Electromagnetic N-Δ transition at high Q^2

Carl E. Carlson

*Department of Theoretical Physics, University of Lund, Sölvegatan 14A, S-223 62 Lund, Sweden
and Physics Department, College of William and Mary, Williamsburg, Virginia 23185**

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The electromagnetic N-Δ transition is studied using perturbative QCD. Several results are obtained. Among them is that at high Q^2, in contrast to low Q^2, the $E2$ multipole amplitude is larger than the $M1$; their ratio is $\sqrt{3}$. Also, the asymptotic Q^2 dependence of N-Δ form factors should be the same as their elastic e-N scattering counterparts, when such counterparts exist. Data for some N-Δ form factors show that they fall faster with Q^2 than the nucleon dipole form, but since we have an underlying theory we can now discover that this is due to kinematic factors in the definition of the form factors in question.

I. INTRODUCTION

There has been continuing interest in electroproduction of the Δ resonance $e+N\to e+\Delta$. At low Q^2, recent interest[1–3] in this reaction has been motivated by the possibility of seeing "deformations" in the quark wave functions of the N and Δ. "Deformations" here means admixtures of spatial states other than the lowest S states into the N and Δ. At high Q^2 we have the possibility of calculating[4] the reaction using perturbative QCD.

The low-Q^2 idea is as follows.[1–3] Three multipole amplitudes, $M1$, $E2$, and $C2$, contribute to the N-Δ transition. Neglecting recoil and deformation,

$$F_{E2}=F_{C2}=0\ ,$$

since both of these require $l=2$ spherical harmonics. The $M1$, which requires just a spin flip, is nonzero and hence dominant. Now a tensor interaction from a pion cloud, gluon exchange, or anywhere will induce some D state in the N and Δ. The size of this admixture can perhaps be measured by measuring the size of the $E2$ and $C2$ amplitudes.

The perturbative QCD result, valid at high Q^2, contrasts with the small $E2$ and $C2$ amplitudes seen at low Q^2. The simplest results, which follow just from hadronic helicity conservations,[5] are that

$$F_{E2}=\sqrt{3}F_{M1}$$

and that the contributions to the cross section from F_{C2} are small.

We can also determine the asymptotic Q^2 dependence of the helicity amplitudes or multipole amplitudes. When N-Δ quantities have analogs in elastic e-N scattering, the asymptotic Q^2 dependence of the analogous quantities is the same. For example, from one of the helicity amplitudes we can naturally define a form factor $G_M^{N\to\Delta}$ which is analogous to the nucleon magnetic form factors G_{Mn} and G_{Mp}, and all these quantities should fall like $1/Q^4$ (modulo $\ln Q^2$ factors) at high Q^2. On the experimental side, checking the Q^2 dependence of N-Δ form factors began before the appearance of QCD. It was found that the form factors studied fell faster with Q^2 than the nucleon dipole form. With hindsight, we can examine the kinematic factors in the definitions of the form factors that were used. We learn using perturbative QCD (PQCD) that the form factors in question should fall faster than $1/Q^4$ at high Q^4. The data[6–9] are not in disagreement with simple expectations from PQCD.

One can go further and obtain the normalization of the various $N\to\Delta$ transition amplitudes at high Q^2 in terms of the distribution amplitudes (meaning wave functions integrated over transverse momenta) of the N and Δ. The distribution amplitude of the Δ is not known. However, the structure of the N-Δ result allows comments about some possible nucleon distribution amplitudes[10,11] which had been gleaned from studying the nucleon elastic form factors.

Some formulas for cross sections and other kinematic matters are given in Sec. II. The PQCD results for N-Δ transitions are presented in Sec. III and we summarize in Sec. IV.

II. KINEMATICS

One of the simple attributes of perturbative QCD is that the helicity of the hadrons is conserved.[5] (This is because the interactions among quarks proceed via gluon or photon exchange, both of which involve vector interactions and vector interactions preserve the quark helicity in the limit that we can neglect quark masses or off-shell effects.) Hence it is convenient to work with helicity amplitudes[12] and we shall define them in the Breit frame. In this frame the photon, N, and Δ momenta are collinear with the incoming N and outgoing Δ having the opposite direction but same magnitude three-momentum.[13]

We let q and N be the photon and nucleon four-momenta, respectively, with $Q^2=-q^2$ and $N\cdot q=m_N\nu$ with ν being the photon energy in the nucleon rest frame; for the N-Δ transition we have $\nu=(Q^2+m_\Delta{}^2-m_N{}^2)/2m_N$. The three independent helicity amplitudes (see Fig. 1) are defined for a helicity $(+\frac{1}{2})$ incoming nucleon and are

$$G_m = \langle \Delta, \lambda' = m - \tfrac{1}{2} \,|\, \epsilon_\mu^{(m)} \cdot J^\mu \,|\, N, \lambda = \tfrac{1}{2} \rangle / 2m_N \ ,$$

where J^μ is the electromagnetic current operator and the factor $1/2m_N$ makes G_m dimensionless. The transverse polarization vectors are

$$\epsilon^{(\pm)} = (0, \mp 1, -i, 0)/\sqrt{2}$$

and $\epsilon^{(0)}$ is normalized and satisfies $\epsilon^{(0)} \cdot q = \epsilon^{(0)} \cdot \epsilon^{(\pm)} = 0$. Since the N and the Δ move in opposite directions angular momentum conservation tells us that $m = \lambda + \lambda'$.

The differential cross section including the possibility of polarized electrons and nucleons is[12,14]

$$\frac{d\sigma}{d\Omega} = \frac{d\Sigma}{d\Omega} + hP\frac{d\Delta}{d\Omega} \ ,$$

where h and P are the electron longitudinal polarization and nucleon polarization, respectively, and

$$\frac{d\Sigma}{d\Omega} = \frac{\sigma_m f_\Delta^{-1}}{1+\tau^*} \{ G_0{}^2 + \tfrac{1}{2}[1 + 2(1+\tau^*)\tan^2\theta/2](G_+{}^2 + G_-{}^2) \} \ ,$$

$$\frac{d\Delta}{d\Omega} = \frac{\sigma_m f_\Delta^{-1}}{1+\tau^*} \{ \sqrt{2}(1+\tau^*)^{1/2} \tan\theta/2 \sin\beta \cos\alpha \, G_+ G_0 + [(1+\tau^*)(1+\tau^* \sin^2\theta/2)]^{1/2} \tan\theta/2 \sec\theta/2 \cos\beta (G_-{}^2 - G_+{}^2) \} \ ,$$

$$\sigma_m \equiv \frac{4\alpha^2 E'^2}{Q^4} \cos^2\theta/2 = \left| \frac{\alpha \cos\theta/2}{2E \sin^2\theta/2} \right|^2 \ ,$$

$$f_\Delta = \frac{E}{E'} \left| 1 - \frac{m_\Delta{}^2 - m_N{}^2}{2m_N E} \right| = 1 + \frac{2E \sin^2\theta/2}{m_N} \ .$$

Here $\tau^* = \nu^2/Q^2 = (Q^2 + m_\Delta{}^2 - m_N{}^2)^2 / 4m_N{}^2 Q^2$ and α and β are the azimuth and polar angles of the nucleon polarization direction in the N rest frame (with the z axis defined by the incoming photon momentum and the x axis lying in the scattering plane).[14]

The same cross sections written in terms of $C2$, $M1$, and $E2$ multipole amplitudes may be obtained with the substitutions[14–16]

$$G_0 = \sqrt{4\pi} \frac{Q}{|\mathbf{q}|} F_{C2} = \frac{\sqrt{4\pi}}{\sqrt{1+\tau^*}} F_{C2} \ ,$$

$$G_+ = \sqrt{4\pi}(F_{M1} + \sqrt{3} F_{E2})/2 \ ,$$

$$G_- = \sqrt{4\pi}(-\sqrt{3} F_{M1} + F_{E2})/2 \ ,$$

where $\mathbf{q}$ is the photon three-momentum in the nucleon rest frame (laboratory frame).

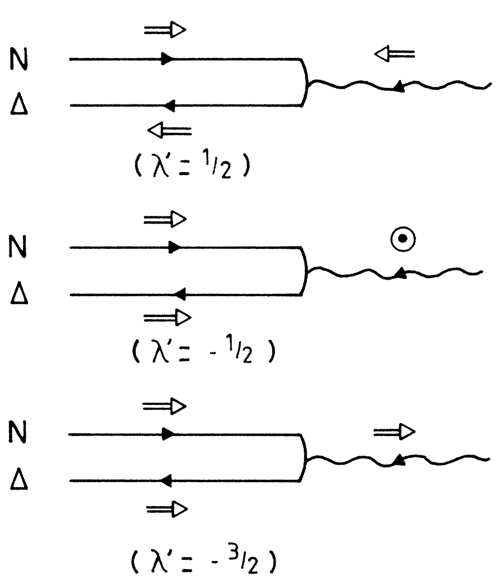

FIG. 1. The three independent helicity amplitudes for electromagnetic N-Δ transitions.

III. RESULTS

A. High-Q^2 relation among multipoles

The most elementary prediction of what happens at high Q^2 follows from the quark, and therefore hadronic, helicity-conserving property of QCD. Since G_+ is the only helicity amplitude with the same helicity for the outgoing Δ as for the incoming N, it is the one that is large and G_0 and G_- are asymptotically zero relative to it. From the latter and from the relations between the helicity and multipole amplitudes immediately follows the result: in our convention,[17]

$$\lim_{Q^2 \to \infty} F_{E2} = \sqrt{3} F_{M1} \ .$$

This in interesting contrast with the nonrelativistic result, where to the extent that the N and Δ have spherically symmetric spatial wave functions and recoil can be neglected, the $E2$ and $C2$ amplitudes are both zero and the $M1$ dominates.

B. Q^2 dependence

We can establish without detailed calculation the Q^2 dependence of the helicity amplitudes, up to factors involving $(\ln Q^2)$. We can do so either by directly calculating the Q^2 dependence of the Feynman diagrams, an example being given in Fig. 2, that underlie the $N \to \Delta$ transition amplitude or by analyzing the diagrams using rules suggested by Vainshtein and Zakharov[18] and used in Ref. 19 on the deuteron form factors. Briefly, if all quark heli-

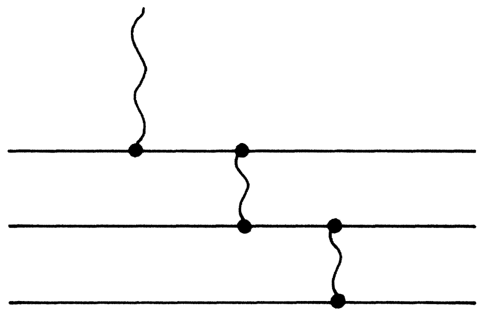

FIG. 2. One of several possible lowest-order diagrams for electromagnetic N-Δ transitions.

cities are conserved, the Q^2 dependence of the whole diagram follows from a $1/Q^2$ for each gluon line, a single Q for each quark line with just a single gluon attached, and no factor of Q for a quark line with two gluons or a gluon and photon attached. For every quark that requires its helicity flipped, we multiply by (m/Q), where m is some mass scale. This gives us at high Q^2,

$$G_+ \sim 1/Q^3 ,$$

$$G_0 \sim (m/Q) G_+ ,$$

$$G_- \sim (m^2/Q^2) G_+ .$$

(Noticing that $|\mathbf{q}| \sim Q^2/m$, we see that the $C2$, $E2$, and $M1$ amplitudes all fall at the same rate asymptotically, namely $1/Q^3$. However, because of the kinematic factors involved, the $C2$ amplitude will not contribute significantly to the cross section at high Q^2.)

At this point we should compare our results to the known results for the nucleon elastic form factors. There is no analog of G_-, and the other two helicity amplitudes G_{+N} and G_{0N} also fall like $1/Q^3$ and $1/Q^4$, respectively. More common are the magnetic and electric form factors and they are directly related to the transverse and longitudinal helicity amplitudes. Getting the important kinematic factors, we have

$$G_{MN} = (2m_N{}^2/Q^2)^{1/2} G_{+N} \sim 1/Q^4 ,$$

$$G_{EN} = G_{0N} \sim 1/Q^4 ,$$

from which also follow $F_{1N} \sim 1/Q^4$ and $F_{2N} \sim 1/Q^6$. ("Scaling" or the constancy of G_{EN}/G_{MN} is thus required at high Q^2; that it works to some level of accuracy at all Q^2 is something extra.)

For the N-Δ process there is a natural analog to the nucleon magnetic form factor, namely,

$$G_M^{N\to\Delta}(Q^2) = (2m_N{}^2/Q^2)^{1/2} G_+(Q^2)$$

and clearly enough this form factor should also fall like $1/Q^4$ asymptotically. So we have shown by example our claim that corresponding form factors have the same asymptotic Q^2 dependence.

What do the data say? The magnetic form factors used in analyzing the data have not, unfortunately, been ones suggested by QCD.

C. Comments on the data

The most common presentation of the data has been in terms of the form factor G_M^*. It was introduced by Ash *et al.*[6] and also used by Bartel *et al.*[7] and Stein *et al.*[8] A few details should help connect us to the experimental papers. G_M^* is given operationally[8] in terms of the cross section for $e + N \to e + \Delta(\to N\pi)$ at the resonance peak:

$$\left.\frac{d\sigma}{d\Omega\, dE'}\right|_{\text{peak}} = \Gamma_T \frac{4\alpha\pi(\nu^2+Q^2)}{\Gamma_R m_\Delta (m_\Delta{}^2 - m_N{}^2)} G_M^{*2}(Q^2) ,$$

where

$$\Gamma_T = \frac{\alpha}{2\pi^2} \frac{m_\Delta{}^2 - m_N{}^2}{2m_N Q^2} \frac{E'}{E} \frac{1}{1-\epsilon}$$

and

$$\epsilon = [1 + 2(1+\tau^*)\tan^2\theta/2]^{-1} .$$

Our previous theoretical cross-section formula was given for a stable Δ. It is simple to convert it for a Δ of width Γ_R (neglecting variations of parameters over the width of the resonance):

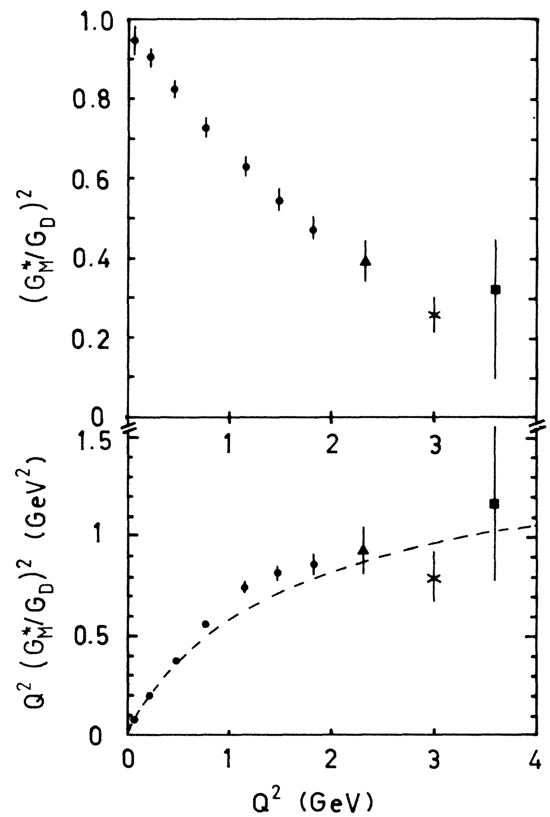

FIG. 3. The data for G_M^* divided by $G_D = 3/(1+0.71\ \mathrm{GeV}^2)^2$ and plotted in two ways vs Q^2. The dots are from Stein *et al.* (Ref. 8), the triangle from Bartel *et al.* (Ref. 7), the cross from Haiden (Ref. 9), and the square from Cone *et al.* (Ref. 9) (extracted following Ref. 21). The dashed curve corresponds to $Q^2/(1+Q^2/1.43\ \mathrm{GeV}^2)$.

$$\frac{d\sigma}{d\Omega}\to\frac{d\sigma}{d\Omega\,dE'}=\frac{d\sigma}{d\Omega}f_\Delta\frac{2m_N m_\Delta \Gamma_R \pi^{-1}}{(W^2-m_\Delta{}^2)^2+\Gamma_R{}^2 m_\Delta{}^2},$$

where W is the $N\pi$ mass from the decaying Δ. This should suffice to allow us to get

$$G_M^{*2}(Q^2)=\frac{2m_N{}^2}{\nu^2+Q^2}(G_+{}^2+G_-{}^2+2\epsilon G_0{}^2)$$

at any Q^2, and for high Q^2

$$G_M^*(Q^2)=\frac{m_N{}^2\sqrt{8}}{Q^2}G_+\sim 1/Q^5 .$$

Data[6-9] for G_M^* divided by the nucleon dipole form

$$G_D(Q^2)=3/(1+Q^2/0.71\ \text{GeV}^2)^2$$

are shown in Fig. 3(a). The G_M^* data falls faster with Q^2 than the nucleon dipole form,[7,8,15,20] as we now expect.[21] Perhaps more interesting is to plot $Q^2(G_M^*/G_D)^2$, as we do in Fig. 3(b), to see if there is any sign of leveling off at high Q^2. The curve in the figure is

$$Q^2/(1+Q^2/M^{*2}) ,$$

where $M^{*2}=1.43\ \text{GeV}^2$.

To summarize, the data is behaving the way that PQCD would lead us to expect. We should still be tentative: the Q^2's involved are not extremely high and we may be in a situation where the threshold effects are more important than in other situations. For example, in the combination $(G_+{}^2+G_-{}^2)$, the former dominates at high Q^2 but the latter is $\frac{3}{4}$ of the total at low Q^2.

D. The normalized PQCD result

The helicity amplitude G_+ is the one which can be most easily calculated at high Q^2 in perturbative QCD. All masses can be set to zero and the leading term obtained. In order to borrow results from elsewhere,[22,23] we work in the infinite momentum frame. One may do the Lorentz transformations to see that

$$2p^+G_M^{N\to\Delta}=\langle\Delta,\lambda'=\tfrac{1}{2}\,|\,J^+\,|\,N,\lambda=\tfrac{1}{2}\rangle ,$$

where p is the momentum of the incoming nucleon, which is moving very fast in the z direction, and $p^+=p^0+p^3$ and $J^+=J^0+J^3$. The bulk of the calculation of the matrix element for $Q\to\infty$ is similar to the nucleon elastic magnetic form factor case. The result may be given[23] as a convolution over the light-cone momentum fractions of the quarks of the distribution amplitudes for the N and Δ and the hard-scattering amplitude T_H:

$$G_M^{N\to\Delta}=\int[dx][dy]\Phi_\Delta(y)T_H(x,y,Q)\Phi_N(x) ,$$

where

$$[dx]=dx_1dx_2dx_3\delta(1-x_1-x_2-x_3) .$$

Also

$$\Phi_\Delta(x)=\phi_\Delta(x)(1/\sqrt{3})\,|\,uud+udu+duu\,\rangle_{\uparrow\downarrow\uparrow}+\text{permutations} ,$$

where ϕ_Δ is symmetric under $x_1\rightleftarrows x_3$ and the analogous expression for the proton has terms antisymmetric as well as symmetric under the same interchange:

$$\Phi_P(x)=\phi_S(x)(1/\sqrt{6})\,|\,2udu-uud-duu\,\rangle_{\uparrow\downarrow\uparrow}+\phi_A(x)(1/\sqrt{2})\,|\,uud-duu\,\rangle_{\uparrow\downarrow\uparrow}+\text{permutations} .$$

We can get

$$Q^4G_M^{p\to\Delta^+}=\left[\frac{16\pi\alpha_s}{3}\right]^2\frac{\sqrt{2}}{3}\int[dx][dy]\{(T_1-T_2)[\phi_\Delta(x)\phi_S(y)+x\rightleftarrows y]+\sqrt{3}T_1[\phi_\Delta(x)\phi_A(y)+x\rightleftarrows y]\} ,$$

where T_1 and T_2 come from T_H and are[23]

$$T_1=\frac{1}{x_3(1-x_1)^2y_3(1-y_1)^2}+\frac{1}{x_2(1-x_1)^2y_2(1-y_1)^2}-\frac{1}{x_2x_3(1-x_3)y_2y_3(1-y_1)} ,$$

$$T_2=\frac{1}{x_1x_3(1-x_1)y_1y_3(1-y_3)} .$$

By isospin, the $n\to\Delta^0$ amplitude is the negative of the $p\to\Delta^+$ one. If we have only symmetric wave functions with $\phi_\Delta=\phi_S$, then

$$G_M^{p\to\Delta^+}=-\sqrt{2}G_{Mn} .$$

If we expand the distribution amplitudes in Appel polynomials[23] as

$$\phi_\Delta(x)=x_1x_2x_3\sum N_i^*\tilde{\phi}_i(x) ,$$

where for the Δ the sum is only over polynomials symmetric under $x_1\rightleftarrows x_3$, then

$$Q^4G_M^{p\to\Delta^+}=\left[\frac{4\pi\alpha_s}{27}\right]^2\frac{\sqrt{2}}{3}\sum E_{ij}N_i^*N_j ,$$

where N_j are the analogous coefficients for the N and the coefficients E_{ij} are given in Table I for i and j less than 5.

Without knowing ϕ_Δ we cannot proceed much further. The N and Δ distribution functions need not be the same; we already know that one of them can have an antisymmetric part but the other cannot. The N-Δ transition can however be a decisive check on one aspect of the nucleon wave function.

It has been suggested that the ratio G_{Mp}/G_{Mn}, which has been measured[24] to 10 GeV^2, requires that the nucleon distribution amplitudes be asymmetric.[10,11] However, for the N-Δ transition form factor one finds significant de-

TABLE I. The coefficients E_{ij} that appear for $G_M^{N\to\Delta}$ when the N and Δ distribution amplitudes are expanded in Appel polynomials. The cases $i=1$ and 4 do not appear for the Δ because these are antisymmetric in x_1 and x_3.

$i \backslash j$	0	1	2	3	4	5
0	-162	$63\sqrt{3}$	-81	45	$-15\sqrt{3}$	-30
2	-81	$-42\sqrt{3}$	18	-90	$11\sqrt{3}$	$\frac{5}{2}$
3	45	$110\sqrt{3}$	-90	170	$-23\sqrt{3}$	$-\frac{65}{2}$
5	-30	$-(35\sqrt{3})/2$	$\frac{5}{2}$	$-\frac{65}{2}$	$41/(6\sqrt{3})$	$\frac{145}{18}$

structive interference between the symmetric and antisymmetric parts of the particular nucleon distribution amplitudes needed to fit G_{Mp}/G_{Mn}. For example, one possibility is $\phi_S+\phi_A=N_3(\phi_3-\phi_1)$ but from Table I it can be seen that it is roughly the sum of N_1 and N_3 that contributes to the N-Δ transition. The Chernyak-Zhitnitsky distribution amplitude[11] is more complicated but leads to the same qualitative result: that $G_M^{N\to\Delta}$ is significantly less than $|G_{Mn}|$ or G_{Mp}.

The data for the N-Δ transition are not at high Q^2 and there may be threshold effects as mentioned earlier, but if we boldly extract an asymptotic normalization we get

$$Q^4 G_M^{N\to\Delta}=\frac{3}{2}\frac{M^*(0.71\ \mathrm{GeV}^2)^2}{m_N}\approx 1.0\ \mathrm{GeV}^4$$

which is about 10% less than $Q^4 G_{Mp}$ in the 5–10-GeV^2 range.

There are several possibilities. One is that $Q^4 G_M^{N\to\Delta}$ will drop significantly in magnitude as we get measurements at higher Q^2. [Figure 3(b) is interesting to look at.] Another possibility is that the ratio G_{Mp}/G_{Mn} may at 10 GeV^2 still be affected by non-PQCD contributions and that consequently our estimate of the antisymmetric part of the wave function is too high. Then $G_M^{N\to\Delta}$ could maintain some value in the range $|G_{Mn}|$ to G_{Mp}. In addition in this case, the ratio G_{Mp}/G_{Mn} should move with Q^2 (and relatively quickly, not on a logarithmic scale) to a value closer to -3.

E. Comments on the soft contributions

The "soft contributions"[25] are the contributions to $G_M^{N\to\Delta}$ that come from low-k_T quarks in the wave function. These contributions will be small at high Q^2, but we would like to see what they are at moderate Q^2. For definiteness let us approximate the low-k_T part of the transverse-momentum wave function with a Gaussian, as in

$$\psi_\Delta(x,k_T)\approx\phi_\Delta(x)(192\pi^4/\alpha^4)\exp\left[-\sum k_{iT}^2/2\alpha^2\right]$$

and use the same value of α for both N and Δ. With $\zeta\equiv Q^2/2\alpha^2$ and $x_{ij}=x_i^2+x_j^2+x_ix_j$ we have, for $p\to\Delta^+$,

$$G_{M,\mathrm{soft}}^{N\to\Delta}=\sqrt{2/3}\int[dx][\phi_\Delta(\phi_S+\sqrt{3}\phi_A)e^{-x_{23}\zeta}-\phi_\Delta\phi_S e^{-x_{13}\delta}]\ .$$

The above expression is zero at zero Q^2 for any wave function in agreement with the expected threshold behavior for this form factor. Also, if the proton and Δ distribution amplitudes are completely symmetric in x_1, x_2, and x_3 then the above soft contributions are zero for *any* Q^2. At last we have a clear example where the soft contributions are decidedly less than the hard-scattering contributions.

More interestingly, the result that $G_{M,\mathrm{soft}}^{N\to\Delta}$ is zero for wave functions that are completely symmetric, coupled with the nonzero result for the hard scattering $G_M^{N\to\Delta}$ in the same case, means that the wave function's tail is not symmetric in x_1, x_2, and x_3. This is quite possible since the hard-gluon exchanges that give the tail are sensitive to the relative helicity of the quarks they connect and thus distinguish the antiparallel quark from the other two. This also shows that a factorizable form of the wave function, $\psi=\phi(x)g(k_T)$, which has the same symmetry among the x_i at all k_T, cannot be right for a completely symmetric $\phi(x)$ and is at least suspect in any other case.

IV. SUMMARY AND CONCLUSIONS

We have the prediction that at high Q^2, $F_{E2}=\sqrt{3}F_{M1}$. If borne out, this will be quite dramatic. The present data on F_{E2}/F_{M1} show it to be well below unity.[26] The expected increase in F_{E2} is in different language a recoil effect. Even if the Δ were spherically symmetric, in the N rest frame it would appear squashed by a Lorentz contraction. The large size of the effect should encourage thinking for lower Q^2 about the effect of recoil as well as about the effect of deformation.

We have also seen the asymptotic Q^2 dependence of the form factors, for example, that $G_M^{N\to\Delta}\sim 1/Q^4$, and have seen that the data are not out of line with expectation.

Finally, the calculation of the normalization of $G_M^{N\to\Delta}$ in terms of the N and Δ distribution amplitudes may suggest interesting trends, namely, that either $Q^4G_M^{N\to\Delta}$ itself will fall or $|G_{Mp}/G_{Mn}|$ will continue to drift upwards as Q^2 becomes somewhat higher.

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*Permanent address.

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[13]For unequal-mass initial and final hadrons, one can distinguish between a Breit frame and a brick wall frame (no energy transfer), where $q=(0,0,0,|\mathbf{q}|)$ and $\epsilon^{(0)}=(1,0,0,0)$. However the latter frame has the property that the N and Δ are moving in the same direction for $Q^2<(m_\Delta{}^2-m_N{}^2)$ but in opposite directions above this Q^2.

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