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Hydrodynamics and large transverse momenta in high-energy collisions

K. Wehrberger
Fachbereich Physik, Universität Marburg, Mainzer Gasse 33, D-3550 Marburg, Federal Republic of Germany

D. Syam and S. Raha
Theoretical Nuclear Physics Division, Saha Institute of Nuclear Physics, 92 Acharya Prafulla Chandra Road, Calcutta 700 009, India
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In high-energy multiproduction collisions, secondaries with transverse momentum (p_T) greater than a few times the pion mass have a p_T distribution which falls off like a power of p_T. This has been interpreted as being due to nonhydrodynamic processes, e.g., constituent interchange or hard scatterings of partons. We show that, on incorporation of transverse expansion with appropriate boundary conditions, a Landau-type hydrodynamic model leads to a falloff like p_T^{-5} for relatively large transverse momenta ($p_T \sim$ a few GeV), contrary to popular expectations.

The problem of multiparticle production in high-energy collisions (hadronic, leptonic, and nuclear) has been the focus of intense investigation for over three decades. Despite the attempts of numerous physicists using several different approaches, no clear-cut understanding of the phenomenon has yet emerged. But there exists a school of thought which holds that hydrodynamic models constitute an acceptable scenario for such processes. The present authors subscribe to this viewpoint. Indeed, in our opinion, one would be hard put to find an alternate scenario which explains as well the many facets of the multiproduction processes simultaneously. There are, of course, several unanswered questions in the hydrodynamic picture too. In this work, we address one such issue, namely, that of the secondary ejectiles with large transverse momenta (p_T) in the central rapidity region. [Thus, they may be safely assumed to come from the central "fireball" (at rest in the c.m. frame).] It is worth noting that the low-p_T region ($0 \lesssim p_T \lesssim 2$–$3\mu$, μ being the pion mass) shows a thermal spectrum and is explained by the thermal distribution at breakup in the hydrodynamic picture. The intermediate-to-high-p_T region (2–$3\mu \leq p_T \lesssim 5$–6 GeV) is an open problem in hydrodynamic models due to the lack of an accurate solution of the three-dimensional (3D) motion, but we expect hydrodynamics to cover this region too. As a matter of fact, this is precisely the contention of this article. The very-high-p_T region ($p_T \gtrsim 6$ GeV) is populated by the very rare hard collisions between partons with large momentum transfers, which can adequately be described by QCD (which enters the hydrodynamic picture only indirectly through the equation of state).

In hydrodynamic models,[1] evolution of the fireball is described by the set of equations $\partial_\mu T^{\mu\nu}(x)=0$, where $T^{\mu\nu}$ is the energy-momentum tensor of the fluid constituting the fireball. For an ideal fluid, $T^{\mu\nu}=(\epsilon+p)u^\mu u^\nu - pg^{\mu\nu}$, the symbols having their usual significance. One must supplement this set of equations with an equation of state for the fluid (e.g., $p=c_0{}^2\epsilon$, c_0 being the velocity of sound) to render the system deterministic. Because of the nonlinear nature of the resulting equations, it is notoriously difficult to solve them exactly. For the simplified case of one-dimensional (1D) motion, however, Landau could obtain an analytical solution which explains the longitudinal-rapidity-distribution data fairly well. Physically, this means that during the initial stage of the evolution, the motion is primarily 1D and even after 3D motion has set in, the longitudinal motion of the fluid is not *drastically* altered. But the Landau solution clearly cannot say anything about the transverse-momentum distribution of the secondaries, as that necessitates a thorough analysis of the full 3D hydrodynamics. On this score, the best one can hope for, as far as analytical studies go, are approximate solutions of the type obtained by Milekhin[2] and Yotsuyanagi.[3] Here we follow closely the treatment, as well as the notation of Yotsuyanagi[3] with the only difference being that his symbol η ($\equiv \ln T/T_0$, T being the instantaneous temperature at a point and T_0 the initial uniform temperature of the fireball) is replaced by ω.

We visualize the transverse evolution of the *central region* of the fireball (having longitudinal rapidity $\alpha \approx 0$ or equivalently $x \approx 0$) as follows. The matter starts flowing out transversely from the edge of the fireball as a simple wave, the front of which moves outward with the velocity of light. The rear end of the simple wave, on the other hand, propagates inward into the fireball with a velocity c_0 (the local speed of sound). At any instant of dimensionless proper time $\tau \equiv (t^2-x^2)^{1/2}/\Delta$ ($< a'/c_0$, $a' \equiv a/\Delta = \sqrt{s}/4m_p >> 1$, where a is the measure of the transverse size of the fireball and Δ the Lorentz-contracted longitudinal length[4]), $\rho=\rho_0 \equiv a' - c_0\tau$ denotes the surface between 1D and 3D motions, where ρ stands for the dimensionless transverse coordinate [$\equiv (y^2+z^2)^{1/2}/\Delta$]. (We are, here and in the following, making use of the cylindrical symmetry of the problem.) Identifying this surface correctly is of primary importance, as this implies the proper boundary conditions to be imposed on the 3D hydrodynamic equations. For $\tau \gtrsim a'/c_0$, there is no region of pure 1D motion left at a given x, since by then the rarefaction wave (the rear end of the simple wave) will have reached the symmetry axis $\rho=0$. Indeed, this, namely, in the identification of the 1D-3D interface, is essentially where we differ from Yotsuyanagi.[3] He considers a situation where the rarefaction wave originating from the surface reaches the symmetry axis and after "bouncing" from it, forms a region of nontrivial flow which then, because of the high pressure inside, expands outward, pushing the trivial simple wave region away. Confining his attention to the nontrivial region alone, he suggests that the 1D-3D interface emanates from the symmetry axis and is described by $\rho \approx c_0\tau$ ($\tau > a'/c_0$). But, from our above arguments, it should be obvious that there does not exist a region of pure 1D flow at this time and thus the manifestly 1D Landau solution, which serves to fix the essential boundary conditions for the full hydrodynamic equations governing the transverse flow, becomes *a priori* invalid. We therefore consider Yotsuyanagi's premises to be unjustified and replace them with our choice for the interface. As we shall see below, this materially affects the transverse-momentum distribution.[5]

As remarked earlier, the full set of 3D relativistic hydrodynamic equations is extremely complicated. The cylindrical symmetry of the problem is helpful in simplifying them to some extent, but it is still not enough. However, Yotsuyanagi showed[3] that in circumstances where the conditions

$$e^{-2\xi}\left|\frac{\partial\omega}{\partial\tau}-\frac{\partial\omega}{\partial\rho}\right| \sim e^{-2\xi}\left|\frac{\partial\xi}{\partial\tau}-\frac{\partial\xi}{\partial\rho}\right| \sim e^{-2\xi}\left|\frac{1}{\tau}-\frac{1}{\rho}\right| << 1 \quad (1)$$

(where ξ is the transverse rapidity) are satisfied, substantial simplification occurs, yielding the following set of tractable equations in place of the full hydrodynamic equations for the central region ($x \approx 0$) of the fireball:

$$\frac{\partial\omega}{\partial\tau}+\frac{\partial\omega}{\partial\rho}+\frac{c_0^2}{1-c_0^2}\left[\frac{1}{\tau}+\frac{1}{\rho}\right]\approx 0 \ ,$$
$$\frac{\partial\xi}{\partial\tau}+\frac{\partial\xi}{\partial\rho}-\frac{c_0^2}{1-c_0^2}\left[\frac{1}{\tau}+\frac{1}{\rho}\right]\approx 0 \ . \quad (2)$$

The appropriate boundary condition for these equations must be that the solution matches the Landau solution on the 1D-3D interface $\rho_0 \equiv a' - c_0\tau$. In terms of the variables ω and ξ, the Landau solution for $x \approx 0$ (or $\alpha \approx 0$) reads (for $\tau >> 1$) $\omega = \omega_1 \equiv -c_0^2 \ln\tau$ and $\xi = 0$ (see Ref. 3 for details). Thus solutions to (2) must satisfy $\xi = 0$ and $\omega = \omega_1$ for $\rho = \rho_0$.

Before proceeding to solve (2), let us first check if (1) is truly satisfied for our case. For $0 < \tau \lesssim a'/c_0$, the simple wave spans the region $a' + \tau \gtrsim \rho \gtrsim a' - c_0\tau$. The corresponding average values of τ and ρ are readily seen to be

$$\bar{\tau} = \frac{a'}{2c_0} \simeq 0.85a', \quad \text{for } c_0^2 = \tfrac{1}{3} \ ,$$

$$\bar{\rho} = \tfrac{2}{3}a'\left[\left[\frac{3}{2}+\frac{1}{2c_0}\right]-\frac{(1+1/2c_0)}{(3+1/c_0)}\right] \simeq 1.3a', \quad \text{for } c_0^2 = \tfrac{1}{3} \ .$$

In a recent work,[6] one of us (K.W.) and Weiner have shown that, for the hydrodynamic evolution, $c_0^2 \sim \frac{1}{3}$ is the appropriate *effective* value to use even in the presence of very strong interactions. Thus, from now on, we shall always set $c_0^2 = \frac{1}{3}$. In that case, we may write $|\bar{\tau} - \bar{\rho}| << a'$, $\bar{\tau} \sim \bar{\rho} \sim a'$; it is then straightforward to see that relation (1) is indeed satisfied for a large range of values of ξ. We are therefore no longer restricted to very large ξ, as was the case in Ref. 3. As a matter of fact, we face certain problems when ξ is too large. On the one hand, for high-p_T particles coming from the front of the simple wave ($\rho \approx a' + \tau$), we must have $\xi >> 1$ to satisfy (1) [see (3) below]. On the other hand, very large ξ might mean a temperature $T < \mu$ (the break-up temperature, in the spirit of the Landau model), implying that the particles are already "frozen out"; i.e., they are no longer contained in the hydrodynamic phase. One can, nevertheless, obtain high-p_T secondaries with ρ close to, but less than, $a' + \tau$. We shall discuss them shortly.

Given the boundary conditions, the solution to (2) is found to be

$$\omega = -\tfrac{1}{2}\ln\left[\frac{(1+1/\sqrt{3})\tau\rho}{a'-(\tau-\rho)/\sqrt{3}}\right]+\tfrac{1}{6}\ln\left[\frac{\tau-\rho+a'}{1+1/\sqrt{3}}\right] \ ,$$
$$\xi = \tfrac{1}{2}\ln\left[\frac{(1+1/\sqrt{3})^2\tau\rho}{[a'-(\tau-\rho)/\sqrt{3}](\tau-\rho+a')}\right] \ , \quad (3)$$

The reader may easily verify that solution (3) with the condition $|\tau-\rho| << a'$, $\tau \sim \rho \sim a'$ satisfies (1).

Relations (3) may be inverted to yield τ and ρ in terms of ω and ξ. For the benefit of the reader who might wish to check the later results, we write down the resulting expressions, which facilitate, for example, the derivation of (6):

$$\rho \approx \frac{a'-(1+1/\sqrt{3})e^{-3(\omega+\xi)}}{2}+\frac{\{[a'-(1+1/\sqrt{3})e^{-3(\omega+\xi)}]^2+4[a'e^{-(3\omega+\xi)}-(1/\sqrt{3})e^{-(6\omega+4\xi)}]\}^{1/2}}{2} \ ,$$
$$\tau \approx -\frac{a'-(1+1/\sqrt{3})e^{-3(\omega+\xi)}}{2}+\frac{\{[a'-(1+1/\sqrt{3})e^{-3(\omega+\xi)}]^2+4[a'e^{-(3\omega+\xi)}-(1/\sqrt{3})e^{-(6\omega+4\xi)}]\}^{1/2}}{2} \ . \quad (4)$$

At breakup, $\omega = \omega_c \equiv \ln\mu/T_0$. From phenomenological analysis[7] of e^+e^- annihilation data and pp data from CERN ISR, we can relate ω_c to the total available energy $\sqrt{s}$:

$$\frac{T_0}{\mu} \approx \left[\frac{\sqrt{s}}{m_p}\right]^{1/2} \ . \quad (5)$$

With $\omega = \omega_c$ in (4), we have, using $\tau \sim \rho \sim a'$,

$$\left[1+\frac{1}{\sqrt{3}}\right]e^{-3(\omega_c+\xi)} \sim a' = \frac{\sqrt{s}}{4m_p} \ .$$

Using (5) and the fact[3] that for $\xi > 1$, $e^{\xi} \sim 2p_T/\mu$, we have

$$\langle p_T\rangle \sim \mu\left[\frac{\sqrt{s}}{m_p}\right]^{1/6} \ , \quad (6)$$

which agrees with earlier estimates[2,8] for the growth of $\langle p_T\rangle$ with $\sqrt{s}$.

The dependence of $\langle p_T\rangle$ on $\sqrt{s}$ does not, however, say anything specific about the high-p_T particles coming from near the front end of the simple wave. We now investigate how close to the surface $\rho = a' + \tau$ we can approach without violating the condition $T \gtrsim \mu$. [Principally, the problem arises because one wants to describe the longitudinal expansion in terms of the Landau solution, which becomes applicable only after some time τ_L for which $(t^2 - x^2)^{1/2} \gtrsim (3-4)\Delta$. By then, a slice near the transverse edge has cooled down below μ.]

Let us write

$$\tau - \rho + a' = \delta a' \quad (\delta << 1) \ ,$$

and choose δ such that ω at this point (ω_δ) is equal to ω_c. Note that $\delta=0$ characterizes the front of the simple wave. For very small δ, we can write, from (3),

$$\omega_\delta \approx -\tfrac{1}{2}\ln\left[\frac{(\tau+a')\tau}{a'}\right]+\tfrac{1}{6}\ln\left[\frac{\sqrt{3}\delta a'}{\sqrt{3}+1}\right] .$$

For $\tau=\bar{\tau}$, this amounts to

$$\omega_\delta \approx -\tfrac{1}{2}\ln\left[\frac{\sqrt{3}}{2}\left(1+\frac{\sqrt{3}}{2}\right)a'\right]+\tfrac{1}{6}\ln\left[\frac{\sqrt{3}\delta a'}{\sqrt{3}+1}\right] . \quad (7)$$

In this calculation, one might (or even ought to) replace $\bar{\tau}$ by τ_L, the value of which is unfortunately unknown. However, $\tau_L<\bar{\tau}$ and hence using τ_L in place of $\bar{\tau}$ would, according to the formulas below, give an even larger $p_T|_\delta$. (Thus we have made here a conservative estimate of $p_T|_\delta$.)

Equating (7) to ω_c, we can solve for δ:

$$\delta \approx \frac{0.4m_p}{\sqrt{s}} . \quad (8)$$

It is hence clear that for ISR energies and beyond, δ is indeed $\ll 1$. For example, $\sqrt{s}=30$ GeV gives $\delta\approx 0.014$; $\sqrt{s}=270$ GeV leads to $\delta\approx 0.0015$.

Now we have near the front end of the simple wave

$$|\tau-\rho| \sim (1-\delta)a' ,$$

which by (4) gives

$$\left(1+\frac{1}{\sqrt{3}}\right)e^{-3(\omega_c+\xi)} \sim \delta a' = \frac{\delta\sqrt{s}}{4m_p} . \quad (9)$$

This yields, for $\xi \gg 1$ [which is required in order that (1) is satisfied for these elements],

$$p_T|_\delta \sim \delta^{-1/3}\mu\left(\frac{\sqrt{s}}{m_p}\right)^{1/6} . \quad (10)$$

The $p_T|_\delta$ given in (10) is a *measure* of the highest p_T for which a hydrodynamic description is expected to work. The value of p_T thus obtained, however, should be trusted only as an order-of-magnitude estimate and not as an absolute upper limit, because of the substitution $\tau=\bar{\tau}$, among other things. For $\sqrt{s}=30$ GeV, $p_T|_\delta \sim 1$ GeV; $\sqrt{s}=270$ GeV gives $p_T|_\delta \sim 3$ GeV.

We are now in a position to calculate the transverse-momentum distribution of the high-p_T ejectiles in the central region of longitudinal rapidity from solution (4). The essential ingredient here is the thermodynamic concept that the number of particles produced at breakup in an element of volume is proportional to the total entropy contained in that volume. Now the local rest frame of the hadronizing element can be related to the c.m. frame by two successive Lorentz boosts, one for the longitudinal motion and the other for the transverse motion. Transformation to the c.m. frame results in the following prescription[3] for the number of ejectiles:

$$dN \propto d\xi\, d\alpha\, \Delta^3\tau\rho\left(-\sinh\xi\frac{\partial\tau}{\partial\xi}+\cosh\xi\frac{\partial\rho}{\partial\xi}\right) .$$

The longitudinal and transverse rapidities may also be related[3] to the longitudinal and transverse momenta of the ejectiles by

$$d\alpha\, d\xi = \frac{dp_L\, dp_T}{E\sqrt{p_T^2+\mu^2}} \underset{p_T\gg\mu}{\sim} \frac{d^3p}{Ep_T^2} .$$

So, for $\xi \gg 1$ and $a' \gg (1+1/\sqrt{3})e^{-3(\omega_c+\xi)}$ [which follows trivially from (9)], we have

$$E\frac{d^3N}{d^3p} \sim \text{const}\times\left(\frac{\mu}{2p_T}\right)^2\left(\frac{\sqrt{s}}{4m_p}\right)^{-3}\left[a'\left(1+\frac{1}{\sqrt{3}}\right)e^{-(6\omega_c+3\xi)}-\frac{1}{\sqrt{3}}\left(1+\frac{1}{\sqrt{3}}\right)e^{-(9\omega_c+6\xi)}\right.$$
$$\left.+\frac{a'^{3/2}}{2}\left(\frac{(1+1/\sqrt{3})e^{-(6\omega_c+5\xi)}-\frac{2}{3}e^{-(6\omega_c+3\xi)}}{e^{-[(3\omega_c+\xi)/2]}}\right)-\frac{a'^{1/2}}{2\sqrt{3}}\left(\frac{(1+1/\sqrt{3})e^{-(9\omega_c+8\xi)}-\frac{2}{3}e^{-(9\omega_c+6\xi)}}{e^{-[(3\omega_c+\xi)/2]}}\right)\right]$$
$$=\text{const}\times\left[C_1\left(\frac{\sqrt{s}}{4m_p}\right)\left(\frac{2p_T}{\mu}\right)^{-5}-C_2\left(\frac{\sqrt{s}}{4m_p}\right)^{3/2}\left(\frac{2p_T}{\mu}\right)^{-8}+C_3\left(\frac{\sqrt{s}}{4m_p}\right)^{3/4}\left(\frac{2p_T}{\mu}\right)^{-13/2}\right.$$
$$\left.-C_4\left(\frac{\sqrt{s}}{4m_p}\right)^{3/4}\left(\frac{2p_T}{\mu}\right)^{-9/2}-C_5\left(\frac{\sqrt{s}}{4m_p}\right)^{5/4}\left(\frac{2p_T}{\mu}\right)^{-19/2}+C_6\left(\frac{\sqrt{s}}{4m_p}\right)^{5/4}\left(\frac{2p_T}{\mu}\right)^{-15/2}\right] , \quad (11)$$

where $C_1\sim 100$, $C_2\sim 450$, $C_3\sim 20$, $C_4\sim 10$, $C_5\sim 80$, and $C_6\sim 35$.

For this result to be physically meaningful, one must require that the differential cross section is positive for any given value of $p_T \lesssim p_T|_\delta$. It can be easily checked that the appropriate lower cutoff on p_T has a weak dependence on $\sqrt{s}$. For example, if $\sqrt{s}=30$ GeV, p_T must be bigger than ~ 150 MeV for this formula to apply; $\sqrt{s}=270$ GeV requires $p_T \gtrsim 250$ MeV. Of course, the requirement $\xi \gg 1$ already implies $p_T \gg \mu$.

Thus we see that for all values of $\sqrt{s}$ of interest in high-energy processes, the secondary particles with transverse momenta in the range $(2\text{–}3)\mu \leq p_T \lesssim 5$ GeV show a rather involved dependence on p_T. For large p_T [up to which a hydrodynamic scenario is expected to work; see discussion following relation (9)], the leading behavior is like p_T^{-5}. For lower values of p_T, the falloff is obviously sharper. In contrast with this, Yotsuyanagi[3] obtained a dependence like p_T^{-8} for all $p_T \gg \mu$. This result, obtained after suppression of one term in the final expression by hand, was mainly due to his choice of the boundary conditions which, we believe, are inappropriate. One should, however, mention here that some recent experiments[9] at CERN SPS apparently show p_T^{-8} behavior up to very large p_T, about 10 GeV. On the other hand, perturbative QCD predicts a dependence like p_T^{-4} for asymptotic values of p_T. According to some estimates,[10] the asymptotic regime starts already at $p_T \gtrsim 6$ GeV. This tallies nicely with our finding that for $p_T \sim 5$ GeV, the leading behavior of the cross section is like p_T^{-5}.

The main objective of this paper is to point out that the observation of a slower falloff at large p_T cannot by itself be interpreted to rule out hydrodynamics as a viable model for high-p_T production, as has been prematurely suggested by some authors. Instead, we suggest that one should look more deeply into this problem and explore the suggestions like leakage,[11] bag pressure,[12] etc., which may modify the transverse-momentum distribution of high-p_T secondaries, within a hydrodynamical framework.

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[1]For a review, see E. I. Diabog *et al.*, Fortschr. Phys. **27**, 313 (1979).

[2]G. Milekhin, Zh. Eksp. Teor. Fiz. **35**, 1185 (1959) [Sov. Phys. JETP **8**, 829 (1959)].

[3]I. Yotsuyanagi, Prog. Theor. Phys. **55**, 539 (1976).

[4]In what follows, $\sqrt{s}$ stands for the available hadronization energy in the collision process. Similarly, m_p, usually identified with the proton mass, is a generic symbol here, standing for a phenomenological Lorentz-contraction parameter. For hadron-hadron (and perhaps also nucleus-nucleus) collisions, the available energy is the total energy less that carried by the leading clusters.

[5]Hama and Pottag have already performed some numerical calculations with these altered boundary conditions. See Y. Hama and F. W. Pottag, University of Sao Paulo Reports No. IF USP/P-479, 1984 and No. IF USP/P-481, 1984 (unpublished).

[6]K. Wehrberger and R. M. Weiner, Phys. Rev. D **31**, 222 (1985).

[7]For e^+e^- annihilation, H. Banerjee *et al.*, Z. Phys. C **14**, 111 (1982). For *pp* collision, see S. Chadha *et al.*, Phys. Rev. D **10**, 2817 (1974).

[8]M. Chaichian *et al.*, Phys. Lett. **50B**, 362 (1974).

[9]UA1 Collaboration, G. Arnison *et al.*, paper presented at 21st International Conference on High Energy Physics, Paris, 1982 (unpublished); see also D. Sivers *et al.*, Phys. Rep. **23C**, 1 (1976).

[10]M. Faessler, Report No. CERN-EP/84-165, 1984 (unpublished).

[11]E. Friedländer and R. Weiner, Phys. Rev. Lett. **43**, 15 (1979); E. V. Shuryak, Phys. Rep. **61**, 71 (1980).

[12]E. V. Shuryak and O. V. Zhirov, Phys. Lett. **89B**, 253 (1980); N. K. Glendenning and T. Matsui, *ibid.* **141B**, 419 (1984).