# Spinless-dyon models of composite leptons and quarks

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This paper includes the following. (1) A general opinion on the study of the possible compositeness of leptons and quarks. Practices as to set many specific standards of naturalness before the understanding of the dynamics are pondered. (2) Two composite models with scalar preons [all preons are dyons of a hidden $U(1)_s$] are introduced. We concentrate on one with six preons $T^a$, $T^b$, $T^c$ and $V^a$, $V^b$, $V^c$, where $a$, $b$, and $c$ are three different $U(1)_s$ dyon charges carried by preons. Their values are not arbitrary in order to match their fundamentalness. $T$ and $V$ are differentiated by a mass difference. Leptons and quarks consist of three preons; vector bosons and Higgs bosons consist of six preons due to a force similar to the covalent bound between hydrogen atoms in an $H_2$ molecule. Possible multiplets are given in a table. The symmetry of the composite system is $SU(4)\times SU(2)_L\times SU(2)_R\times U(1)_F$. (3) The symmetry-breaking pattern of the model is basically fixed by requiring that exact degeneracies and symmetry survive to the maximal extent. (4) The slow rate of proton decay and $e$, $\mu$ universality are reached. The Kobayashi-Maskawa matrix is discussed. (5) Vector bosons other than gauge bosons are predicted. $W'_L$ and $W'_R$ interact with leptons 3 times stronger than with quarks. (6) Problems and uncertainties in the models, once met, are explicitly discussed.

## I. INTRODUCTION

To seek for the ultimate and universal is one of the most profound pursuits of human beings, especially of physicists. It is a long march to reach this aim and in this long march we have already passed many milestones, many layers: objects (e.g. stones, trees, etc.)→chemically classified substances (e.g. NaCl, $H_2O$, etc.)→atoms →nuclei→nucleons→quarks and leptons. In this way we have explored deeper and deeper structures of matter. A long-lasting controversy about this exploration is whether there is a last stop, or a fundamental layer. If there is one, have we already reached this layer? In particular, the question we now face is whether quarks and leptons are fundamental point particles. And, if there is no such last stop, do we feel confident falling from one layer to another deeper one without an end? Now again we are in a difficult time: although there is no evidence which favors the compositeness of leptons and quarks, it seems, to many physicists, that the present standard quark-lepton picture with $3+2+1$ [i.e., $SU(3)\times SU(2)\times U(1)$ gauge] interactions is far from satisfactory. Many problems are not even questioned within the standard framework.[1] For example, why are there spin-0, -$\frac{1}{2}$, and -1 fundamental particles rather than only one or two different spins? Why are there three different kinds of interactions, gauge interactions, Yukawa interactions, and Higgs self-interactions? Why are there specific representations of gauge groups (singlets, doublets, and triplets)? Why are there specific symmetry-breaking patterns [SU(3) unbroken, $SU(2)\times U(1)\rightarrow U(1)$]? Why are there some global symmetries [e.g., possible[2] $U(1)_A$]? Why are there masses and mixing angles and other coupling constants taking specific values? Why are there generations? And so on. While grand-unification theories[3,4] intend to answer a few of these problems, composite theories are aimed to solve most of them. In this sense, the latter is much more ambitious. From the history of the quark model,[5] one may believe that the two approaches might be cooperative.

Though we are not sure if quarks and leptons are composites, much work on their possible compositeness has appeared.[3,6–22] Many interesting opinions on the philosophy in the study of the possible compositeness of leptons and quarks have been raised. It is almost agreed that the characteristic of a lepton or quark, comparing it to the other composite systems studied, is expressed by the following inequality:

$$m/\Lambda \ll 1 ,$$

where $m$ is the mass of a lepton or quark and $\Lambda^{-1}$ its size. The most naive (also model-independent) way of measuring the sizes of leptons or quarks is to measure their form factors, which typically give the lower limit of the $\Lambda$ bound by the maximal energies of accelerators:

$$\Lambda \gtrsim 100 \text{ GeV} .$$

Some more sophisticated but sometimes model-dependent estimates[9] give $\Lambda > 1$ TeV. However, there are quite different opinions on the sizes of gauge and Higgs bosons: if they are composites, they might be either smaller or larger than leptons and quarks. Excited leptons and quarks, or gauge bosons, if discovered (e.g., spin-$\frac{3}{2}$ leptons and spin-2 bosons), may yield important information about their compositeness, yet may not help to specify the value of $\Lambda$. The energy difference between the excited and ground states $\Delta E$ may be of the same order as $\Lambda$ in the hydrogen-atom case (where $\Delta E \simeq R_d{}^{-1}$) or unrelated as in the oscillator case [where $\Delta E = (k/m)^{1/2}$, $\Lambda = (km)^{1/4}$]. While the inequality for leptons and quarks is agreeable among physicists, how to realize it is quite arguable.[10–12] Without an understanding of a strong binding system, such as a lepton or quark (if it is a composite), it is perhaps not wise to set a specific standard for naturalness in getting the inequality, although physicists have useful ambiguous common sense about what is natural. Since we have always been getting simpler pictures of nature when we go to deeper layers of structure, it is difficult to imagine that the next layer, the preon layer, is more complicated than the present known leptons and quarks, except that the presently discovered leptons and quarks are just a negligible part of their entire spectrum. Electrons and photons have so far survived as point particles from a few previous composite studies (e.g., the studies of atoms, nuclei, and nucleons). This fact naturally leads people to wonder whether it is possible that Higgs bosons only or gauge bosons only are composites while the others are not at this stage. Of course, there are also some other possible choices. An interesting hint might be obtained from the following comparison. In the early 1960s, bootstrap theories and the eightfold way were modestly successful. However, in these theories all hadrons were relevant, but not leptons and photons. Then it turned out that the quark model was a composite model of hadrons. Now, in the $3+2+1$ unified gauge theory, all leptons and quarks, all gauge bosons, and at least one Higgs doublet boson are indispensable parts. Perhaps their scales of fundamentalness (or compositeness) should not be so different. However, the role of gravity is still ambiguous in this hint. A quite subtle question on compositeness is whether it is possible to make the effective interactions of a composite system satisfy gauge principles. Putting it another way, the question would be whether gauge principles are immortal—therefore we must reserve all the $3+2+1$ gauge interactions at the lepton-quark layer to the preon layer—or they are just special phenomena which appear at this layer (the lepton-quark layer)—therefore we may need not do so.[13] A related question is, then, whether the non-Abelian gauge binding for the hadrons should be repeated for the binding of leptons and quarks if the latter are composites of preons. At last, a technical question is as follows: Once a composite model is set, which among the problems listed in the first paragraph are the easiest to solve? We doubt that the mass problem could be included in the list of easy problems.

After these general discussions, we turn to describing specific composite models[14–22] with spinless dyons as constituents. Previous results are reviewed in Secs. II and III to make the paper self-contained. The simplest composites are discussed with small changes, corrections and new understandings. Two different models are introduced in parallel. Starting from Sec. III, we concentrate on one model which is defined in Sec. II as the $A$ model. After the discussion of the symmetry property of the model in Sec. III, we discuss the possible covalent force to explain how gauge bosons and Higgs bosons are bound in Sec. IV. We prove that no gauge or Higgs bosons can mediate proton decay; therefore we slow down the proton decay rate by allowing only decay forbidden by the Okubo-Zweig-Iizuku rule. Possible Higgs multiplets and non-gauge-vector particles are given in a table (Table II). Section V addresses the possible generation mechanism of the model. Universality, elimination of flavor-changed neutral currents, and Kobayashi-Maskawa (KM) mixings are discussed. We shall find that this $A$ model corresponds to realistic physics in many aspects. However, we would rather call the $A$ model a toy model which is useful for at least presenting our tendency in respect to questions and problems raised before. At the same time, we shall not overlook the chance of the model becoming a realistic one. We shall state it explicitly whenever (in the paper) we meet a problem or uncertainty. Section VI is especially devoted to some problems in the model.

## II. THE SPINLESS-DYON MODELS

Using the strong U(1) magnetic force as the binding force for preons to make composite leptons and quarks and bosons was first suggested by Pati.[15] Pati then suggested that this U(1) should be a hidden U(1) different from electromagnetism.[17] For our further convenience, we shall call this U(1) "sound U(1)" and its quantum a "phonon," with electriclike and magneticlike charges.[16] The purpose of introducing $U(1)_s$ binding is, first, to minimize the number of preons and gauge bosons at the constituent level and, second, to guarantee fewer low-energy exotic leptons and quarks.[15] It was argued,[21,22] from the zero-energy solution of the Dirac equation in a $\alpha=1$ Coulomb potential,[23] that we may get light composites from heavy constituents by U(1) binding and build up the chiral symmetry from none. The inverse size of the zero-energy solution is about $\Lambda \sim 10 m_e$, where $m_e$ is the mass of the charged fermion. Koh, Pati, and Rodriguez[16] have also found that the U(1) binding seems to work well for both the Harari-Shupe-type model[14] and the Pati-Salam model.[1,6] In general, we refer to composite models which have fewer properties of composites built into constituents as arithmetic models (or $A$ models) and those with many properties of composites built into constituents, block models (or $B$ models). The atomic model is an example of an $A$ model in which one cannot read out the periodic table of elements from the electron. The quark model is an example of a $B$ model in which properties of hadrons have obvious marks of the flavors of quarks. We do not know whether a composite lepton or quark (if it is composite) should belong to the $A$ or $B$ model. It is obvious that the Harari-Shupe model[14] looks like an $A$ model and the Pati-Salam model[3,6] like a $B$ model. Using the U(1) sound binding, these models should be read as the following.[16,17]

Preons in the $A$ model:

$$\begin{aligned} &T^a,\ T^b,\ T^c\ \ (\text{masses}=M_T)\ , \\ &V^a,\ V^b,\ V^c\ \ (\text{masses}=M_V)\ , \\ &(M_T \neq M_V)\ . \end{aligned} \tag{1}$$

Preons in the $B$ model:

$$W_1^a,\ C_1^b,\ S^c,\ W_2^a,\ C_2^b,\ C_3^b,\ C_4^b\ , \tag{2}$$

(masses of $C_1$, $C_2$, and $C_3$ are equal), where the indices $a$, $b$, and $c$ are the sound charges (all nonzero) of the preons which satisfy the condition

$$a+b+c=0\ . \tag{3}$$

The magnitude of the masses of preons will be discussed later. In many cases they are very heavy. Note there are some features in models (1) and (2) which compare with their original forms.[3,6,14] They are (1) the introduction of a hidden U(1) sound force between preons, (2) preons are scalars (see later) instead of fermions, and (3) there are no electromagnetic interactions at the preon level, neither are there any weak or color interactions.

As we have mentioned, in general, these U(1) sound charges $a$, $b$, and $c$ may have two components, electric and magnetic charges, and thus may be dyons.[24,25] For these dyon charges, the Dirac quantization condition must be met; that is, for two particles with $a=(a_1,\ a_2)$ and $b=(b_1,b_2)$ charges, respectively,

$$a\times b\equiv a_1b_2-a_2b_1=4\pi q\ , \tag{4}$$

where $q$ is an integer or half-integer. It is also proved that the system gains a spin $|q|$ in addition to possible conventional angular momenta of the system.[26] Figure 1 shows how a system with static electrically and magnetically charged particles gets a spin diagramatically. The preons might be fermions or scalars, but for reason of economy we assume they are scalars since the sound field can serve the necessary spin $\frac{1}{2}$ when $|q|=\frac{1}{2}$. Incidentally, this minimizes the degrees of freedom of preons. The spin-statistics theorem should guarantee automatically that composite spin-$\frac{1}{2}$ particles satisfy the Fermi-Dirac statistics.[27,28] It is argued[20] that for a neutral three-spinless-dyon system the spin of the ground state is $\frac{1}{2}$, if $|a\times b|=2\pi$. Spin-$\frac{3}{2}$ bound states must involve orbital angular momentum excitations; therefore, they are very

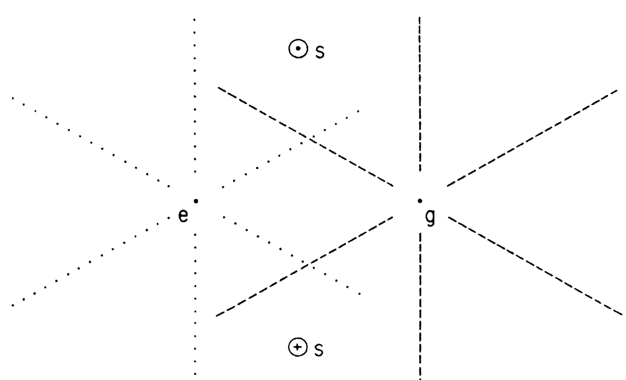


FIG. 1. Separate infinitely massive magnetic monopole $g$ and electrically charged particle $e$ have a spin from the electromagnetic field. Dashed lines are magnetic-field lines and dotted lines are electric-field lines. The direction of the momentum of the field $\mathbf{S}=(1/4\pi)\mathbf{E}\times\mathbf{B}$ is perpendicular to the axis through $e$ and $g$. The direction of the angular momentum is from $e$ to $g$.

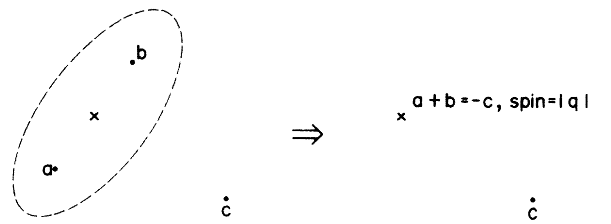


FIG. 2. The spin of the three-dyon system in the ground state, when $a+b=-c$, equals the spin of the two-dyon subsystem $a$ and $b$, which has spin $|q|$ and total charge $-c$. This subsystem and the third charge $c$ together make a pure Coulomb binding system.

heavy. The spin formation of a three-dyon system is illustrated in Fig. 2.

Since the values of sound charges $a$, $b$, and $c$ are very big, in the normal case only sound-neutral systems can be seen. The simplest sound-neutral composites may consist of two or three preons and are called duons and trions, respectively.

Trions in the $A$ model are

$$\begin{aligned}&T^aT^bT^c,\ V^aV^bV^c\ ,\\&T^aV^bV^c,\ T^bV^cV^a,\ T^cV^aV^b\ (\text{spin}=\tfrac{1}{2})\ ,\\&V^aT^bT^c,\ V^bT^cT^a,\ V^cT^aT^b\ .\end{aligned} \tag{5}$$

Duons in the $A$ model are

$$\begin{aligned}&T^a\overline{T}{}^a,\ T^b\overline{T}{}^b,\ T^c\overline{T}{}^c\\&V^a\overline{V}{}^a, V^b\overline{V}{}^b, V^c\overline{V}{}^c\end{aligned}\ (\text{spin parity}=0^{++})\ , \tag{6a}$$

$$\begin{aligned}&T^a\overline{V}{}^a,\ T^b\overline{V}{}^b,\ T^c\overline{V}{}^c\ ,\\&V^a\overline{T}{}^a,\ V^b\overline{T}{}^b,\ V^c\overline{T}{}^c\ .\end{aligned} \tag{6b}$$

Trions in the $B$ model are

$$W_\alpha C_i S\ \ (\text{spin}\ \tfrac{1}{2})\ (\alpha=1,2,\ i=1,2,3,4)\ . \tag{7}$$

Duons in the $B$ model are

$$W_\alpha\overline{W}_\alpha\ , \tag{8a}$$

$$C_i\overline{C}_i\ \ (\text{spin parity}=0^{++})\ , \tag{8b}$$

$$S\overline{S}\ . \tag{8c}$$

Those exotics such as $TT\overline{T}$, etc., in the $A$ model or $WWS$, etc., in the $B$ model are not sound neutral; therefore, they must be very heavy. We see that the U(1) binding is much more efficient than a non-Abelian binding in eliminating unwanted bound states such as spin-$\frac{3}{2}$ light fermions and exotics.

Of course, the sound charges $a$, $b$, and $c$ are the most fundamental quantities in such kinds of composite models with U(1) sound binding. If these models are taken seriously, there should be no arbitrariness in the assignment of the values of $a$, $b$, and $c$. Then the only natural way of fixing these fundamental charges under the condition, Eq. (3), is[11]

$$a = g_0(1,0),\quad b = g_0\left[-\frac{1}{2},\frac{\sqrt{3}}{2}\right],\quad c = g_0\left[-\frac{1}{2},-\frac{\sqrt{3}}{2}\right]. \tag{9}$$

In the electric and magnetic charge plane (Fig. 3), the three charges form an equilateral triangle. As we know, calling the U(1) charges electric or magnetic is subject to an uncertainty owing to the dual rotation invariance of the U(1) gauge theory.[25] Therefore the positions of the three dyon charges $a$, $b$, and $c$ are absolutely equal. In addition, owing to the Dirac quantization condition, Eq. (4), $g_0$ is fixed as (when $q=\frac{1}{2}$)

$$g_0 = (4\pi/\sqrt{3})^{1/2}. \tag{10}$$

There are no free parameters in the assignment of these fundamental quantities, which is what we expected. The inner products of any pairs of these charges measure the strength of the Coulomb interactions within the pairs. They are

$$\begin{aligned} a\cdot a &= b\cdot b = c\cdot c = 4\pi/\sqrt{3}, \\ a\cdot b &= b\cdot c = c\cdot a = -2\pi/\sqrt{3}. \end{aligned} \tag{11}$$

Therefore the "binding strength" between a preon and an antipreon is twice that between two differently charged preons which resemble the ratio of binding strengths between $q\bar{q}$ and $qq$ in the quark model. Incidentally, the critical value[23] $\alpha$ of the Coulomb potential for the Klein-Gordon equation is $\frac{1}{2}$. The difference between $1/\sqrt{3}$ (and $1/2\sqrt{3}$) in the dyon models here and $\frac{1}{2}$ might have something to do with the additional spin $|q|$. Perhaps it is also due to the fact that in the Klein-Gordon equation with a Coulomb potential the source of the potential is static, while for the composite system in the dyon model the source itself is moved. The dynamics of the three-dyon system is very difficult to solve because of two reasons: (1) the U(1) theory with a magnetic monopole (in general, dyons) is nonlocal; there is a string[24,25] between particles with charges $a\times b\neq 0$; (2) the three-body problem itself is notoriously difficult. Instead, we make the following assumption.

*Assumption 1.* Ground-state neutral three-dyon com-

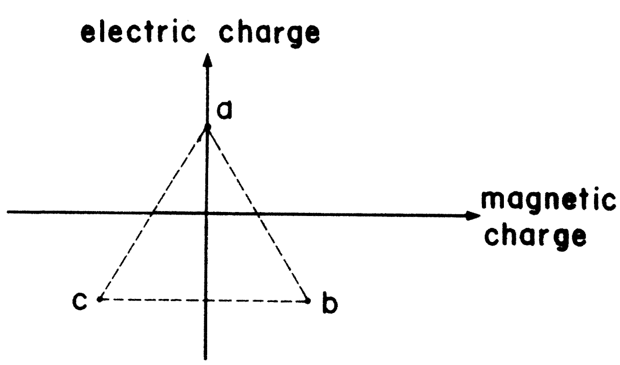


FIG. 3. In the two-dimensional charge plane, the three charges $a$, $b$, and $c$ form an equilateral triangle.


FIG. 4. Binding diagrams of (a) duon and (b) trion. Number of lines between preons indicates the strength of Coulomb interaction. The single lines in trion (b) can also be understood as topological strings.

posites (trions) are massless, and their inverse sizes are about $\Lambda\sim 10M$ where $M$ is the typical mass of preons.

The binding diagrams for trions and duons are separately given in Fig. 4. The binding lines in a trion can also be seen as strings due to the topological property of the magnetic charge.[24] The low-lying composite systems made of dyons are sound neutral, which is extremely important for avoiding potentially serious contradictions between the model and real physics. Still, contradictions may arise owing to possible sound dipole moments. Of course, the dipole moments are two-component quantities. Fortunately, it is easy to prove, in the equally massive three-dyon case, that the dipole moments are strictly zero. The point is, when the dyons are equally massive, the Hamiltonian operator $\hat{H}(a,b,c)$ of the three-dyon system must be symmetric under $S_3^c$, the permutations of the three charges $a$, $b$, and $c$. Therefore, the bound states of the three dyons must be in a pure representation of $S_3^c$. On the other hand, the operator of the dipole moment of the three-dyon system must also be $S_3^c$ symmetric, in addition to being linear in respect to the three charges; therefore, we have

$$\mu_{nn'} = \langle n\,|\,\hat{\mu}(a,b,c)\,|\,n'\rangle = (a+b+c)\eta_n\delta_{nn'}, \tag{12}$$

where $n$ denotes states in different representations of $S_3^c(S, M_1, M_2$, and $A$, see Appendix B). $\eta_n$ is a constant depending on the dynamics. Combining with Eq. (4), we have $\mu_{nn'}=0$. It is argued[19] (see Sec. V) that it might happen that the difference between $e$, $\mu$, and $\tau$ is due to the difference in $S_3^c$ properties. In that case, Eq. (12) means

$$\mu \to e + \text{phonon}$$

is forbidden at the tree level (e.g., in the $A$ model). For trions which consist of differently massive preons, Eq. (12) is not valid; however, $\mu_{nn'}$ should be very small, perhaps proportional to $(\Delta M/M)(m_F/\Lambda^2)$ where $\Delta M/M$ is the mass difference over the average mass of preons, which is due to Eq. (12); and $m_F/\Lambda^2$ is the mass of trions (fermions) over the square of the binding scale, which is due to the suppression of anomalous moments caused by the extremely relativistic binding.[9] Therefore, it is very difficult for phonons to scatter from leptons and quarks, once the preons are heavy enough.

## III. SYMMETRY PROPERTIES OF THE $A$ MODEL (REFS. 21 AND 22)

It has been pointed out that the exact symmetry of the $A$ model, Eq. (1), is

$$[U(1)]^6 \times S_3^c , \tag{13}$$

where six U(1)'s correspond to the separate conservations of the numbers of each kind of preon $N_\sigma^\alpha$ ($\alpha = T$ or $V$, $\sigma = a,b$, or $c$). These conservation laws are due to the property of the U(1) gauge interaction between preons. The $S_3^c$ group is all of the permutations among the three kinds of charge discussed before. This symmetry is due to both the symmetric charge assignment and the equal massiveness of three $T$'s and that of three $V$'s. When we only consider neutral composites, we have $N^a = N^b = N^c$ (where $N^\sigma = N_T^\sigma + N_V^\sigma$) owing to Eq. (3). Two conservation laws become trivial and only four nontrivial ones need to be considered. We can choose these four nontrivial U(1)'s as

$$N \equiv \tfrac{1}{3} \sum_{\alpha,\sigma} N_\alpha^\sigma , \tag{14}$$

$$B - L \equiv \tfrac{2}{3}(I_3^a + I_3^b + I_3^c) , \tag{15}$$

$$T_3 = \tfrac{1}{2}(I_3^b - I_3^a) , \tag{16}$$

$$T_8 = \frac{1}{2\sqrt{3}}(2I_3^c - I_3^a - I_3^b) , \tag{17}$$

where

$$I_3^\sigma = \tfrac{1}{2}(N_T^\sigma - N_V^\sigma) . \tag{18}$$

In the limit when $M_T - M_V \to 0$, the global symmetry of the constituents is actually $[SU(2)]^3 \times [U(1)]^3 \times S_3^c$ where $[SU(2)]^3$ has operators in Eq. (18) as its Cartan operators. The trions in Eq. (5) are in (2,2,2) representation of $[SU(2)]^3$ and duons in Eq. (6) are in (1 + 3,1,1) + (1,1 + 3,1) + (1,1,1 + 3). Since $[SU(2)]^3$ has nothing to do with the physics, we hope this group will be dynamically broken. If this happens (the difficulty here, see Sec. VI), the duons in Eq. (6b) will become Goldstone bosons. Fortunately $[SU(2)]^3$ is not an exact symmetry owing to $M_T \neq M_V$; Higgs bosons in Eq. (6b) should be pseudo-Goldstone bosons with a mass proportional to $|M_T - M_V|$. The quantum numbers of trions and duons in respect to operators defined in Eqs. (14)–(17) are given in Table I. In this table we also show quantum number $Q$, which is

$$Q \equiv N + \tfrac{1}{2}(B - L) = \tfrac{1}{3} N_T \equiv \tfrac{1}{3} \sum_\sigma N_T^\sigma . \tag{19}$$

We have chosen a specific set of eight trions to work with in Table I. Actually there are $2^{8-1}$ possible choices of making a set of eight. The set in Table I is the only choice which satisfies the condition

$$\mathrm{Tr} N^3 = 0, \quad \mathrm{Tr} N O_1 O_2 = 0 , \tag{20}$$

where $O_1$ and $O_2$ are any operators in Eqs. (15)–(17) and (19). The philosophy is, according to assumption 1, that the trions are massless; therefore, they are chiral symmetric, if their interactions with duons respect the symmetry. In order to have interactions which respect the chiral symmetry, the only possibility is to suppose that the original vectorlike global symmetry $N$ becomes chiral;

$$N \to N_L + N_R . \tag{21}$$

Because only for the quantum number $N$, a consistent way to define the chiral transformation of duons exists; that is

$$N_L = N_R = 0 \quad \text{(for duons)} . \tag{22}$$

TABLE I. Simplest composites: trions and duons.

| Structure | $N$ | $Q$ | $B-L$ | $\lambda_3$ | $\sqrt{3}\lambda_8$ | Name |
|---|---|---|---|---|---|---|
| $V^aV^bV^c$ | $\frac{1}{2}$ | 0 | $-1$ | 0 | 0 | $\nu$ |
| $\bar{T}^a\bar{T}^b\bar{T}^c$ | $-\frac{1}{2}$ | $-1$ | $-1$ | 0 | 0 | $e$ |
| $V^aT^bT^c$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $u^a$ |
| $T^aV^bT^c$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $u^b$ |
| $T^aT^bV^c$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | $-1$ | $u^c$ |
| $\bar{T}^a\bar{V}^b\bar{V}^c$ | $-\frac{1}{2}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $d^a$ |
| $\bar{V}^a\bar{T}^b\bar{V}^c$ | $-\frac{1}{2}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $d^b$ |
| $\bar{V}^a\bar{V}^b\bar{T}^c$ | $-\frac{1}{2}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | 0 | $-1$ | $d^c$ |
| $T^\sigma\bar{T}^\sigma$ | 0 | 0 | 0 | 0 | 0 | |
| $V^\sigma\bar{V}^\sigma$ | 0 | 0 | 0 | 0 | 0 | |
| $V^a\bar{T}^a$ | 0 | $-\frac{1}{3}$ | $-\frac{2}{3}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\phi_-^a$ |
| $V^b\bar{T}^b$ | 0 | $-\frac{1}{3}$ | $-\frac{2}{3}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\phi_-^b$ |
| $V^c\bar{T}^c$ | 0 | $-\frac{1}{3}$ | $-\frac{2}{3}$ | 0 | $-1$ | $\phi_-^c$ |

After the chiralization of the global symmetry, Eq. (20) will prevent the symmetry from being broken by triangle anomalies. Since the eight trions in Table I are in two different irreducible representations of $[SU(2)]^3$, to work with this set means to break $[SU(2)]^3$ dynamically.

It is worth noting that strict degeneracies appear in the following three triplets:

$$T^aV^bV^c,\ T^bV^cV^a,\ T^cV^aV^b\ , \tag{23}$$

$$V^aT^bT^c,\ V^bT^cT^a,\ V^cT^aT^b\ , \tag{24}$$

$$\phi^a,\ \phi^b,\ \phi^c\ (\phi^\sigma=T^\sigma\bar{V}^\sigma)\ . \tag{25}$$

This fact is guaranteed by the $S_3^c$ symmetry.

The global symmetry of the composites might be larger than U(1) and $S_3^c$, since, at least, there are three exact triplets. If this is so (which is very difficult to prove, see Sec. VI), the maximal symmetry of the composites cannot be larger than

$$G_s=SU(4)\times SU(2)_L\times SU(2)_R\times U(1)\ \text{factors}, \tag{26}$$

because the duons cannot be put into larger groups. If $G_s$ is the global symmetry of the composites, then fermions will be in (4,2,1) + (4,1,2) representation of SU(4)×SU(2)×SU(2), and $\phi_\sigma$ and $\bar{\phi}_\sigma$ will be in (6,1,1). We shall discuss duons $T^\sigma\bar{T}^\sigma$ and $V^\sigma\bar{V}^\sigma$later on. Of course, $G_s$ is not an exact symmetry of the model, since there is a mass difference between $T$ and $V$ preons.

## IV. COVALENT BOUND STATES: GAUGE BOSONS AND HIGGS BOSONS (REFS. 21 AND 22)

To discuss gauge bosons, there are two possibilities. One possibility easily to be thought of is that the vector bosons are $p$-wave states of duons. However, the binding force between a pair of preons in a duon is exactly the Coulomb type. As we discussed at the beginning, the excitations in the Coulomb potential have the energy about the binding scale $\Lambda$, which is too large for the standard gauge bosons. Therefore we must appeal for the other possibility, i.e., to make gauge bosons by a multipreon system. We think the best choice is to suppose that vector bosons are a six-valence-preon system[14] (sixions). How can one make a sixion that can be divided into neutral subsystems such as trions or duons? At a first glance, this seems impossible. However, it might be possible. Comparing to the covalent bond between hydrogen atoms in a hydrogen molecule, we may get some hint. Let us denote the positions of the atoms by numbers and the spins of electrons belonging to the atoms by arrows, then we can write the formula for the molecule as

$$\frac{1}{\sqrt{2}}[H^\uparrow(1)H^\downarrow(2)-H^\downarrow(1)H^\uparrow(2)]=H_2\ . \tag{27}$$

The other possible state

$$\frac{1}{\sqrt{2}}[H^\uparrow(1)H^\downarrow(2)+H^\downarrow(1)H^\uparrow(2)]$$

is dissociated or unbound, so is $H^\uparrow(1)H^\uparrow(2)$. Note here that both hydrogen atoms are neutral systems. The energy needed to hit an electron off an $H_2$ is almost the same as that from an H atom. The binding energy of the molecule is of the order of $\frac{1}{100}$ of that of the atom. That means the atom is a relatively stable component in the molecule, though an electron of one atom has a certain chance to be shared by the other atom.

Now, in our composite model, the trions are very stable because of strings and strong bindings between differently charged preons. A huge amount of energy transfer must be involved in a process involving the breaking down of a string. Based on the analysis of the hydrogen molecule, we suggest the following rules for the covalent binding between two trions (temporarily, we consider only one generation).

(1) There must be more than one possible choice of pairs of trions, which have the same net preon content.

(2) These pairs of trions must have the same quantum number in respect to the symmetries discussed.

(3) If the above two conditions are satisfied, at least one linear combination of trion pairs is a bound state and at least one other is not.

These three rules are the contents of our assumption 2. Although the assumption does not give a rule to choose one when there are many possible linear combinations of trion pairs, to aquire the bound states being in a multiplet of $G_s$ will settle down the uncertainties in most cases. According to these rules, we would be able to write gauge bosons by formulas in which only leptons and quarks appear, similar to Eq. (27). Here, two trions may have little chance to share their constituents. We shall identify gauge bosons in the $G_s$ gauge model as the following: The left-handed $W$ bosons are

$$\begin{aligned} W_{L\mu}^-&=\frac{1}{2}\left[\sum_\sigma \bar{u}_L^\sigma\gamma_\mu d_L^\sigma+\bar{\nu}_L\gamma_\mu e_L\right],\\ W_{L\mu}^0&=\frac{1}{2\sqrt{2}}\left[\sum_\sigma(\bar{u}_L^\sigma\gamma_\mu u_L^\sigma-\bar{d}_L^\sigma\gamma_\mu d_L^\sigma)\right.\\ &\qquad\left.+\bar{\nu}_L\gamma_\mu\nu_L-\bar{e}_L\gamma_\mu e_L\right], \end{aligned} \tag{28}$$

and the formulas for right-handed $W$ bosons are similar. Gluons are

$$g_a^b=\tfrac{1}{2}(\bar{u}^a\gamma_\mu u^b+\bar{d}^a\gamma_\mu d^b)\ \ (a\neq b),\ \text{etc.} \tag{29}$$

(Note here, left and right chiralities are equally entering, the normalization constant is $1/\sqrt{4}$ instead of $1/\sqrt{2}$.) Leptoquarks are

$$X_\mu^\sigma=\frac{1}{\sqrt{4}}(\bar{e}\gamma_\mu u^\sigma+\bar{\nu}\gamma_\mu d^\sigma)\ . \tag{30}$$

The $B-L$ gauge boson is

$$Y_\mu=\frac{1}{\sqrt{48}}\left[\sum_\sigma(\bar{u}^\sigma\gamma_\mu u^\sigma+\bar{d}^\sigma\gamma_\mu d^\sigma)-3(\bar{e}\gamma_\mu e+\bar{\nu}\gamma_\mu\nu)\right]. \tag{31}$$

It is amusing that in these expressions the interactions of these vector bosons with fermions are explicitly fixed and are exactly in the gauge way; i.e., for non-Abelian interactions, there is only one coupling constant in respect to different fermion multiplets. For example, there are four $SU(2)_L$ triplets in the $W_L^+$ formula of Eq. (28). This happens because all of them have essentially the same $\bar{V}\bar{V}\bar{V}TTT$ of preon contents, and also because we require $W_L$ to be a singlet of SU(4). However, to show that the gauge-boson self-couplings (both cubic and quartet terms) are also controlled by the same coupling constant and the masslessness of gauge bosons needs to be done; this work has not yet been done and must be done to prove that those vector bosons are effective gauge bosons. We would think that the gauge behavior of the vector bosons is perhaps possible. Nature seems to have a "feedback system" to adjust the coupling constants of vector bosons in order to have them behave well in all ranges of energy from zero to the binding scale $\Lambda_c$ of the covalent bond which is much above the masses of these vector bosons.[29] The interactions of sixions and leptons and quarks can be represented by "rubber diagrams" (Fig. 5). To remind us of the difference of interactions between leptons and quarks and covalent bond bosons from that among hadrons, we use a wavy double line when two leptons and quarks meet and make a fundamental boson (which is bound by covalent force), instead of a hadronic meson (which is bound by gluons). Similar diagrams have been used in QCD where the gluon is represented by a double quark line.[30]

There are many scalar bosons which we are not going to discuss in detail. Most of them are in representations of bilinear forms of (4,2,1) and (4,1,2). One thing worth noting is related to a potential proton decay mediator and rule 2. One would perhaps think that the linear combinations, such as ($\gamma$ matrices are omitted)

$$(u^a d^b - u^b d^a) + (\bar{u}^c \bar{e} - \bar{\nu}\bar{d}^c) , \tag{32}$$

may be a bound state, had we not considered the global symmetry SU(4) of composites. However, even without the restriction of SU(4), Eq. (32) could not be a bound state because, under the $a \leftrightarrow b$ exchange, the first term is antisymmetric while the second symmetric. This is a beautiful feature of the model. The dissociation of the binding formula, Eq. (32), means that the model does not enjoy SU(5) and larger grand-unification symmetries. As we know, Eq (32) is a necessary bound member, if composites possess SU(5) symmetry. Instead of (32), the allowed bound state is a (6,1,1) scalar multiplet with the following components:

$$\frac{1}{\sqrt{4}}(u^a d^b - u^b d^a), \;\; \frac{1}{\sqrt{4}}(u^c e - \nu d^c) . \tag{33}$$

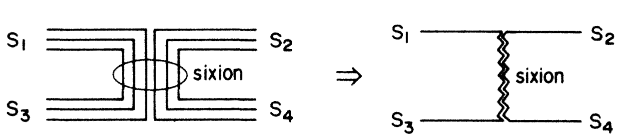


FIG. 5. Interaction between trions and sixion bosons can be expressed by Zweig diagrams with wavy double lines. In each vertex, leptons and quarks may or may not change flavor or colors.

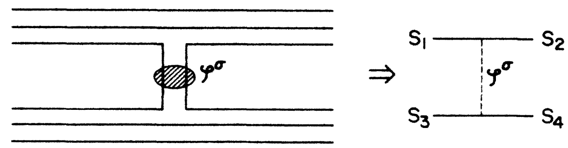


FIG. 6. $\phi^\sigma$ mediate the Okuba-Zweig-Iizuku-rule-forbidden processes.

This multiplet does not contribute to proton decay; therefore it need not be extremely heavy. Actually, there are no sixions which may contribute to proton decay. The only proton-decay mediator is the duon multiplet $(\phi^\sigma, \bar{\phi}^\sigma)$ (see Fig. 6). However, this process is forbidden by the Okuba-Zweig-Iizuku rule, because to couple $\phi^\sigma$ with trions we must break the string. The coupling constant $\lambda$ between $\phi^\sigma$ and fermions becomes very small: $\lambda \sim m_F/\Lambda$. The corresponding proton decay rate is suppressed by both $\lambda^4$ and ${m_p}^4/{m_\phi}^4$, i.e.,

$$\Gamma_p \sim m_p \frac{{m_p}^4}{{m_\phi}^4}\lambda^4 , \tag{34}$$

which is extremely small, if we also have $m_p \ll m_\phi$. Phenomenology of this proton-decay process is analyzed in Ref. 31 in a general context.

All possible covalent bound states, sixions, are listed in Table II. Multiplets above double lines are divided into two parts by a dotted line. The first part are linear combinations of lepton and quark pairs with different flavors [i.e., $SU(3)\times SU(2)_L \times SU(2)_R$ quantum numbers]. At least one component in each multiplet in the second part is not made of different flavor pairs, but different spin pairs, e.g., $\epsilon_{\alpha\beta}\nu_L^\alpha e_L^\beta$, etc. The gauge boson (1,1,1) couples to fermion numbers. The Lorentz structure of each covalent bound state is given in the last column of the table. Readers who are interested in how to make light composites from chiral fermions should consult Ref. 11. Since different bound states may have different binding energies and zero-point wave functions, etc., it does not seem wise to calculate the relative strength of their coupling constants before solving the dynamical problems. However, if one accepts the arbitrary assumption that "at the binding scale $\Lambda_c$ of sixions, all coupling constants are equal," he will obtain $\sin^2\theta_W = \frac{3}{8}$ at $\Lambda_c$. Of course this number cannot be taken seriously. Perhaps the differences among coupling constants are no larger than an order of magnitude. It is difficult to understand why there are very different Yukawa coupling constants (e.g., $G_t/G_e \gtrsim 6\times 10^4$) by simple argument, except perhaps serious dynamical calculations. The Higgs bosons in Table II may give the following symmetry-breaking pattern:

$$SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_F$$

$$\xrightarrow{10^{10}\text{ GeV}} SU(3)_c \times SU(2)_L \times U(1)$$

$$\xrightarrow{300\text{ GeV}} SU(3)_c \times U(1)_{em} . \tag{35}$$

TABLE II. Possible sixion boson multiplets.

| Multiplets | Typical component | Lorentz structure | Fermion number |
|---|---|---|---|
| (1,1,1) | $\bar{\nu}\nu+\bar{e}e+\Sigma(\bar{d}^{\sigma}d^{\sigma}+\bar{u}^{\sigma}u^{\sigma})$ | $\bar{\psi}\gamma_\mu\psi\equiv\bar{\psi}_L\gamma_\mu\psi_L+\bar{\psi}_R\gamma_\mu\psi_R$ | 0 |
| (15,1,1) | Eqs. (29)–(31) | $\bar{\psi}\gamma_\mu\psi$ | 0 |
| (1,3,1) | Eq. (28) | $\bar{\psi}_L\gamma_\mu\psi_L$ | 0 |
| (1,1,3) | Eq. (28) | $\bar{\psi}_R\gamma_\mu\psi_R$ | 0 |
| (1,2,2) | $\bar{u}d+\bar{\nu}e$, etc. | $\bar{\psi}_L\psi_R$ and $\bar{\psi}_R\psi_L$ | 0 |
| (6,1,1) | Eq. (33) | $\psi^TC\psi$ | 2 |
| (6,2,2)[a] | Eq. (33) | $\psi_R^T\gamma_\mu\psi_L$ | 2 |
| (10,3,1) | $\nu\nu$ | $\psi_L^TC\psi_L$ | 2 |
| (10,1,3) | $\nu\nu$ | $\psi_R^TC\psi_R$ | 2 |
| (6,3,1) | Eq. (33) | $\psi_L^TC\psi_L$ | 2 |
| (6,1,3) | Eq. (33) | $\psi_R^TC\psi_R$ | 2 |
| $W_L'$ triplet | Eqs. (37) and (38) | $\bar{\psi}_L\gamma_\mu\psi_L$ | 0 |
| $W_R'$ triplet | Eqs. (37) and (38) | $\bar{\psi}_R\gamma_\mu\psi_R$ | 0 |

[a]Ungauged vector bosons owing to being in representations other than adjoint ones. Compare Ref. 34. (6,2,2) should be easily dissociated.

The first breaking is realized by a nonzero vacuum expectation value (VEV) of the (10,1,3) multiplet, the second by (1,2,2). In order to get the measured Weinberg angle $\sin^2\theta_W$ and perhaps also known $\alpha/\alpha_s$ at low energies,[32] the scale $\Lambda_c$ of covalent binding (or the inverse size of the covalent bound states) must be large enough, perhaps[33]

$$\Lambda_c > 10^{10}\ \text{GeV} \tag{36}$$

and this scale also fixes the scale for the first symmetry breaking in Eq. (35). If we only had one restriction (e.g., $\alpha/\alpha_s$), we would be able to suppose this scale much smaller. Of course, $\Lambda\sim 10M$ is much larger than $\Lambda_c$. The symmetry breaking (35) seems intended to break all approximate symmetry of the composite model while reserving strict degeneracies Eqs. (23)–(25) untouched, and as many exact symmetries [Eq. (13)] of the composite model as possible. The breaking down of the exact symmetry $B-L$ (e.g., $N_V$ conservation) is really a "tragedy" in this hierarchy. This phenomenon needs further investigation. The composite model somehow (as shown) fixes leptons and quarks, gauge bosons, Higgs bosons, and the symmetry-breaking pattern (including the direction of the vacuum, e.g., $Q=0$, color-singlet components but no others get a VEV). Then one can work with an effective grand-unified gauge theory and analyze possible phenomenology with less arbitrarines.

In Table II, we also list two possible vector-boson multiplets which cannot be filled in the group $G_s$. The formula of the vector-boson multiplets are ($\gamma$ matrices are omitted)

$$W_L^{0'}=\left[\sum_\sigma(\bar{u}^{\sigma}_Lu^{\sigma}_L-\bar{d}^{\sigma}_Ld^{\sigma}_L)-3(\bar{\nu}_L\nu_L-\bar{e}_Le_L)\right]\Big/\sqrt{24} \tag{37}$$

and, according to $SU(2)_L$ symmetry

$$W_L^{+'}=\left[\sum\bar{d}^{\sigma}_Lu^{\sigma}_L-3\bar{e}_L\nu_L\right]\Big/\sqrt{12}\ . \tag{38}$$

We are not going to repeat the formula for right-handed ones. Equation (37) is similar to Eq. (31) in respect to preon constituents; this is the reason why we guess that it perhaps is a new vector boson. $W_L'$ and $W_R'$ cannot be put into SU(4) representations; still they might be bound states. That can happen in two cases. One case is if instead of SU(4), SU(3)×U(1) is the symmetry of the composites. In this case, all other sixion multiplets will decompose into SU(3)×U(1) representations.[34] Another possibility is that only the lowest-lying composites can fully fill a representation of the group; the higher-lying multiplets are not complete because some members are not bound at all. Symmetry breaking of SU(4) becomes clearer, for higher-lying states. In any case, $W'$ couples to leptons three times stronger than to quarks; it is a wrong component under SU(4) transformation; therefore, it is impossible to be gauged. $W'$ must be fatter than other vector bosons and before its bad behavior appears at a certain high energy (much smaller than $\Lambda_c$), it dissociates and the SU(4) symmetry recovers to some extent, i.e., no incomplete multiplets, and no wrong components.

There are perhaps also five preon composites and other composite systems. We are not going to give details here because they are relatively complex.

## V. GENERATIONS AND MIXINGS (REFS. 19 AND 21)

Although masses and mixings can be formally discussed in an effective gauge theory, it is more interesting to discuss them from the composite point of view. The former may only give some restrictions on possibilities; the latter should be able to fix all arbitrary parameters in principle, though it is difficult in practice. Here we would like to present an example of getting hierarchy and mixings. Of course, as we warned before, this is just a toy model.

In Ref. 19, we classify the splittings in the spectrum of a three-body system into three classes (e.g. the equally massive three-body linear oscillator). The first is governed by the main quantum numbers (e.g., $N$ for the oscillator system). The second, which is also smaller, is governed by the angular momentum number with the same main quantum number (e.g., when introducing $L^2$-dependent forces into the oscillator system). The third, the smallest, is governed by $S_3$ quantum numbers with the same main and angular momentum numbers (e.g., when introducing mass differences in the oscillator system). It might be that the third type of splitting in the three-dyon system is responsible for the generation phenomenon. That is, the lowest-lying three states of the trion have the same main, angular, and parity quantum numbers but different $S_3$ properties: One is in the symmetric representation and the other two are in the mixed representation. If the three dyons are equally massive, the mixed representation (a doublet) will be degenerate. Now the problem is whether the symmetric representation is the lowest state or the mixed. In Ref. 19 we suppose that the symmetric is the lowest. Now let us try the other possibility: the mixed is the lowest. Then our first-order lepton mass matrix (e.g., of $e$, $\mu$, and $\tau$) is

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A \end{bmatrix} . \qquad (39)$$

The familiar example of the complicated systems, whose ground state does not enjoy the lowest degeneracy, is the iron magnet. A three-body spin system with the Hamiltonian

$$H = \sum_{i<j} \mathbf{S}_i \cdot \mathbf{S}_j \qquad (40)$$

is another example, whose ground state is in the $S_3$ doublet instead of singlet.

The wave functions of the three generations in the equally massive case (e.g., $TTT$ or $VVV$) are (Appendix A)

$$U = \frac{1}{\sqrt{2}}[Y'(12,3)(\tfrac{1}{12})^{1/2}(bca - cba - acb + cab - 2abc + 2bac) + Y''(12,3)(\tfrac{1}{4})^{1/2}(bca + cba - acb - cab)], \qquad (41)$$

$$V = \frac{1}{\sqrt{2}}[-Y'(12,3)(\tfrac{1}{4})^{1/2}(bca - cba + acb - cab) + Y''(12,3)(\tfrac{1}{12})^{1/2}(bca + cba + cab + acb - 2abc - 2bac)] , \qquad (42)$$

$$W = (\tfrac{1}{6})^{1/2} Y(12,3)(abc + bac + bca + cba + cab + acb) , \qquad (43)$$

where wave functions are factorized into space parts $Y$, $Y'$, and $Y''$, and particle name parts (e.g., $abc$, etc.). We name the dyons by their dyon charges. The positions of the names in each term are very important because they correspond to the preon's coordinates $\mathbf{r}_1$, $\mathbf{r}_2$, or $\mathbf{r}_3$, which are simplified in $Y$'s by 1, 2, and 3. $12 = (1/\sqrt{2})(\mathbf{r}_1 - \mathbf{r}_2)$. $Y(12,3)$ is symmetric under permutations of the three coordinates. $Y'(12,3)$ and $Y''(12,3)$ make a mixed representation of $S_3$; $Y'(12,3) = -Y'(21,3)$, $Y''(12,3) = Y''(21,3)$. All three wave functions are symmetric under the simultaneous permutation of $a,b,c$ and 1,2,3 for the sake of bosonic statistics of the preons. $U$ is antisymmetric under $a \leftrightarrow b$ exchange, while $V$ is symmetric. It is worth noting that both $U^2 + V^2$ and $\overline{U}U + \overline{V}V$ are symmetric under $S_3^c$. Therefore, when entering the formula of, say $W^+$, they should appear in combination as

$$(\overline{e}\nu_e + \overline{\mu}\nu_\mu) + \eta\overline{\tau}\nu_\tau . \qquad (44)$$

Only $\eta$ is arbitrary because the third term itself is symmetric under $S_3^c$. We must put $\eta = 1$ in order to get a good behavior of the vector particle up to higher energies of about $10^{10}$ GeV. Since leptons are in pure $S_3^c$ representations, there are no mixings among leptons; in addition, the first two leptonic generations are massless; therefore the $e,\mu$ universality is perfect. It is interesting that in Eq. (44), potentially "maybe" terms like $\overline{e}\nu_\mu$, etc. are forbidden to enter into $W^+$ due to the $S_3^c$ symmetry. The same rule applies to neutral currents and thus forbids flavor-changed neutral-current interactions.

Now, let us turn to the quark sector. To discuss masses and mixings, we should stick to one color of quarks, say those with color $c$, i.e., $T^aT^bV^c$ for up quarks and $\overline{V}^a\overline{V}^b\overline{T}^c$ for down quarks. In both cases, $a$ and $b$ quarks are equally massive. The Hamiltonian of the composite quarks is symmetric under the exchange $a \leftrightarrow b$, though not symmetric under $S_3^c$,

$$\hat{H}_q(a,b,c) = \hat{H}_q(b,a,c) . \qquad (45)$$

The eigenfunctions of Eq. (45) are not in the pure representation of $S_3^c$ because $S_3^c$ is not the symmetry of the Hamiltonian. However, in order to get $W^+$ boson purely symmetric under $S_3^c$ (because the "leptonic part" of $W$ is so) the quarks entering the formula of $W^+$ must be written in the "ideal" wave function corresponding to the $M_T = M_V$ limit, which is denoted by the subindex 0

$$\overline{d}_0 u_0 + \overline{s}_0 c_0 + \overline{b}_0 t_0 . \qquad (46)$$

From the physical states to the ideal states, there is a transformation; this is where the mixing comes in. Once the transformation is unitary, the neutral current of quarks will keep the original diagonal form. Therefore the Glashow-Iliopolous-Maiani (GIM) mechanism[35] works. The same mixings happen in the interactions of $X^\sigma$ [Eq. (30)] but not in $g_a^b$ and $Y_\mu$. The mass matrix on the bases of $S_3^c$ eigenstates is

$$\begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & \beta \\ 0 & \beta^* & \gamma \end{bmatrix} , \tag{47}$$

where

$$M_{nn'} = \langle n \mid H_q(a,b,c) \mid n' \rangle \quad (n,n'=U,V,W) . \tag{48}$$

We have $|\gamma| \gg |\beta|$, $|\epsilon_1|$, $|\epsilon_2|$. $\epsilon_1 \neq \epsilon_2$ for the same reason[36] that causes the splitting between $\Lambda$ and $\Sigma^0$. If it happens that $0 \sim |\epsilon_1| < |\epsilon_2|$ for down quarks, then there must be $0 \sim |\epsilon_2| < |\epsilon_1|$ for up quarks, because in the two cases, the $a,b$ preons have a different relationship with the $c$ preon in respect to the masses. For example, if $M_T > M_V$, there are two light preons in down quarks, while there are two heavy preons in up quarks.

In the above discussion of quark mixings, we did not consider that the physical states for unequal-mass preons may not be expanded completely in the bases of the lowest-lying pure $S_3^c$ states. A formal calculation including this consideration is given in Appendix B. When excited states other than the lowest-lying three are energetically located far away, their contribution should be very small. However, since $\epsilon_1$ and $\epsilon_2$ are very small, the KM matrix[37] is very sensitive to the corrections to the first $2\times 2$ block of the mass matrix.

## VI. SUMMARY AND DISCUSSIONS

Two years ago it seemed impossible to seriously work with such a compact model as the $A$ model discussed in the preceding sections.[21] There were too many problems: the hydrogen atom would collapse since all six net preon numbers in it are zero; there were no consistent rules for writing down sixions; there was no way to reproduce the GIM mechanism, etc. Now we find, as discussed in this article, that the model itself has some features which may hint at the possible, consistent solutions of these difficulties. However, as we have pointed out many times, most of our ideas argued are not well supported. The main reason for this situation is that we cannot solve the dynamical problem of a three-dyon bound system.

One of these arguments is worth putting special attention to. We have supposed that the symmetry of the preons $[SU(2)]^3 \times [U(1)]^3 \times S_3^c$ goes through a "magic" dynamical process and becomes the symmetry of composites, $G_s = SU(4)\times SU(2)\times SU(2)\times U(1)$. $G_s$ seems to be a good symmetry if only lowest-lying trions and sixions are considered. SU(5) and larger symmetries are not suitable. However, when we consider the six flavor neutral duons, the symmetry $G_s$ is also in question. Let us consider these duons in the equal-mass limit ($M_T = M_V$). They are

$$\eta_0 = \frac{1}{\sqrt{6}} \sum_\sigma (T^\sigma \overline{T}^\sigma + V^\sigma \overline{V}^\sigma) , \tag{49}$$

$$\eta_{I_3} = \frac{1}{\sqrt{6}} \sum (T^\sigma \overline{T}^\sigma - V^\sigma \overline{V}^\sigma) , \tag{50}$$

$$\eta_{\lambda_3} (\tfrac{1}{4})^{1/2} (T^a \overline{T}^a + V^a \overline{V}^a - T^b \overline{T}^b - V^b \overline{V}^b) , \tag{51}$$

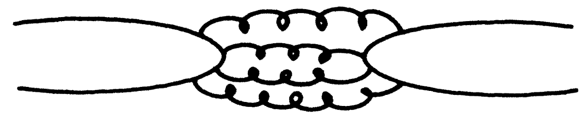

FIG. 7. The mixing diagram for neutral duons. At least two phonons are involved.

$$\eta_{\lambda_8} = (\tfrac{1}{12})^{1/2} (T^a \overline{T}^a + T^{T^b} \overline{T}^b + V^a \overline{V}^a + V^b \overline{V}^b - 2T^c \overline{T}^c - 2V^c \overline{V}^c) , \tag{52}$$

$$\eta_{I_3\lambda_3} = \tfrac{1}{4} (T^a \overline{T}^a - V^a \overline{V}^a - T^b \overline{T}^b + V^b \overline{V}^b) , \tag{53}$$

$$\eta_{I_3\lambda_8} = (\tfrac{1}{12})^{1/2} (T^a \overline{T}^a - V^a \overline{V}^a + T^b \overline{T}^b - V^b \overline{V}^b - 2T^c \overline{T}^c + 2V^c \overline{V}^c) . \tag{54}$$

It is easy to prove that, including all order of sound interactions, $\eta_0$ is heavier than the other five and the latter five are always degenerate (see Appendix C and Figs. 7 and 8) therefore one can choose any linear combinations of the five as physical eigenstates. The trouble is that they are members of SU(2) triplet, SU(3) octet and $SU(2)\times SU(3)$ $(3\times 8)$-plet, however, with all charges vanishing. Since they make incomplete representations of SU(3) and $SU(2)_W$, it becomes suspect that $SU(3)_c$ and $SU(2)_W$ (not yet even chiralized) are good symmetries of the composites. However, it is equally suspect that these $\eta$'s break SU(3) or SU(2) symmetry. Writing them in the form

$$\eta_i = \left[\frac{6}{7}\right]^{1/2} \left[ p^i \overline{p}^i - \tfrac{1}{6} \sum_j p^j \overline{p}^j \right], \quad i = (\sigma,\alpha) , \qquad \sum_i \eta_i = 0 , \tag{55}$$

we find, considering the interaction of all $\eta_i$ with duons $\phi^\sigma$ (Fig. 8), they cannot split $\phi^\sigma$. They cannot split the other degeneracies [including those, when $M_T = M_V$, in the $SU(2)_W$ multiplets] either. This might have something to do with the following equation:

$$\sum_\alpha {H_\alpha}^2 \propto I . \tag{56}$$

That is, the summation of all squared Cartan operators in a fundamental representation of an SU($n$) algebra is pro-

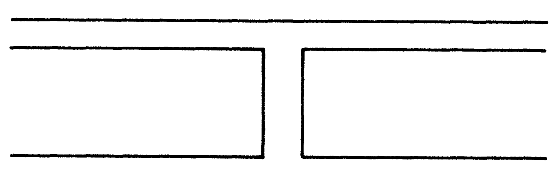

FIG. 8. Duons can also interact among themselves by this diagram.

portional to unity. Besides, these particles interact with quarks and leptons and sixion bosons very weakly as we argued before. It becomes very difficult to find any physical effects of their existence.

## ACKNOWLEDGMENTS

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## APPENDIX A: GENERATIONS FOR THE EQUAL-MASS PREON CASE

In Ref. 19 we have discussed a possible mechanim to produce generations. The main idea is described in Sec. V. We are now going to reproduce wave functions for different generations in order to give details of the calculation and make some corrections on coefficients appearing in the final formulas of Ref. 19.

The equal-mass three-dyon neutral system might have a ground state, which is in the mixed representation of the $S_3$ group, the permutation group of three coordinates:

$$\begin{matrix} Y'_{E_0|q|m}(\mathbf{r}_{12},\mathbf{R}_3) \\ Y''_{E_0|q|m}(\mathbf{r}_{13},\mathbf{R}_3) \end{matrix} \quad (\text{doublet of } S_3)\ . \tag{A1}$$

Near this ground state, there might be an $S_3$ symmetric "$S_3$ excitation"

$$Y_{E_1|q|M}(\mathbf{r}_{12},\mathbf{R}_3)\ , \tag{A2}$$

where $E_0$ and $E_1$ are energies, $|q|$ and $m$ denote the spin and its $z$ component of the system, respectively. $\mathbf{r}_{12}=(1/\sqrt{2})(\mathbf{r}_1-\mathbf{r}_2)$ and $\mathbf{R}_3=(1/\sqrt{6})(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3)$ are the relative coordinates. Except the space part (A1) or (A2), the total wave function also has a particle name part, similar to the flavor part in the quark model. Since $S_3^c$ is a good symmetry for the three-body system, the name part wave function should be able to be classified according to $S_3^c$ representations:

$$S=\frac{1}{\sqrt{6}}(abc+bac+bca+cba+cab+acb)\ . \tag{A3}$$

For $M$:

$$\begin{aligned} &(\tfrac{1}{12})^{1/2}(bca-cba-acb+cab-2abc+2bac)\equiv M^1\ ,\\ &(\tfrac{1}{4})^{1/2}(bca+cba-acb-cab)\equiv M^2\ . \end{aligned} \tag{A4}$$

For $N$:

$$\begin{aligned} &-(\tfrac{1}{4})^{1/2}(bca-cba+acb-cab)\equiv N^1\ ,\\ &(\tfrac{1}{12})^{1/2}(bca+cba+cab+acb-2abc-2bac)\equiv N^2; \end{aligned} \tag{A5}$$

$$A=(\tfrac{1}{6})^{1/2}(abc-bac+bca-cba+cab-acb)\ . \tag{A6}$$

To meet the bosonic statistics of preons, we must have

$$\text{space part}\times\text{ name part}=\text{symmetric.} \tag{A7}$$

There are three ways to meet the statistics condition for space wave functions (A1) and (A2):

$$\begin{aligned} U&=\frac{1}{\sqrt{2}}[Y'(12,3)M^1+Y''(12,3)M^2]\ ,\\ V&=\frac{1}{\sqrt{2}}[Y'(12,3)N^1+Y''(12,3)N^2]\ ,\\ W&=Y(12,3)S\ . \end{aligned} \tag{A8}$$

Their explicit forms are given in the text. The energies of states $U$ and $V$ are exactly the same as $E_0$. It is not certain, like the generation mechanism itself, whether $E_1>E_0=0$ or $E_0>E_1=0$. The latter possibility was taken in Ref. 19, while here we suppose $E_0=0$.

## APPENDIX B: GENERATIONS AND MIXINGS FOR THE INEQUALLY MASSIVE PREON SYSTEMS

Let us make the following wave functions by equal-mass preon bound-states solutions:

$$\psi_1^0=\frac{1}{\sqrt{2}}\sum_E A_E{}^1[Y'_{E|q|m}(12,3)M^1+Y''_{E|q|m}(12,3)M^2]\ , \tag{B1}$$

where the summation is over all possible states with the same parity, spin $|q|$, its $z$ component $m$, and $S_3$ properties. Coefficients $A_E{}^1$ are chosen to minimize the value

$$E_1^0\equiv\langle\psi_1^0|\hat{H}(M_T,M_V)|\psi_1^0\rangle=\text{minimum.} \tag{B2}$$

where $\hat{H}$ is that in Eq. (45). Similarly,

$$\psi_2^0=\frac{1}{\sqrt{2}}\sum_E A_E{}^2[Y'_{E|q|m}(12,3)N^1+Y''_{E|q|m}(12,3)N^2] \tag{B3}$$

and

$$E_2^0\equiv\langle\psi_2^0|\hat{H}(M_T,M_V)|\psi_2^0\rangle=\text{minimum} \tag{B4}$$

and

$$\psi_3^0=\sum_E A_E{}^3Y_{E|q|m}S\ , \tag{B5}$$

$$E_0{}^3\equiv\langle\psi_3^0|\hat{H}(M_T,M_V)|\psi_3^0\rangle=\text{minimum.} \tag{B6}$$

The mass matrix for down quarks is

$$M^d=\begin{bmatrix} E_1^0 & 0 & 0\\ 0 & E_2^0 & \beta\\ 0 & \beta^* & E_3^0 \end{bmatrix}, \tag{B7}$$

that for up quarks is

$$M^u=\begin{bmatrix} E_1^{0'} & 0 & 0\\ 0 & E_2^{0'} & \beta'\\ 0 & \beta^{*'} & E_3^{0'} \end{bmatrix}, \tag{B8}$$

where

$$M_{ij}=\langle \psi_i^0 \,|\, \hat{H}(M_T,M_V) \,|\, \psi_J^0 \rangle \ . \tag{B9}$$

As discussed in the text,

$$(\,|E_1^0|-|E_2^0|\,)(\,|E_1^{0'}|-|E_2^{0'}|\,)<0 \ . \tag{B10}$$

The way quarks enter the formula of $W^+$ should be

$$\frac{1}{\sqrt{3}}\sum_i \bar{\psi}_{id}^{\,0}\psi_{ju}^0 \ . \tag{B11}$$

Both (B8) and (B9) can be diagonalized by unitary matrices and the KM matrix may be obtained by the standard way,[37] although not a realistic one.

## APPENDIX C: MASSES OF NEUTRAL DUONS

The mass matrix can be written as (in the $M_T=M_V$ limit, $n=6$)

$$\begin{bmatrix} X+Y & Y & Y & \cdots \\ Y & X+Y & Y & \cdots \\ Y & Y & X+Y & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \quad (n\times n \text{ matrix}) \ . \tag{C1}$$

After diagonalization, it becomes

$$\mathrm{diag}[X+(n-1)Y,X,X,\ldots\,] \ , \tag{C2}$$

where $X$ is exactly the mass of $T^a\bar{V}^a$, if one does not consider the possibility that $T^a\bar{V}^a$ may become Goldstone bosons.

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