$B \rightarrow K l^+ l^-$ with four generations: Rates and CP violation

G. Eilam,* J. L. Hewett, and T. G. Rizzo

Ames Laboratory and Department of Physics, Iowa State University, Ames, Iowa 50011

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In the standard model with three generations the branching ratios for $B \rightarrow Kl^+l^- + X$ $(B \rightarrow Kl^+l^-)$ are 1.9×10^{-6} (0.9×10^{-6}) and 2.4×10^{-6} (1.2×10^{-6}) for $m_t = 40$ and 80 GeV, respectively, with very little uncertainty, and the maximal *CP*-violation asymmetry is of the order of 1.5%. As demonstrated here in the standard model with four generations, these results may change drastically with a possibility of an increase by more than an order of magnitude in rates and *CP*-violation asymmetries. The uncertainty is now large for both quantities.

Circumstantial evidence, though very far from being convincing at present, has accumulated lately-at least according to some authors-in favor of a fourth generation of quarks and leptons.1 With two more phases, three extra mixing angles, and two unknown masses as compared with the three-generation case, results for quark processes are expected to suffer from large uncertainties. That fact is actually a blessing since in the three-generation scenario there are precise predictions, which are perhaps on the verge of being in disagreement with experiment, such as^{2,3} ϵ'/ϵ . The three-generation model may also predict effects which are too small to be observable in the near future, such as CP violation in rare charged B decays.^{4,5} Here we discuss rates and CP-violation asymmetries for a particular rare B decay which enjoys relative immunity from strong-interaction corrections⁵ and which will-both soon and not so soon—be searched for,⁶ namely, $B \rightarrow K l^+ l^$ with l = e or μ .

In the three-generation case the branching ratio is calculated with confidence⁵ (long-distance corrections and ignorance regarding some of the mixing angles and the phase cause no major problem) to be 1.9×10^{-6} and 3.2×10^{-6} for $B \rightarrow K l^+ l^- + X$ and 0.9×10^{-6} and 1.6×10^{-6} for $B \rightarrow K l^+ l^-$, when m_t is 40 and 240 GeV, respectively. However, the *CP* asymmetry which should manifest itself as a nonzero difference between the partial decay rates for B^+ and B^- is uncertain in the threegeneration case. Based on present information regarding the Kobayashi-Maskawa⁷ (KM) angles, the asymmetry has an upper limit of ~1.5%. The experimentally relevant number for measuring an asymmetry is defined as

$$N \equiv Ba^2 , \qquad (1)$$

where B and a are the branching ratio and asymmetry, respectively. N for three generations is at most 5×10^{-10} for $B \rightarrow K l^+ l^- + X$, and 2×10^{-10} for $B \rightarrow K l^+ l^-$, almost independently of m_t , thus rendering a measurement of the *CP* asymmetry in this clean reaction a remote possibility, even for the Superconducting Super Collider (SSC) where $10^7 - 10^8$ "useful" B's are expected to be available.⁶ The branching ratio itself, being of order 10^{-6} in the threegeneration case, can in principle be measured even before the hoped-for advent of the SSC. In the standard model with four generations, as will be shown below, one can achieve rates, *CP*-violation asymmetries, and *N* values which are much higher than those obtained without the fourth generation. In view of the certainty in the calculation with only u,c,t quarks in the loop, it becomes crucial to search for this decay in forthcoming experiments.

Let us now describe our results. We generalize the calculation of Ref. 5 for $b \rightarrow sl^+l^-$ with three generations of quarks in the loop for this "electromagnetic penguin" process (the lowest-order diagrams are loop diagrams), using the same notation. Denoting the fourth generation up quark by t', the matrix element for the quark level process $b \rightarrow sl^+l^-$ is

$$V = -i \left(G_F / 2\sqrt{2} \right) (\alpha / \pi) G_1 \bar{u}_2 \gamma^{\mu} (1 - \gamma_5) u_1 \bar{u}_4 \gamma_{\mu} v_3 , \qquad (2)$$

where

$$G_1 = A_c(F_1^c - F_1^{t'}) + A_u(F_1^u - F_1^{t'}) + A_t(F_1^t - F_1^{t'})$$
(3)

with $A_j = U_{sj} U_{jb}^{\dagger}$ (j = u, c, t, t'), the U's being elements of the KM four-generation matrix⁸ given by

TABLE I. Values of the form factor F_1^j (j=t,t') as a function of m_j .

m_j (GeV)	F_1^j	
40	0.65	
60	0.31	
80	0.07	
160	-0.44	
240	-0.68	
300	-0.80	
400	-0.95	
500	-1.06	

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(4)



where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, $s_{\delta i} = \sin\delta_i$, $c_{\delta i} = \cos\delta_i$ $(0 \le \theta_i \le \pi/2, 0 \le \delta_i \le 2\pi)$, and $A_u + A_c + A_t + A_{t'} = 0$. The form factors F_i^j are given by their $k^2 \simeq 0$ values $(k^2$ being the invariant mass of the lepton pair) for j = t, t'; in Table I these form factors⁹ are shown for $40 \le m_j \le 500$ GeV and $M_W = 82$ GeV. For j = u, c the form factors are

$$F_{i}^{j}(k^{2}) = -2e_{j} \int_{0}^{1} dy \int_{0}^{1-y} dx \frac{y(x-1)+2x(x-1)}{y+\hat{m}_{j}^{2}(1-y)-\hat{k}^{2}x(1-x)} , \qquad (5)$$

where $\hat{k}^2 = k^2 / M_W^2$, $\hat{m}_j^2 = m_j^2 / M_W^2$, and $e_j = \frac{2}{3}$. Obviously, only for $k^2 > 4m_j^2$ will Im $F_1^j \neq 0$.

We equate the inclusive rate $B \rightarrow Kl^+l^- + X$ to the quark level process $b \rightarrow sl^+l^-$. Then, squaring V in Eq. (2), and integrating over the three-body phase space we find

$$\Gamma(B \to K l^+ l^- + X) = \frac{G_F^2 m_b^5}{192\pi^3} \left[\frac{\alpha}{2\pi}\right]^2 \sum_{i=1}^{18} a_i \hat{I}_i^{\text{in}}, \quad (6)$$

where

$$\hat{I}_{i}^{\text{in}} = \int_{z_{\min}}^{1} dz (1-z)^{2} (1+2z) I_{i} ,$$

$$z_{\min} = 4(m_{l}/m_{b})^{2}$$
(7)

with the superscript "in" standing for inclusive. The angular coefficients a_i and the integrands I_i are given in Table II (note that for three generations, only ten terms survive⁵). For the *CP*-violation asymmetry defined as

$$a = \frac{\Gamma_b - \Gamma_{\overline{b}}}{\Gamma_b + \Gamma_{\overline{b}}} \tag{8}$$

we obtain

$$a = \frac{a_5(\hat{I}_5^{\text{in}} - \hat{I}_{10}^{\text{in}}) + a_{12}\hat{I}_{12}^{\text{in}} + (a_{14} + a_{15})\hat{I}_{14}^{\text{in}}}{a_5(\hat{I}_5^{\text{in}} - \hat{I}_{10}^{\text{in}}) + a_{12}\hat{I}_{12}^{\text{in}} + (a_{14} + a_{15})\hat{I}_{14}^{\text{in}} - \Gamma_b}$$
(9)

with Γ_b as in Eq. (6).

Before exhibiting our numerical results for the inclusive reaction, let us turn to the exclusive decay $B \rightarrow K l^+ l^-$.

As shown in Ref. 5, the results for $B \rightarrow K l^+ l^-$ are identical to those in Eqs. (6) and (9) with the following changes: All inclusive integrals \hat{I}_i^{in} $(i=1,\ldots,18)$ are now replaced by the exclusive integrals

TABLE II. Values of the angular coefficients a_i (where $A_j = U_{sj}U_{jb}^{\dagger}$) and integrands I_i appearing in the inclusive and exclusive rates [see Eqs. (7) and (10)].

i	a _i	I _i
1	$(\operatorname{Re}A_c)^2$	$({\bf Re}F_1^c - F_1^{t'})^2$
2	$2A_u \operatorname{Re}A_c$	$(\text{Re}F_{1}^{c}-F_{1}^{t'})(\text{Re}F_{1}^{u}-F_{1}^{t'})$
3	A_{μ}^{2}	$({\rm Re}F_1^u - F_1^{t'})^2$
4	$(\mathrm{Im}A_c)^2$	$(\mathrm{Im}F_1^c)^2$
5	$-2A_{u} \text{Im}A_{c}$	$(\operatorname{Re}F_{1}^{u}-F_{1}^{t'})\operatorname{Im}F_{1}^{c}$
6	$(\mathrm{Im}A_c)^2$	$({\rm Re}F_1^c - F_1^{t'})^2$
7	$(\mathbf{Re}A_c)^2$	$(\mathrm{Im}F_1^c)^2$
8	$2A_u \text{Re}A_c$	$\mathrm{Im}F_{1}^{c}\mathrm{Im}F_{1}^{u}$
9	A_{μ}^{2}	$(\mathrm{Im}F_{1}^{u})^{2}$
10	$2A_u \operatorname{Im} A_c$	$(\operatorname{Re}F_1^c - F_1^{t'})\operatorname{Im}F_1^{u}$
11	$2A_{\mu}\text{Re}A_{\mu}$	$(\operatorname{Re}F_{1}^{u}-F_{1}^{t'})(F_{1}^{t}-F_{1}^{t'})$
12	$2A_{\mu} \text{Im}A_{t}$	$ImF_{1}^{u}(F_{1}^{t}-F_{1}^{t'})$
13	$2 \operatorname{Re} A_c \operatorname{Re} A_t$	$(\operatorname{Re}F_{1}^{c}-F_{1}^{t'})(F_{1}^{t}-F_{1}^{t'})$
14	$2 \operatorname{Re} A_c \operatorname{Im} A_t$	$\text{Im}F_{1}^{c}(F_{1}^{t}-F_{1}^{t'})$
15	$-2 \operatorname{Im} A_c \operatorname{Re} A_t$	$ImF_{1}^{c}(F_{1}^{t}-F_{1}^{t'})$
16	$2 \operatorname{Im} A_c \operatorname{Im} A_t$	$(\operatorname{Re}F_{1}^{c}-F_{1}^{t'})(F_{1}^{t}-F_{1}^{t'})$
17	$(\mathbf{Re}\boldsymbol{A}_t)^2$	$(F_1^t - F_1^{t'})^2$
18	$(\mathrm{Im}A_t)^2$	$(F_1^t - F_1^{t'})^2$

$$\hat{I}_{i}^{\text{ex}} = \frac{1}{2} \int_{z_{\min}}^{1} dz \, (1-z) I_{i}$$
(10)

with the same integrands I_i as in Eq. (7).

As indicated above, the proliferation of parameters in the four-generation case introduces large uncertainties, while the existing constraints for the three-generation case yield a precise value for the branching ratio once m_t is known. We therefore took the following approach: For representative values of m_t and $m_{t'}$ the KM angles were varied subject to the experimental constraints¹⁰

$$U_{ud} = 0.9729 \pm 0.0012, \quad U_{us} = \begin{cases} 0.231 \pm 0.003 \\ 0.221 \pm 0.002 \end{cases},$$
$$|U_{cs}| = 0.85 \pm 0.15, \quad |U_{cd}| = 0.24 \pm 0.03 , \quad (11)$$
$$|U_{cb}| = 0.06 \pm 0.02, \quad U_{ub} \le 0.01 .$$

Because of the conflict of the data analyses for the value of U_{us} we took U_{us} to lie in the range $0.219 \le U_{us} \le 0.234$. We also took the value of U_{ud} to be fixed at 0.9729 due to the small error. We then maximized, separately for the inclusive and for the exclusive decays, first the *CP*- violation asymmetry a, then the branching ratio B, and finally N [see Eq. (1)] which is the experimentally relevant number for any attempt at measuring the asymmetry a; all the maximizations were performed independently from each other. The values of a, B, N, and the angles are given in Tables III–V, where, for comparison, the last three rows display the three-generation results.⁵

One notes in Tables III-V a large variation in the results for the observables a, B, N, from values much lower than those for the three-generation case to much higher values. While the uncertainty in B is 2 orders of magnitude (from around 10^{-7} - 10^{-5} or so), the results for *a*, *N* span a much wider range. It is important to observe that branching ratios higher than 10^{-5} are possible for the inclusive, as well as for the exclusive case. Such high rates are certainly excluded in the three-generation case, and are accessible in forthcoming experiments. Furthermore N [see Eq. (1)] can be higher than 10^{-8} , which is about 2 order of magnitude higher than its maximal value for three generations; for possible nonzero measurements of the CP-violation asymmetry a, we should therefore await the SSC with $10^7 - 10^8 B$ decays. With only three generations, there seems to be no prospect for detecting an asymmetry even at the SSC.

Finally let us emphasize, that any result or limit from existing and future experiments for both the branching ratio and the *CP*-violation asymmetry should be more than welcome in view of our ignorance regarding the existence

TABLE III. Values of the asymmetry *a*, the branching ratio *B*, *N* as defined in Eq. (1), and KM angles for the inclusive and exclusive reactions $B \rightarrow Kl^+l^- + X$, $B \rightarrow KL^+l^-$, respectively, for $m_t = 40$ GeV, $m_{t'} = 160$ GeV. The columns *a* maximized, *B* maximized, *N* maximized, denote results for an independent maximalization of *a*, *B*, *N*, respectively. For comparison, the last three rows display the three-generation results from Ref. 5.

		Inclusive			Exclusive	
Four generations	a maximized	B maximized	N maximized	a maximized	B maximized	N maximized
a	11.6%	0.856%	9.01%	16.4%	0.551%	12.2%
В	7.54×10^{-7}	1.35×10^{-5}	3.85×10^{-6}	2.10×10^{-7}	6.92×10^{-6}	1.88×10^{-6}
N	1.02×10^{-8}	9.90×10 ⁻¹⁰	3.13×10 ⁻⁸	5.63×10 ⁻⁹	2.10×10^{-10}	2.78×10^{-8}
$\operatorname{Sign}_{\delta_1}$			-	+		
$\operatorname{Sign}_{\delta_2}$			+	-	+	+
$\operatorname{Sign}_{\delta_3}$	+	+	+	+	+	+
s _{δ1}	-0.763	-0.261	-0.146	-0.919	-0.336	-0.338
s _{b2}	-0.469	0.936	0.644	-0.937	-0.227	0.784
s _{o3}	0.115	-0.831	0.681	0.200	0.328	0.675
<i>s</i> ₁	0.231	0.231	0.231	0.231	0.231	0.231
<i>s</i> ₂	0.392	0.402	0.431	0.388	0.427	0.419
S ₃	0.112	0.223	0.242	0.143	0.286	0.225
S 4	0.938	0.687	0.582	0.710	0.719	0.566
S 5	0.948	0.985	0.985	0.967	0.989	0.985
<i>s</i> ₆	0.337	0.137	0.223	0.195	0.050	0.219
Three generations						
$\max(a)$		1.56%			1.48%	
В		1.86×10^{-6}			9.36×10^{-7}	
max(N)		4.53×10 ⁻¹⁰			2.05×10^{-10}	

		Inclusive			Exclusive	
Four generations	a maximized	B maximized	N maximized	a maximized	B maximized	N maximized
a	11.9%	0.144%	10.2%	15.6%	0.162%	12.9%
В	6.19×10^{-7}	7.77×10 ⁻⁶	1.11×10 ⁻⁶	3.04×10^{-7}	3.84×10 ⁻⁶	5.54×10^{-7}
Ν	8.70×10 ⁻⁹	1.62×10^{-11}	1.15×10^{-8}	7.42×10^{-9}	1.01×10^{-11}	9.91×10 ⁻⁹
Sign c_{δ_1}	—	_		-		
Sign c_{δ_2}		+			+	
Sign c_{δ_3}	+	+	+	+	+	+
Sδ1	0.211	0.677	0.563	0.211	0.677	0.562
s _{δ2}	-0.428	0.657	0.387	-0.428	0.657	0.387
<i>s</i> δ ₃	-0.397	-0.912	-0.235	-0.397	-0.912	-0.235
s ₁	0.231	0.231	0.231	0.231	0.231	0.231
<i>s</i> ₂	0.416	0.391	0.434	0.416	0.391	0.434
<i>s</i> ₃	0.281	0.274	0.288	0.281	0.274	0.289
<i>s</i> ₄	0.876	0.749	0.665	0.876	0.749	0.665
S 5	0.990	0.991	0.989	0.990	0.991	0.989
<i>s</i> ₆	0.177	0.091	0.196	0.177	0.091	0.196
Three generations						
max(a)		1.42%			1.35%	
В		2.38×10^{-6}			1.20×10^{-6}	
$\max(N)$		4.81×10^{-10}			2.18×10^{-10}	

TABLE IV. Same as Table III, except for $m_t = 80$ GeV, $m_{t'} = 160$ GeV.

of further generations, other extensions of the standard model, or mixing in the quark sector. If a fourth generation has an impact on nuclear β decays¹ and on K decays,¹ then it should certainly influence B decays. $B \rightarrow K l^+ l^-$ is especially important since we do not have any direct experimental indication for (1) higher-order corrections in the standard model, and the process discussed here is a pure loop process with a non-Abelian triple coupling becoming more prominent as m_t , $m_{t'}$ increase and (2) *CP* violation outside the neutral-*K* system. Note that observation of *CP* violation in the neutral-*B* system,¹¹ although certainly important, may be a boring

Inclusive Exclusive Four generations a maximized **B** maximized N maximized a maximized **B** maximized N maximized а 12.9% 0.376% 8.22% 16.1% 0.504% 10.6% B 9.32×10^{-7} 2.06×10^{-5} 5.65×10^{-6} 4.83×10^{-7} 1.03×10^{-5} 2.89×10^{-6} N 2.89×10^{-10} 1.56×10^{-8} 3.82×10^{-8} 1.25×10^{-8} 2.61×10^{-10} 3.22×10^{-8} Sign c_{δ_1} ++----------Sign c_{δ_2} _____ +++ +____ Sign c_{δ_3} + +++++0.601 -0.336-0.3380.601 -0.335 -0.338S₈₁ -0.2270.777 0.784 0.777 Sδ2 -0.2270.784 -0.0670.328 0.675 -0.067 So3 0.328 0.675 0.231 0.231 0.231 0.231 s_1 0.231 0.231 0.389 0.427 0.419 0.389 0.419 s_2 0.427 0.153 0.286 0.225 0.153 0.225 53 0.286 0.741 0.719 0.566 0.741 84 0.719 0.566 0.972 0.985 \$ 5 0.989 0.972 0.989 0.985 0.254 0.050 0.219 **s**₆ 0.254 0.050 0.219 Three generations max(a)1.56% 1.48% 1.86×10^{-6} 9.36×10^{-7} B 4.53×10^{-10} 2.05×10^{-10} max(N)

TABLE V. Same as Table III, except for $m_t = 40$ GeV, $m_{t'} = 300$ GeV.

repetition of mass mixing for kaons, while the charged B system has the potential to be the first system to help solve the puzzle of the origin of CP violation.^{5,6} We expect the same maximum values for the asymmetry a, for other (strong) penguin-induced rare charged-B decays such as $B_u \rightarrow K\pi$, $K\phi$; i.e., they should be at most of the order of 15% with four generations. The rates for such processes—which are subject to a high degree of uncertainty even for the three-generation case—should have upper limits which are not that influenced by the presence of a fourth generation, since the non-Abelian triple cou-

- *On leave of absence from the Physics Department, Technion, Haifa, Israel.
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pling which causes most of the effect¹² is absent in the "strong penguins."

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