$D^0 \to \phi \bar{K}^0$ decay in the algebraic approach to nonleptonic weak decays

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Contrary to the expectation of the naive spectator model, $D^0 \to \phi \bar{K}^0$ decay has recently been observed with an appreciable branching ratio. In this paper, it is however pointed out that the sizable rate of $D^0 \to \phi \bar{K}^0$ decay is, in fact, no surprise in the newly developed algebraic approach to nonleptonic weak interactions and it can be estimated from the well-known rate of $K_S \to 2\pi$ decays. The value of $\Gamma(D^0 \to \phi \bar{K}^0)$ thus obtained is in good agreement with experiments. The approach uses a hard-pseudoscalar-meson extrapolation instead of a soft one and deals with long-distance physics in earnest, by using the algebraic method developed in the framework of asymptotic flavor symmetry which nevertheless maintains a close connection with quark-line diagrams.

I. INTRODUCTION

According to recent experiments,[1] the decay $D^0 \to \phi \bar{K}^0$, which proceeds only through the W-exchange diagram[2] in the naive quark model, has a sizable decay rate. In the traditional QCD-inspired approach, this fact implies that the color suppression expected in the perturbative approach[3] does not work in quasi-two-body decay as well as for the $D^0 \to \bar{K}^0 \pi^0$ and the W-exchange diagram (Fig. 1) has to play a significant role.[4] However, the naive W-exchange diagram without nonperturbative gluon corrections will undergo helicity suppression as in the $\pi \to e\nu$ decay. Therefore, it seems that nonleptonic decays of hadrons (at least of D^0) have to be treated in a nonperturbative way. On the other hand, the experimental fact[5,6] that the $D^0 \to K^0 \bar{K}^0$ decay, which can occur also predominantly through the W-exchange diagram, is suppressed relative to the $D^0 \to K^+ K^-$ decay although both of them are Cabibbo-angle-suppressed decays suggests that the cancellation of the amplitudes expected under $SU_f(3)$ symmetry works in the former decay. However, if this is the case, it is not easy to explain the experimental data,[7] $\Gamma(D^0 \to K^+ K^-)/\Gamma(D^0 \to \pi^+ \pi^-) \simeq 4$, a well-known puzzle

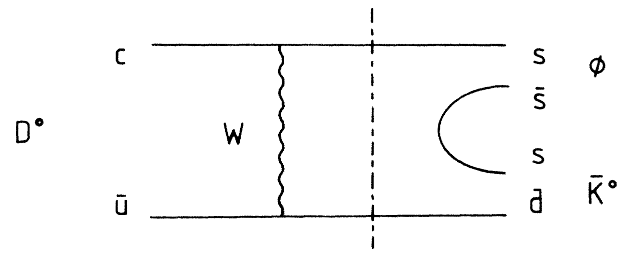

FIG. 1. The $D^0 \to \phi \bar{K}^0$ decay through the W-exchange diagram. The intermediate states are ordinary $(Q\bar{Q})$ meson states.

in the Cabibbo-angle-suppressed decays of the D^0 meson. In all, the understanding of nonleptonic D^0 decays from the conventional approach is at best in confusion. It is, therefore, useful to add a fresh point of view towards the unified description of nonleptonic weak decays of hadrons. In this paper, we discuss the newly observed $D^0 \to \phi \bar{K}^0$ decay, continuing our previous studies[8] which use an algebraic approach with a hard-pseudoscalar-meson extrapolation. The method can be partly viewed as an innovation of the old current algebra which is marred by the serious ambiguity associated with the use of the soft-meson approximation. We have replaced the soft-meson approximation by the much milder hard-meson extrapolation which is executed in the infinite-momentum frame (IMF). The asymptotic hadron matrix elements of H_w appearing in the amplitudes thus obtained are then severely constrained by the sum rules obtained in the algebraic approach[8] developed in QCD and electroweak theory. In this theoretical framework, though *still approximate*, a reasonable unified description of $K_S \to 2\pi$ and $D^0 \to \bar{K}\pi$ decays has previously been obtained. In this paper, we show that the $D^0 \to \phi \bar{K}^0$ can also be accommodated well in the same scheme.

II. BRIEF REVIEW OF THEORETICAL FRAMEWORK AND METHOD

We briefly explain the theoretical framework and the method by which asymptotic constraints on the relevant matrix elements of H_w are derived. First of all, the algebras (equal-time commutation relations) involving the generators of underlying symmetry groups of QCD (i.e., the vector charges V_α and axial-vector charges A_α) are regarded as the *constraints* upon the world of observable hadrons which we solely deal. [Recall in this connection the successful current-algebra calculation of $g_A(0)$.] It is also important to note that these algebras are valid in broken symmetries. For V_α we introduce the well-tested concept of asymptotic flavor symmetry—the creation and annihilation operators of *physical* (i.e., "in" or "out") hadrons do transform *linearly* under flavor-$SU_f(N)$ transformation generated by V_α but only in the infinite-momentum limit of the $SU_f(N)$ multiplet where multiplet masses do not play a role. The inevitable and important effect of symmetry breaking, i.e., flavor particle mixing is taken into account in this asymptotic limit. Vacuum annihilation by V_α is used only among the states of flavor multiplets with infinite momenta (vacuum annihilation by lightlike charges). For the algebras involving the charge V_α, the application of asymptotic flavor symmetry immediately yields broken-flavor-symmetry sum rules, where physical masses and mixing parameters of hadrons play a crucial role. Our asymptotic sum rules are then shown to be explicitly *compatible* with the Gell-Mann—Okubo multiplet mass splittings including flavor particle mixing (see Ref. 8). For the algebras involving axial-vector charges [the safest charge to be dealt with is of course the SU(2) charge such as $A_{\pi^+} \equiv A_1 + iA_2$, etc.], only asymptotic flavor-symmetry contents of the algebra are requested to be realized levelwise or blockwise. This ansatz which synthesizes, at the observable hadronic level, the two distinct but equally important aspects of quarks, i.e, current and constituent quarks, has also been successful in producing dynamical constraints which are in agreement with many important experiments. A comprehensive review on the above algebraic approach has recently been given in Ref. 8. We stress the fact that we always deal only with the physical "in" or "out" hadrons. No virtual particles are involved in the computation.

III. CONSTRAINTS ON THE ASYMPTOTIC MATRIX ELEMENTS OF H_w

We start by briefly recapitulating the derivation of important constraints[8,9] upon the asymptotic single-particle ground-state-meson matrix elements of the weak Hamiltonian $H_w = H_w^{PC} + H_w^{PV}$. We pick out, for example, the algebras

$$[[H(0,-),A_{\pi^-}],A_{\pi^+}] = [[H(0,-),V_{\pi^-}],V_{\pi^+}] ,$$

$$[[H(0,-),A_{\pi^+}],A_{\pi^-}] = [[H(0,-),V_{\pi^+}],V_{\pi^-}] ,$$

and

$$[H_w^{PV(PC)},A_\alpha] = [H_w^{PC(PV)},V_\alpha]$$

with $\alpha = \pi^{\pm,0}$, where $H(0,-)$ denotes the effective weak Hamiltonian (in the limit of infinite W-boson mass) describing the (charm-conserving and strangeness-changing) process with $\Delta C = 0$ and $\Delta S = -1$. In broken $SU_f(3)$, we then insert these algebras between physical ground-state-meson states $\langle M(\alpha,\mathbf{p},\lambda)|$ and $|M(\beta,\mathbf{p}',\lambda)\rangle$, where $\alpha = \pi^{+,0},\eta,\eta',\rho^{+,0},\phi,\omega$ and $\beta = K^{+,0},K^{*+,0}$ with $\mathbf{p} = \mathbf{p}' \to \infty$ and helicity λ. Then, the right-hand side (RHS) of these equations involving vector changes V_α takes definite expressions which contain only asymptotic single-particle ground-state-meson matrix elements of H_w with the use of asymptotic $SU_f(3)$ symmetry. On the left-hand side (LHS) we insert a complete set of single-particle meson states with $\mathbf{p} \to \infty$ between the operators (H_w and A_α's) and group them into blocks (i.e., $Q\bar{Q}$ and possibly $QQ\bar{Q}\bar{Q}$ meson states labeled by levels). We then require that asymptotic flavor-symmetry contents of the algebra expressed on the RHS of these equations should be realized on the LHS in a simple manner, i.e., blockwise (or levelwise). Note that no saturation assumption is introduced. We have found that this procedure can, in fact, explain many important dynamical relations of hadrons[8] which include almost all the successful SU(6) result. For the present problem, the realization requirement for the ground-state $Q\bar{Q}$-meson sector yields,[8] among others,

$$\sqrt{2}\langle \pi^0(\mathbf{p})|H_w^{PC}|K^0(\mathbf{p}')\rangle + \langle \pi^+(\mathbf{p})|H_w^{PC}|K^+(\mathbf{p}')\rangle = 0$$

and

$$\sqrt{2}\langle \rho^0(\mathbf{p})|H_w^{PC}|K^{*0}(\mathbf{p}')\rangle_{\lambda=0} + \langle \rho^+(\mathbf{p})|H_w^{PC}|K^{*+}(\mathbf{p}')\rangle_{\lambda=0} = 0 ,$$

with $\mathbf{p} = \mathbf{p}' \to \infty$, which demonstrate that the ground-state-meson matrix elements of $H(0,-)$ do satisfy the *asymptotic* $|\Delta\mathbf{I}| = \frac{1}{2}$ rule. We also obtain SU(6)-type *asymptotic* constraints such as

$$\langle \pi^0(\mathbf{p})|H_w^{PC}|K^0(\mathbf{p}')\rangle = \pm\langle \pi^0(\mathbf{p})|H_w^{PV}|K^{*0}(\mathbf{p}',\lambda=0)\rangle$$

and

$$\langle \pi^+(\mathbf{p})|H_w^{PC}|K^+(\mathbf{p}')\rangle = \pm\langle \pi^+(\mathbf{p})|H_w^{PV}|K^{*+}(\mathbf{p}',\lambda=0)\rangle$$

with $\mathbf{p} = \mathbf{p}' \to \infty$. Charm counterparts of these constraints in broken $SU_f(4)$ have also been obtained. Here we list only the constraints which will be used in this paper.

(i) The *asymptotic* $|\Delta\mathbf{I}| = \frac{1}{2}$ rule and its charm counterpart:

$$\langle \pi^+|H_w^{PC}|K^+\rangle + \sqrt{2}\langle \pi^0|H_w^{PC}|K^0\rangle = 0 , \quad (3.1a)$$

$$\langle \pi^+|H_w^{PV}|K^{*+}\rangle + \sqrt{2}\langle \pi^0|H_w^{PV}|K^{*0}\rangle = 0 , \quad (3.1b)$$

$$\langle \bar{K}^0|H_w'^{PC}|D^0\rangle + \langle \pi^+|H_w'^{PC}|F^+\rangle = 0 , \quad (3.1c)$$

$$\langle \bar{K}^0|H_w'^{PV}|D^{*0}\rangle + \langle \pi^+|H_w'^{PV}|F^{*+}\rangle = 0 . \quad (3.1d)$$

(ii) The SU(6)- and SU(8)-like *asymptotic* constraints:

$$\langle \pi^+|H_w^{PC}|K^+\rangle = \pm\langle \pi^+|H_w^{PV}|K^{*+}\rangle , \quad (3.2a)$$

$$\langle \bar{K}^0|H_w'^{PC}|D^0\rangle = \pm\langle \bar{K}^0|H_w'^{PV}|D^{*0}\rangle . \quad (3.2b)$$

(iii) *Asymptotic* $SU_f(4)$ parametrization for the matrix elements of H_w:

$$\langle \bar{K}^0|H_w'^{PC}|D^0\rangle = -\cot\theta_C\langle \pi^+|H_w^{PC}|K^+\rangle . \quad (3.3)$$

Here θ_C denotes the Cabibbo angle and all the above matrix elements are evaluated in the infinite-momentum frame (IMF). H_w' denotes $H(-,-)$ with $\Delta C = \Delta S = -1$.

IV. COMPUTATION AND NUMERICAL EVALUATION

In order to write the amplitude in terms of the matrix elements of H_w taken between two hadron states with infinite momenta such as given by Eqs. (3.1)—(3.3), we have developed a hard-pseudoscalar- (PS) meson technique in the IMF (Ref. 8). Using this extrapolation which is much milder compared with the old soft-pion approximation, we can describe the amplitude (which is symmetrized with respect to the two final PS mesons) for

$$P_1(p_1)\to P_2(p_2)+P_3(q) \tag{4.1}$$

approximately as[8]

$$M(P_1\to P_2P_3)\simeq M_{\rm ETC}(P_1\to P_2P_3)+M_S(P_1\to P_2P_3)\ . \tag{4.2}$$

We have evaluated the amplitude in the limit $\mathbf{q}\to 0$ and $\mathbf{p}_1=\mathbf{p}_2\to\infty$ which *effectively* achieves the limit $q_\mu\to 0$ ($\mu=0,1,2,3$), without assuming the masslessness of the PS meson. However, it should be noted that terms such as $(q\cdot p_1)$ can now remain finite after the above limiting procedure. Therefore the equal-time-commutator (ETC) part and the surface term, which was set to be zero in the old soft-meson limit but now *survives* in the present hard-meson extrapolation, are given by Eqs. (4.3) and (4.4) below,

$$M_{\rm ETC}(P_1\to P_2P_3)=-i\{(2f_{P_2})^{-1}\langle P_3\,|\,[V_{\bar P_2},H_w^{\rm PC}]\,|\,P_1\rangle+(2f_{P_3})^{-1}\langle P_2\,|\,[V_{\bar P_3},H_w^{\rm PC}]\,|\,P_1\rangle\}\ . \tag{4.3}$$

$M_{\rm ETC}$ is the usual equal-time-commutator part which now has to be evaluated in the IMF, enabling us to use asymptotic flavor symmetry:

$$\begin{aligned}M_S(P_1\to P_2P_3)\equiv&\lim_{\mathbf{q}\to 0,\mathbf{p}_1\to\infty}[i(2f_{P_2})^{-1}q_\mu T_\mu^{(2)}+i(2f_{P_3})^{-1}q_\mu T_\mu^{(3)}]\\
=&\,i(2f_{P_2})^{-1}\Big[\sum_n[(m_3{}^2-m_1{}^2)/(m_n{}^2-m_1{}^2)]\langle P_3\,|\,A_{\bar P_2}\,|\,n\rangle\langle n\,|\,H_w^{\rm PV}\,|\,P_1\rangle\\
&+\sum_l[(m_3{}^2-m_1{}^2)/(m_l{}^2-m_3{}^2)]\langle P_3\,|\,H_w^{\rm PV}\,|\,l\rangle\langle l\,|\,A_{\bar P_2}\,|\,P_1\rangle\Big]\\
&+i(2f_{P_3})^{-1}\Big[\sum_n[(m_2{}^2-m_1{}^2)/(m_n{}^2-m_1{}^2)]\langle P_2\,|\,A_{\bar P_3}\,|\,n\rangle\langle n\,|\,H_w^{\rm PV}\,|\,P_1\rangle\\
&+\sum_{l'}[(m_2{}^2-m_1{}^2)/(m_{l'}{}^2-m_2{}^2)]\langle P_2\,|\,H_w^{\rm PV}\,|\,l'\rangle\langle l'\,|\,A_{\bar P_3}\,|\,P_1\rangle\Big]\ .\end{aligned} \tag{4.4}$$

Here

$$T_\mu^{(j)}=i\int d^4x\,\langle P_k(p_2)\,|\,T[A_\mu^{(j)}(x),H_w(0)]\,|\,P_1(p_1)\rangle e^{-iqx}$$

($j=2,k=3$ and $j=3,k=2$). $A_\mu^{(j)}(x)$ denotes the axial-vector current which transforms like $\bar P_j$, and f_{P_i} the decay constant of the pseudoscalar meson P_i. The summation $\sum$ is extended over all the possible single-particle states. In Eq. (4.3) we have used the well-known commutation relation

$$[A_\alpha,H_w^{\rm PC(PV)}]=[V_\alpha,H_w^{\rm PV(PC)}]\ . \tag{4.5}$$

The surviving surface term Eq. (4.4) corresponds to the diagrams drawn in Fig. 2 and involves only asymptotic single-particle meson matrix elements of H_w and axial-vector *charges* A_α.

Now, we apply the same technique also to the quasi-two-body decay:

$$P_1(p_1)\to V(p_2)+P_2(q)\ , \tag{4.6}$$

where V denotes a vector meson. Then the amplitude is written as [using the method which led to Eq. (4.2)]

$$M(P_1\to VP_2)\simeq M_{\rm ETC}(P_1\to VP_2)+M_S(P_1\to VP_2)\ , \tag{4.7}$$

where

$$M_{\rm ETC}(P_1\to VP_2)=-i\{(2f_{P_2})^{-1}\langle V\,|\,[V_{\bar P_2},H_w^{\rm PV}]\,|\,P_1\rangle-(2f_{P_1})^{-1}\langle V\,|\,[V_{P_1},H_w^{\rm PV}]\,|\,\bar P_2\rangle\}\ , \tag{4.8}$$

$$\begin{aligned}M_S(P_1\to VP_2)=&\,i(2f_{P_2})^{-1}\Big[\sum_n[(m_V{}^2-m_1{}^2)/(m_n{}^2-m_1{}^2)]\langle V\,|\,A_{\bar P_2}\,|\,n\rangle\langle n\,|\,H_w^{\rm PC}\,|\,P_1\rangle\\
&+\sum_l[(m_V{}^2-m_1{}^2)/(m_l{}^2-m_V{}^2)]\langle V\,|\,H_w^{\rm PC}\,|\,l\rangle\langle l\,|\,A_{\bar P_2}\,|\,P_1\rangle\Big]\\
&-i(2f_{P_1})^{-1}\Big[\sum_n[(m_V{}^2-m_2{}^2)/(m_n{}^2-m_2{}^2)\langle V\,|\,A_{P_1}\,|\,n\rangle\langle n\,|\,H_w^{\rm PC}\,|\,\bar P_2\rangle\\
&+\sum_l[(m_V{}^2-m_2{}^2)/(m_l{}^2-m_V{}^2)]\langle V\,|\,H_w^{\rm PC}\,|\,l\rangle\langle l\,|\,A_{P_1}\,|\,\bar P_2\rangle\Big]\ .\end{aligned} \tag{4.9}$$

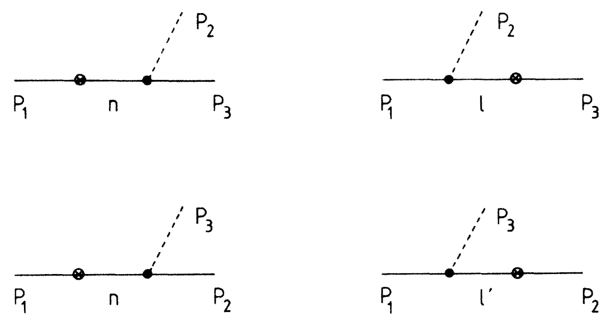

FIG. 2. Single-hadron intermediate state contribution to the surface term $M_S(P_1\to P_2+P_3)$ in Eq. (4.4). ⊗ denotes the weak vertex.

In Eqs. (4.8) we have again used the commutation relation, Eq. (4.5).

In the expression of Eq. (4.9), the amplitude is antisymmetrized[10] with respect to the exchange of two PS-meson momenta $-q\leftrightarrow p_1$ in the crossed channel, in anticipation of the use of asymptotic $SU_f(4)$ symmetry. Recall that our key constraint Eq. (3.3) was obtained[8,9] from the equal-time commutator $[H(-,-),V_{D^0}]=\cot\theta_C H(0,-)$ with $V_{D^0}\equiv V_{11}-iV_{12}$, using asymptotic $SU_f(4)$. Our amplitude can then smoothly reach its flavor-symmetry limit, when all the quark masses are set to be equal or equal to zero.

We now choose to relate the $D^0\to\phi\bar{K}^0$ to the $K_S^0\to\pi^+\pi^-$ decay. We think that this procedure is least ambiguous compared with other dynamical comparisons between the processes involving $\Delta C=\Delta S=-1$ and $\Delta C=0$ and $\Delta S=-1$, because of reasons which will be clarified later. For this purpose, we first rewrite Eq. (4.2) as

$$M(P_1\to P_2P_3)\simeq M_{\rm ETC}(P_1\to P_2P_3)[1+r(P_1\to P_2P_3)] . \tag{4.10}$$

Here $r(P_1\to P_2P_3)$ is defined by

$$r(P_1\to P_2P_3)=M_S(P_1\to P_2P_3)/M_{\rm ETC}(P_1\to P_2P_3) , \tag{4.11}$$

and provides a measure of the importance of the contribution of the surface term relative to the ETC term. Keeping *only* the ground-state $Q\bar{Q}$-meson contribution in the surface term M_S, we have obtained the following result in Ref. 8. Substituting Eqs. (3.1a)—(3.2b) into Eq. (4.10) together with Eqs. (4.3), (4.4), and (4.11), and using *asymptotic* (not exact) $SU_f(4)$ symmetry for the matrix elements of the charges A_π and A_K, we obtain[8]

$$M(D^0\to K^-\pi^+)\simeq M_{\rm ETC}(D^0\to K^-\pi^+)(1+0.5) , \tag{4.12a}$$

$$M(K_S^0\to\pi^+\pi^-)\simeq M_{\rm ETC}(K_S^0\to\pi^+\pi^-)(1+0.2) , \tag{4.12b}$$

where

$$M_{\rm ETC}(D^0\to K^-\pi^+)=i(2f_K)^{-1}\langle\bar{K}^0\,|\,H_w^{\rm PC}\,|\,D^0\rangle , \tag{4.13a}$$

$$M_{\rm ETC}(K_S^0\to\pi^+\pi^-)=-i(\sqrt{2}f_\pi)^{-1}\langle\pi^+\,|\,H_w^{\rm PC}\,|\,K^+\rangle . \tag{4.13b}$$

Here, we have chosen the positive sign in Eqs. (3.2a) and (3.2b), and used[11] $\langle\pi^+\,|\,A_{\pi^+}\,|\,\rho^0\rangle\simeq+1.0$. It should be noted that in Eqs. (4.12a) and (4.12b), the ETC term is larger than the surface term where only the ground-state-meson contribution is retained. Later we find that the same trend persists also for the $D^0\to\phi\bar{K}^0$ decay. From Eqs. (4.12a) and (4.12b) with Eqs. (4.13a), (4.13b), and (3.3), we obtain[8] $\Gamma(D^0\to K^-\pi^+)/\Gamma(K_S^0\to\pi^+\pi^-)\simeq4.4$ by using[12] $f_\pi\simeq f_K$. Although it is smaller by a factor $\simeq2$ than the observed ratio[13] $\Gamma(D^0\to K^-\pi^+)/\Gamma(K_S^0\to\pi^+\pi^-)\,|_{\rm expt}\simeq10$, this result may be considered reasonable in view of the approximation made. Namely the contribution of higher excited states to the surface term of the $D^0\to K^-\pi^+$ decay is expected to be substantial (while it is not to that of the $K\to2\pi$ decays) and will explain the above-mentioned discrepancy. Next, we make a crude estimate on the ratio of the amplitudes given by (in the same approximation)

$$R_M(D^0\to\phi\bar{K}^0)\equiv M(D^0\to\phi\bar{K}^0)/M(K_S^0\to\pi^+\pi^-)$$
$$\simeq R_{\rm ETC}(D^0\to\phi\bar{K}^0)+R_S(D^0\to\phi\bar{K}^0) , \tag{4.14}$$

where $R_{\rm ETC}$ and R_S are defined, respectively, by

$$R_{\rm ETC}(D^0\to\phi\bar{K}^0)=M_{\rm ETC}(D^0\to\phi\bar{K}^0)/M(K_S^0\to\pi^+\pi^-) , \tag{4.15}$$

$$R_S(D^0\to\phi\bar{K}^0)=M_S(D^0\to\phi\bar{K}^0)/M(K_S^0\to\pi^+\pi^-) . \tag{4.16}$$

The term in $M_{\rm ETC}$ $(D^0\to\phi\bar{K}^0)$,

$$i(2f_D)^{-1}\langle\phi\,|\,V_{D^0}\,|\,\bar{D}^{*0}\rangle\langle\bar{D}^{*0}\,|\,H_w^{\rm PV}\,|\,K^0\rangle ,$$

is dropped since $\langle\phi\,|\,V_{D^0}\,|\,\bar{D}^{*0}(\mathbf{p})\rangle$ with $\mathbf{p}\to\infty$ vanishes in the ideal ω-ϕ mixing limit. (Our theoretical framework predicts[8] without any other assumption that the 1^{--} nonet is very close to the ideal limit.) Thus we find

$$M_{\rm ETC}(D^0\to\phi\bar{K}^0)=-i(2f_K)^{-1}\langle\bar{K}^{*0}\,|\,H_w^{\rm PV}\,|\,D^0\rangle$$

in the ideal ω-ϕ mixing limit. Since the ETC term does exist, in the present approach there is *no* reason to suspect that $D^0\to\phi\bar{K}^0$ decay is suppressed.

The ground-state-meson contribution to $M_S(D^0\to\phi\bar{K}^0)$, which corresponds to the diagrams drawn in Fig. 3, is given by

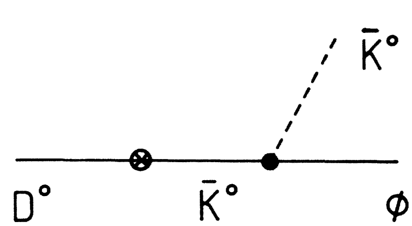

FIG. 3. Ground-state-meson contribution to the surface term $M_S(D^0\to\phi\bar{K}^0)$. ⊗ denotes the weak vertex.

$$M_S^{(L=0)}(D^0 \to \phi \bar{K}^0) = i(2f_K)^{-1}[(m_\phi{}^2 - m_D{}^2)/(m_K{}^2 - m_D{}^2)]\langle \phi \,|\, A_{K^0} \,|\, \bar{K}^0 \rangle \langle \bar{K}^0 \,|\, H_w^{\rm PC} \,|\, D^0 \rangle \; . \tag{4.17}$$

Again $M_S^{(L=0)}(D^0 \to \phi \bar{K}^0)$ does not contain the term proportional to $(2f_D)^{-1}$ in the ideal ω-ϕ mixing limit, since the asymptotic matrix element $\langle \phi \,|\, A_{D^0} \,|\, \bar{D}^0 \rangle$ vanishes in the limit. Neglecting contributions of higher excited states to the surface term (for justification, see the remark below), choosing the same positive sign in Eqs. (3.2a) and (3.2b) as used in Eqs. (4.12a) and (4.12b), assuming $f_\pi \simeq f_K$ and using Eqs. (3.3), (4.12b), and (4.13b), we estimate $R_{\rm ETC}(D^0 \to \phi \bar{K}^0) \simeq 2.6$ and $R_S(D^0 \to \phi \bar{K}^0) \simeq 1.2$ and hence $R_M(D^0 \to \phi \bar{K}^0) \simeq 3.8$. Thus we obtain the decay rate $\Gamma(D^0 \to \phi \bar{K}^0) \simeq 1.9 \times 10^{10}$ sec^{-1} by using as an input the well-known value $\Gamma(K_S^0 \to \pi^+\pi^-)_{\rm expt} = 0.7689 \times 10^{10}$ sec^{-1}. Our result reproduces well the recently measured branching ratio[1] $B(D^0 \to \phi \bar{K}^0) \sim 1\%$, which corresponds to the decay rate $\Gamma(D^0 \to \phi \bar{K}^0) \sim 2 \times 10^{10}$ sec^{-1} with the use of the world average[13] of the lifetime of the D^0 meson, $\tau(D^0) = (4.35 \pm 0.32) \times 10^{-13}$ sec.

We may remark here about the accuracy of the above result which is based on asymptotic flavor symmetry, where the mixings between ground-state mesons and their excited states are neglected for simplicity. The $SU_f(4)$ mass formula for the ground-state mesons under consideration, obtained using the same asymptotic $SU_f(4)$ symmetry and neglecting *intermultiplet* flavor mixings, contain[8] only about 10% error when compared with experiments. As mentioned in Ref. 12, we obtain $f_\pi \simeq f_K$ in the same approximation.

So far, we have neglected higher excited-state contributions to the surface term. Here we show qualitatively that the approximation is reasonable as far as the cases of $K_S^0 \to \pi^+\pi^-$ and $D^0 \to \phi \bar{K}^0$ under consideration are concerned. As for the two-body decays of the K meson, the ETC term $M_{\rm ETC}$ contributes dominantly while the surface term M_S is minor, i.e., $|\, r^{(L=0)}(K_S^0 \to \pi^+\pi^-) \,| \simeq 0.2$ as shown in Eq. (4.12b) and the contributions of higher excited states are kinematically suppressed due to the mass-dependent factors in Eq. (4.4). Thus we can safely neglect the contributions of higher excited states, as long as we do not treat small effects such as, for example, the deviations from the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule. In the case of $D^0 \to \phi \bar{K}^0$, we choose to discuss from a diagrammatical viewpoint. There is, in fact, a nice intimate correspondence between the result of present algebraic approach and the simple quark-line diagrams. For details, see Ref. 14. The s channel of the $D^0 \to \phi \bar{K}^0$ decay proceeds only through the W-exchange diagram (Fig. 1), as is well known. Although the crossed channels can involve multiquark (for example, $QQ\bar{Q}\bar{Q}$) meson states as the intermediate states, the crossed-channel contributions will be small due to the mass factors in Eq. (4.4), since the square of the multiquark-meson masses are much larger than those of ground-state-meson masses. Therefore, the important contribution to the surface term $M_S(D^0 \to \phi \bar{K}^0)$ will come from the W-exchange diagram (Fig. 1), in which the effective weak vertex is given by[15]

$$\langle (s\bar{d})_L \,|\, H_w \,|\, D^0 \rangle \; , \tag{4.18}$$

where L denotes the level to which the relevant $(s\bar{d})$ mesons belong. The matrix element of Eq. (4.18) corresponds to the diagram in Fig. 4 in which all the flavors of quarks participating in the weak vertex are different. In this case the matrix element of Eq. (4.18) can be written as

$$\langle (s\bar{d})_L \,|\, H_w \,|\, D^0 \rangle \sim |\, \Psi_L^*(0) \Psi_D(0) \,| \tag{4.19}$$

in the nonrelativistic quark model. Here $\Psi_L(0)$ and $\Psi_D(0)$ denote the values of the wave functions of the $(s\bar{d})_L$ meson and D meson at the origin. When $L \neq 0$, the RHS of Eq. (4.19) vanishes since $\Psi_L(0) = 0$ in the nonrelativistic limit. Hence the matrix element of Eq. (4.18) will be small in general if $L \neq 0$. One can also produce a similar result from the realization involving higher L states. The ETC term $M_{\rm ETC}$, of course, does not include any higher excited states. Therefore, we could neglect, in effect, the contributions of higher excited states to the decays of $K_S^0 \to \pi^+\pi^-$ and $D^0 \to \phi \bar{K}^0$. In other quasi-two-body decays of the D^0 meson, the situation is different; i.e., multiquark hadron states, if they exist, can take part in M_S through the so-called spectator diagram and can give sizable contributions. A measure of the exotic resonance contribution may be estimated crudely from $\Gamma(F^+ \to \phi \pi^+)_{\rm expt}$. The reason is simple. Multiquark mesons can contribute to the amplitude for the process through the first term on the RHS of Eq. (4.9), which will be enhanced by the mass-dependent factor since the mass of the possible exotic (i.e., $u\,s\,\bar{d}\,\bar{s}$) meson is expected to be close to the F^+ meson mass. On the contrary, because of the Okubo-Zweig-Iizuka rule,[16], ordinary $(Q\bar{Q})$ mesons contribute to the surface term of the amplitude only through the third term on the RHS of Eq. (4.9), which is relatively suppressed by the mass-dependent factor.

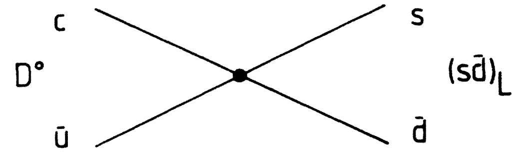

FIG. 4. Quark-line diagram corresponding to the matrix element $\langle (s\bar{d})_L \,|\, H_w \,|\, D^0 \rangle$.

V. CONCLUSION

We conclude the following. (i) The predicted value of the decay rate for $D^0 \to \phi \bar{K}^0$ is sizable and is reasonably consistent with present experiment, and the ratio of the rates of $K_S \to 2\pi$ and $D^0 \to \phi \bar{K}^0$ decays can be understood rather unambiguously. (ii) A unified general description of $K \to 2\pi$, $D \to \bar{K}\pi$, and other quasi-two-body decays of D mesons seems feasible in the present approach which deals with long-distance physics in earnest. As shown in this paper it is indeed possible to obtain a rather unambiguous unified description of the $D^0 \to \phi \bar{K}^0$ and $K_S \to 2\pi$ decays. However, contributions of multiquark hadron states to M_S have to be estimated for other processes.

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[1]ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. **158B**, 525 (1985); Mark III Collaboration, R. Baltrusaitis *et al.* Phys. Rev. Lett. **55**, 150 (1985); CLEO Collaboration, C. Bebek *et al.*, *ibid.* **56**, 1893 (1986).

[2]I. Bigi and M. Fukugita, Phys. Lett. **91B**, 121 (1980). It has recently been argued that the $D^0 \to \phi \bar{K}^0$ decay may proceed without the W-exchange diagram, if *inelastic* final-state interactions are important [see J. F. Donoghue, Phys. Rev. D **33**, 1516 (1986)].

[3]N. Cabibbo and L. Maiani, Phys. Lett. **73B**, 418 (1978).

[4]The hypothesis of the W-exchange dominance and the analyses of D meson decays based on it are seen, for example, in H. Fritsch and P. Minkowski, Phys. Lett. **90B**, 455 (1980); S. P. Rosen, Phys. Rev. Lett. **44**, 4 (1980); M. Bander, D. Silberman, and A. Soni, *ibid.* **44**, 7 (1980); K. Terasaki, Prog. Theor. Phys. **66**, 988 (1981).

[5]R. H. Schindler, in *Proceedings of the XXII International Conference on High Energy Physics, Leipzig, 1984*, edited by A. Meyer and E. Wieczorek (Akademie der Wissenschaften der DDR, Institute für Hochenergiephysik, Zeuthen, Berlin, 1984), p. 171.

[6]D. Hitlin, Caltech Report No. CALT-68-1230, 1985 (unpublished).

[7]It was first observed by the Mark II Collaboration and was confirmed recently by the Mark III Collaboration [R. H. Schindler *et al.*, Phys. Rev. D **24**, 78 (1981); R. M. Baltrusaitis *et al.*, Phys. Rev. Lett. **55**, 150 (1985); **55**, 639(E) (1985)].

[8]S. Oneda and K. Terasaki, Prog. Theor. Phys. Suppl. **82**, 1 (1985). See also K. Terasaki, S. Oneda, and T. Tanuma, Phys. Rev. D **29**, 456 (1984) and Ref. 9.

[9]T. Tanuma, S. Oneda, and K. Terasaki, Phys. Rev. D **29**, 444 (1984).

[10]T. Tanuma, S. Oneda, and K. Terasaki, Phys. Lett. **110B**, 260 (1982).

[11]The magnitude of $\langle \pi^+ | A_{\pi^+} | \rho^0 \rangle$ is estimated from $\Gamma(\rho \to 2\pi)_{\text{expt}} \simeq 158$ MeV using PCAC (partial conservation of axial-vector current).

[12]From the equal-time charge—charge-density commutator, $[A^0_{\pi^+}(0), V_{K^0}] = A^0_{K^+}(0)$, and asymptotic flavor symmetry, we obtain $f_\pi = f_K$ (see Ref. 8) in the approximation in which we neglect mixings between ground-state mesons and higher-lying states (radially excited states, etc.). Since, in this paper, we also neglect the effect of such mixing, we use $f_\pi = f_K$ in our estimate.

[13]E. H. Thorndike, in *Proceedings of the Twelfth International Symposium on Lepton and Photon Interactions at High Energies, Kyoto, 1985*, edited by M. Konuma and K. Takahashi (Research Institute for Fundamental Physics, Kyoto University, Kyoto, 1985), p. 406.

[14]K. Terasaki, Nuovo Cimento A (to be published).

[15]Radially excited states may not couple strongly to the ground-state mesons via A_π. For example, the $\rho'(1600)$ decays dominantly into $\rho\pi\pi$ through $A_1\pi$ while the $\pi\pi$ mode is relatively minor.

[16]S. Okubo, Phys. Lett. **5**, 165 (1963); G. Zweig, CERN Report No. TH401, 1964 (unpublished); J. Iizuka, K. Okada, and O. Shito, Prog. Theor. Phys. **35**, 1061 (1966).