Kobayashi-Maskawa type of hard-*CP*-violation model with three-generation Majorana neutrinos

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Within the framework of the Kobayashi-Maskawa (KM) type of hard *CP*-violation model with three-generation Majorana neutrinos, we point out that on-shell *CP*-violation phenomena (i.e., *CP*-violating effects taking place in on-shell processes), which are characteristic of Majorana neutrinos, can only occur in total-lepton-number-conserving reactions, and are unobservably small. Off-shell *CP*-nonconserving effects which arise from gauge bosons are undetectable, but those which are mediated by Higgs bosons could be seen in certain rare decays. It is emphasized that *CP*-odd effects intrinsic to Majorana behavior depend not only on the two *CP*-violating Majorana phases but also on the KM phase. We then demonstrate why the KM model, which has rich implications in the hadronic sector, leads to no observable *CP*-violating effects in leptonic processes (except in neutrino oscillations) directly related to the *CP*-odd KM phase.

I. INTRODUCTION

Up to now, T symmetry (or equivalently CP symmetry, if CPT holds) in the leptonic sector has been tested only in the muon decay,[1] and in the electric dipole moment of the electron[2] and the muon.[3] Although these measurements yield null results, it is pertinent to ask what *CP*-violating effects in leptonic processes are expected from our present understanding of CP violation in hadronic systems and whether it is feasible to measure these effects experimentally.

In the Kobayashi-Maskawa (KM) model,[4] CP nonconservation in the six-quark sector arises from the complex charged-current gauge couplings induced from the complex Yukawa couplings. For some time, the task set for the KM model was to understand why the phase of the product of KM matrix elements $U_{ud}U_{cs}U_{cd}^*U_{us}^*$ was so small $\sim 10^{-3}$ in order to fit the *CP*-violating parameter ϵ. Nowadays, because of the smallness of the elements U_{cb} and U_{ub} inferred from the b-quark lifetime measurement, and of the B parameter of $K^0-\bar{K}^0$ (about $\frac{1}{3}$), it turns out that the magnitude of $|U_{us}U_{cb}U_{cs}^*U_{ub}^*|\sin\phi$, where ϕ is the phase of $U_{us}\cdots U_{ub}^*$, may be too small to explain ϵ even if the phase ϕ is large.[5] This, together with the too-large prediction for ϵ'/ϵ, may signal new physics of CP violation beyond the KM model. Nevertheless, fruitful *CP*-nonconservation phenomena in the KM model has been studied in various systems.[6]

Since neutrinos could have masses, it is natural to extend the KM model to leptons so that CP violation can be induced from the complex lepton mixings as in the quark case. In the spirit of the KM model, the masses of Majorana neutrinos are generated from spontaneous symmetry breaking; thus, additional Higgs fields are required in the theory. As a result, hard CP violation will not completely reside in the charged gauge interactions. Furthermore, for three-generation Majorana neutrinos there are three *CP*-violating phases in the leptonic sector: one is the usual KM phase, the other two are Majorana phases defined in Majorana self-conjugation conditions; the latter has received much attention in recent years.[7–12]

In Sec. II we discuss direct and indirect *CP*-violating effects characteristic of Majorana neutrinos based on the observation that physical quantities should not depend on the phase convention chosen for charged-lepton fields. Sections III and IV present arguments to explain why in contrast with the quark sector, the KM model does not lead to observable *CP*-violating effects directly related to the KM phase (except in neutrino oscillations). CP violation in neutrino oscillations is briefly discussed in Sec. V. Models which can generate large *CP*-odd phenomena are briefly surveyed in Sec. VI. Section VII comes to conclusions.

II. DIRECT AND INDIRECT *CP*-VIOLATING EFFECTS CHARACTERISTIC OF MAJORANA BEHAVIOR

To begin with, let us consider three-generation massive Majorana neutrinos. In the spirit of the KM model, masses of Majorana neutrinos are generated through spontaneous symmetry breaking as in the case of charged leptons, thus additional Higgs triplet and/or singlet are inevitably needed in the theory, whereas the lepton number can be either explicitly or spontaneously broken. Since now there are two or more Higgs fields which can develop vacuum expectation values (VEV's), CP violation can arise not only from the complex Yukawa couplings but also from the relative phases of VEV's. The latter possibility occurs if the lepton number is an explicitly broken symmetry; complex trilinear terms in the Higgs potential and hence complex VEV are allowed. Thereby, lepton number must be spontaneously broken in order to have the KM type of hard CP violation.[13] Owing to the additional complex Yukawa couplings coming from the interactions of leptons with the Higgs triplet and/or singlet, CP violation in the KM model with Majorana neutrinos will not reside completely in the charged gauge interactions. For example, in the Gelmini-Roncadelli model,[14] the same complex lepton mixing matrix in the gauge current will appear in the Yukawa interaction of the charged Higgs H^+, and ϕ^{++}, as elaborated recently in Ref. 12. In the model of Chikashige, Mohapatra, and Peccei,[15] one can have CP nonconservation in the neutral Yukawa couplings with Majorana neutrinos.

Suppose the Majorana mass eigenfields satisfy the Majorana self-conjugation conditions

$$\nu_i^c = \nu_i \quad (i=1,2,3) \ . \tag{1}$$

Then, in contrast with the quark case, the analogous leptonic unitary mixing matrix contains three CP-violating angles[7,8] which cannot be rotated away as the phases of Majorana fields are fixed by Eq. (1). The mixing matrix can be parametrized as

$$U=\begin{bmatrix} c_1 & s_1c_3e^{-i\alpha} & s_1s_3e^{i\beta} \\ -s_1c_2e^{-i\alpha} & c_1c_2c_3-s_2s_3e^{i\delta} & (c_1c_2s_3+s_2c_3e^{i\delta})e^{-i(\alpha-\beta)} \\ -s_1s_2e^{-i\beta} & (c_1s_2c_3+c_2s_3e^{i\delta})e^{i(\alpha-\beta)} & c_1s_2s_3-c_2c_3e^{i\delta} \end{bmatrix}, \tag{2}$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, and θ_i are the lepton mixing angles. The extra two CP-violating angles α and β are intrinsic to Majorana neutrinos as they can be absorbed as the Majorana phases in the redefined Majorana conditions.

$$\nu_1^c=\nu_1, \quad \nu_2^c=e^{2i\alpha}\nu_2, \quad \nu_3^c=e^{2i\beta}\nu_3 \ . \tag{3}$$

In this convention the unitary mixing matrix has only one CP-violating angle, the KM phase δ as in the quark case, and CP violation characteristic of Majorana behavior is hidden in the Majorana conditions (3) (Ref. 10). When CP is conserved, Majorana phases $\alpha,\beta=0$ or $\pi/2$ depending on the intrinsic CP parity of ν_2 and ν_3 relative to that of ν_1. *Therefore, even if CP is conserved, some of the matrix elements in Eq. (2) could be purely imaginary.* Although propagators of neutrinos and Feynman rules for vertices are different in the parametrizations (1) and (3), physical results should be the same.[10] In this paper we adopt the first parametrization.

As mentioned before, besides the gauge interaction, CP violation can also occur in the Yukawa interaction of the charged Higgs bosons and in the neutral Yukawa couplings with Majorana neutrinos. For our later purposes we just consider the former case since it will give some interesting CP-violating phenomena. In terms of the mass eigenstates, the relevant Yukawa interactions are given by[12]

$$\mathcal{L}_Y^{\text{charged}} = \left[\frac{\sqrt{2}}{u}\sin\alpha\,\bar{\nu}UD_l l_R + \frac{1}{v}\cos\alpha\,\bar{\nu}D_\nu U l_L \right] H^+ - \frac{1}{\sqrt{2}v}\bar{l}_R^c U^T D_\nu U l_L \phi^{++} + \text{H.c.} \ , \tag{4}$$

$$\cos\alpha = u/(u^2+2v^2)^{1/2} \ ,$$

where D_l and D_ν are diagonal mass matrices for charged leptons and neutrinos, respectively, $u \simeq 246$ GeV and v are vacuum expectation values of the Higgs-boson doublet and triplet, respectively, and the successful experimental relation $M_W = M_Z\cos\theta_W$ requires that $v \ll u$. Masses of the singly charged H^+ and doubly charged ϕ^{++} are given by[14]

$$m_\phi{}^2 \simeq 2m_H{}^2 = \tfrac{1}{2}\lambda_4(u^2+2v^2) \simeq \lambda_4/(2\sqrt{2}\,G_F) \ , \tag{5}$$

where λ_4 is the coefficient of one of the terms in the Higgs-boson potential which gives mass to both charged Higgs bosons.

CP-violating effects in the KM model with Majorana neutrinos can be classified into two categories depending on whether they are characteristic of Majorana behavior. In the first category, CP-violation phenomena are independent of CP-violating Majorana phases. Hence in this case not only the charged leptonic fields but also neutrino mass eigenfields are defined up to arbitrary phases. Since the observed physical quantities must be free of the rephasing ambiguity, it turns out that CP-odd effects due to complex mixings are proportional to the imaginary parts of the rephasing invariant[16]

$$I_{\gamma k} = U_{\alpha i}U_{\beta j}(U_{\alpha j}U_{\beta i})^* \quad (\alpha,\beta,\gamma \text{ and } i,j,k \text{ cyclic}) \tag{6}$$

or of the products of (6) (Ref. 17) where α (i) denotes the leptonic (neutrino) state. There are nine of the invariants $I_{\gamma k}$ and all of them have the same imaginary components (even the sign)

$$\text{Im}I = c_1c_2c_3s_1{}^2s_2s_3\sin\delta \tag{7}$$

owing to the unitarity of the KM matrix.[16]

By definition CP-nonconserving effects in the second class depend on the Majorana phases α and β. Unlike the previous case, the phases of Majorana particles are now fixed and the only phase freedom comes from the charged leptonic states. It turns out that the second-class rephasing invariant which contains Majorana phases and has the imaginary part is of the form

$$J_{\gamma k} = m_i m_j U_{\alpha i}U_{\beta j}^*(U_{\alpha j}U_{\beta i}^*)^* \quad (\alpha,\beta,\gamma \text{ and } i,j,k \text{ cyclic}) \ , \tag{8}$$

where the neutrino masses must appear to characterize the Majorana feature. (In Sec. IV we shall give an example of a rephasing invariant which involves six matrix elements.) Obviously, the imaginary parts of J vary from case to case. To illustrate CP-odd effects of the second class, let us consider the decay $\mu \to e\nu_1\nu_2$. Since a Majorana particle is identical with its antiparticle, there is an additional Feynman diagram, Fig. 1(b), whose interference with Fig. 1(a) gives rise to a term proportional to

$$m_1m_2U_{e1}U_{\mu 2}^*(U_{e2}U_{\mu 1}^*)^* \ , \tag{9}$$

where m_1 and m_2 come from the Majorana mass inser-

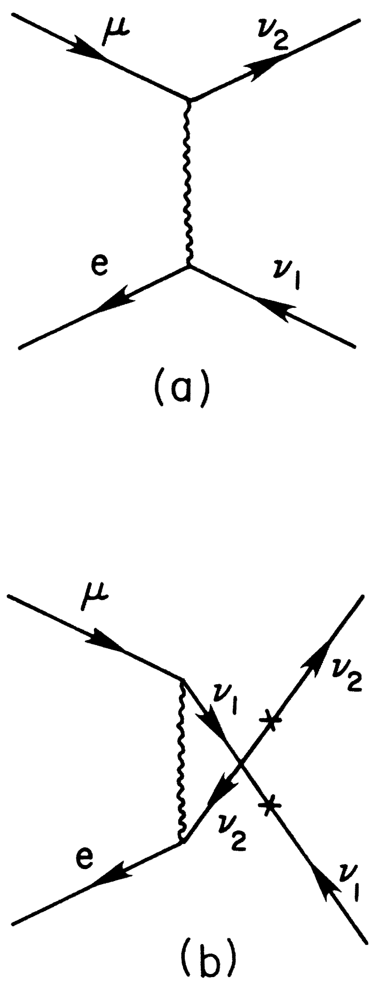

FIG. 1. Feynman diagrams for $\mu \to e \nu_1 \nu_2$. (b) occurs when neutrinos are Majorana particles.

tions necessary for the interference. The CP-violating effect associated with the imaginary part of (9) can be realized as a T-odd triplet-product correlation $\boldsymbol{\sigma}_\mu \cdot \mathbf{p}_\nu \times \boldsymbol{\sigma}_\nu$, where $\mathbf{p}_\nu$ and $\boldsymbol{\sigma}_\nu$ are the momentum and polarization vector, respectively, of the detected neutrino in the rest frame of the muon.[18] However, this T-odd correlation is suppressed by a factor of $(m_1 m_2 / m_\mu^2)$ with respect to other measurable quantities in $\mu \to e \nu_1 \nu_2$, and hence is ridiculously small.

From Eq. (8) we make the following observations.

(1) Direct second-class CP-violating effects take place only in the total-lepton-number-conserving processes, not in the $|\Delta L| = 2$ reactions. Moreover, they are suppressed by factors of $(m_\nu / m_l)^2$, so they are unobservably small.

(2) The KM phase δ appears in all rephasing invariants $J_{\gamma k}$. Therefore, *in contrast with the two-generation case, CP-violation phenomena intrinsic to Majorana neutrinos can occur even if Majorana phases are not genuine CP-violating phases.* This means that measurements of the second-class CP-odd effects, if not impossible, do not necessarily reflect a clean evidence of nontrivial Majorana phases being different from 0 or $\pi/2$. The only exceptional case is the decay $\mu \to e \nu_1 \nu_2$: if $c_2 c_3 \gg s_2 s_3$, the aforementioned triple-product correlation will be proportional to $\sin 2\alpha$, and hence gives a clean signal that α is a genuine CP-odd angle.

We have another second-class rephasing invariant:

$$K = \left| \sum_i m_i U_{\alpha i} U_{\beta i} \right|^2 \quad (|\Delta L| = 2) . \tag{10}$$

Since K is real, no direct CP-violating phenomena can be observed in the $|\Delta L| = 2$ processes (L being the total lepton number), which is consistent with the previous observation. In spite of this, indirect tests of CP violation can be provided by reactions involving $|\Delta L| = 2$ neutrino propagations. A well-known example is the neutrinoless double-β decay, where one measures the average neutrino mass:

$$\langle m_\nu \rangle = | m_1 c_1{}^2 + m_2 s_1{}^2 c_3{}^2 e^{2i\alpha} + m_3 s_1{}^2 s_3{}^2 e^{2i\beta} | . \tag{11}$$

We notice that both the KM phase and the mixing angle θ_2 do not emerge in Eq. (11). Another example is the (μ^-, e^+) conversion; the amplitude is proportional to

$$| m_1 c_1 c_2 - m_2 c_3 (c_1 c_2 c_3 - s_2 s_3 e^{i\delta}) e^{2i\alpha} - m_3 s_3 (c_1 c_2 s_3 + s_2 c_3 e^{i\delta}) e^{2i\beta} | . \tag{12}$$

In neutrino-antineutrino oscillations, the measurable quantity is similar to Eq. (7) except that an additional time-evolution factor $\exp(-iEt)$ is invoked for each neutrino mass eigenstate.

In general, it is also extremely difficult to tell whether or not the Majorana phases α and β are intrinsic CP-violating angles from the measurements of total-lepton-number-nonconserving processes. For instance, consider the $(\beta\beta)_{0\nu}$ decay, and suppose $\langle m_\nu \rangle < m_1$ and $\cos\theta_1 \sim 1$; this will not imply unambiguous CP violation with Majorana neutrinos since ν_2 and/or ν_3 with opposite CP parity to ν_1 can mimick the same result.[19] Furthermore, the KM phase appears in all $|\Delta L| = 2$ reactions except in neutrinoless double-β decays. Therefore, unless all mixing angles, neutrino masses, lepton-number-violating quantities, as well as the KM phase are well determined, it would be a formidable task to identify CP nonconservation special to Majorana neutrinos.

III. ON-SHELL *CP*-VIOLATING EFFECTS

In the previous section we have discussed CP-violating effects which are characteristic of Majorana neutrinos. They are of order $(m_\nu / m_l)^2$, and hence are unobservable in practice. In this section we focus on the first-class CP-odd effects. A crucial difference between the phenomena of CP violation in quark and leptonic systems is that the KM phase in the complex quark mixings can be observed only in the hadronic processes owing to quark confinement. This accounts for the main disparity between quarks and leptons in the KM model. Let us first briefly review what CP-violating phenomena are expected in the hadronic sector. CP violation can manifest itself in the mass matrix which mixes the neutral-meson pair P^0-$\bar{P}^0$, and in the decay amplitude, while T violation can exhibit itself in the electrical dipole moment and T-odd correlations. (Since the pseudoscalar meson does not carry spin, the T-odd correlation is usually measured in the semileptonic decays.) For leptons, unfortunately, there is no analogous K^0-$\bar{K}^0$ mixing, which is the only CP-violation phenomenon being seen thus far. The lepton's electric dipole moment (EDM) vanishes to two-loop diagram of order $G_F{}^2$ in the KM model, as in the case of the neutron.[20] But unlike the EDM of the neutron, leptons do not receive QCD corrections. As a result, CP-violating effects in the KM model with neutrinos can take place only in the decay amplitudes and in the T-odd correlations; both are measured by the following generic CP asymmetry resulting from the different interference effects between particle and antiparticle channels:

$$\Delta=\frac{\Gamma(l\to f)-\bar{\Gamma}(\bar{l}\to\bar{f})}{\Gamma(l\to f)+\bar{\Gamma}(\bar{l}\to\bar{f})}\ , \tag{13}$$

where $l(f)$ is the lepton (final) state, and $\bar{l}\to\bar{f}$ is the CP-conjugate channel of $l\to f$ and Γ denotes any observable quantity. A familiar example of CP asymmetry is the partial-decay-rate differences between particle and antiparticle. The T-odd correlation is normally measured by the differential-cross-section difference. For instance, the transverse polarization of the electron perpendicular to the plane of $\mathbf{p}_e$ and σ_μ, say the xy plane, in $\mu\to e\nu\bar{\nu}$ decays is a signal of T violation. In this case, $\Gamma(\bar{\Gamma})$ denotes the differential cross section along the $z(-z)$ direction. It is known that in order to achieve CP asymmetry there must exist two or more amplitudes with different weak-interaction phases and different nonweak-interaction phases which are not affected by CP inversion. For simplicitly we write

$$A(l\to f)=g_1M_1+g_2M_2\ , \tag{14}$$

where g_i are generic weak couplings. With CPT theorem it follows that

$$|A(l\to f)|^2-|\bar{A}(\bar{l}\to\bar{f})|^2=4\,\mathrm{Im}(g_1^*g_2)\mathrm{Im}(M_1M_2^*)\ . \tag{15}$$

This means that unless the amplitudes M_1 and M_2 have a nontrivial phase difference, the CP-odd terms in $|A(l\to f)|^2$ are canceled by that in $|\bar{A}(\bar{l}\to\bar{f})|^2$. The nonweak-interaction phases can occur in the following ways.[21]

(1) Final-state interactions. As we know, isospin phase shifts for final-state elastic scattering amplitudes are required by unitarity of the S matrix. Final-state interactions are particularly troublesome for heavy-meson decays due to resonances and rescattering effects. For weak decays of leptons, the final-state interaction is of the electromagnetic type, and the desired absorptive part comes from the vertex loop diagram. The electromagnetic-interaction phase for the final state is small and is at most of the order of α/π (α being the fine-structure constant).

(2) Absorptive part attributed from the Feynman loop integral. A perfect example is the timelike penguin diagram [Fig. 3(c)] which we shall discuss later. The final-state interaction for leptons actually belongs to this category.

(3) The time-evolution factor $\exp(-iEt)$. This happens, for example, when an initial weak eigenstate is prepared at $t=0$, and the decomposed mass eigenstates then oscillate with different time evolution factors.

It is easily seen that in order to have on-shell CP asymmetries (i.e., CP-violation phenomena in on-shell processes or at the tree level), it is necessary to consider leptonic decays involving at least two cascade sequences: namely, five-body decays, so that each tree graph contains four KM matrix elements. As an example, consider the decay $\tau^-\to\mu^-\mu^+e^-\nu\bar{\nu}$; the interference of Fig. 2(b) with Fig. 2(a) has a weak coupling given by Eq. (7). This decay is however dominated by $\tau\to e\nu\bar{\nu}\gamma$ followed by a muon-antimuon pair production from the virtual photon. As a result, the CP asymmetry in this example is estimated to be of the order of

$$\Delta\sim\left[\frac{\alpha}{4\pi G_F{}^2m_\tau{}^4}\right]^{-1}\mathrm{Im}I\times(\text{relative phase due to electromagnetic interaction})\ . \tag{16}$$

How large is $\mathrm{Im}I$ as given by Eq. (7)? The analogous quantity in the quark sector is about[5] 10^4. In the leptonic system, it is found to be $\lesssim10^{-5}$ in the Fritzsch model of the lepton mixing matrix.[22] Even if the relative phase attributed from the electromagnetic final-state interactions

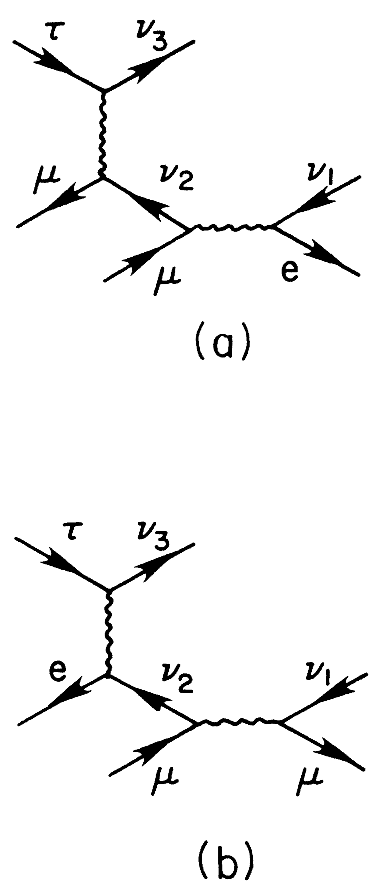

FIG. 2. Two Feynman diagrams for $\tau\to e\mu\bar{\mu}\bar{\nu}_1\nu_3$ which can generate a relative CP-violating phase among them.

is taken to be $\sim\alpha/\pi$, and $\mathrm{Im}I$ is assumed to be the optimistic value $\sim 10^{-5}$, it still turns out that

$$\Delta \sim 10^{-13}$$

which is too small to be detected; other five-body decay modes are no better than this. We thus conclude that the on-shell CP asymmetry first occurs in five-body decays of leptons, but it is practically an academic problem.

The situation is drastically changed in hadronic systems. To illustrate this, let us consider the exclusive two-body decays $B_c^- \to K^- \bar{D}^0$, in analogue to lepton three-body decays. The W-annihilation diagram [Fig. 3(b)], which has no counterpart in the leptonic sector, has a weak phase different from that of the spectator diagram, Fig. 3(a). Since the form factors of W annihilation contain an absorptive part due to resonances,[23] it is clear that a partial-rate asymmetry in $B_c^- \to K^- \bar{D}^0$ versus $B_c^+ \to K^+ D^0$ can result from the interference between the two tree diagrams: Figs. 3(a) and 3(b) (Ref. 24). Moreover, the timelike penguin diagram, Fig. 3(c), where the momentum squared k^2 of the virtual gluon is ≥ 0, has the desirable absorptive part when $k^2 \geq 4m^2$ (m being the quark mass in the loop).[25] Again, the interference of this penguin diagram with Fig. 3(a) will give a noticeable CP asymmetry in the partial-rate difference, as elaborated in great detail in Ref. 26. It has been estimated by Chau and the author that CP-violating effects in partial-rate asymmetries in some two-body decay channels of B systems can be as large as 10% (Ref. 26), and that the number of events[27] needed to see this sort of CP asymmetry is about 10^6–10^7, which is not unaccessible in the future experiments. Large CP asymmetries can also be generated in certain five-body decays of the b quark, as discussed in Ref. 28.

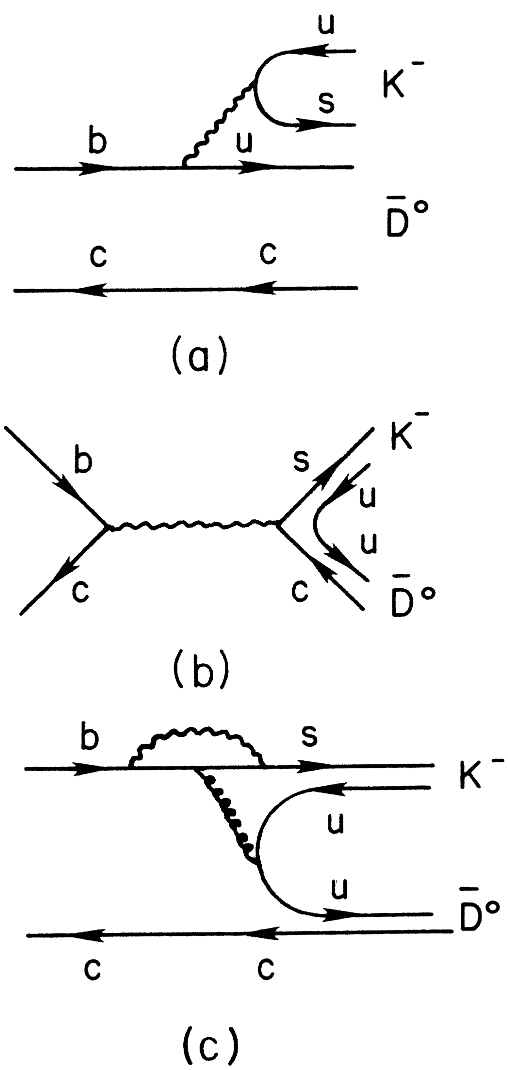

FIG. 3. Quark diagrams for $B_c \to K^- \bar{D}^0$. (a), (b), and (c) are, respectively, the spectator, W-annihilation, and timelike penguin diagrams.

IV. OFF-SHELL CP-VIOLATING EFFECTS

In order to manifest the KM phase in three- or four-body decays of leptons, obviously one has to consider the loop effect so that all three neutrinos or charged leptons will get involved. A nice example is the decay $\mu^- \to e^- e^- e^+$; the interference of the magnetic-dipole amplitude [Fig. 4(a)] with the box diagram, Fig. 4(b) gives

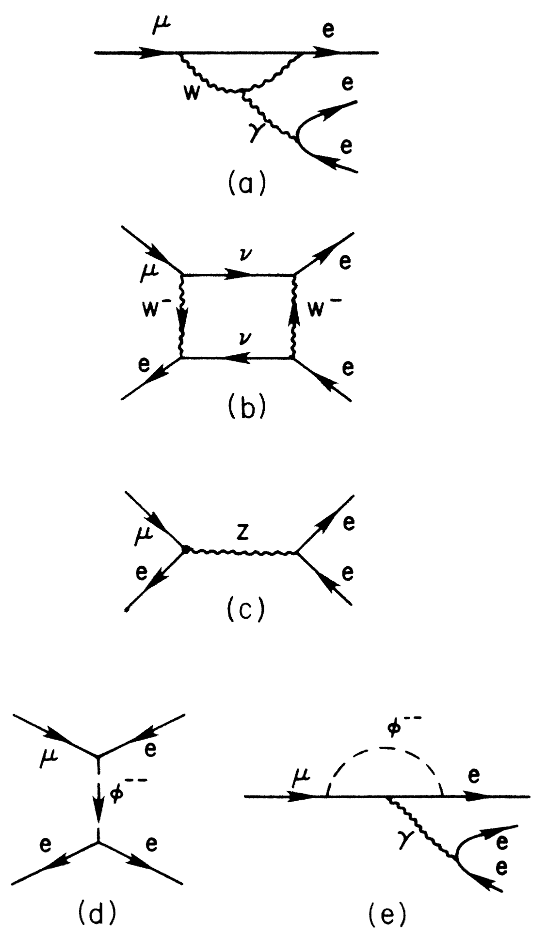

FIG. 4. Feynman diagrams for $\mu \to 3e$. In the case of Dirac neutrinos, only the first three diagrams contribute. For Majorana neutrinos, there are additional contributions from (d) and (e), as well as (a)–(c) with W^+ replaced by H^+ and ϕ^{++}.

rise to a T-odd triplet-product correlation $\sigma_\mu \cdot \mathbf{p}_1 \times \mathbf{p}_2$, where $\mathbf{p}_1$ and $\mathbf{p}_2$ are the momenta of the identical electrons.[29] Also, when CP is conserved, the radiation in the radiative decay $\nu_2 \to \nu_1 + \gamma$ will be of pure electric (magnetic) dipole when Majorana neutrinos ν_2 and ν_1 have the same (opposite) CP parity.[30] In other words, a simultaneous presence of $E1$ and $M1$ radiation is not allowed by CP invariance if neutrinos are of Majorana type.

The lepton-family-number-violating reactions due to charged gauge interactions are, however, severely suppressed by the Glashow-Iliopoulus-Maiani (GIM) mechanism[31] and by the smallness of neutrino masses. The GIM suppression factor for Fig. 4(a) is given by

$$\left| \sum_i U_{ei}^* U_{\mu i} m_i^2 \right| \Big/ M_W^2 \tag{17}$$

while for the box diagram, Fig. 4(b), it is

$$\left| \sum_i U_{ei}^* U_{\mu i} m_i^2 \ln(M_W^2/m_i^2) \right| \Big/ M_W^2 . \tag{18}$$

Because of the unitarity of the mixing matrix, the Glashow-Iliopoulus-Maiani (GIM) cancellation will be complete if all neutrinos have the same masses. For $m_i \sim 10$ eV, the loop amplitude is thus suppressed by at least a factor of 10^{20}. Consequently, the branching ratio of the rare decay, say $\mu \to e\gamma$, is hopelessly small $< 10^{-43}$. Moreover, the CP-violating effect in $\mu \to 3e$ is suppressed by the smallness of the relative weak phase between (17) and (18); it vanishes to the leading approximation that $\ln(M_W/m_i)$ is a constant independent of the neutrino states, as noticed in Ref. 29. For $\nu_2 \to \nu_1 + \gamma$, the GIM suppression factor becomes

$$\left| \sum_l U_{l2} U_{l1}^* m_l^2 \right| \Big/ M_W^2 \tag{19}$$

which is much better than the previous case, as neutrino particles are replaced by charged leptonic states. Even so, the lifetime of radiative decay through the W-loop diagram is too long. Choosing (100 MeV)^2 for the factor in parentheses in (19), the standard calculation leads to[30,32]

$$\tau \sim 10^{35} \text{ yr} \tag{20}$$

for $m_2 = 15$ eV, $m_1 = 0$, which is almost 20 orders of magnitude longer than the radiative lifetime, $\tau < 10^{16}$ yr, as suggested by the astrophysical observations.[32] So we conclude that off-shell CP-violating effects (i.e., CP-violating phenomena through loop effects) which arise from charged gauge currents are unobservable.

For Majorana-type neutrinos, a dramatic effect can happen in the presence of charged Higgs bosons. From Eq. (4) we see that the Yukawa coupling of ϕ^{++} and H^+ has a term proportional to m_ν/v, which is not necessarily small owing to the astrophysical constraint[33] $v < 9$ keV. As a result, the GIM suppression factor for Fig. 4(e) and Fig. 4(a) with W^+ replaced by H^+ is the same as Eq. (17) except that $v\lambda_4^{1/2}$ is in place of M_W, which is larger than the suppression factor (17) by a factor of $(M_W^2/\lambda_4 v^2) \gtrsim 10^{14}$. As for the $\mu \to 3e$ decay, it can even proceed through the tree diagram mediated by the doubly charged ϕ^{++} [Fig. 4(d)] (Ref. 34). The amplitude of $\mu^- \to e_1^- + e_2^- + e_3^+$ from Figs. 4(d), 4(e), and 4(a) with W^+ replaced by H^+ is given by[35]

$$A(\mu \to 3e) = 4G_F \left[\alpha \bar{e}_{3R}^c \mu_L \bar{e}_{1L} e_{2R}^c + \frac{\beta}{q^2} \bar{e}_1 \sigma_{\mu\nu} q^\nu \left[m_\mu \frac{1+\gamma_5}{2} + m_e \frac{1-\gamma_5}{2} \right] \mu \bar{e}_3^c \gamma^\mu e_2^c - (1 \leftrightarrow 2) \right] , \tag{21}$$

where

$$\alpha = \frac{1}{2\sqrt{2}\lambda_4} \frac{M_{ee}^* M_{e\mu}}{v^2} , \tag{22}$$

$$\beta = \frac{5}{6\sqrt{2}\lambda_4} \left[\frac{\alpha}{4\pi} \right] \frac{\langle m_\nu^2 \rangle_{e\mu}}{v^2} , \tag{23}$$

$$M_{\alpha\beta} = \sum_i U_{\alpha i} U_{\beta i} m_i , \quad \langle m_\nu^2 \rangle_{e\mu} = \sum_i U_{ei}^* U_{\mu i} m_i^2 . \tag{24}$$

q_μ is the four-momentum of the virtual photon, λ_4 is the parameter explained in Sec. II, and use has been made of

$$\bar{e}^c \gamma_\mu e^c = -\bar{e} \gamma_\mu e . \tag{25}$$

In Eq. (21), α is obtained from Eq. (4) and the result for β is taken over from Ref. 36. From the current experimental bounds $B(\mu \to 3e) < 2.4 \times 10^{-12}$ (Ref. 37) and $B(\mu \to e\gamma) < 1.7 \times 10^{-10}$ (Ref. 38), we obtain

$$|\alpha| < 8 \times 10^{-7}, \quad |\beta| < 9 \times 10^{-8} . \tag{26}$$

This together with Eqs. (22), (23), and the bound on v yields

$$\begin{aligned} |M_{ee}^* M_{e\mu}| &< 2 \times 10^2 \lambda_4 \ \text{eV}^2 , \\ |\langle m_\nu^2 \rangle_{e\mu}| &< 2 \times 10^4 \lambda_4 \ \text{eV}^2 . \end{aligned} \tag{27}$$

It is then straightforward to show that the interference which arises from Eq. (21) gives a T-violating correlation $\text{Im}(\alpha\beta^*) \sigma_\mu \cdot \mathbf{p}_1 \times \mathbf{p}_2$. CP asymmetry in this correlation is thus of the form

$$\Delta \sim \text{Im}(\alpha\beta^*)/|\alpha|^2 \tag{28}$$

and is proportional to the imaginary part of

$$L = \left[\sum_i U_{ei}^2 m_i \right] \left[\sum_j U_{ej}^* U_{\mu j}^* m_j \right] \left[\sum_k U_{ek}^* U_{\mu k} m_k^2 \right] \tag{29}$$

which is a second-class rephasing invariant involving six mixing matrix elements. Consequently, not only the KM CP-odd angle but also Majorana phases are responsible for the T-violating correlation in $\mu \to 3e$. The prediction of Δ is, however, hampered by our lack of knowledge of mixing angles, neutrino masses, and CP-violating phases. Nevertheless, if this CP asymmetry turns out not unreasonably small, one could have a chance to probe *CP*-violating effects in $\mu\to 3e$, $\tau\to 3e$, $3\mu, ee\mu, e\mu\mu$, as these rare decays can proceed through the charged-Higgs-boson exchanges.

How about the radiative decay $\nu_2\to\nu_1+\gamma$? The contribution from the charged Higgs boson H^+ proportional to $\cos^2\alpha$ term [see Eq. (4)] (Ref. 39) has a GIM factor

$$\frac{\sum_l U_{l2}U_{l1}^* m_l^2(m_1 m_2/m_l^2)}{\lambda_4 v^2} . \tag{30}$$

Comparing this with Eq. (19), it is evident that the enhancement factor $(M_W^2/\lambda_4 v^2)$ for $\mu\to 3e$ is despoiled here by the small neutrino masses, $(m_1 m_2/m_l^2)$. Even with a light H^+ (from DESY PETRA experiments, $m_H > 21$ GeV, this corresponds to $\lambda_4 > 1.4\times 10^{-2}$), the enhancement of $\nu_2\to\nu_1+\gamma$ due to the H^+ exchange is far too small to explain the observed radiative lifetime.[32] *CP* violation in $\nu_2\to\nu_1+\gamma$ is therefore undetectable within the framework of the KM model.

V. NEUTRINO OSCILLATIONS

Thus far, it appears that in the KM model the only promising place to see *CP*-violating effects lies in the rare decays mediated by charged Higgs; indirect *CP* nonconservation can be tested, although it is difficult, in the $|\Delta L|=2$ processes. In all cases, the KM phase always emerges together with the phases α,β intrinsic to Majorana neutrinos. So, can we conclude that there is no *CP*-violating phenomenon directly related to the KM *CP*-odd angle? Recall that in Sec. III it is pointed out that a time-evolution factor $\exp(-iEt)$ can provide the nonweak-interaction phases necessary for generating *CP* asymmetry. This is realized in neutrino-oscillation experiments[40] where one measures the time-dependent transition asymmetry

$$a(t)=\frac{P(\nu_\alpha(t)\to\nu_\beta)-P(\overline{\nu}_\alpha(t)\to\overline{\nu}_\beta)}{P(\nu_\alpha(t)\to\nu_\beta)+P(\overline{\nu}_\alpha(t)\to\overline{\nu}_\beta)} \tag{31}$$

with weak eigenstates ν_α and $\overline{\nu}_\alpha$ prepared at $t=0$. Although the result for (31) is pretty well known,[41] we still give a short and different derivation. Writing

$$A(\nu_\alpha\to\nu_\beta)=\delta_{\alpha\beta}+\sum_i U_{\alpha i}U_{\beta i}^*[\exp(-i\Delta_{ik}-1)] , \tag{32}$$

where $\Delta_{ij}=\frac{1}{2}(\delta m^2)_{ij}t/E$, $(\delta m^2)_{ij}=m_i^2-m_j^2$, it follows immediately that

$$\begin{aligned} P(\alpha\to\beta)-P(\overline{\alpha}\to\overline{\beta}) &= 4\sum_k \mathrm{Im}I_{\gamma k}\sin\Delta_{ij} \quad (\alpha,\beta,\gamma \text{ and } i,j,k \text{ cyclic}) \\ &= 4\,\mathrm{Im}I(\sin\Delta_{12}+\sin\Delta_{23}+\sin\Delta_{31}) , \end{aligned} \tag{33}$$

where use has been made of Eqs. (6) and (7). The empirical result uncovered in Ref. 41 that the off-diagonal probability differences are identical in all three available channels $\nu_e\to\nu_\mu$, $\nu_\mu\to\nu_\tau$, and $\nu_\tau\to\nu_e$ has a simple explanation here: it is attributed to the fact that *CP*-violating effects must be rephasing-invariant and the mixing matrix is unitary.

Unlike the aforementioned *CP*-odd phenomena, *CP*-violating effects in neutrino oscillations could be significant since the smallness of δm^2 can be offset by an appropriate choice of t/E. Therefore, in the KM model of *CP* violation with three-generation neutrinos, neutrino oscillation is the sole place that *CP* nonconservation can appreciably manifest itself directly through the KM phase.

In contrast with the leptonic sector, the KM model has rich phenomenological implications in hadronic systems even in the absence of the P^0-$\overline{P}^0$ mixing (i.e., no *CP* violation in the mass matrix). Owing to the $\Delta S=1$ penguin diagram, various *CP* asymmetries are expected in nonleptonic decays of hyperons,[42] charged kaons,[43] and in tagged neutral kaon beams. In particular, large *CP* asymmetries can be seen in the partial-rate differences in two-body decays of charged and neutral B mesons since there *CP*-violating phases can occur directly "on shell."

VI. OTHER MODELS

The observation of any *CP*-violating effects in leptonic processes which cannot manifest themselves in the KM model, such as the electric dipole moment and the T-odd transverse polarization of the electron in μ decay, will indicate new physics of *CP* violation beyond the KM model. Hence, it is worthwhile to briefly survey and discuss models which can exhibit large *CP*-nonconserving effects. The salient features of this sort of models are the following: (a) it contains a gauge boson or a scalar field which couples to both left-handed and right-handed leptons with different phases and (b) it should lead to *CP*-violation phenomena which do not vanish with neutrino masses.

We illustrate these features by considering the electric dipole moment of the electron d_e. It is easily seen that *if the first requirement is not satisfied,* d_e *will vanish to one-loop diagram.* At the one-loop level, a nonvanishing d_e is always proportional to the mass of the leptonic state in the loop since a mass insertion is necessary to match chiralities. In order to obtain a large d_e, it is obvious that the lepton in the loop cannot be the light neutrino state; this can be accomplished via the interactions of leptons with the following: (1) a neutral gauge boson Y^0 [Fig. 5(a)] (an example of this is the horizontal gauge model[44]); (2) a neutral scalar [Fig. 5(b)] or a doubly charged scalar field (examples are Lee's model[45] and Zee's model[13]); (3) a right-handed charged gauge boson which mixes with W_L [Fig. 5(c)] as advocated by the left-right-symmetric model.[46] If natural flavor conservation is not imposed to models (1) and (2), it is possible to obtain a large d_e from the τ intermediate state. In case (3), d_e receives dominant contributions from the heavy Majorana neutrino state.

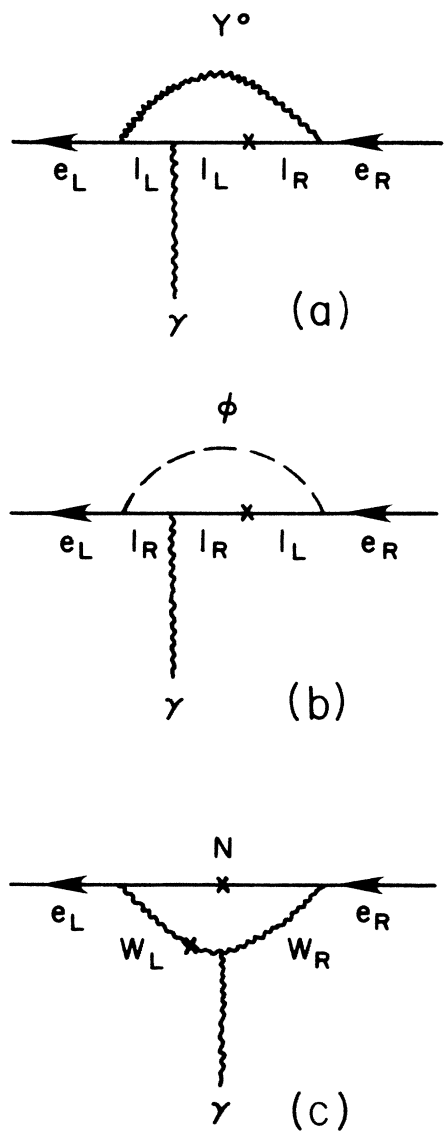

FIG. 5. Electron's electric dipole moment generated at the one-loop level in various models.

For further detailed calculations of the EDM and other *CP*-violating effects in leptonic systems within the framework of various gauge models of *CP* nonconservation, the reader is referred to Ref. 47.

Right-handed currents necessary for a large lepton's electric dipole moment are also inevitable for manifesting *CP*-violating effects which are not allowed in the KM model. It was pointed out in Refs. 18 and 32, respectively, that models with right-handed currents can introduce a *T*-odd correlation in the muon decay and account for the radiative lifetime of $\nu_2 \rightarrow \nu_1 + \gamma$. Finally, it should be stressed that in most models of *CP* violation, additional Higgs-boson multiplets need to be introduced. Consequently, unless there is a principle or symmetry guarantees relative real vacuum expectation values, the nature of *CP* violation in general is not "hard."

VII. CONCLUSIONS

We have studied the extension of the KM model of *CP* violation to three-generation lepton families. If neutrinos are of Majorana type, the leptonic mixing matrix contains the *CP*-odd phases: one KM phase and two Majorana phases defined in Majorana self-conjugation conditions. In the spirit of the KM model, additional Higgs multiplets are needed to generate the masses for Majorana neutrinos. As a result, *CP* violation will not reside completely in the charged gauge interactions. In summary our conclusions are the following.

(1) Direct *CP*-nonconserving effects characteristic of Majorana behavior can only occur in total-lepton-number-conserving $|\Delta L| = 0$ processes; they are suppressed by factors of $(m_\nu/m_l)^2$, and hence are unobservably small.

(2) Indirect *CP* violation intrinsic to Majorana neutrinos can be tested in $|\Delta L| = 2$ reactions, such as $(\beta\beta)_{0\nu}$ decays and (μ^-, e^+) conversion. Since *CP*-violating phases all appear together in both $|\Delta L| = 0$ and 2 transitions, a clean evidence of Majorana phases being genuine *CP*-odd angles generally requires that all mixing angles, neutrino masses, as well as the KM phase be well determined.

(3) No on-shell *CP*-nonconservation phenomena (i.e., *CP*-violating effects taking place in on-shell processes) can be detected in leptonic systems excepted in neutrino oscillations. By contrast, large *CP* asymmetries can exhibit in nonleptonic decays of *B* mesons even in the absence of $B^0 - \bar{B}^0$ mixing.

(4) Neutrino oscillation is the only place where *CP* violation can appreciably manifest itself through the KM phase. Therefore, except in neutrino oscillations, *CP* violation in the manner of Kobayashi and Maskawa will be practically just of academic interest if neutrinos are Dirac particles.

(5) Off-shell *CP*-violating effects which arise from charged gauge bosons are undetectable; however, those which are mediated by Higgs bosons could be seen in certain rare decays. *CP* violation in the radiative decay $\nu_2 \rightarrow \nu_1 + \gamma$ is undetectable in this model. In hadronic systems, various off-shell *CP* asymmetries in nonleptonic decays of hyperons and charged kaons are predicted in the KM model.

(6) The observation in the future of *CP*-nonconserving effects, which are unobservably small in the KM Model, such as the electric dipole moment of the electron or a *T*-odd correlation in the muon decay, will indicate new physics of *CP* violation beyond the KM model. Indeed, there already exists such a hint in the K^0-$\bar{K}^0$ system. We have briefly surveyed models which can exhibit large *CP*-violating effects in leptonic systems.

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[1]F. Corriveau *et al.*, Phys. Lett. **129B**, 260 (1983).
[2]M. A. Player and P. G. H. Sandars, J. Phys. B 3, 1620 (1970); P. G. H. Sandars and R. M. Sternheimer, Phys. Rev. A **11**, 473 (1975); T. G. Vold *et al.*, Phys. Rev. Lett. **52**, 2229 (1984). A new experiment which is sensitive to the level $d_e \sim 10^{-28}$ e cm is discussed by W. Bialek, J. Moody, and F. Wilczek, Phys. Rev. Lett. **56**, 1623 (1986).
[3]J. Bailey *et al.*, Phys. Lett. **68B**, 191 (1977); J. Phys. G **4**, 345 (1978).
[4]M. Kobayashi and K. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[5]For a review, see P. Langacker, in *Proceedings of the 1985 International Symposium on Lepton and Photon Interactions at High Energies, Kyoto,* edited by M. Konuma and K. Takahashi (Research Institute for Fundamental Physics, Kyoto University, Kyoto, 1985), p. 186; A. J. Buras, in *Proceedings of the International Europhysics Conference on High-Energy Physics, Bari, 1985,* edited by L. Nitti and G. Preparata (Laterza, Bari, 1985), p. 1035.
[6]For a summary, see L. L. Chau, Prog. Theor. Phys. Suppl. **85**, 147 (1986).
[7]S. M. Bilenky, J. Hosek, and S. T. Petcov, Phys. Lett. **94B**, 495 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D **23**, 1666 (1981).
[8]M. Doi *et al.*, Phys. Lett. **102B**, 323 (1981).
[9]J. Bernabeu and P. Pascual, Nucl. Phys. **B228**, 21 (1983).
[10]A. Barroso and J. Maalampi, Phys. Lett. **132B**, 355 (1983).
[11]C. W. Kim and H. Nishiura, Phys. Rev. D **30**, 1123 (1984); H. Nishiura, Phys. Lett. **157B**, 442 (1985).
[12]J. Bernabéu, A. Pich, and A. Santamaría, Z. Phys. C **30**, 213 (1986).
[13]Of course, one can also construct a hard *CP*-violation model of non-KM type. An example is the model of A. Zee [Phys. Rev. Lett. **55**, 2382 (1985)] in which he introduces a doubly charged scalar singlet and a scalar triplet which has no vacuum expectation value. Neutrino masses in this model are generated at the one-loop level.
[14]G. B. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981).
[15]Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Lett. **98B**, 265 (1981).
[16]D.-d. Wu, Phys. Rev. D **33**, 860 (1986); C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); O. W. Greenberg, Phys. Rev. D **32**, 1841 (1985).
[17]In the quark sector, this can happen in *CP* asymmetries in five-body decays of the b quark. Hence, not all *CP*-violating effects are proportional to $\mathrm{Im}I$.
[18]M. Doi *et al.*, Prog. Theor. Phys. **67**, 281 (1982).
[19]L. Wolfenstein, Phys. Lett. **107B**, 77 (1981).
[20]E. P. Shabalin, Sov. J. Nucl. Phys. **31**, 864 (1980). It is easy to check that both charged Higgs bosons H^+ and ϕ^{++} do not induce the lepton's electric dipole moment at the one-loop level.
[21]For a review, see W. Y. Keung, in *Proceedings of the Oregon Meeting,* Annual Meeting of the Division of Particles and Fields of the American Physical Society, Eugene, 1985, edited by R. C. Hwa (World Scientific, Singapore, 1986), p. 207.
[22]J. L. Hewett and T. G. Rizzo, Phys. Rev. D **33**, 1519 (1986); see also A. Bottino, C. W. Kim, H. Nishiura, and W. K. Sze, *ibid.* **34**, 862 (1986).
[23]Another way to see a nontrivial strong-interaction relative phase between Figs. 3(a) and 3(b) is based on the observation that the final state $K^- \bar{D}^0$ in W annihilation must be in the $I=0$ state, whereas the same final state in Fig. 3(a) can be in $I=0$ and 1 states.
[24]J. Bernabeu and C. Jarlskog, Z. Phys. C **8**, 233 (1981).
[25]M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).
[26]L. L. Chau and H. Y. Cheng, Phys. Rev. Lett. **53**, 1037 (1984); Phys. Lett. **165B**, 429 (1985).
[27]This *CP* asymmetry is in general measured by $\mathrm{Im}I$/(mixing angles of the spectator diagram). At first sight, one may argue that it is much easier to detect large *CP*-violating effects in the KM-angle doubly suppressed B-decay channels. However, this is offset by the number of events, $n \gtrsim (B\Delta^2)^{-1}$ needed for measuring partial-decay-rate differences, where B is the branching ratio.
[28]A. B. Carter and A. I. Sanda, Phys. Rev. D **23**, 1567 (1981).
[29]S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D **16**, 152 (1977).
[30]P. Pal and L. Wolfenstein, Phys. Rev. D **25**, 766 (1982); J. Nieves, *ibid.* **26**, 3152 (1982).
[31]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).
[32]J. Maalampi, K. Mursula, and M. Roos, Phys. Rev. Lett. **56**, 1031 (1986).
[33]D. S. P. Dearborn, D. N. Schramm, and G. Steigman, Phys. Rev. Lett. **56**, 26 (1986).
[34]Heavy neutrinos in the model of Ref. 15 do not help to enhance $\mu \to 3e$ because of the very small admixture of ν_L.
[35]This is to be compared with Eq. (A1) of Ref. 29.
[36]A Pich, A. Santamaría, and J. Bernabeú, Phys. Lett. **148B**, 229 (1984).
[37]W. Bertl *et al.*, Nucl. Phys. **B260**, 1 (1985).
[38]Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984).
[39]Because of the stringent constraint on $v < 9$ keV (Ref. 33), the term proportional to $\sin\alpha\cdot\cos\alpha$ is no longer the dominant contribution, as advocated in Ref. 30.
[40]The first discussion of *CP* violation in neutrino oscilations was due to N. Cabibbo, Phys. Lett. **72B**, 333 (1978).
[41]V. Barger, K. Whisnant, and R. J. N. Phillips, Phys. Rev. Lett. **45**, 2084 (1980); see also S. Pakvasa, in *High Energy Physics—1980,* proceedings of the XX International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP Conf. Proc. No. 68) (AIP, New York, 1981), p. 1164.
[42]L. L. Chau and H. Y. Cheng, Phys. Lett. **131B**, 202 (1983); T. Brown, S. F. Tuan, and S. Pakvasa, Phys. Rev. Lett. **51**, 1823 (1983).
[43]C. Avilez, Phys. Rev. D **23**, 1124 (1981); H. Y. Cheng, Phys. Lett. **129B**, 357 (1983); B. Grinstein, S. J. Rey, and M. B. Wise, Phys. Rev. D **33**, 1495 (1986).
[44]M. B. Gavela and H. Georgi, Phys. Lett. **119B**, 141 (1982).
[45]T. D. Lee, Phys. Rev. D **8**, 1226 (1973); Phys. Rep. **9C**, 148 (1974). An analogous Lee's model of *CP* violation in the leptonic sector has been constructed by C. O. Escobar and V. Pleitez, Ann. Phys. (N.Y.) **146**, 289 (1983).
[46]R. N. Mohapatra, and J. C. Pati, Phys. Rev. D **11**, 566 (1975); G. Senjanovic and R. N. Mohapatra, *ibid.* **12**, 1502 (1975); J. W. F. Valle, Phys. Lett. **138B**, 155 (1984); J. F. Nieves, D. Chang, and P. B. Pal, *ibid.* **33**, 3324 (1986).
[47]H. Y. Cheng, Phys. Rev. D **28**, 150 (1983).