(n_0/3)R^2$ this corresponds to

$$\frac{3A}{n_0}\frac{dn_u}{dT}+\frac{3n_u}{n_0}\frac{dA}{dT}=\left.\frac{dN_u}{dT}\right|_{\rm evap}-3\Gamma\frac{n_s}{n_0}AT^{-3} . \tag{19}$$

The net evaporation rate of a nugget, dA/dT, is

$$\frac{dA}{dT}=\sum_h\left[\frac{dN_{\bar h}}{dT}-\frac{dN_h}{dT}\right]B_h+\left.\frac{dN}{dT}\right|_{\rm abs} , \tag{20}$$

with a notation as in Eq. (17). The constants B_h are the baryon numbers (Table I). Note that kaon emission contributes to dN_u/dT, but not to dA/dT.

At a given temperature, n_s and n_u are related to the chemical potentials μ_s and μ_u via integration of the Fermi distribution. Since we have assumed that $n_0=n_s+2n_u$ is constant, there are only two unknown quantities in Eq. (19), e.g., $A(T)$ and $n_u(T)$. Thus for given initial values at temperature $T_{\max}$, $A(T_{\max})$, and $n_u(T_{\max})$, Eqs. (19) and (20) can be integrated numerically.

B. Results

Figure 4 shows the results of some numerical calculations of strange nugget evaporation. Integration was started at $T_{\max}=50$ MeV for a number of different $A_{\max}=A(T_{\max})$, with absorption given by $\Delta\Omega_n h^2=1$, allowing for diffusion but not convection. s-quark decay rates are taken from Ref. 10.

Solid curves are results for $m_s=0$, $\alpha_c=0$ (no QCD corrections), $n_0=7.62\times10^{38}$ cm^{-3} [corresponding to a baryon number density of (125 MeV)^3]. In this case, nuggets with $A_{\max}\lesssim10^{45}$ evaporate completely, nuggets with $A_{\max}=10^{46}$ are reduced to a few percent of their initial size, whereas nuggets with $A_{\max}=10^{48}$ lose less than 40% of their initial baryon number.

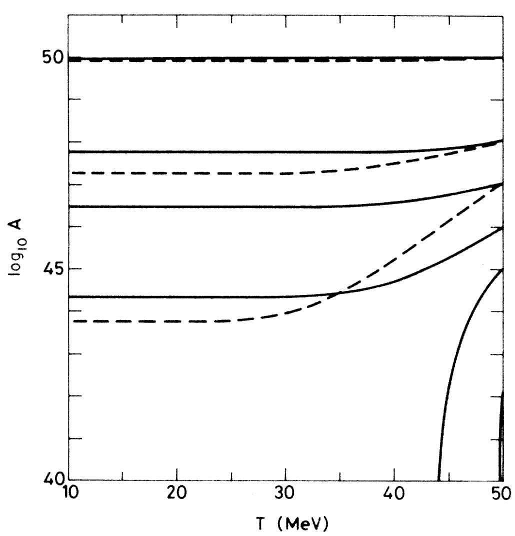

FIG. 4. The baryon number of strange nuggets as a function of the temperature of the Universe. The model for evaporation is described in Sec. IV. Nuggets of different sizes are followed from $T=50$ MeV to evaporation effectively stops at $T\approx20$ MeV. The solid lines are results for $\alpha_c=0$, dashed lines for $\alpha_c=0.1$, both using $m_s=0$ (see text for further details).

For a typical integration the surface layer quickly settles into a state with almost temperature independent μ_u and μ_s [the results are therefore not sensitive to the choice of $n_u(T_{\max})$]. For example in the case with $A_{\max}=10^{48}$ μ_u increases from 173 MeV at $T=50$ to 191 MeV at $T=20$ MeV, whereas μ_s increases from 313 to 344 MeV. That they both increase reflects finite-temperature effects.

At temperatures above 20 MeV, the chemical potentials in the surface layer are mainly governed by a competition between emission of neutrons, protons, and Λ, and emission of $\bar K^0$, K^-. Emission of n, p, and Λ reduce n_u, whereas $\bar K^0$, K^- emission, by removing thermal $\bar d$ and $\bar u$, effectively supplies d and u quarks to the nugget. The surface layer settles into a quasistationary composition where these two rates are equal. As can be seen from Fig. 1, this corresponds to μ_u slightly below 200 MeV, as confirmed by the numerical computations mentioned above. Thus the effective binding energy of a nucleon is $I_n=m_n-3\mu_u\gtrsim350$ MeV.

The speed of contraction of a nugget is so high as to prevent diffusion and convection from playing any role until T is somewhat below 20 MeV. Thus the effectively emitting surface layer is very thin, $D_0\sim1$ F [as mentioned previously, the results are insensitive to the choice of D_0 in Eq. (13)]. At $T\approx15-17$ MeV convection and diffusion has time to establish. At the same time, the rate of absorption of nucleons exceeds the emission rate, even for a homogeneous nucleon background ($\Delta=1$). If nucleons, as might be expected, are overabundant near the surface of the nugget ($\Delta>1$) (re)absorption will dominate at a higher temperature (see Fig. 2).

The reliability of our numerical model breaks down at this point. Absorption as well as diffusion and convection all tend to increase μ_u and μ_d relative to μ_s in the surface layer, leading to a decrease in the nucleon binding energy and hence an increase in the nucleon emission rate. However, the change in effective binding energy is not dramatic. Without convection, we find that the net evaporation below $T=20$ MeV is negligible in spite of diffusion, and convection is not likely to alter this significantly, since it can be seen from Fig. 3 that a lowering of I stops the convection at a given temperature.

A number of simplifying assumptions are inherent in the results described so far. These will be discussed in the following. Most of them are likely to have led to an overestimate of the net evaporation, so it is very likely that even nuggets with $A_{\max}$ below 10^{46} are able to survive the evaporation. However, the physics involved is extremely complicated, and the situation is far from settled.

T_{max} was taken to be 50 MeV in the calculations presented in Fig. 4. This value is rather arbitrary, though motivated by the estimates in Ref. 8 of when neutrino heating of the nuggets becomes efficient. Choosing a higher $T_{\max}$ increases the total emission. For $T_{\max}$ as high as 100 MeV, nuggets with $A_{\max}\lesssim10^{51}$ evaporate (whereas choosing T_{max} as low as 20 MeV would mean that almost all nuggets would survive).

$T_s = T_u$, that is, the temperature of the Universe and the temperature within the nugget were assumed to be equal. This is not correct, since the nugget is cooled by the emission of hadrons. Reheating must occur and takes place primarily by neutrino absorption. This balance of heating versus cooling was discussed by Alcock and Farhi.[8] Their results seem to imply that we have *overestimated* the value of A_{max} for surviving nuggets by 2–3 orders of magnitude by our choice of $T_s = T_u$. Thus $A_{max} \gtrsim 10^{43}$ may survive.

$m_s = 0$ was assumed in Fig. 4 resulting in an underestimate of μ_s for given n_s. Recent estimates[9] of m_s point to values around 100–150 MeV. Thus with $\mu_s \sim 300$–350 MeV our neglect of m_s is not very severe (at most a factor of 10 error in A). Our results are inadequate if $m_s > 200$ MeV.

$n_0 = constant$ was used to simplify calculations. The correct assumption would have been constant pressure, but as explained previously, these assumptions are equivalent when one quark flavor outnumbers the other, as is the case for s quarks in our numerical calculations. In most cases the pressure in the surface layer changes by only a few percent. The bag pressure corresponding to our choice of n_0 is approximately (140 MeV)^4. Our conclusions are not very sensitive to reasonable changes of n_0. A 30% change of n_0 leads to less than one order of magnitude change of A.

Coulomb barriers have been neglected. This is not always warranted (as shown in Ref. 7 it is exactly this Coulomb barrier that may influence nucleosynthesis), but the net evaporation rates are not likely to be changed by more than a factor of 2, since three of the important particles, n, Λ, and $\bar{K}^0$ are electrically neutral.

$\mu_u \equiv \mu_d$ is consistent with the neglect of Coulomb barriers, given that $\mu_u = \mu_d$ from the beginning. This assumption has been motivated previously.

QCD effects may be important for the conclusions. The solid curves in Fig. 4 do not include QCD corrections. To illustrate the potential importance of these corrections the dashed curves show results from similar calculations including first-order QCD effects at *zero* temperature, with $\alpha_c = 0.1$, where $\mu_i = (1 + 8\alpha_c / 3\pi)\pi^{2/3} n_i^{1/3}$ for massless quarks. The chemical potentials are increasing functions of α_c for fixed density. The resulting decrease in binding energy I_h explains the more efficient evaporation visible in Fig. 4. Increasing α_c from 0 to 0.1 in the model raises the value of A_{max} for nuggets that survive by an order of magnitude. A detailed study of QCD corrections including finite-temperature effects and quark masses has not been attempted. This is a major uncertainty in the present investigation.

Reabsorption of emitted hadrons (or their decay products) has not been considered in the numerical calculations. The calculations presented in Fig. 4 assumes instantaneous removal of emitted particles. This is certainly wrong, as discussed previously. Even the (inadequate) pressure equilibrium assumption used in Ref. 8 leads to some reabsorption, thus allowing smaller nuggets to survive. We have certainly overestimated the amount of evaporation by our neglect of reabsorption in the numerical model.

V. DISCUSSION

It should be obvious from this paper that a detailed treatment of strange nugget evaporation, given that nuggets were created in the first place, involves plenty of complicated physics. We have tried to model some of the relevant processes, making a number of simplifying assumptions. From this we can draw two main conclusions.

(1) Reabsorption of previously emitted hadrons may significantly reduce the net evaporation of strange nuggets, thereby enabling nuggets with low A to survive; (2) even when reabsorption is neglected, it seems possible for nuggets (much) smaller than the horizon scale, e.g., nuggets with $A_{max} \sim 10^{46}$, to survive until the present time.

Many assumptions are hidden behind these conclusions, but as discussed in the previous section, most of the assumptions seem to overestimate the evaporation. Thus we feel that it is fair to conclude, that strange quark nuggets *may* survive from the early Universe and could be a candidate for cold dark matter in the Universe today.

In many respects this paper complements the thorough investigations by Alcock and Farhi (Ref. 8). The most interesting new results in our study concern the importance of (re)absorption, and the estimate of the effective binding energy of hadrons, which turns out to be of order 350 MeV, due to the establishment of a quasistationary evaporation with equilibrium between kaon and nucleon emission.

Reabsorption of previously emitted hadrons may by far exceed the estimates obtained by Alcock and Farhi.[8] They assumed pressure equilibrium in the nuclear gas outside a nugget, but as seen from Eq. (9), there is not enough time available to remove the emitted nucleons. The local density near the surface and, as a result, the amount of reabsorption will be much larger than estimated using pressure equilibrium. We have not attempted to make a proper treatment of the complicated hadron transport, but if removal of hadrons is governed by diffusion, reabsorption may well prevent evaporation of nuggets for virtually all values of A_{max} (see Sec. II).

Absorption of the cosmological background of nucleons (not included in Alcock and Farhi's treatment) dominates over the emission when the temperature decreases to between 10 and 20 MeV. Thus even without local nucleon density enhancements, evaporation will stop at these temperatures.

The effective binding energy of a baryon in a nugget will significantly exceed the 20 MeV used as a standard value in Ref. 8, and even the upper limit of 100 MeV mentioned there. This is due to the relative slowness of diffusion, convection, and s-quark conversion in the very thin outer layer of the nugget taking part in hadron emission. n and p evaporation leads to an increase of the local s-quark density, whereas thermally produced $\bar{u}$ and $\bar{d}$ remove s quarks in the form of $\bar{K}^0$ and K^-. A quasistationary equilibrium is obtained, with $\mu_u \lesssim 200$ MeV, corresponding to a neutron binding energy $I_n \gtrsim 340$ MeV. The large μ_s allows emission of strange baryons, but our numerical models indicate that evaporation is sufficiently slowed down to allow nuggets with $A \gtrsim 10^{46}$ to survive.

Our numerical models involve a number of simplifying assumptions that reflect the complexity of the problem. For instance the chemical potential may be off by perhaps 10% as a result of inadequate treatment of pressure equilibrium and charge neutrality. Our results should therefore not be taken as canonical, but rather as an indication, that there *is* hope for the survival of strange nuggets, in contrast with the conclusion in Ref. 8.

In a recent paper, Schaeffer, Delbourgo-Salvador, and Audouze[12] have studied emission versus absorption of nucleons in order to calculate the influence of nuggets on big-bang nucleosynthesis. Their results differ markedly from those of this paper (and the calculations of Alcock and Farhi). The differences are a result of the authors' choice of emission rates. In analogy with α decay in nuclear physics, they choose an emission rate of the form $\lambda_{\rm em}=(\eta c/R)(1/A)e^{-I/T}$, where c is the speed of light and η is a constant of order 0.1. The term $\eta c/R$ represents the frequency of attempts for nucleons to go out of the nugget.

There are however major differences between the emission of α particles from a nucleus and emission of hadrons from a macroscopic nugget. "Quasibaryons" (three quarks moving along for a while) are constantly created throughout a nugget, but due to the strong interactions with surrounding quarks, only those "quasihadrons" that are made less than a mean free path (of order 1 F) from the surface have a chance of escaping.

The emission rate may be derived in analogy with α decay but η cannot be treated as a constant. Instead, it depends on A through an integral over the distribution of quasihadrons.

A detailed calculation of this sort seems rather difficult, but in a first approximation the term $\eta c/R$ should be replaced by $bR^2\lambda c/\lambda=bR^2c$, where b is an unknown constant, and λ the mean free path of a quasihadron, so that $R^2\lambda$ is the volume near the surface from which particles can be emitted. This changes the A dependence of the emission rates from $dN/dt\,|_{\rm emission}\propto A^{-1/3}$, as used in Ref. 12 to $dN/dt\,|_{\rm emission}\propto A^{2/3}$ as is also derived on a different basis in Ref. 8 and this paper.

Unfortunately, this also means that the predictions concerning nucleosynthesis in Ref. 12 are inadequate. Furthermore it seems fair to say that quantitative predictions of big-bang nucleosynthesis are severely complicated by the presence of quark nuggets. This is due to the inhomogeneous baryon distribution surrounding the nuggets. Even though the net emission is turned into net absorption when $T \lesssim 15$–20 MeV, as demonstrated in Sec. III, previously emitted hadrons will have an inhomogeneous distribution[3–5] when nucleosynthesis takes place at $T\sim 0.1$ MeV. Thus the standard big-bang nucleosynthesis scenario may be ruined (as shown in Refs. 3–5 this may be the case due to clouds of baryons resulting from the QCD phase transition, even if nuggets are never created).

Quark nuggets, if created after the QCD phase transition, *may* survive evaporation in the early Universe, but detailed calculations of the resulting distribution of these dark matter candidates and their influence on nucleosynthesis, are yet to be performed. In doing such calculations, and probably also in treatments of nugget formation, one should recognize the differences between strange matter in bulk and the active surface layer.

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