PHYSICAL REVIEW D VOLUME 34, NUMBER 9 1 NOVEMBER 1986

Studies of the hydrodynamic evolution of matter produced in fluctuations in $\bar{p}p$ collisions and in ultrarelativistic nuclear collisions. II. Transverse-momentum distributions

M. Kataja and P. V. Ruuskanen
Physics Department, University of Jyväskylä, Seminaarinkatu 15, SF-40100, Jyväskylä, Finland

L. D. McLerran
Physics Department MS-106, Fermilab, P.O. Box 500, Batavia, Illinois 60510

H. von Gersdorff
Department of Physics and Astronomy, University of Minnesota at Minneapolis—St. Paul, Minneapolis, Minnesota 55455
(Received 28 May 1986)

We study solutions to the hydrodynamic equations appropriate for ultrarelativistic nuclear collisions. We find that the matter produced in such collisions spends time $t > 30$ fm/c at temperatures larger than 150 MeV. The transverse momentum of protons, kaons, and pions is computed in the central region of ultrarelativistic nuclear collisions. Assuming Bjorken's initial conditions for the hydrodynamic equations, and a bag-model equation of state, we show that the transverse-momentum distribution as a function of dN/dy does reflect properties of the equation of state. We demonstrate that such a distribution approximately scales as a function of $(1/A)dN/dy$. The relation between p_t and dN/dy is shown to be significantly altered under different assumptions about the equation of state. The transverse-momentum distribution of heavy hadrons is shown to be much enhanced relative to that of light pions. These distributions are little changed by differences in the assumptions about the initial transverse density and velocity profile. We are unable to fit the observed correlation between p_t and dE/dy observed in the Japanese-American Cooperative Emulsion Experiment.

I. INTRODUCTION

In this second paper of a series of two, we use the methodology explained in the previous paper in an attempt to compute the transverse-momentum distributions of hadrons as they might be produced in ultrarelativistic nuclear collisions. Arguments have been put forth by Shuryak,[1] Shuryak and Zhirov,[2] and by van Hove[3] that the transverse momentum as a function of multiplicity reflects general properties of the equation of state of high-temperature hadronic matter. These arguments, when substantiated by detailed computation, might allow an experimental determination of the properties of high-temperature hadronic matter.

The argument originally proposed by Shuryak is quite simple.[1,2] In the expansion of a fluid, the matter in the fluid does work on particles as they are pushed into the vacuum. This work is caused by a pressure difference between the vacuum and the matter in the fluid. As the energy density increases, the pressure increases, and therefore the transverse momentum of secondary particles increases. For example, the multiplicity of particles reflects the energy density achieved in a collision. Therefore, the degree to which the average transverse momentum increases as a function of multiplicity reflects properties of the equation of state.

The correlation between multiplicity and transverse momentum most dramatically reflects the equation of state in the case that there is a first-order phase transition. As has been emphasized by van Hove,[3] the transverse momentum may not rise so steeply for energy densities appropriate for a first-order phase transition. During a first-order transition, the energy density increases during the mixed phase, while the pressure remains constant. For equations of state appropriate to describe quark-gluon matter, this region of constant pressure may occur for energy densities which vary by an order of magnitude. The transverse-momentum distribution does not change much for this range of energy densities since the work done by the fluid on particles while expanding into the vacuum remains roughly constant, since the pressure remains constant.

There are of course many reservations concerning the above argument. In fact for a quark-gluon plasma, there is a slight increase in the energy per degree of freedom relative to that of an ideal pion gas. Probably the greatest gap in this argument, however, and the gap which is hardest to fill without the detailed computations which we present, is the following. As the energy density increases, the system spends more time in a mixed phase than as a low-pressure pion gas. Therefore, as the energy density increases, we expect a continued increase in the average transverse momentum due to a larger time-averaged value of the pressure.

The correlation between transverse momentum and multiplicity may be most easily seen by considering the example of the expansion of a spherical droplet of fluid, initially at rest.[4,5] Such an initial condition might arise as a fluctuation in $\bar{p}p$ collision. For such a system, initially in a volume V, the total energy and total entropy are conserved. Therefore, the quantities E/S, the energy per degree of freedom, and E/V, where V is the initial volume, are conserved by the equation of motion. Up to a constant which may be determined from geometry and by relating entropy to multiplicity, E/S is the average transverse momentum. The total energy of the secondaries may also be measured, and if the initial volume is known, the initial energy density may be experimentally determined. Notice that the total energy is easily related to the total multiplicity once the energy per particle is known. The relation between E/S and E/V may also be theoretically determined. The result of such a computation for a bag model is shown in Fig. 1. At low densities, $E/S \sim T$ since the energy per degree of freedom is that of a pion gas. At high densities, E/S is again $\sim T$ with roughly the same proportionality constant, since the energy per degree of freedom is almost independent of the constituents for a massless ideal gas. At energy densities appropriate to the phase transition, the energy per degree of freedom remains approximately constant as the hadron gas melts into a plasma.

For ultrarelativistic nuclear collisions, the geometry is not so simple as was the case for $p\bar{p}$ collisions. For systems of infinite transverse extent, the solutions of Bjorken[6] determine the space-time evolution of matter in the central region. These equations may also be generalized to the fragmentation region.[7,8] Unfortunately, the issue of measuring the properties of the equation of state by studying the transverse-momentum distribution of secondaries is not possible without allowing for transverse expansion. The formalism for treating transverse expansion of a quark-gluon plasma has been developed by Baym *et al.*[9] and by Białas, Czyż, and Kolawa.[10] Their method is applicable for the central region of head-on nuclear collisions of equal-A nuclei. They have made explicit computations for the case of an ideal-gas expansion.

The generalization of their results to the case where a fluid transversely expands and undergoes a phase transition is not entirely straightforward. As matter transversely rarefacts into the phase-transition region, a transverse rarefaction shock develops.[11,12] In order to treat this case, in a previous paper[13] (hereafter denoted as I) we presented an algorithm originally due to Boris and Book[14] which has been applied in the past to relativistic nuclear collisions by workers at Frankfurt.[15,16] We also discussed the general initial conditions, the method of solution of hydrodynamic equations, and decoupling. In this paper we shall make extensive use of the results presented in I.

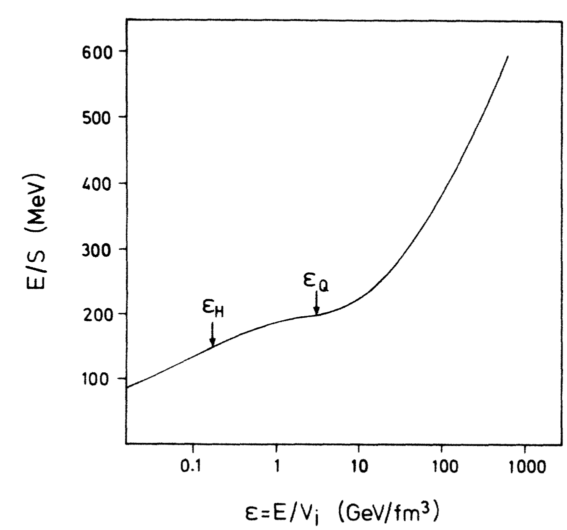

FIG. 1. E/S vs E/V for the bag-model equation of state with the parameters given in Sec. III.

This second paper presents results of explicit solutions to the hydrodynamic equation appropriate for the conditions in the central region of ultrarelativistic nuclear collisions. In Sec. II we discuss qualitative and quantitative features of our scenario for initial conditions and subsequent expansion of matter. We discuss the interplay between transverse and longitudinal expansion. We describe the shock wave which appears in the transverse rarefaction, and will show in Sec. III that our numerical solution properly reproduces the known features of this shock as gleaned from analytic studies. We also qualitatively analyze the correlation between transverse momentum and multiplicity for various nuclear sizes A.

In Sec. III we present the results of our computations for the hydrodynamic evolution of matter as it might be produced in ultrarelativistic nuclear collisions. We explicitly compute the p_t distributions of hadrons. We argue that the correlation between p_t and dN/dy is fairly independent of the details of the assumptions concerning the matter distributions and the initial conditions. We show that this correlation does in fact measure properties of the equation of state. We show that the correlation approximately scales as a function of $(1/A)dN/dy$. We also compute the effects of a quark-gluon plasma upon the kaon and proton spectrum. We present the actual p_t distribution for pions for a variety of values of dN/dy and for pions, kaons, and nucleons together for a typical value of the multiplicity density in head-on U-U collisions.

In Sec. IV we attempt to compare our results with the experimental results from the Japanese-American Cooperative Emulsion Experiment (JACEE). We find that our results are unable to fit the rapid rise in p_t seen in these experiments. We attempt to critically assess this discrepancy.

In Sec. V we briefly summarize our results.

II. INITIAL CONDITIONS AND QUALITATIVE FEATURES OF THE FLOW

We shall give in this section a qualitative and semiquantitative discussion of how the matter flows in ultrarelativistic nuclear collisions, and of what qualitative features exist in the p_t spectrum of hadrons as a function of dN/dy. We also discuss the expansion time of the matter produced in these collisions and the shock wave which appears in the transverse rarefaction of the matter. In addition, the interplay between transverse and longitudinal expansion, as well as general features of the correlation between multiplicity and average transverse momentum are described.

We will consider a situation where the hot hadronic matter expands longitudinally according to the scaling solution but is initially at rest in the transverse direction. Because of pressure gradients, transverse flow will develop, the details depending on the initial conditions, the equation of state, and on other assumed dynamical properties like the nucleation rate if a first-order phase transition is possible. Here we shall assume the bag-model equation of state and a "fast nucleation scenario" which means that the phase transition is assumed to proceed through the equilibrium mixed phase where pressure stays constant. This has a profound effect on the evolution of transverse flow and consequently the resulting p_t distribution is not related in a simple way to the initial temperature T_i or the initial energy density ϵ_i.

During the expansion energy is lost from any slice of matter in a given rapidity interval dy through the work which is done in the longitudinal expansion. Since for an ideal massless relativistic gas, entropy conservation implies gas particle number conservation in this slice, the energy per particle will decrease. In principle mass effects in the pion gas phase could lead to a decrease in particle number but the processes to achieve this are probably too slow when the pion mass is important.[17] This might be also compensated by the entropy production during various stages of the expansion. In terms of $\langle p_t \rangle$, the coupling to the longitudinal expansion means that the final value is usually less than what one would expect on the basis of initial temperature.

The effect of the phase mixture on the flow is twofold. On one hand, it slows down the transverse expansion as compared to the ordinary transverse rarefaction wave in simple relativistic fluid, leading to longer decoupling times. On the other hand, when the initial energy density is in the mixed phase interval, it provides a mechanism to drive the transverse expansion through the formation of a rarefaction shock wave at the interface of mixed phase and hadron gas. Since the strength of the shock depends on the energy density of the mixed phase, the flow velocity at decoupling increases with increasing energy density even though the initial temperature is fixed to T_c in this interval. As a result the p_t distribution broadens and $\langle p_t \rangle$ increases throughout the whole phase-transition region contrary to the naive expectations.

This conclusion should hold even in the case without homogeneous mixed phase when the initial matter would consist of plasma droplets embedded in hadron gas. As the droplets deflagrate the resulting pions will get the vacuum energy of the bag constant in addition to the thermal energy and consequently their average energy will be higher than that of the pions originally in hadron phase. It should be noted that even though $\langle p_t \rangle$ grows throughout the phase transition region $T_i = T_c$, the growth is much slower than in the case of no phase transition when the initial matter would consist of pions.

When $T_i > T_c$ the temperature drops to T_c in time $\tau_Q = \tau_i (T_i / T_c)^3$ due to the longitudinal expansion alone. If $\tau_Q \ll 3^{1/2} R_A$ the ordinary rarefaction wave has little effect and from τ_Q on the system developed essentially as if the initial energy density had been ϵ_Q. Consequently the growth of $\langle p_t \rangle$ is slow until τ_Q becomes comparable with $3^{1/2} R_A$. Then the ordinary rarefaction wave has time to evolve and a strong transverse flow builds up. Depending on the details of the decoupling this may even lead to shorter decoupling times for some parts of the matter when initial temperature increases.

The above arguments about the decrease of average energy per particle in the final state due to longitudinal expansion hold only for pions. For the heavier particles the situation may be different if they are able to stay in local thermal equilibrium and follow the collective flow. This is because their thermal motion even in the high-temperature initial state may, due to their mass, be small compared to the velocity of final flow which is dominated by the (almost) massless pions. As a result the p_t distribution of heavy particles should broaden more than that of the pions when going from hadron collisions to heavy-ion collisions. This expectation is verified by numerical results.

III. RESULTS AND THEIR DEPENDENCE ON THE PARAMETERS

In this section we present the numerical results on the flow and the p_t distributions and discuss the details of their dependence on the initial conditions and the main parameters of the model, the critical temperature T_c, the decoupling temperature $T_{\rm dec}$, and the radius R_A of the colliding nuclei. The number of quark flavors is taken to be 2.5 to account roughly for the mass of strange quark. With pions alone in the hadron phase, the ratio of the number of degrees of freedom in the plasma and hadron gas is 14.1. Most results are for $T_c = 200$ MeV which gives $B = 0.8$ GeV/fm^3 for the bag constant. Initial time τ_i and temperature T_i are related through $\tau_i = C / T_i$, where $C = 250$ MeV fm$/c$ as explained in I.

It actually turns out that for the flow and the hadronic p_t distributions the values of τ_i and T_i separately are not very important when $T_i > T_c$. The primary quantity is $\tau_i T_i^{\,3}$ which is a measure of total entropy dS/dy or, because of the approximate entropy conservation during the expansion, of the total multiplicity dN/dy. Incidentally, $\tau_i T_i^{\,3}$ also fixes the value of τ_Q, which is the essential time scale in the build up of the transverse flow. Since in most of the cases we consider, τ_Q is much less than the transverse size of the system, there is little transverse expansion before forming a mixed phase, and therefore little dependence in the flow upon T_i or τ_i. The value of T_i is, on the other hand, essential from the point of view of dilepton production and possible other signals which depend on the whole space-time history of the collision.[18] Here the discussion of dependence of different quantities on initial conditions can always be taken to refer to the total multiplicity.

If the initial energy density is less than ϵ_H, the critical energy density of the hadron gas, transverse flow will be weak. All that happens is that longitudinal expansion cools the matter and the p_t distribution is essentially the thermal distribution at $T_{\rm dec}$ with $\langle p_t \rangle = 2.12 T_{\rm dec}$. When ϵ_i grows across the phase transition region from $\epsilon_H = 0.2$ GeV/fm^3 to $\epsilon_Q = 3.8$ GeV/fm^3, the rarefaction shock gets stronger, the flow velocity of the outcoming hadrons increases and the p_t distribution broadens. Figures 2(a) and

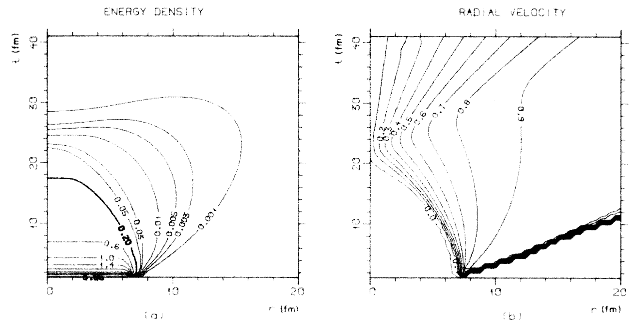

FIG. 2. Curves of (a) constant energy density (GeV/fm^3) and (b) constant radial velocity for $T_i = T_c = 200$ MeV and $\epsilon_i = \epsilon_Q = 3.8$ GeV/fm^3. The velocity curves near the light cone are an artifact of numerical approximations and have no physical significance.

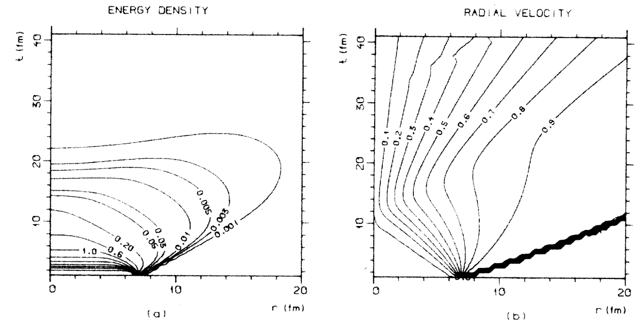

FIG. 4. As in Fig. 3 but without phase transition. To achieve the same multiplicity $(1/A)dN/dy = 4.2$ with the pion gas, the initial temperature is $T_i = (14.1)^{0.25}$ and $T_c = 480$ MeV.

2(b) show the contour plots of the energy density and the radial velocity for the flow when $\epsilon_i = \epsilon_Q$. The shock front follows closely the $\epsilon = \epsilon_H$ contour and is clearly seen as a jump in radial velocity. As the longitudinal expansion dilutes the energy density the velocity of the shock front increases[11] and this is seen as a bending of the shock region toward the time axis.

In Figs. 3(a) and 3(b) the same plots are shown for $T_i = 350$ MeV. One can see how the ordinary rarefaction wave starts from the surface but since $\tau_Q = 3.8$ fm/c, it cannot reach the inner parts of the matter before turning into the slowly propagating rarefaction shock. As a result the transverse flow is not dramatically increased from that in Fig. 2 and the growth of $\langle p_t \rangle$ is moderate as will be seen soon. This is possible because the decoupling time increases and the matter can lose a large part of its initial thermal energy as work in the longitudinal expansion. From here on the ordinary rarefaction becomes more important and $\langle p_t \rangle$ starts to grow faster with multiplicity.

To exhibit how the phase transition affects the flow, we show in Fig. 4 the flow for the pion gas with the same multiplicity as for the flow in Fig. 2. In the pion gas the transverse expansion is faster and a much stronger flow builds up than is the case for a mixed phase. This shows up quantitatively in the values of the average transverse momentum which for the pions in these two cases are 495 and 690 MeV with and without a phase transition.

From the flow quantities the p_t distributions for the pions can be calculated as explained in I and for the massless pions the results as a function of multiplicity are shown in Fig. 5. Qualitatively the distributions for largest multiplicities differ from thermal distributions in the sense that they do not exhibit exponential falloff as the thermal distributions would do in the shown p_t range.

The effect of the phase transition on the dependence of the p_t spectra is illustrated in Fig. 6, where the average value of p_t is shown both with and without phase transition as a function of multiplicity for iron-iron collisions with $R_A = 4.2$ fm. The rapid growth in the case of no

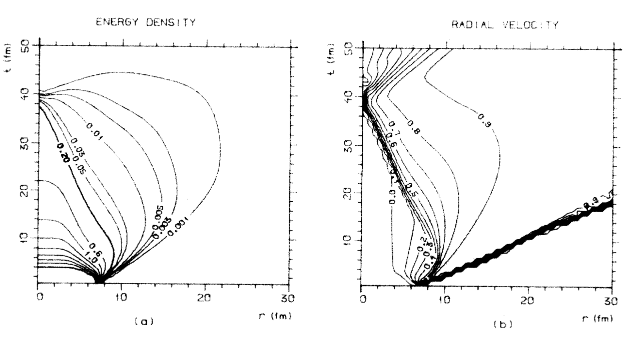

FIG. 3. As in Fig. 2 but for $T_i = 350$ MeV and $\tau_i = 0.71$ fm/c corresponding to $\epsilon_i = 28$ GeV/fm^3.

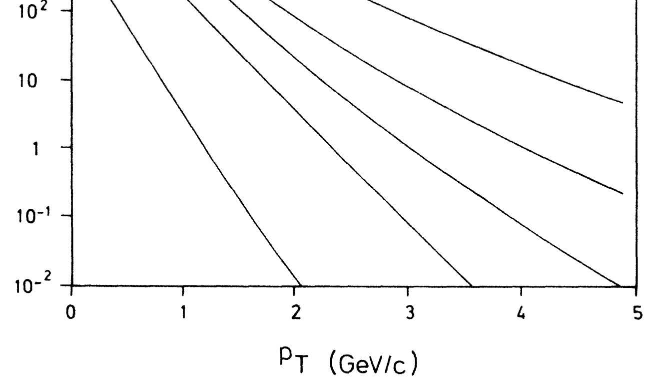

FIG. 5. p_t distributions for (massless) pions in U-U collisions ($R_A = 6.8$ fm) for different values of multiplicity $(1/A)dN/dy$.

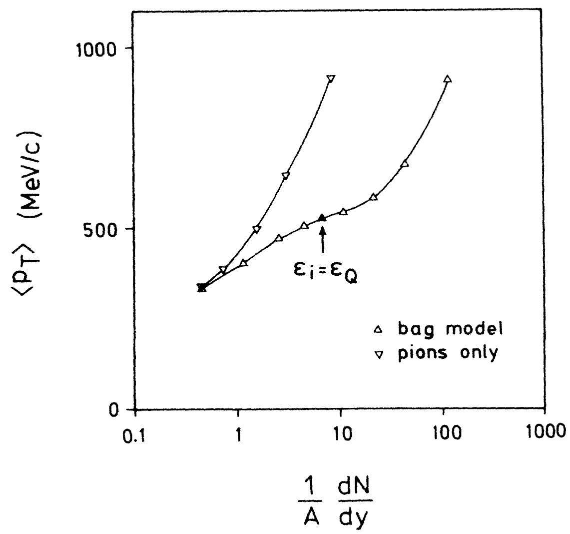

FIG. 6. Average p_t with and without phase transition. In the latter case the matter is assumed to be produced as hot pion gas. At the point indicated by $\epsilon=\epsilon_Q$, pure plasma state at T_c is reached at τ_i.

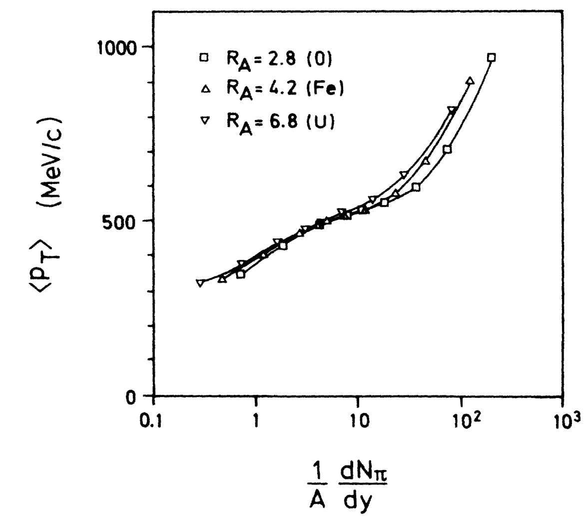

FIG. 7. Average p_t as a function of normalized multiplicity $(1/A)dN/dy$ for U-U ($R_A=6.8$), Fe-Fe ($R_A=4.2$), and O-O ($R_A=2.8$ fm) collisions.

phase transition reflects the fact that with the pion degrees of freedom available only, high initial temperatures are needed to raise the multiplicity. The highest point corresponds to temperature $T_i=550$ MeV at $\tau=1$ fm/c. According to the present ideas about the reasonable values of T_c, this is probably not a realistic temperature for the hadron phase. We return to this question later when discussing the JACEE data.[19] The fact that in the case of a phase transition the growth of $\langle p_t \rangle$ is slowest around $\epsilon_i=\epsilon_Q$ shows how effective the rarefaction shock is in delivering energy into the transverse flow. When the shock attains its full strength at $\epsilon_i=\epsilon_Q$ (the place is indicated by ϵ_Q in Fig. 6) the growth of $\langle p_t \rangle$ almost stops until the ordinary rarefaction wave in the plasma has enough time to develop.

Next we consider the effect of the size of the colliding nuclei on $\langle p_t \rangle$. It turns out that $\langle p_t \rangle$ as a function of $(1/A)dN/dy$ exhibits approximate scaling. Qualitatively this can be understood as follows: For large A, a fixed value of $(1/A)dN/dy$ implies larger initial temperatures than for small A. On the other hand the transverse flow develops faster and decoupling is reached sooner for small R_A, so that longitudinal expansion has less of a diluting effect upon the transverse-momentum distribution. The cancellation of these opposite effects then leads to the aforesaid scaling which is illustrated in Fig. 7.

It should be emphasized that in a scenario where the matter undergoes scaling expansion in the longitudinal direction, this expansion has a crucial effect on the final values of $\langle p_t \rangle$. In the case of no transverse motion the final p_t distribution is always the thermal distribution at $T=T_{\rm dec}$ and the role of the longitudinal motion is to cool the matter to this temperature through the work in the expansion. Even with the transverse motion taking place some part of the initial thermal energy is always lost into the longitudinal work. As a result $\langle p_t \rangle$ is, in general, reduced during the hydrodynamic stage from its initial value, which for massless quanta is $\langle p_t \rangle = 2.12 T_i$. The effect of the longitudinal work in our explicit calculations is shown in Fig. 8 where the final values of $\langle p_t \rangle$ are plotted together with $\langle p_t \rangle$ of the initial thermal distribution. As explained earlier, there are uncertainties in the initial time and consequently in the initial temperature, which would be reflected as an uncertainty in the initial $\langle p_t \rangle$. Still, since we are looking at $\langle p_t \rangle$ as a function of multi-

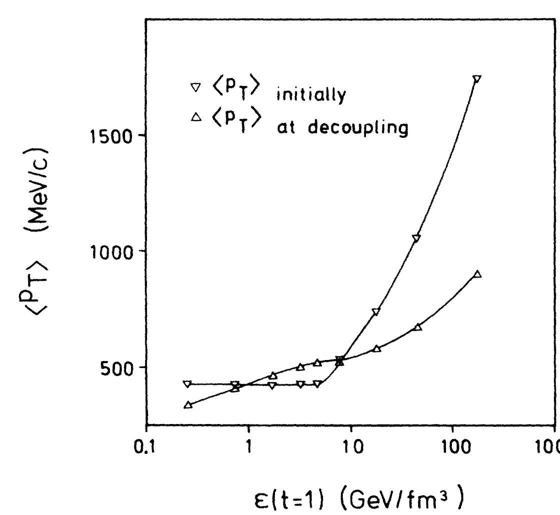

FIG. 8. Comparison of average p_t at initial time, i.e., the earliest time with thermal equilibrium, with no transverse flow, and at the decoupling time when the flow is fully developed but the matter has cooled down to $T_{\rm dec}$.

plicity which means fixing of $\tau_i T_i^3$, a factor of 2 uncertainty in τ_i would cause a 20% uncertainty in T_i, only.

In the actual nuclear collisions at high but finite energies the longitudinal work should result as an energy transfer from the central region to the fragmentation regions[8] and should be studied through the systematics of the $A(R_A)$ and dN/dy dependence of the shapes of both p_t and y distributions. Note that in spherical expansion $\langle p_t \rangle$ is conserved as a consequence of the symmetry throughout the expansion. In that situation $\langle p_t \rangle$ directly reflects T_i. Such situations might occur due to fluctuations in the initial conditions. On the other hand, the possible high T_i fluctuations with small or zero velocity (longitudinal) gradient may not be spherically symmetric. Instead they might exhibit Landau-type initial conditions which would still lead to faster expansion in the longitudinal direction. Since, especially for large nuclei, one would expect the spherical fluctuations to be embedded in longitudinally expanding matter and interacting with it, it may be difficult to understand large $\langle p_t \rangle$ values from the point of view of hydrodynamics.

In this paper we assume that the density of heavier particles is low enough so that their effect on hydrodynamic evolution of the matter can be neglected. This seems reasonable because they affect only the hadron gas stage of the hydrodynamic expansion and because of the mass suppression one would expect the effect to be less than 10% to the equation of state. (The fraction of kaons between T_c and T_{dec} is of the order of 30% or less and for that part the pressure could be lowered roughly by 30% giving a combined effect of $0.3\times0.3=0.1$ or 10%.) If we further assume that the heavy particles follow the overall flow, we obtain the p_t distributions of kaons and nucleons by simply calculating the flux of $n_i u^\mu$ through the decoupling surface as explained in detail in I. n_i, $i=K,N$, are the ideal-gas densities with zero chemical potential for the kaons and nucleons at the decoupling temperature and u^μ is the four-velocity of flow.

The results are shown in Fig. 9 together with the p_t distribution of massless pions for U-U collisions with $(1/A)dN/dy=13.5$. The normalization corresponds to one internal degree of freedom for each particle species. The spectra differ qualitatively from each other showing clearly the strong boost given by the transverse flow to the heavy particles. The dependence of $\langle p_t \rangle$ on the (total) multiplicity is shown in Fig. 10 for the three particle species exhibiting the stronger growth of $\langle p_t \rangle$ for the heavier particles. These features of the dependence of p_t on the mass of the particle and the multiplicity of events provide an experimental test for the existence of the collective flow in the final state of heavy-ion collisions.

In our computations, the kaon and nucleon distributions which are used to compute the emission of these particles through the decoupling surface were taken to be ideal-gas distribution functions with zero chemical potential. We might have allowed for a constant chemical potential which would change the relative normalization of the presented distributions. It would, however, change neither the shape of the distributions nor the average transverse momentum. If, as is expected for kaons, the chemical potential for strangeness is time dependent, then this algorithm is not strictly correct. The qualitative and semiquantitative nature of the enhancement of heavy particles at large p_t is quite general, and we expect that our approximate treatment reflects this effect. We next consider how the results depend on the different basic parameters, which up to now have been kept fixed.

(i) Decoupling temperature T_{dec}. When T_{dec} is lowered

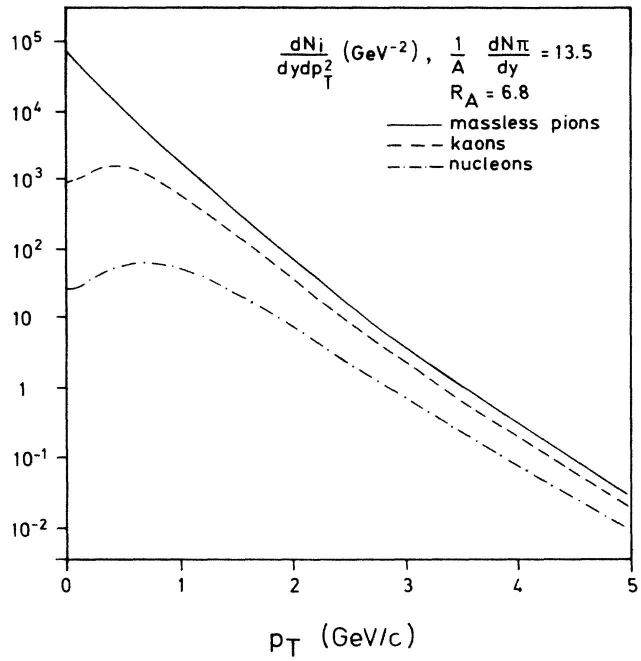

FIG. 9. Comparison of the shapes of the p_t spectra of (massless) pions, kaons, and nucleons in U-U collisions for $(1/A)dN/dy=13.5$. For normalization, see the text.

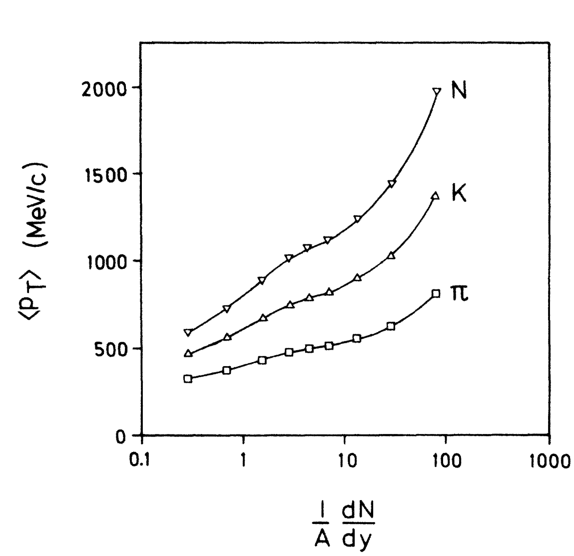

FIG. 10. Average p_t for pions, kaons, and nucleons as a function of normalized multiplicity $(1/A)dN/dy$ in U-U collisions ($R_A=6.8$ fm).

to 100 MeV there is a small increase (10 MeV/c or less) in $\langle p_t \rangle$ except for the lowest initial energy density $\epsilon_i = \epsilon_H$, where the longitudinal cooling still dominates over the transverse cooling. Compared to the other uncertainties, those coming from the uncertainty of the value of T_{dec} can be considered unimportant.

(ii) Critical temperature T_c. The value of critical temperature affects the p_t distributions in two ways, it changes the time scales and contributes to the initial energy density (for fixed multiplicity) through the bag constant. Table I shows the situation in the case of $T_i = 500$ MeV. These values correspond to the same initial entropy and essentially the same $(1/A)dN/dy$. (The final values of dN/dy differ a little because the details of the flow differ and this results in small differences in entropy production associated with the shock wave rarefaction.)

One can see from these results that $\langle p_t \rangle$ is not very sensitive to the precise value of T_c. In order to get a significant increase in $\langle p_t \rangle$ the value of T_c must be given values which are quite high at least from the point of view of using the simple bag-model equation of state. For example, $T_c = 400$ MeV leads to $B = 14$ GeV/fm^3, and at such high temperatures pions alone would not give a good approximation of the (degrees of freedom of) hadron gas.

(iii) Initial time τ_i and temperature T_i. As has been already discussed, the changes of τ_i and T_i are not important as long as $dN/dy \sim \tau_i T_i^3$ is kept fixed. At the initial time $v_r = 0$ and since τ_Q is now fixed, the differences for different τ_i arise from the evolution of the flow in the time interval $\delta\tau_i$ which one would expect to be of the order of 1 fm or less. This is a short time interval from the perspective of the total expansion time.

(iv) Shape of the initial densities. If the initial temperature distribution is not a step function, there will be a transverse pressure gradient (except where $T = T_c$) and the transverse acceleration takes place also in the matter. This will strengthen the flow and shorten the decoupling time. Taking the initial temperature distribution as

$$T(r) = T_0[1-(r/R_A)^2]^\kappa \tag{1}$$

we have considered three cases with $\kappa = 0$, $\frac{1}{6}$, and $\frac{1}{3}$. Roughly speaking for $\kappa = \frac{1}{6}$, the production of final-state matter would be proportional to the number and for $\kappa = \frac{1}{3}$ to the square of the number of colliding pairs of nucleons at the distance r from the collision axis. For $R_A = 6.8$ and $(1/A)dN/dy = 27.5$ fm, $T_0 = 500$, 572, and 630 MeV and the resulting values of p_t are 630, 657, and 697 MeV/c, respectively. The effect is quite moderate.

(v) The effect of initial transverse velocity. In all the results which have been discussed above $v_r = 0$, initially. It is clear that $\langle p_t \rangle$ can be arbitrarily enhanced if strong initial transverse flow is assumed. In the head-on collisions this, however, does not sound physically justified; one would rather expect that with the exception of the outer surface, the produced matter would be transversally at rest. A calculation with $\kappa = \frac{1}{6}$, $R_A = 6.8$ fm, $(1/A)dN/dy = 27.5$, and initial v_r growing linearly from 0 at $r = 0$ to 0.3 at the surface gives $\langle p_t \rangle = 765$ MeV as compared to 660 MeV for $v_r = 0$ at $\tau = \tau_i$. While it is conceivable that the initial flow might be stronger close to the surface, it is difficult to understand how in head-on collisions, a significant collective motion in the transverse direction could be present initially. Calculations with different multiplicities show that the effect grows with multiplicity. The change in numbers is less than 20% for the above numbers with rather high multiplicity.

(vi) The effects due to uncertainties in decoupling. Figure 11 shows the $\langle p_t \rangle$ from three different decoupling calculations. In flow decoupling the contribution from the collective motion to $\langle p_t \rangle$ is included, only. As a result it gives smaller values than the particle decoupling. The dependence on increasing initial temperature is, however, similar in all cases and the flow decoupling gives a nice independent check of the calculations. The third curve in Fig. 11 corresponds to pions with mass. Strictly speaking it is not consistent to include the mass in the decoupling calculation since the hydrodynamics is calculated with massless pions. However, the effect of the mass to the equation of state and the hydrodynamic calculation is small but is not that negligible to $\langle p_t \rangle$ (of the order of 15% to the right direction from the point of view of JACEE data). Most of this effect is due however to the fact that for massive pions, the relationship between multiplicity and entropy is altered. This effect is presumably less important if pion number is treated as conserved when the effect of pion mass is important.

In summary, we have shown that there is a strong correlation between $\langle p_t \rangle$ and dN/dy which reflects properties of the equation of state of hadronic matter. We have shown that these correlations may be computed up

TABLE I. Average transverse momentum of pions for different values of critical temperature. Multiplicity is fixed to $(1/A)dN/dy = 27.5$ and $R_A = 6.8$ fm.

T_c (MeV)	160	200	300	400
$\langle p_t \rangle$ (MeV/c)	592	632	817	980

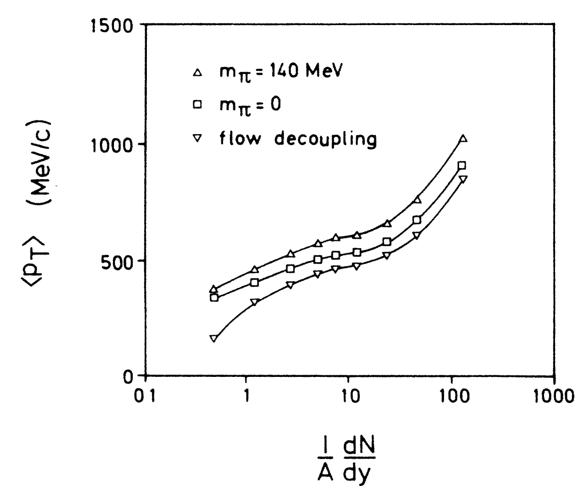

FIG. 11. Comparison of the results for average p_t from different decoupling algorithms.

to a fair number of theoretical uncertainties. These uncertainties are all at the 10—20% level or less, and hopefully the sum of their contributions is equally small. In the future, computations may allow a more precise determination of these uncertainties, and a hopefully more reliable estimate of the correlation. Given the stability of our numerical results to such changes, we believe that this correlation will survive closer inspection.

IV. COMPARISON WITH JACEE DATA AND DISCUSSION

The data taken by the JACEE cosmic-ray collaboration for p_t vs dE_t/dy is shown in Fig. 12. We have here only included the data for nucleus-nucleus collisions. In this plot, the energy density $\langle \epsilon \rangle$ at initial time $\tau_0 = 1$ fm/c is defined as

$$\langle \epsilon \rangle = \frac{3}{4\pi} \langle m_t \rangle \left. \frac{dN}{dy} \right|_{\rm ch} A_{\rm min}^{-2/3} \tag{2}$$

according to the formula of Bjorken.[6] All quantities in this relationship are experimentally measurable parameters. In order to compare our hydrodynamic results with these data, we have calculated from the numerical results of $\langle p_t \rangle$ and dN/dy for massless pions with different initial conditions an "initial energy density" at $\tau_0 = 1$ fm/c according to the above formula. It should be noted that as compared to the hydrodynamic calculation, this result always underestimates the initial energy density because it has no reference to the work during the expansion. Since the data is mainly for light nuclei, we use $R_A = 4.2$ fm.

At a value of $\langle \epsilon \rangle$ of the order of 1—10 GeV/fm^3, there is a sharp break in the p_t distribution. This break occurs in a plot which involves a variety of nuclei, and in fact primarily light nuclei. The interpretation of the lower axis as an energy density is therefore not on a very sound footing. Nevertheless, the occurrence of this break at an

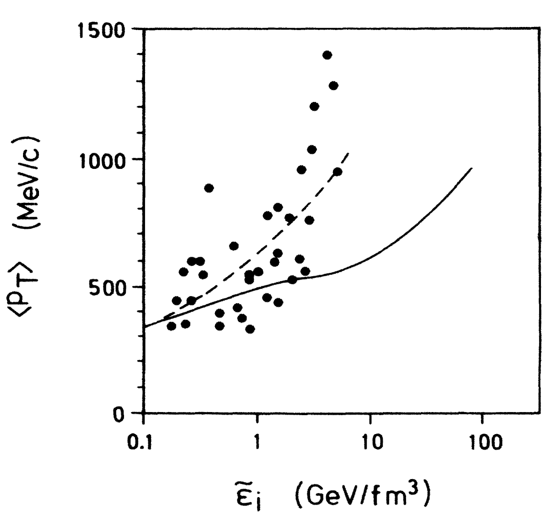

FIG. 12. Comparison of the hydrodynamic results for average p_t with the data of the JACEE[19] group.

"energy density" where one expects a quark-gluon plasma, and the above arguments for enhancements in the p_t spectrum due to hydrodynamic flow make it tempting to interpret this plot as evidence for the existence of a quark-gluon plasma.

In Ref. 4 it was argued that such a behavior might be reproduced if it was assumed that the plasma was formed as a result of spherically symmetric fluctuations in a volume of radius $1/p_t$. While such a model does fit the data on p_t versus $\langle \epsilon \rangle$, it suffers from lack of evidence that the distribution appears in any sense to correspond to a spherical fluctuation. Also, even for light nuclei, it is not clear why small volume (as compared to the nuclear size) fluctuations would dominate the whole event. We shall therefore in this paper, try to analyze the JACEE data assuming a boost-invariant matter distribution, as is expected to be the case for typical collisions of ultrarelativistic heavy nuclei.

The result of such an attempted comparison is shown in Fig. 12. As can be seen by the plot, the JACEE data rises too sharply for such an interpretation. Inclusion of the effects from pion mass and initial transverse velocity would enhance the calculated curve, but the shape of the curve would be almost unaffected. It is obviously the steep rise of $\langle p_t \rangle$ in a very narrow range of ϵ which is very difficult to obtain in scaling hydrodynamics with phase transition. There is some possibility that the data might fit by an ideal pion gas, with no plasma, as shown in the figure. At the energy densities of interest, the use of an ideal pion gas equation of state is probably not valid, since the pions most likely strongly interact, and contributions from strange particles and resonances are important.

There are several reasons why such a comparison might not be meaningful. First, in the absence of more detailed data which would establish whether or not the matter produced in such collisions was fluctuation dominated or not, we do not know if for these small nuclei, we are in fact using a correct space-time model to describe the collisions. Also, it has not been established that for such small nuclei, there is any evidence for hydrodynamic flow.

Another possibility is that the physics which produces this large p_t enhancement is not that of a quark-gluon plasma. For example, minijets may be important. To check this possibility, the azimuthal angular distribution of matter produced in such processes must be studied in detail, and systematic comparison of model computations which include jets must be made.

It is tempting to assume that perhaps the particles which generate the p_t enhancement are particles heavier than pions, and therefore acquire a larger p_t enhancement. Unfortunately, the experiment measures the distribution of photons to compute dE_t/dy. These photons should arise primarily from the decay of π^0's. If there was another substantial source of photons, it might be possible to explain the data. Another possibility which we do not consider is that the JACEE data is incorrect.

To summarize, using a conventional space-time picture of heavy-ion collisions, we are unable to explain the observed correlation in the JACEE data between p_t and dN/dy.

V. CONCLUSIONS

In this paper, we have applied a conventional space-time picture of heavy-ion collisions together with a hydrodynamic computation of the matter produced in such collisions, to describe the transverse momentum of particles, and to semiquantitatively determine the lifetime of this matter. We have found that the matter probably lives long enough, at least for heavy nuclei, to justify such a treatment. We have also shown that the transverse-momentum distributions do in fact reflect properties of the equation of state. It is of course premature to claim that these computations present signals for a quark-gluon plasma because there are many unforeseen possibilities. On the other hand, if there is any hope of obtaining information on the equation of state from heavy-ion collisions, collective phenomenon must be observed experimentally. This makes the systematic study of the dependence of the p_t spectra on multiplicity and the size of the colliding nuclei a very important experimental issue. Differences in the behavior of the spectra between pions and heavier particles should also show up if the collective flow exists. In order to achieve the above goals, a good understanding of background processes is necessary.

Our conclusions on these points leave some room for optimism, but it would be useful to have further theoretical studies in several areas to make the conclusions firmer. For example, to understand the possibilities for finding evidence of collective flow, a truly $(3+1)$-dimensional code should be employed to study the correlations between the central region and the fragmentation regions or between the transverse momentum and rapidity distributions. Also an equation of state which allows for massive pions is needed because the issue of decoupling is not clearly defined. More realistic equations of state should be used to further estimate the uncertainties in these computations. There should also be a cascade analysis. Such an analysis might determine the degree to which the plasma evolution is fluctuation dominated or a smooth hydrodynamic evolution. The effect of density fluctuations due to phase separation in the mixed phase could be determined.

[1]E. V. Shuryak, Phys. Rep. **61**, 71 (1980).
[2]E. V. Shuryak and O. Zhirov, Phys. Lett. **89B**, 253 (1980); Yad. Fiz. **28**, 485 (1978) [Sov. J. Nucl. Phys. **28**, 247 (1978)].
[3]L. van Hove, Phys. Lett. **118B**, 138 (1982).
[4]J. Kapusta, S. Pratt, L. McLerran, and H. von Gersdorff, Phys. Lett. **163B**, 253 (1985).
[5]K. Redlich and H. Satz, Phys. Rev. D **33**, 3747 (1986).
[6]J. D. Bjorken, Phys. Rev. D **27**, 140 (1982).
[7]K. Kajantie and L. McLerran, Phys. Lett. **119B**, 203 (1982); Nucl. Phys. **B214** 261 (1983).
[8]K. Kajantie, R. Raitio, and P. V. Ruuskanen, Nucl. Phys. **B222**, 152 (1983).
[9]G. Baym, B. Friman, J.-P. Blaizot, M. Soyeur, and W. Czyż, Nucl. Phys. **A407**, 541 (1983).
[10]A. Białas, W. Czyż, and A. Kolawa, Acta Phys. Pol. **B15**, 229 (1984).
[11]B. Friman, K. Kajantie, and P. V. Ruuskanen, Nucl. Phys. **B266**, 468 (1986).
[12]S. Pratt, Phys. Rev. D **33**, 1314 (1986).
[13]H. von Gersdorff, L. McLerran, M. Kataja, and P. V. Ruuskanen, Phys. Rev. D **34**, 794 (1986), hereafter denoted as I.
[14]J. P. Boris and D. L. Book, J. Comput. Phys. **11**, 38 (1973); **20**, 397 (1976); D. L. Book, J. P. Boris, and K. Hain, *ibid.*, **18**, 248 (1975).
[15]G. Graebner, Ph.D. thesis, Frankfurt University.
[16]G. Buchwald, G. Graebner, J. Theis, S. Maruhn, W. Greiner, and H. Stöcker, Phys. Rev. Lett. **52**, 1594 (1984); Nucl. Phys. **A418**, 625 (1984).
[17]S. Gavin, Nucl. Phys. **A435**, 826 (1985).
[18]K. Kajantie, M. Kataja, L. McLerran, and P. V. Ruuskanen, Phys. Rev. D **34**, 811 (1986).
[19]T. H. Burnett *et al.*, Phys. Rev. Lett. **50**, 2062 (1983).