# The $R^2$ cosmology: Inflation without a phase transition

Milan B. Mijić
*Theoretical Physics, California Institute of Technology, Pasadena, California, 91125*
*and Theoretical Physics Laboratory, BKI, Belgrade, Yugoslavia*

Michael S. Morris
*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125*

Wai-Mo Suen
*Department of Physics, University of Florida, Gainesville, Florida 32611*
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A pure gravity inflationary model for the Universe is examined which is based on adding an $\epsilon R^2$ term to the usual gravitational Lagrangian. The classical evolution is worked out, including eventual particle production and the subsequent join to radiation-dominated Friedmann behavior. We show that this model gives significant inflation essentially independent of initial conditions. The model has only one free parameter which is bounded from above by observational constraints on scalar and tensorial perturbations and from below by both the need for standard baryogenesis and the need for galaxy formation. This requires $10^{11} < \epsilon^{-1/2} < 10^{13}$ GeV.

## I. INTRODUCTION

The inflationary universe model,[1,2] in which the Universe has undergone a long period of exponential expansion, has successfully explained many problems in the standard Friedmann cosmology. A particularly attractive feature is that the model provides a mechanism to generate the small-scale density fluctuations in the Universe which are needed to seed galaxy formation.[3,4] They are the zero-point fluctuations of the quantum fields which get pushed into the classical regime by the large expansion.

In the standard picture of inflation this exponential expansion of the Universe is driven by the false vacuum energy density of a Higgs field which acts like an effective cosmological constant in the Einstein equations. Many different underlying particle physics theories have been proposed. The most popular of these are the Coleman-Weinberg model,[5] Witten's model with a logarithmic potential,[6] and the $N=1$ supergravity version of Nanopoulos *et al.* and Linde.[7]

These proposals, though, are not without their problems. First, one has to typically introduce a scalar "inflaton" field which is postulated especially for this purpose. This makes the whole scenario less plausible in that it is less natural. Second, to achieve a large enough inflation, suitable reheating after the inflation, and to make the material fluctuations small enough to be consistent with observation, relevant couplings or masses in the suggested models all have to be fine-tuned in one way or another. An even more serious problem has been pointed out by Mazenko, Unruh, and Wald.[8] A quantum field which is violently fluctuating in its high-temperature symmetric state may not settle into the false vacuum state as the Universe cools. This then may invalidate the whole picture of vacuum-energy-driven inflation. Although the problem might be circumvented again by fine-tuning the parameters involved,[9] it is reasonable to assert that the idea of inflation is very attractive while the "standard" models which generate the inflationary phase by a false vacuum energy density are less satisfying.

Is it possible to inflate the Universe by a different mechanism? Linde[10] has proposed in his chaotic inflation scenario that the inflation may be a direct result of large fluctuations of quantum fields in the very hot primordial universe. In the Planck regime, a scalar field $\phi$ will tend to be excited to large values so that its energy density inside some domain will be of order Planck. If $\phi$ has a very flat potential, i.e., a small "restoring force," it will remain roughly at the fluctuated value for a comparatively long time and hence drive an essentially exponential expansion. Linde has shown that in a $\lambda\phi^4$ theory there will be a classically tractable sufficient inflation when $\lambda < 10^{-2}$ (for more details see Linde[2]). However, two new questions immediately appear which a cosmology based on chaotic inflation must answer: what is the underlying particle model and what determines the initial fluctuations? Without these one has neither a complete nor a realistic model of chaotic inflation. This is one thread leading to the present work.

A second thread leads from the fact that within different frameworks one is repeatedly led to consider an action containing terms of quadratic or higher order in the curvature tensor. We will discuss this point more fully in Sec. VI. It is important to understand the implication of these higher derivative terms on the evolution of the early Universe. In this work we will restrict our attention to terms which are quadratic. They can be written as

$$\alpha R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}+\beta R_{\mu\nu}R^{\mu\nu}+\gamma R^2=\epsilon R^2+\zeta C_{\mu\nu\lambda\sigma}C^{\mu\nu\lambda\sigma}+\eta\chi_E$$

 

(where $\chi_E$ is the density of the Euler number for the manifold and $C$ is the Weyl tensor). When we consider a Robertson-Walker metric (homogeneous and isotropic universe)[11]

$$ds^2=-dt^2+a^2(t)\left[\frac{dr^2}{1-\kappa r^2}+r^2(d\theta^2+\sin^2\theta\, d\phi^2)\right] \quad (1.1)$$

(here $\kappa = +1, -1$, or 0—although, unless otherwise indicated, we will be studying the case $\kappa = 0$). This metric is conformally flat so that the $C^2$ term vanishes. The effective gravitational Lagrangian density yielding the evolution of the Universe is then given by

$$L=R+\epsilon R^2 . \quad (1.2)$$

The evolution equation for $R$ determined by (1.2) can be written as

$$\ddot{R}+3H\dot{R}+\frac{1}{6\epsilon}R=0 , \quad (1.3)$$

where the dot denotes a coordinate time derivative $(=d/dt)$ and $H$ is the Hubble parameter $(H=\dot{a}/a)$. Thus $R$ behaves like a damped harmonic oscillator with the restoring force given by $1/6\epsilon$. If $\epsilon$ is large, the potential is flat and $R$ takes up the role of the inflation-driving field. The aim of this paper is to study the cosmology based on this model. We show the range of initial data and the allowed value of $\epsilon$ so that inflation can be realized in this curvature-squared model in a manner consistent with observational constraints. We consider now the generic evolution of the Universe to be divided into four regimes. (i) There may be a quantum phase in which the Universe begins its Lorentzian life—as described in the wave-function picture[12]—with some expectation values for the initial conditions but continues with strong fluctuations for some time. The classical evolution only becomes meaningful after fluctuations around the average trajectory have become small. Whether this subsequent classical evolution is applicable to the Universe as a whole or just an homogeneous "bubble" part of it (as in Linde's chaotic inflation picture) we expect to be answered by a proper quantum treatment at very early times. (ii) At the start of the classical evolution there will quite generally be an inflationary phase of superluminal expansion in which the Hubble parameter decays linearly in time with small slope. (iii) When the Hubble parameter hits zero and bounces back the Universe goes into an oscillation phase in which it is reheated as material fields are excited by the oscillating geometry. (iv) There will be a final Friedmann phase in which our now matter-content-dominated model is joined to standard cosmology. We will exhibit and explain the inflationary solution, discuss reheating of the Friedmann universe, and the generation and evolution of scalar and tensor perturbations. These considerations all place constraints on the parameters of the model.

The effect of higher derivative terms on the evolution of the early universe has been studied by many authors. Zeldovich and Pitaevskii[13] have discussed the possibility of avoiding the initial singularity by including the higher-order term. Starobinsky[14] has shown that the quantum corrections for a conformally invariant free field will modify the Einstein equations with higher-order terms such that an unstable de Sitter solution will result. Whitt[15] points out that the evolution equation for an $R+\epsilon R^2$ Lagrangian admits primordial inflation. Hawking and Luttrell[12] have also shown that the wave function of the Universe for this Lagrangian is peaked about classical trajectories which exhibit an exponential expansion. In fact, the initial motivation for our work comes from the desire to understand and investigate in detail the inflationary phase displayed in the numerical solution of Hawking and Luttrell's wave function.

Parallel to conducting our discussion directly in the physical space-time we will make use of the fact that this theory can be rewritten as pure Einstein gravity plus matter in a conformal space-time. Whitt[15] has shown that by a transformation, $\tilde{g}_{\mu\nu} = (1+2\epsilon R)g_{\mu\nu}$, we can discuss the theory as Einstein gravity described by $\tilde{g}_{\mu\nu}$ plus a scalar field $R$ (which is the scalar curvature in the physical space), with minimal coupling to gravity by means of the equation

$$\tilde{R}_{\tilde{\mu}\tilde{\nu}}-\tfrac{1}{2}\tilde{g}_{\tilde{\mu}\tilde{\nu}}\tilde{R}=8\pi G\tilde{T}_{\tilde{\mu}\tilde{\nu}}(R) , \quad (1.4a)$$

where

$$\tilde{T}_{\tilde{\mu}\tilde{\nu}}=\frac{6\epsilon^2}{8\pi G(1+2\epsilon R)^2}\left[\partial_{\tilde{\mu}}R\,\partial_{\tilde{\nu}}R - \tilde{g}_{\tilde{\mu}\tilde{\nu}}\left[\frac{1}{2}\partial^{\tilde{\sigma}}R\,\partial_{\tilde{\sigma}}R+\frac{R^2}{12\epsilon}\right]\right] . \quad (1.4b)$$

Here, the scalar field $R$ can be given an action

$$S[R]=\int d^4x\; 6\epsilon^2\sqrt{-\tilde{g}}\,(1+2\epsilon R)^{-2}\left[\partial_{\tilde{\sigma}}R\,\partial^{\tilde{\sigma}}R+\frac{R^2}{6\epsilon}\right] . \quad (1.5)$$

In this conformal picture—as we are working with standard Einstein gravity — we already have some known tools which provide for us both insight and a good check on the less familiar behavior of the full fourth-order model. We will appreciate its full power in evaluating scalar and tensor perturbations.

In Sec. II we consider the classical evolution of a flat ($\kappa = 0$) Robertson-Walker universe under the influence of an $R^2$ term in the effective Lagrangian. In Sec. III we then treat in greater detail the exit from the inflationary phase, the reheating of the Universe, and the subsequent join to Friedmann behavior. Next, in Secs. IV and V, we estimate the generation of gravitational wave and scalar perturbations in the model. In Sec. VI we display some present constraints on and possible origins for $\epsilon$. Finally, conclusions are presented in Sec. VII.

Throughout this work we use units in which $\hbar = c = k_B = 1$. We measure all quantities in Planck units so that the gravitational constant $G$ is equal to $1 l_{\rm Pl}{}^2$ (where $l_{\rm Pl}$ denotes the Planck length).

## II. CLASSICAL EVOLUTION

We begin discussion of the Universe and its evolution at the time when it emerges from the Planck era. The

Universe would then be filled with relativistic particles of violently fluctuating energy density and its space-time geometry, too, would be violently fluctuating. However, a region not too big compared to the Planck size could be approximately isotropic and homogeneous and could then be described by the Robertson-Walker metric (1.1). For simplicity we consider only the case $\kappa = 0$. We follow the evolution of this small region with the classical equations of motion derived from the Lagrangian density (1.2).

It is straightforward to write down the field equation for the effective gravitational Lagrangian density (1.2) with a cosmological constant term and matter field terms added:[12,15]

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}+2\epsilon[R(R_{\mu\nu}-\tfrac{1}{4}g_{\mu\nu}R)+R_{;\kappa\lambda}(g^{\kappa\lambda}g_{\mu\nu}-\delta^{\kappa}_{\mu}\delta^{\lambda}_{\nu})]=8\pi G T_{\mu\nu}\ . \quad (2.1)$$

For the most part in this paper we will set $\Lambda = 0$ (except briefly in Sec. VI) and we will always use a perfect cosmological fluid expression for $T_{\mu\nu}$:

$$T_{\mu\nu}=(\rho+p)u_\mu u_\nu+\rho g_{\mu\nu}\ , \quad (2.2)$$

where $p = \rho/3$ (a relativistic equation of state) and $\mathbf{u} = \partial/\partial t$ (comoving four-velocity). It is simple to verify that the left-hand side of (2.2) is divergence-free so that energy-momentum conservation is still given by

$$T^{\mu\nu}_{;\nu}=0\ , \quad (2.3a)$$

which implies

$$\rho \sim \frac{1}{a^4}\ , \quad (2.3b)$$

as in the standard Einstein cosmology.

There are only two nonvacuous field equations. The $t$-$t$ component of (2.1) can be written as

$$\dot{R}=\frac{1}{12}\frac{R^2}{H}-RH-\frac{H}{2\epsilon}+\frac{4\pi}{3}\frac{G}{\epsilon}\frac{\rho}{H}\ , \quad (2.4)$$

and the contraction of (2.1) gives

$$\ddot{R}+3H\dot{R}+\frac{1}{6\epsilon}R=0\ . \quad (2.5)$$

The relations of $R$ and $H$ to the scale factor $a(t)$ are given by

$$R=6\dot{H}+12H^2 \quad (2.6)$$

and

$$H=\dot{a}/a\ . \quad (2.7)$$

Equations (2.4)–(2.7) are then a complete set for describing the classical evolution of the Universe.

Next we notice that with $\rho$ given by (2.3b), Eq. (2.4) is the first integral of (2.5). Therefore the system we have left is equivalent to a third-order differential equation in the scale factor $a(t)$. We set the time coordinate origin so that our analysis begins at $t=0$, which is the time the classical evolution begins to make sense. A complete set of initial conditions for the system is then given by $\rho_i$, $a_i$, $H_i$, and $R_i$ (the subscript $i$ will be used to denote quantities at $t=0$). We first assume for simplicity the matter term on the right-hand side of Eq. (2.4) to be negligible (that is, $\rho_i \approx 0$)—we shall insert its contribution at a later point. Now the initial size $a_i$ of the small homogeneous domain does not enter the dynamical equations and it relates coordinate length to physically measured length at $t = 0$ [the equation for $a(t)$ is trivially integrated in terms of $H(t)$]. We will take $\epsilon$ to be a free parameter, since before appeal to a higher theory it can be regarded as a new fundamental constant subject to experimental verification. So, one way to phrase the question that this paper addresses is what are the allowed ranges of $\epsilon$ and the initial data, $H_i$ and $R_i$, so that the non-Einstein term will produce a sensible inflation, give sufficient expansion to solve the horizon and flatness problems, command an exit from the inflationary phase, yield a reheating temperature high enough to not thwart standard baryogenesis but low enough to avoid the grand-unified-theory (GUT) phase transition and its associated monopole problem, and finally deliver the correct material and gravitational perturbation spectrum and magnitude?

We study first the classical evolution by means of Eqs. (2.4)–(2.7). To ensure the classical validity of the evolution we will think of $H_i$ and $R_i$ to be both less than or on the order of the Planck scale. We may combine Eqs. (2.4) and (2.6) to derive a master equation for the classical evolution with zero matter content:

$$\ddot{H}-\frac{1}{2}\frac{1}{H}\dot{H}^2+3H\dot{H}+\frac{1}{12\epsilon}H=0\ . \quad (2.8)$$

The remaining dependence on the parameters $H_i$, $R_i$, and $\epsilon$ can then be discussed as follows.

(A) $\epsilon > 0$, $R_i > 0$, and $H_i > 0$. We will show that this is the only case that will be of interest so that we will consider it in detail.

(i) First, we look at the case where $R$ starts at roughly its maximum value, that is $\dot{R}(t=0) = 0$. Then Eq. (2.4) relates $R_i$ and $H_i$ by

$$R_i=6H_i^2\left[1+\left[1+\frac{1}{6\epsilon H_i^2}\right]^{1/2}\right]\ . \quad (2.9)$$

The typical behavior of $H(t)$ for this case is shown in Fig. 1. There is a long phase in which $H$ decreases linearly in time with a small slope. This slope may be estimated from Eq. (2.8). For $\epsilon \geq 1$ and $H \geq 1/6\sqrt{6\epsilon}$ we have

$$\dot{H}\approx-\frac{1}{36\epsilon}\ . \quad (2.10)$$

Hence the total expansion in the scale factor of the Universe after this linear near–de Sitter phase is given by

$$a(t_{H\approx 0})=a_i e^{18\epsilon H_i^2}\ . \quad (2.11)$$

Although this linearly decaying Hubble parameter means that the expansion is slower than that of an exact de Sitter phase, a comoving length will still get pushed outside of the horizon as $a(t)H(t)$ is increasing in time. This is sometimes called generalized inflation.[16] To obtain a cosmologically significant expansion—say a factor of $e^{75}$ (cf. Linde[2])—we see that we need only to have $\epsilon H_i^2 \geq 4.2$, a perfectly natural value in our picture. This explicitly is the sought-for inflation in the model. When

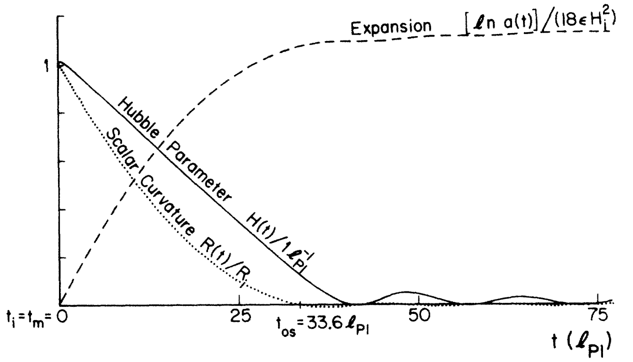


FIG. 1. A model cosmology for $\epsilon = 1 l_{Pl}^2$, $H_i = 1 l_{Pl}^{-1}$, and $\dot{R}_i = 0$ [corresponding to (i) of case (A) of the text so that $R_i \approx 12.5 l_{Pl}^{-2}$]—showing typical behavior of the Hubble parameter $[H(t)/1 l_{Pl}^{-1}]$, the normalized scalar curvature $[R(t)/R_i]$, and the inflation-normalized number of expansion $e$ foldings $[\ln a(t)/18\epsilon H_i^2]$. This plot has been generated from a numerical integration of the field equations (2.4)—(2.7) with zero initial matter content. The Hubble parameter displays a clean separation between the linear inflationary phase and the subsequent oscillation phase at $t_{os} = 36\epsilon H_m - [1/(2\omega)] \approx 33.6 l_{Pl}$ [cf. Eq. (2.23)]. The slight initial rise in $H(t)$ is real since at the start $\dot{H} = \frac{1}{6}(R - 12H^2) > 0$. For models with a much higher value of the parameter $\epsilon$ (we are observationally constrained to $\epsilon > 10^{11} l_{Pl}^2$) the linear phase is stretched out to a shallow slope and the subsequent oscillations are correspondingly reduced in both amplitude and frequency.

$H$ finally gets small, as shown in Fig. 1, it switches from the linearly decaying phase into a damped oscillation. This oscillation will be seen to reheat the Universe.

(ii) What if $R_i \gg 6H_i^2\{1+[1+(6\epsilon H_i^2)^{-1}]^{1/2}\}$? From Eqs. (2.4) and (2.6) it is clear that both $R$ and $H$ will increase rapidly:

$$R \approx \frac{1}{12}\frac{R_i^2}{H_i} t \tag{2.12a}$$

and

$$H \approx \frac{1}{6}\int R\, dt \approx \frac{1}{144}\frac{R_i^2}{H_i} t^2 . \tag{2.12b}$$

Therefore $12H^2$ will catch up with $R$ at $t_m$:

$$t_m \approx 5.2 \left[\frac{H_i}{R_i^2}\right]^{1/3} \tag{2.13}$$

and

$$H_m \equiv H(t_m) \approx 0.2 \left[\frac{R_i^2}{H_i}\right]^{1/3} . \tag{2.14}$$

Then by Eq. (2.6), $\dot{H}$ will change sign and then go into the linear decaying phase of the previous case (i). The total expansion accumulated during the initial rapidly rising period is negligible:

$$\int H\, dt \approx \tfrac{1}{400} .$$

We can thus perfectly well regard $H(t_m)$ and $R(t_m)$ as the initial values from which the linear phase begins.

(iii) If $R_i \ll 6H_i^2\{1+[1+(6\epsilon H_i^2)^{-1}]^{1/2}\}$, then

$$\dot{R} \approx -\frac{H}{2\epsilon} \tag{2.15a}$$

and

$$\dot{H} \approx -2H^2 . \tag{2.15b}$$

Both $H$ and $R$ will fall rapidly. For a typical value of $H$ there will not be sufficient inflation before it bounces at zero. The Universe will go into the oscillation phase without having been inflated.

(B) $H_i < 0$. From Eq. (2.8) we can see that as $H \to 0$, $\dot{H}$ must also go to zero so that $\dot{H}^2/H$ is finite. Therefore, $\ddot{H}$ is negative if $H$ approaches zero on the negative side. Thus when $H$ hits zero it will bounce back and remain negative (on the other hand, a positive $H$ will remain positive for the same reason). For the case $H_i < 0$ the domain in consideration will always be contracting until it collapses back to the Planck regime.

(C) $R_i < 0$, $H_i > 0$. From Eq. (2.6) $H$ will be decreasing rapidly as long as $R$ is negative. Since $H$ has to remain positive as argued in case (B), $R$ will have to cross zero and become positive. Again, typically the total expansion in the initial period will be negligible and we arrive back at case (A).

(D) $\epsilon < 0$. From Eq. (2.5) we see that when $\epsilon$ is negative we have an antirestoring force. Indeed, it is easy to see that when $H_i$ is positive, the solution will go into a linearly increasing form asymptotic to a slope

$$\dot{H} = -\frac{1}{36\epsilon} > 0 ,$$

which is physically unacceptable. When $H_i$ is negative $H(t)$ will be decreasing and will not be interesting as described under case (B).

We conclude that (i) $\epsilon$ has to be positive to give a finite period of inflation (note that tachyonic solutions would also exist if $\epsilon$ were negative[17]). (ii) To study the inflation we only have to study the case with positive $H_i$. The inflation occurs during a period when $H$ decreases linearly with a slope $-1/36\epsilon$. The total expansion factor in this phase is given by Eq. (2.11) [with $H_i$ replaced by $H_m$ in (ii) or (iii) of case (A)]. (iii) The linearly decaying $H(t)$ will bounce into an oscillation phase when it approaches zero. These descriptions of the evolution have been verified numerically.

Now we return to consider the contribution of the material term which we neglected in Eq. (2.4). By Eq. (2.3b) the energy density $\rho$ of the relativistic particles evolves inversely proportional to $a^4$. It is then clear that once the inflationary era begins $\rho$ will be quickly red-shifted away. Thus by Eq. (2.4) the effect of $\rho$ on the evolution is just to give $R$ an initial kick. That is, if $\rho_i$ is large while $H_i$ and $R_i$ are of order 1, then $R$ will quickly rise to

$$\left[\frac{16\pi}{\epsilon}\rho_i\right]^{1/2}$$

in a short time. The subsequent evolution is then given by

(ii) of case (A).

It is nice to see the inflationary solution also by considering the conformal picture. In the conformal picture the classical background consists of gravity described by a scale factor $\tilde{a}(\tilde{t})$ and a spatially homogeneous scalar field $R(\tilde{t})$. They evolve according to

$$\frac{d^2R}{d\tilde{t}^2}-\frac{2\epsilon}{1+2\epsilon R}\left[\frac{dR}{d\tilde{t}}\right]^2+3\tilde{H}\frac{dR}{d\tilde{t}}+\frac{R(\tilde{t})}{6\epsilon(1+2\epsilon R)}=0 \tag{2.16}$$

and

$$\tilde{H}^2=\frac{\epsilon^2}{(1+2\epsilon R)^2}\left[\left[\frac{dR}{d\tilde{t}}\right]^2+\frac{R^2}{6\epsilon}\right], \tag{2.17}$$

where $d\tilde{t}=(1+2\epsilon R)^{1/2}dt$. It is easy to see that there is a consistent solution for $\epsilon R \gg 1$:

$$\tilde{H}=\frac{1}{2\sqrt{6\epsilon}} \tag{2.18}$$

and

$$R(\tilde{t})=R_i-\frac{\tilde{t}}{3\epsilon\sqrt{6\epsilon}} . \tag{2.19}$$

Transforming back we find a linearly decreasing Hubble parameter as discussed above. The fact that in the conformal picture one has a solution as nice as the de Sitter solution makes the prospect for further analysis very promising.

From now on we consider only the case (A) above since the other cases either lead back to it or are uninteresting and we will refer to the inflated region as "the Universe." In the linear phase, we have, by comparing terms in Eq. (2.8),

$$\left|\frac{1}{2}\frac{1}{H}\dot{H}^2\right| \ll |3H\dot{H}| . \tag{2.20}$$

As $H$ decreases and becomes small the inequality sign will eventually flip and we will go over to the oscillatory phase. Equation (2.8) then becomes

$$\ddot{H}-\frac{1}{2}\frac{1}{H}\dot{H}^2+\frac{1}{12\epsilon}H=-3H\dot{H}\approx 0 . \tag{2.21}$$

If one neglects the $3H\dot{H}$ term in Eq. (2.21), the solution is given easily by

$$H(t)=\text{const} \times \cos^2\omega t , \tag{2.22}$$

where

$$\omega \equiv \frac{1}{\sqrt{24\epsilon}} .$$

To do better in approximation and in particular to obtain the damping for the amplitude we have to include the presently neglected term. We do this by substituting a form for $H(t)$ which is $H = f(t)\cos^2\omega t$ and then finding $f(t)$ in the approximation that the damping is slow $\dot{f}^2/f \approx 0$, $f\ddot{f} \approx 0$. The initial value of $f$ is determined by matching on to the linear phase—that is, requiring the two terms in (2.20) to be equal at $t = t_{os}$, the time the oscillation phase begins. When this has been accomplished we determine the following approximate analytic form for the whole classical evolution of the Universe in the absence of matter fields:

$$H(t)\approx\begin{cases} H_m-\frac{1}{36\epsilon}(t-t_m) , & t_m<t\le t_{os} , \\ \left[\frac{3}{\omega}+\frac{3}{4}(t-t_{os})+\frac{3}{8\omega}\sin 2\omega(t-t_{os})\right]^{-1}\cos^2\omega(t-t_{os}) , & t_{os}\le t , \end{cases} \tag{2.23}$$

where $\dot{R}(t_m) = 0$, $\omega \equiv (1/\sqrt{24\epsilon})$, and $t_{os} = 36\epsilon H_m + t_m - [1/(2\omega)] \approx 36\epsilon H_m$ . A simple approximate solution for $a(t)$ in the oscillatory phase can be obtained by integrating the $H$ averaged over a few cycles:

$$a(t)\approx\begin{cases} a_m e^{H_m(t-t_m)+(t_m/72\epsilon)(2t-t_m)-(t^2/72\epsilon)} , & t_m<t\le t_{os} \\ a_{os}\left[1+\frac{\omega(t-t_{os})}{4}\right]^{2/3} , & t_{os}\le t , \end{cases} \tag{2.24}$$

where $a_{os} \equiv a_m \exp(18\epsilon H_m{}^2-\frac{1}{12})$. In the oscillation phase $R$ is essentially $6\dot{H}$ [cf. Eq. (2.6)] so that we have

$$R(t)\approx\begin{cases} 6\left[2H_m{}^2-\frac{1}{36\epsilon}-\frac{H_m}{9\epsilon}(t-t_m)+\frac{2}{36\epsilon^2}(t-t_m)^2\right] , & t_m<t<t_{os} , \\ -6\left[\frac{3}{\omega}+\frac{3}{4}(t-t_{os})+\frac{3}{8\omega}\sin 2\omega(t-t_{os})\right]^{-1}\omega\sin 2\omega(t-t_{os}) , & t_{os}<t . \end{cases} \tag{2.25}$$

Notice that $a(t)$ and $H(t)$ are matched at $t = t_{os}$ whereas $R(t)$ is not—otherwise we would have had an exact solution. It is important that the oscillatory phase depends only on the parameter $\epsilon$ for size and shape—the oscillatory solution has no dependence on the initial conditions except in the time the phase begins (at $t_{os} \approx 36\epsilon H_m$). Equation (2.24) shows that the scale factor expands like a matter-dominated universe: $a(t) \propto t^{2/3}$—as in the post-inflationary phase of the Starobinsky model[14] where it is known as the "scalaron" phase.

## III. REHEATING OF THE UNIVERSE DURING THE INFLATION/FRIEDMANN INTERPHASE

These oscillations will excite the material fields and reheat the Universe. To estimate the reheating, we consider the simple case of a scalar field $\phi$ satisfying

$$g^{\mu\nu}\phi_{;\mu\nu}=0 \ . \tag{3.1}$$

The energy density of the scalar particles produced can be easily determined. Let

$$\phi=\int d^3k(\hat{a}_k u_k+\hat{a}^{\dagger}_k u_k^*) \tag{3.2a}$$

and

$$u_k(x,t)=\frac{1}{(2\pi)^{3/2}}\frac{1}{a}\chi_k(t)e^{ikx} \ , \tag{3.2b}$$

where $\hat{a}_k$ and $\hat{a}^{\dagger}_k$ are the usual annihilation and creation operators. In terms of the conformal time $\eta \equiv \int_0^t a^{-1}dt$, $\chi_k$ satisfies[18]

$$\frac{d^2\chi_k}{d\eta^2}+k^2\chi_k=V\chi_k \ , \tag{3.3a}$$

where

$$V\equiv\tfrac{1}{6}a^2R \ . \tag{3.3b}$$

As we shall see, the typical wave number $k$ which enters our calculation is much bigger than one whereas $V$ is of order one at early times ($\eta \sim 0$). Therefore, the wave is essentially living on a flat background at early times and the positive-frequency mode is then given by

$$\chi_k{}^{(i)}\approx\left[\frac{1}{\sqrt{2k}}e^{-ik\eta}\right] . \tag{3.4}$$

Now we follow Zeldovich and Starobinsky[19] and rewrite (3.3) as an integral equation:

$$\chi_k(\eta)=\chi_k{}^{(i)}+\frac{1}{k}\int_0^{\eta}V(\eta')\sin(k\eta-k\eta')\chi_k(\eta')d\eta' \ . \tag{3.5}$$

For a first-order iteration, we substitute $\chi_k{}^{(i)}$ in the integrand of (3.5) for $\chi_k(\eta')$. At asymptotically late times the Universe will be flat again and the positive frequency mode function is again given by (3.4). Hence the Bogoliubov coefficient describing the particle production is given by

$$\beta_{kk'}=\delta_{kk'}\frac{-i}{2k}\int_0^{\infty}V(\eta')e^{-2ik\eta'}d\eta' \tag{3.6}$$

and the coordinate energy density $\mathbf{p}\cdot(\partial/\partial\eta)$ (where $p \equiv$ momentum per unit comoving volume) is given by

$$p_\eta=\frac{\pi}{(2\pi)^3}\int_0^{\infty}d\eta\int_0^{\infty}d\eta' V(\eta)V(\eta')\int_0^{\infty}dk(ke^{2ik(\eta'-\eta)}) \ . \tag{3.7}$$

Note that prior to the inflation $V=\frac{1}{6}a^2R$ is many orders of magnitude less than its value during the oscillating phase. Also $V$ becomes small after the Universe goes into the radiation-dominated Friedmann phase [cf. Eq. (3.17) below]. Thus we can drop the surface terms in evaluating (3.7) and arrive at

$$p_\eta=\frac{1}{8}\frac{1}{(2\pi)^2}\int_0^{\infty}d\eta\frac{dV}{d\eta}\int_0^{\infty}d\eta'\frac{V(\eta')}{\eta'-\eta} \ . \tag{3.8}$$

We restrict attention to a case where $V(\eta)=F(\eta)\sin(k'\eta)$ and the amplitude $F(\eta)$ for the oscillation is only slowly varying in time, which is the case for our present model. Then with $k'\eta \gg 1$, Eq. (3.8) gives approximately the energy production rate

$$\frac{dp_\eta}{d\eta}\approx\frac{1}{32\pi}k'F^2(\eta)\cos^2k'\eta \tag{3.9}$$

$$\approx\frac{k'a^4}{1152\pi}\bar{R}^2 \ . \tag{3.10}$$

Here $\bar{R}$ denotes the scalar curvature (2.25) with a $\pi/2$ phase shift in the oscillating factor and the scale factor $a(t)$ is given by (2.24). The proper energy density,

$$\rho \equiv \frac{1}{a^3}\mathbf{p}\cdot(\partial/\partial t) \ ,$$

is determined by

$$\frac{d\rho}{dt}=-4\rho H+\frac{1}{a^5}\frac{dp_\eta}{d\eta}=-4\rho H+\frac{\omega\bar{R}^2}{1152\pi} \ , \tag{3.11}$$

where $\omega = k'/a = 1/\sqrt{24\epsilon}$ is the angular frequency of the oscillation in proper time and is given by (2.23).

When the final term in (3.11) vanishes at late times we have $d(\rho a^4)/dt = 0$ as radiation with an equation of state $p = \frac{1}{3}\rho$ should give. When the $\bar{R}^2$ term is nonzero the equation of state is modified. The pressure of the particles is determined by Eqs. (2.3a) and (3.11) to be

$$p=\frac{1}{3}\rho-\frac{\omega}{1152\pi}\frac{\bar{R}^2}{H} \ . \tag{3.12}$$

The complete field equations with the back reaction of the particle generation included can be estimated by putting this $p$ and $\rho$ [Eq. (3.11) and (3.12)] back into the field equations.[20] The $t$-$t$ part of Eq. (2.1) becomes

$$H^2+2\epsilon(HR-\tfrac{1}{12}R^2+RH^2)=\frac{8\pi}{3}G\frac{N}{a^4}\times\int_{t_{os}}^{t}\frac{\omega}{1152\pi}\bar{R}^2a^4dt = \frac{8\pi}{3}G\rho_{\text{matter}}(t) \tag{3.13}$$

and the trace of Eq. (2.1) gives

$$\ddot{R}+3H\dot{R}+\frac{1}{6\epsilon}R = \frac{4\pi GN}{\epsilon}\left[\frac{\omega\bar{R}^2}{1152\pi H}\right], \quad (3.14)$$

where we have inserted a factor $N$ which denotes the number of fields that can be excited by the cosmological oscillation (since massless conformal fields will not be excited this $N$ will be less than the total number of particles in the theory).

The right-hand side of Eq. (3.13), $8\pi GN\rho_{\rm matter}/3$, can be estimated using (2.24) and (2.25). Not too long after the Universe has come into the oscillation phase, say at $t-t_{\rm os} \sim 10/\omega \approx 10\sqrt{24\epsilon}$ we have $\rho \approx 6\times 10^{-7}N/\epsilon^2$, which corresponds to a reheating temperature of

$$T_r \approx 3\times 10^{-2}/\sqrt{\epsilon} = 4\times 10^{17}\ {\rm GeV}\left[\frac{\epsilon}{1 l_{\rm Pl}{}^2}\right]^{-1/2} . \quad (3.15)$$

If $\epsilon$ is not too much bigger than one this particle production time scale may be shorter than the thermalization of the particle content. Still, the reheating temperature, $T_r$, is a useful characterization of the reheating energy (we will, however, show that $\epsilon$ must be indeed large). If this temperature were higher than the GUT phase transition temperature, we would be left with the monopole problem. If this temperature were too cool then baryogenesis may no longer go through. We will return to this point shortly.

When $t-t_{\rm os} \gg 1/\omega$, the time dependence of $\rho_{\rm matter}$ is given by

$$\rho_{\rm matter}(t) \approx \frac{3}{5}\frac{32}{1152\pi}\frac{N\omega^3}{(t-t_{\rm os})} .$$

If we now neglect the back reaction, $H^2$ at late times is given by (2.23) to be

$$H^2 \sim \frac{4}{9}\frac{1}{(t-t_{\rm os})^2} . \quad (3.16)$$

Hence at $(t-t_{\rm os}) \approx 1200\ \epsilon^{3/2}/GN$ the term on the right-hand side of (3.11) will be comparable with $H^2$ and the matter produced will begin to have a significant dynamical effect on the evolution of the Universe. The solution of Eq. (3.13) gradually goes over to a radiation-dominated Friedmann expansion with

$$H \propto \frac{1}{2t}\ ,\ R=0\ ,\ a \propto t^{1/2},\ \text{and}\ \rho \propto 1/t^2. \quad (3.17)$$

However, the transition from the oscillation phase to the radiation-dominated phase will be slow even after $8\pi G\rho_{\rm matter}/3$ is comparable to $H^2$ as a numerical integration of Eq. (3.13) shows. We estimate the time it takes for the Friedmann phase to begin by taking roughly 10 times this value so that the time the Friedmann phase begins is given by $t_F \geq t_{\rm os}+12\ 000\ \epsilon^{3/2}/GN$. The energy density will then be

$$\rho(t_F) \leq 4\times 10^{-9}GN^2/\epsilon^3 \quad (3.18)$$

and the Friedmann universe thus begins with the temperature

$$T_F \leq 1\times 10^{17}\ {\rm GeV}\left[\frac{\epsilon}{1\ l_{\rm Pl}{}^2}\right]^{-3/4} N^{1/4} . \quad (3.19)$$

Notice that the ways $T_r$ and $T_F$ depend on $\epsilon$ are different. It is clear that any constraint on $T_F$ will not be significant. There are important constraints on $T_r$, however. It must be higher than $10^{10}$–$10^{12}$ GeV so that gauge and Higgs bosons can be created and baryogenesis can proceed in the usual way, but lower than any GUT phase transition temperature $\sim 10^{16}$ GeV so that the monopole problem can be avoided.[2] Equation (3.15) then requires $\epsilon$ to be in the range

$$10^3\ l_{\rm Pl}{}^2 < \epsilon < 10^{12}\text{–}10^{15}\ l_{\rm Pl}{}^2 . \quad (3.20)$$

These bounds will be tightened when we consider perturbations generated in the inflationary phase. We summarize the classical evolution of the Universe as follows.

(i) A homogeneous and isotropic region near the Planck time with a Hubble parameter $H_m$ will expand with a linearly decreasing $H$ for a total expansion factor $\sim \exp(18\epsilon H_m{}^2)$.

(ii) Particles will be created during the oscillation phase. The total expansion factor during this time will be

$$\sim \exp\left[\int_{t_{\rm os}}^{t_{\rm os}+12\ 000\epsilon^{3/2}/GN} H\ dt\right] \approx 70\left[\frac{\epsilon}{NG}\right]^{2/3} .$$

(iii) The Universe will then go over to a radiation-dominated Friedmann phase with the temperature $T_F$ given by Eq. (3.19). To red-shift this to the present value of 3 K we must have an expansion factor

$$\left[\frac{T_F}{3\ {\rm K}}\right] \approx \left[5\times 10^{29}\left[\frac{G}{\epsilon}\right]^{3/4} N^{1/4}\right] .$$

Therefore, the total expansion since the Planck era is obtained by multiplying the expansion factors under (i), (ii), and (iii) and it should be greater than the present horizon size, where $1/H_0 \sim 10^{60} l_{\rm Pl}$. This requires in terms of the expansion factor

$$e^{18\epsilon H_m{}^2} \geq 2\times 10^{29}\left[\frac{H_m}{1 l_{\rm Pl}{}^{-1}}\right] \quad (3.21)$$

(where we have dropped a very weak dependence on $\epsilon$, and have set $N \approx 100$ as a typical value—also with a weak dependence). The expansion factor is very sensitively depending on $\epsilon H_m{}^2$ so that unless the initial parameter, $H_m$, is fine-tuned the left-hand side of Eq. (3.21) is likely to be very much bigger than $10^{29}$. We thus expect to have much more inflation than is necessary.

## IV. GRAVITATIONAL WAVE GENERATION

It is crucial to study the generation of gravitational waves in the model since it is well known that inflation close to the Planck time tends to yield excessive gravitational wave generation.[16,21] In the transverse-traceless gauge, a gravitational wave can be expressed in terms of a scalar amplitude, $h$. For a wave with wave number $k$ the metric can be written as

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + he_{ij})dx^i dx^j , \quad (4.1)$$

where $i,j = 1,2,3$ and $e_{ij}$ is the polarization tensor satisfying both the transverse condition $e_{ij}k^j = 0$ and the traceless condition $e_i{}^i = 0$. The field equation (2.1) then reduces to

$$\ddot{h} + \left[ 3H + \frac{1}{6} \frac{\epsilon R^2}{(1+2\epsilon R)H} \right] \dot{h} - \frac{1}{a^2} \partial_i{}^2 h = 0 . \quad (4.2)$$

The second term in the large parentheses is due to the presence of the $\epsilon R^2$ term in the gravitational Lagrangian. Other than this term $h(t)$ satisfies the same equation as an ordinary scalar field in a Robertson-Walker background. Since Eq. (4.2) is second-order in the space-time derivatives, the quantization can proceed in the usual way. We construct an action $S$ from which (4.2) can be derived:

$$S = \int d^4x \sqrt{-g} L , \quad (4.3a)$$

where

$$L = (1+2\epsilon R) g^{\mu\nu} \partial_\mu h \partial_\nu h \quad (4.3b)$$

[here we use the background metric of Eq. (1.1) (with $\kappa = 0$) to compute the quantities $\sqrt{-g}$, $g_{\mu\nu}$, and $R$]. The quantization condition is then

$$\left[ h(t,x), \frac{\partial L}{\partial \dot{h}}(t,y) \right] = iG \frac{\delta^3(x-y)}{a^3} . \quad (4.4)$$

For $L$ given by Eq. (4.3b) we have

$$[h(t,x), \dot{h}(t,y)] = iG \frac{\delta^3(x-y)}{a^3(1+2\epsilon R)} \quad (4.5)$$

[note that the additional factor of $1/(1+2\epsilon R)$ in the normalization enters because of the $\epsilon R^2$ term]. It is straightforward to check that the evolution equation preserves this commutation relation.

If $h$ is composed of modes of more than one wave vector, it can be written as

$$h(t,x) = \int d^3k (\hat{a}_k h_k e^{ikx} + \hat{a}_k^\dagger h_k{}^* e^{-ikx}) , \quad (4.6)$$

with the creation and annihilation operators satisfying the usual relations

$$[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta^3(k-k') , \text{ etc.} \quad (4.7)$$

Then Eqs. (4.4) and (4.5) determine the normalization for (4.6):

$$h_k \dot{h}_k^* - h_k^* \dot{h}_k = \frac{iG}{(2\pi)^3 a^3 (1+2\epsilon R)} . \quad (4.8)$$

The evolution equation for $h_k$ is then

$$\ddot{h}_k + \left[ 3H + \frac{2\epsilon R^2}{H(1+2\epsilon R)} \right] \dot{h}_k + \frac{k^2}{a^2} h_k = 0. \quad (4.9)$$

Now we consider a wave with wavelength equal to or smaller than the present horizon size, $1/H_0$. If the expansion factor in the linear phase is much greater than the minimum requirement (3.21) [cf. also text following (3.21)] the wave number $k$ of these waves will be much greater than 1. On the other hand, the term inside the large parentheses in Eq. (4.9) is of order 1 as $t \to 0$ and is thus negligible compared to $k/a$. Once again we are considering a wave evolving on essentially a flat background. Thus, the initial mode function can be chosen as

$$h_k = h_k{}^{(i)} \exp \left[ -ik \int \frac{dt}{a} \right] \quad (4.10a)$$

and the normalization $h_k{}^{(i)}$ is determined by Eq. (4.5) to be

$$| h_k{}^{(i)} | = \frac{\sqrt{G}}{\sqrt{2k}\,(2\pi)^{3/2} a (1+2\epsilon R)^{1/2}} . \quad (4.10b)$$

In the linear phase $a(t)$ is rapidly increasing so that the wave is soon well outside the horizon (i.e., $k \ll aH$) and the third term in Eq. (4.9) becomes negligible so that $h_k$ approaches a constant. This constant can be estimated by extrapolating (4.10) to the horizon crossing time. $h_k$ then remains at this value until it finally reenters the horizon in the Friedmann phase. This "freezing out" of the gravitational waves often goes by the name of amplification[22] since it is amplification above the adiabatic behavior [Eq. (4.10)]. The amplitude of the gravitational wave of wave number $k$ at reentry is thus given by

$$A_k = (2\pi k)^{3/2} | h_k(t_{\rm hc}) | = \frac{\sqrt{G} H(t_{\rm hc})}{\sqrt{2}[1+2\epsilon R(t_{\rm hc})]^{1/2}} , \quad (4.11)$$

where $t_{\rm hc}$ denotes the initial horizon crossing time in the linear phase. At that time $\dot{H} \sim -1/36\epsilon$, so we have

$$\epsilon R(t_{\rm hc}) \sim 12\epsilon H^2(t_{\rm hc}) . \quad (4.12)$$

We assume waves which reenter the horizon at late times have left the horizon during the inflationary epoch so that $2\epsilon R(t_{\rm hc}) \gg 1$ and

$$A_k \approx \frac{1}{\sqrt{2}} \frac{\sqrt{G}}{\sqrt{24\epsilon}} . \quad (4.13)$$

Notice that the spectrum is flat. Comparing to the $\Delta T/T$ limit for the microwave anisotropy[21] we have

$$A_k \approx \left[ \frac{\Delta T}{T} \right] \le \sqrt{7} \times 10^{-4} \quad (4.14)$$

or

$$\epsilon \ge 3 \times 10^5 l_{\rm Pl}{}^2 , \quad (4.15)$$

which tightens up the bound (3.20) somewhat. Unlike usual inflationary models, it turns out that the microwave measurements constrain not the value of $H(t_{\rm hc})$ but rather the value of $\epsilon$. This is due to the fact that the quantization condition (4.5) is modified by the curvature-squared coupling.

In the conformal picture we arrive at the result quite easily due to the de Sitter background. Note that the conformal transformation maps backgrounds, but leaves the perturbations unchanged: $A = \tilde{A}$, so we have by conventional means

$$A_k = \tilde{A}_k \approx \sqrt{4\pi G} \tilde{H} , \quad (4.16)$$

which leads to $\epsilon > 7\times 10^6 l_{\rm Pl}{}^2$, agreeing with the above limit (4.15) to the order of approximation we are using.

Note that in this picture one matches the amplitude at $\tilde{a}\tilde{H} = k$, while the true perturbation crosses the physical horizon at $aH = k$. However, the difference between the two is $O(\dot{R}/R)$ so that with the same accuracy one has obtained the de Sitter solution one can safely evaluate the perturbation at $\tilde{a}\tilde{H} = k$.

A comparison between the two pictures sheds more light in understanding why the final result does not depend on $H_{\rm hc}$ as in the usual case. In the standard calculation one can estimate the amplitude of the wave by requiring that the expectation value of the total energy of waves within the horizon equals the zero point energy of quantum fluctuations,[22] $E = (1/2)\omega = (1/2)(k/a)$:

$$\frac{1}{H^3}\langle\rho\rangle = E \; . \tag{4.17}$$

The amplitude of the wave at the horizon crossing is obtained by extrapolating this relation to $t_{\rm hc}$, which gives $A \propto H_{\rm hc}$. Now in conformal space where the gravity is pure Einstein and the stress tensor for gravitational waves has the usual form one imposes

$$\frac{1}{\tilde{H}^3}\langle\tilde{\rho}\rangle = \tilde{E} \; . \tag{4.18}$$

However, this relation is not conformally covariant as $\tilde{H} \approx \Omega^{-1/2}H$, $\tilde{E}=\Omega^{-1/2}E$, and $\tilde{\rho} = \Omega^{-1}\rho$ [here $\Omega$ is the conformal factor $= (1+2\epsilon R)$]. So, in terms of the physical $H$ and $R$ this relation reads

$$\frac{1}{H^3}\langle\rho\rangle = \frac{E}{\Omega} \; . \tag{4.19}$$

Since $\Omega = (1+2\epsilon R) \approx 24\epsilon H_{\rm hc}{}^2$ we have that the Hubble parameter drops from the final answer.

## V. SCALAR PERTURBATIONS

As is usual in inflationary models, rather stringent constraints on the model parameters arise from present observational limits on scalar perturbations. In our model scalar perturbations are generated by quantum fluctuations in the scalar curvature around background values. A major obstacle to evaluating these fluctuations is that we are dealing with a fourth-order gravity in which the quantization is not easy. We thus avoid the problem by working in the conformal picture. In the conformal picture there is a neat separation of the degrees of freedom and the background is the de Sitter solution so that our result is easily obtained. From the action (1.5) we obtain a field equation for $\delta R$ which is full of nonlinearities. However, we may make use of the fact that during the inflationary epoch $\epsilon R$ is large ($\epsilon R \approx 12\epsilon H^2 \geq 20$, where physical quantities are without tildes, conformal quantities have tildes) and the field equation reduces in this exponential expansion phase to

$$\frac{d^2(\delta R)}{d\tilde{t}^2} + 3\tilde{H}\frac{d(\delta R)}{d\tilde{t}} - \tilde{a}^{-2}(\partial_i{}^2\delta R) = 0 \; . \tag{5.1}$$

That is, $\delta R$ evolves like a minimally coupled scalar field. However, it is not really one, as can be seen by its stress tensor. We may use the stress-energy tensor given by Eq. (1.4b) to find the background energy density and pressure during this expansion phase (when the matter content is negligible):

$$\tilde{\rho} = \tilde{T}_{\tilde{t}\tilde{t}} = \frac{1}{64\pi G\epsilon}\frac{1}{\left[1+\frac{1}{2\epsilon R}\right]^2}\left[1+6\epsilon\left[\frac{1}{R}\frac{dR}{d\tilde{t}}\right]^2\right] \tag{5.2a}$$

and

$$\tilde{p} = \frac{\tilde{T}_{\tilde{x}\tilde{x}}}{\tilde{a}^2} = \frac{-1}{64\pi G\epsilon}\frac{1}{\left[1+\frac{1}{2\epsilon R}\right]^2}\left[1-6\epsilon\left[\frac{1}{R}\frac{dR}{d\tilde{t}}\right]^2\right] \; . \tag{5.2b}$$

For a scalar wave perturbation of wave number $k$, we can find the linear and quadratic corrections to the energy density:

$$\delta\tilde{\rho} = \delta\tilde{\rho}^{(1)} + \delta\tilde{\rho}^{(2)} \; , \tag{5.3a}$$

where, in particular, to leading order in $1/(\epsilon R)$ we have

$$\delta\tilde{\rho}^{(2)} = \frac{3}{16\pi G}\left[\frac{k}{\tilde{a}}\right]^2\left[\frac{\delta R}{R}\right]^2 \; . \tag{5.3b}$$

Now we proceed in determining the mean-square quantum fluctuations of $\delta R$ (i.e., for waves much shorter than horizon) from the fact that their energy is just the zero-point energy. That is,

$$\frac{1}{\tilde{H}^3}\langle\delta\tilde{\rho}^{(1)} + \delta\tilde{\rho}^{(2)}\rangle = \tilde{E} = \frac{1}{2}\frac{k}{\tilde{a}} \; . \tag{5.4}$$

We evaluate Eq. (5.4) using (5.3) for scales much shorter than the horizon. The expectation value $\langle\delta\tilde{\rho}^{(1)}\rangle$ is zero and we obtain

$$\langle\delta R^2\rangle = \frac{8\pi G}{3}\left[\frac{k}{\tilde{a}}\right]^2 R^2 \; . \tag{5.5}$$

Finally, we extrapolate this to the horizon crossing of the fluctuation, where it is physically matched to the classical post-horizon-crossing amplitude by $|\delta R_{\rm hc}|^2 = 2\langle\delta R^2\rangle$, so

$$|\delta R_{\rm hc}| = \frac{R_{\rm hc}}{3}\left[\frac{2\pi G}{\epsilon}\right]^{1/2} = 4\left[\frac{2\pi G}{\epsilon}\right]^{1/2} H_{\rm hc}{}^2 \; . \tag{5.6}$$

Now we may determine the metric potential $\tilde{A}$ due to a classical wave of amplitude $|\delta R_{\rm hc}|$ using the "time-lag" method of Guth and Pi:[3]

$$\tilde{A} \sim \frac{\delta(\tilde{a}^2)}{\tilde{a}^2} = 2\tilde{H}\delta\tilde{t} = 2\tilde{H}\left|\frac{dR}{d\tilde{t}}\right|^{-1}|\delta R_{\rm hc}| \; . \tag{5.7}$$

If we now plug (5.6) into (5.7) we obtain

$$\tilde{A} \sim \frac{2}{3}\left[\frac{2\pi G}{\epsilon}\right]^{1/2}(18\epsilon H_{\rm hc}{}^2) \; . \tag{5.8}$$

We stress that this is the asymptotic value of the metric perturbation at the end of the inflationary phase and therefore gives the magnitude of the inhomogeneities in the subsequent Friedmann evolution.

Alternatively, we proceed more cautiously using the gauge-invariant formalism of Brandenberger and Kahn.[4] We neglect the effect of sources outside the horizon so that we may use a quantity, $\tilde{\zeta}$, as a conserved gauge-invariant expression between horizon crossings:

$$\tilde{\zeta}=\frac{2}{3}\frac{\left[\tilde{\Phi}_H+\tilde{H}^{-1}\frac{d\tilde{\Phi}_H}{d\tilde{t}}\right]}{(1+\tilde{p}/\tilde{\rho})}+\tilde{\Phi}_H\left[1+\frac{2}{9}\left[\frac{k}{\tilde{a}\tilde{H}}\right]^2\frac{1}{(1+\tilde{p}/\tilde{\rho})}\right], \quad (5.9)$$

where $\tilde{\Phi}_H$ is now a gauge-invariant metric potential given by

$$\tilde{\Phi}_H=4\pi G\tilde{a}^2\nabla^{-2}\left[\tilde{T}_{\tilde{t}\tilde{t}}^{(1)}-3\tilde{a}\frac{d\tilde{a}}{d\tilde{t}}\nabla^{-2}\tilde{T}_{\tilde{t},\tilde{j}}^{\tilde{j}\ (1)}\right]. \quad (5.10)$$

Here $\nabla^{-2}$ is the inverse Laplacian and $\tilde{T}_{\tilde{\mu}\tilde{\nu}}^{(1)}$ is the first-order perturbation in the stress energy. We may calculate from (1.4b) to leading order in $1/(\epsilon R)$ (that is, during the inflationary epoch after the horizon crossing so that the wave is fully classical),

$$\tilde{T}_{\tilde{t}\tilde{t}}^{(1)}=\delta\tilde{\rho}^{(1)}\approx\frac{1}{64\pi G\epsilon}\frac{1}{\epsilon R}\frac{\delta R}{R} \quad (5.11a)$$

and from the stress energy (1.4b) we find, again to leading order (this term is the same order as the first, contrary to Brandenberger and Kahn[4])

$$\tilde{T}_{\tilde{t},\tilde{j}}^{\tilde{j}\ (1)}\approx\left[\frac{k}{\tilde{a}}\right]^2\left[\frac{dR}{d\tilde{t}}\right]\delta R\ . \quad (5.11b)$$

We have then at the horizon crossing of Eq. (5.9),

$$\tilde{\zeta}_{\rm hc}=\left[\frac{20}{3}\frac{\delta\tilde{\rho}^{(1)}}{(\tilde{\rho}+\tilde{p})}+\frac{2\tilde{H}^{-1}\frac{d\delta\tilde{\rho}^{(1)}}{d\tilde{t}}}{(\tilde{\rho}+\tilde{p})}+\frac{3\delta\tilde{\rho}^{(1)}}{\tilde{\rho}}\right], \quad (5.12)$$

where $\delta\tilde{\rho}^{(1)}$ is now calculated in (5.11a) from the classical amplitude $|\delta R_{\rm hc}|$ in Eq. (5.6). We may find $\tilde{\zeta}_{\rm hc}$ by putting (5.11a) and (5.11b) into (5.12):

$$\tilde{\zeta}_{\rm hc}=39\epsilon\,|\delta R_{\rm hc}|=\frac{26}{3}\left[\frac{2\pi G}{\epsilon}\right]^{1/2}(18\epsilon H_{\rm hc}^2)\ . \quad (5.13)$$

This fixes $\tilde{\zeta}$ at the initial horizon crossing, which quantity is roughly conserved until reentry. At the reentry of the scale of interest, the Universe will be in a matter-dominated Friedmann phase ($\tilde{p}=0$) and we may use the Friedmann equation at reentry, $\tilde{H}^2=\frac{8}{3}\pi G\tilde{\rho}$, to find

$$\tilde{\Phi}_H(\tilde{t}_{\rm reentry})=\frac{3}{2}\frac{\delta\tilde{\rho}^{(1)}}{\tilde{\rho}}\ . \quad (5.14)$$

We may now drop the tildes at reentry since during this late phase the conformal factor is $\approx 1$. We have

$$\zeta_{\rm reentry}\approx\tfrac{35}{9}\Phi_H(t_{\rm reentry})\approx\tilde{\zeta}_{\rm hc} \quad (5.15)$$

and finally the metric potential after reentry is

$$\Phi_H(t_{\rm reentry})\approx\frac{78}{35}\left[\frac{2\pi G}{\epsilon}\right]^{1/2}(18\epsilon H_{\rm hc}^2)\ . \quad (5.16)$$

We see that $\Phi_H(t_{\rm reentry})\sim\tilde{A}$ [here $\tilde{A}$ is given by Eq. (5.8)] to within numerical factors. In the $H_{\rm hc}^2$ factor we have some weak scale dependence in the perturbation spectrum. In fact, the spectrum is scale invariant up to a logarithmic term as in the case of standard inflation. We calculate this dependence in the following way—at both the initial and final horizon crossings we have in the physical space

$$a_{\rm hc}H_{\rm hc}=k=a_{\rm reentry}H_{\rm reentry}\ . \quad (5.17)$$

We plug into this our evolution law (2.23)–(2.24) assuming of course that the initial horizon crossing occurs during the linear inflationary phase of the model and we obtain

$$H_{\rm hc}=H_0\left[\frac{k_{\rm reentry}}{k_0}\right]e^{18\epsilon H_{\rm hc}^2}(3.3\times10^{31})\times\left[\frac{\epsilon}{G}\right]^{-1/12}N^{-5/12}, \quad (5.18)$$

where $H_0$ is the Hubble parameter today (we use $H_0$ = 50 km/sec Mpc$^{-1}$ = $3\times10^{-61}l_{\rm Pl}^{-1}$ and $k_0$ is the scale which crosses the horizon today). From this equation we may directly exhibit the logarithmic scale dependence of the perturbations:

$$\frac{\tilde{A}_2}{\tilde{A}_1}\doteq1-\frac{1}{18\epsilon H_{\rm hc}^2}\ln\left[\frac{k_2}{k_1}\right]. \quad (5.19)$$

We note that Eq. (5.18) for a given scale of observational interest completely fixes the horizon crossing Hubble parameter in terms of the model parameter $\epsilon$. That is, the metric potential, $\tilde{A}$, given by Eq. (5.8), again for a given scale, is only dependent on $\epsilon$. Scales which are inside the horizon today are bounded by the microwave anisotropy limit[21] so that $\tilde{A}\le\sqrt{7}\times10^{-4}$ and $k_{\rm reentry}/k_0=1$. We have

$$H_{\rm hc}(k_0)\approx5\times10^{-6}l_{\rm Pl}^{-1}\ \text{and}\ \epsilon>1\times10^{11}l_{\rm Pl}^2\ . \quad (5.20)$$

If we want this primordial spectrum of density fluctuations to be a successful seed for galaxy formation and we use a standard value for the scalar perturbation amplitude of $\sim10^{-4}$, then essentially our bound in (5.20) would change into an equality. If, however, we choose a different scenario[23] that is less constraining in which $\tilde{A}>10^{-6}$ for scales $k_{\rm reentry}/k_0\approx150$, we have

$$H_{\rm hc}(k_{\rm cluster})\approx2\times10^{-8}l_{\rm Pl}^{-1}\ \text{and}\ \epsilon<7\times10^{15}l_{\rm Pl}^2\ . \quad (5.21)$$

The bound (5.20) tightens up (3.20) considerably—although this number is only to be taken as very rough.

Notice also that $18\epsilon[H_{hc}(k_0)]^2 \approx 45$ so that the early evolution for $H(t) > H_{hc}(k_0) \sim 5\times10^{-6} l_{Pl}^{-1}$ is irrelevant to all present observation. Putting it another way, with initial conditions of order Planck our model predicts that the Universe has been expanded something like $2\times10^{12}$ $e$ foldings and the observable part of the Universe will be the same for many future generations.

The scales which cross the horizon at $H_{hc} > H_b \equiv 1/(12\sqrt{2\pi G\epsilon})$ have perturbations bigger than one today. From Eq. (5.20),

$$H_b \leq 10^{-3} l_{Pl}^{-1} .$$

If $H_m > H_b$ that simply means that one has at scales much larger than the present horizon fluctuations which cannot be treated in linear theory. Of course, $H_m$ can as well be less than $H_b$—it is bounded below only by $H_{hc}(k_0)$. The requirement that the perturbations are small at the initial horizon crossing so that the use of perturbation theory is justified leads to only a very weak constraint on $\epsilon$—well within our other bounds.

Interestingly, all these numbers tell us that there is one characteristic mass scale present in the theory as $H_{hc}(k_0) \sim \epsilon^{-1/2} \sim 10^{-6} l_{Pl}^{-1}$. Perturbations in an inflationary model with a massive scalar inflaton have been considered by Halliwell and Hawking,[24] using the full wave-function formalism. They found that compatability with observation restricts this mass to be less than $10^{14}$ GeV. As we have seen, the scalar curvature does obey an equation for a massive scalar field of mass $\sim 1/\sqrt{6\epsilon}$. So we see that despite the unusual self-couplings present in the $\epsilon R^2$ theory, the physical analogy works remarkably well.

Finally, from Eqs. (4.13) and (5.8) the neat result follows that the contribution to the microwave anisotropy of the scalar fluctuations overpowers that from gravitational waves by a factor $18\epsilon[H_{hc}(k_0)]^2 \sim 45$. This is the reason that the bound on $\epsilon$ is much tighter from considering scalar perturbations.

## VI. PRESENT BOUNDS ON $\epsilon$ AND POSSIBLE ORIGINS

It may seem that the condition $\epsilon > 10^{11} l_{Pl}^2$ places a very large unnatural limit on $\epsilon$, which in terms of Planck units it does. We would like to point out that in terms of any presently measured curvature this is really quite small.

We can manipulate the field equation (2.1) in the usual way to get

$$\Lambda - 4\pi G(\rho_S + 3p_S) = 3H_0^2(\sigma_0 - q_0) , \tag{6.1}$$

where

$$\rho_S \equiv \frac{3\epsilon}{4\pi G}\left[\frac{R^2}{12} - \dot{R}H - RH^2\right] \tag{6.2a}$$

and

$$p_S \equiv \frac{\epsilon}{4\pi G}\left[\ddot{R} + 2\dot{R}H + \frac{R^2}{12} - RH^2 - \frac{R\kappa}{a^2}\right] . \tag{6.2b}$$

This is the usual equation which is used to set a limit on the cosmological constant $\Lambda$ in terms of the presently observed $H_0$ (the Hubble parameter), $\sigma_0$ (density parameter), and $q_0$ (deceleration parameter). If we assume $\Lambda = 0$ we thus obtain a cosmological limit on $\epsilon$:

$$\epsilon \leq 10^{120} l_{Pl}^2 . \tag{6.3}$$

Similarly one can consider a limit on $\epsilon$ by asserting that $\epsilon R$ is small in all horizon-exterior curvatures encountered presently in our Universe. We may use for $R$ typically $M/r^3$ and go to the gravitational radius of a black hole. Then $\epsilon R \ll 1$ requires only

$$\epsilon \ll 10^{77}\left[\frac{M}{M_\odot}\right]^2 l_{Pl}^2 . \tag{6.4}$$

This, of course, is a bit of a swindle because a black hole is also a solution of $\epsilon R^2$ gravity[15] so that $R = 0$ and $\epsilon$ will have no effect. We conclude, though, that $\epsilon = 10^{11} l_{Pl}^2$ in terms of any presently encounterable curvature is very small.

We have not as yet addressed the question of the origin of the $\epsilon$ term. Basically, there are three ways that one might imagine it arising. First, it may be that the full fourth-order theory should be postulated as fundamental. Such a form is naturally suggested if one thinks about gravity as the gauge theory of the Poincaré group.[25] Furthermore, the $\epsilon R^2$ terms in the field equations violate the strong energy condition so that the initial singularity might be avoided.[13] It has also been shown that such a theory is renormalizable.[25] And the long-standing objection that it is nonunitary might not be true.[26] Second, it may be a remnant from some more fundamental theory. For instance, in superstring theory the Lagrangian of the point-particle limit of the ten-dimensional full string theory contains the following terms:[27]

$$R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} + aR^{\mu\nu}R_{\mu\nu} + bR^2 ,$$

where $a$ and $b$ are constants. After compactification this leads to

$$L = R + \left[\frac{a+1}{3} + b\right]\frac{GV_6}{\phi}R^2 , \tag{6.5}$$

where $V_6$ is the compactified volume of the six "other" dimensions and $\phi$ is the vacuum expectation value of a scalar field known as the dilaton. We see that this might directly give us an $\epsilon R^2$ behavior even classically in the Lagrangian with a completely determined $\epsilon$. However, the highly preferred values[28] for $a$ and $b$ are $a = -4$, $b = 1$ and then $\epsilon = 0$ at the classical level and there is no $R^2$ term in superstring theory.

Nevertheless, $\epsilon$ should also be expected to arise in a third way—as a quantum effective action correction to the bare theory. Here, the specific fields will contribute to its value. Indeed, this is the approach of Starobinsky.[14] As a quantum correction term $\epsilon$ would be given by

$$\epsilon \sim G\ln\left[\frac{\Lambda_{\text{high cutoff}}}{\Lambda_{\text{low cutoff}}}\right] \tag{6.6}$$

and we would again be forced to consider a higher, more complete theory to fix $\epsilon$.

## VII. CONCLUSION

We thus conclude that at a classical level a cosmology based on the $R + \epsilon R^2$ Lagrangian generically has an inflationary phase with a linearly decreasing Hubble parameter. The total number of expansion $e$ foldings during this phase is $\sim 18\epsilon H_m{}^2$ (if $\dot{R}_i = 0$, then $t_i = 0 = t_m$). After the linear decaying phase $H(t)$ bounces off zero and the Universe goes into an oscillatory phase. The total expansion is sufficient to solve the horizon and flatness problems if $18\epsilon H_m{}^2 > 75$. At the classical level this is a natural and consistent model that relies solely on a modified gravity for its dynamics. Here, the quadratic correction to the Hilbert-Einstein action would be expected to be present somewhat independently of the specific form of the matter Lagrangian (although a value for $\epsilon$ must necessarily come from a higher theory).

The post-inflation oscillatory phase yields a maximal reheating temperature which is small:

$$T_r \approx 1.2\times 10^{12}\ \text{GeV} \left[ \frac{\epsilon}{10^{11} l_{\text{Pl}}{}^2} \right]^{-1/2},$$

in any case very much below any expected GUT phase transition so that the monopole problem is avoided by the $\epsilon R^2$-driven expansion. Standard baryogenesis still may go through at this temperature but the details of this on the nonstandard background will require further attention. Finally, there is a join to a Friedmann phase at a temperature

$$T_F \leq 6\times 10^{8}\ \text{GeV} \left[ \frac{\epsilon}{10^{11} l_{\text{Pl}}{}^2} \right]^{-3/4} N^{1/4},$$

when the evolution goes over to a radiation-dominated expansion.

Gravitational waves and scalar perturbations both yield bounds on the parameters of the model when we must set them small so as not to disturb the isotropy of the microwave background. The bound from gravitational waves is $\epsilon > 10^6 l_{\text{Pl}}{}^2$ with no restriction on $H_{\text{hc}}$ as would occur for the standard inflationary scenario. This spectrum of gravitational waves is scale invariant. However, the scalar perturbations give the much tighter bound of $\epsilon \geq 10^{11} l_{\text{Pl}}{}^2$ and this in turn implies that the perturbation scale which reenters the horizon today must cross the horizon at $H_{\text{hc}}(k_0) \sim 10^{-6} l_{\text{Pl}}{}^{-1}$—that is, at a late stage of the extremely long linear phase. The spectrum of scalar perturbations has only logarithmic dependence on the scale. If one wants baryogenesis to proceed in the usual way there is an upper bound $\epsilon < 10^{15} l_{\text{Pl}}{}^2$. A similar bound follows from a comparison between galaxy formation and the microwave anisotropy in models of galaxy formation with cold dark matter.[23] However, these considerations both carry their own difficulties so that we place somewhat less emphasis here on the upper bound. Our condition of sufficient inflation requires that $H_m > 10^{-5} l_{\text{Pl}}{}^{-1}$—that is, we find that our model would work for essentially all reasonable initial conditions. We thus conclude that the $\epsilon R^2$ model satisfies all requirements for a realistic inflationary model as long as $\epsilon$ is large enough.

To investigate the very early phase we have attempted a preliminary wave-function calculation by solving the Wheeler-DeWitt equation to WKB approximation subject to a tunneling boundary condition in the manner of Vilenkin.[29] We thus obtain peak values for the wave function assuming a closed ($\kappa = +1$) universe of $\langle a \rangle \sim 0.056 l_{\text{Pl}}$, $\langle R \rangle \sim 3800 l_{\text{Pl}}{}^{-2}$, and $\langle H \rangle \sim 18 l_{\text{Pl}}{}^{-1}$ independent of $\epsilon$ (the details of that calculation will be reported in subsequent work). We interpret these as typical of the tunneling values for the Universe into the Lorentzian/classically allowed regime. Also, the peak is not very strong so that these numbers end up only as bounds. That is, we might say

$$R_i \gtrsim 4000 l_{\text{Pl}}{}^{-2},$$

$$a_i \sim 0.06 l_{\text{Pl}} \left[ \frac{R_i}{4000 l_{\text{Pl}}{}^{-2}} \right]^{-1/2},$$

and

$$H_i \sim 20 l_{\text{Pl}}{}^{-1} \left[ \frac{R_i}{4000 l_{\text{Pl}}{}^{-2}} \right]^{1/2}$$

(and $t_i = t_m = 0$). These numbers are sufficiently distant from the horizon crossing of interesting perturbations that the wave function offers no conflict with our lower bound on $H_m$. We thus find the classical evolution to be generally independent of initial conditions. The one remaining question is whether or not there will be a long quantum gap separating the tunneling point from the onset of the classical model. That is, are quantum fluctuations large for an extended period during early times? This of course must be answered by the wave function itself. Also, after doing this further calculation we can determine whether the inflated portion of our present Universe is the whole Universe or only a fluctuated bubble part of it as in Linde's chaotic inflation picture. We note now only that the initial parameters preferred above indicate that the tunneled universe is strongly quantum.

The model we are considering has a lot in common with the Starobinsky model.[14] While our work was carried on, papers by Starobinsky,[30] Kofman, Linde, and Starobinsky,[31] and Vilenkin[29] appeared from which we also learned about earlier work.[32] All of these papers treat the Starobinsky model in considerable detail so that we would like to comment here about similarities and differences between cosmologies based on Eq. (1.2) and the Starobinsky model and also to discuss our results in relation to this other work.

Starobinsky considers a model in which the one-loop quantum corrections to the matter stress-energy tensor of a conformally coupled scalar field are used as source terms for the Einstein equations. At the Lagrangian level this introduces a new parameter, $H_S$, and a new term to Eq. (1.2), $(1/H_S{}^2)R^2\ln(R/\mu)$, where $\mu$ is some renormalization scale. The important point is that $H_S$ is completely fixed by the number of degrees of freedom which give quantum corrections:[29] for example, $H_S \sim 0.7 l_{\text{Pl}}{}^{-1}$ for minimal $SU_5$. There is an exact de Sitter solution in this case with $H_S$ being the initial Hubble parameter. This solution is shown to be unstable—offering an exit from the inflationary phase. Vilenkin[29] has shown by a wave-function calculation that there will be sufficient inflation in the de Sitter phase. For an initial $H_i$ not bigger than $H_S$, $H(t)$ will decrease in time. When $H(t) \ll H_S$ the decrease will be linear with time[32] and the subsequent evolution should be the same as in the present $R^2$ model. In comparison to the Starobinsky model, our work shows that the initial de Sitter phase is not necessary. We have shown that a generic solution of the field equations will have sufficient inflation based solely on the $R^2$ term. We have analyzed the reheating in the oscillation phase, showing that it is characterized by two different temperatures. The reheating temperature, $T_r$, is much higher than the temperature, $T_F$, when the Friedmann phase begins. We have analyzed the metric perturbations both in the conformal picture and using the direct approach. The results obtained essentially agree with those obtained in the Starobinsky model.[29,32] These results indicate that the part of the expansion which is relevant for present observation happens at $H(t) < 10^{-5} l_{\rm Pl}{}^{-1}$ and cannot be due to the de Sitter phase of the Starobinsky model. Finally we note that as the two models have very different early stage evolution the wave function calculation yields very different initial parameters.

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