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# Classical repulsive gravity and broken Lorentz symmetry

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A four-dimensional quasi-Riemannian theory of gravity, with local SO(3) invariance, is used as a simple model to describe gravity under the hypothesis of broken Lorentz gauge symmetry, and to show that in this case the sign of the force between two masses may depend on their relative distance. Considering in particular the field of a static, central macroscopic source, we find however that the gravitational interaction experienced by a macroscopic test body may become repulsive only immediately close to the Schwarzschild surface.

## I. INTRODUCTION

The possible existence of long-range repulsive interactions between macroscopic bodies ("antigravity") has been recently investigated in the framework of supersymmetric and multidimensional unified theories.[1,2] In particular, considering an effective four-dimensional theory in which fermionic matter interacts via long-range scalar, vector, and tensor fields, it has been shown[1] that the sign of the total force between two massive objects may be dependent on their *relative velocity,* and that for suitable values of the vector and scalar charges this possibility is not experimentally ruled out.

A repulsive long-range force is called in Ref. 1 antigravity; however, this term would seem to be not completely appropriate when the total force is of the scalar-vector-tensor type, which acts differently on particles and antiparticles,[1] unlike the usual pure-gravity interaction which couples universally to the total energy of a body, irrespective of its internal content of elementary components.

The aim of this paper is to show that if Lorentz invariance is not an exact local gauge symmetry for the gravitational interaction, the sign of the pure gravity force between two massive bodies may become dependent also on their *relative distance.* In this case one may obtain a repulsive force also in the field of a static and spherically symmetric source, without dragging effects due to rotations; and since all the bodies are repelled with the same acceleration, irrespective of their composition, in agreement with the equivalence principle, the term antigravity would seem more properly used to describe this effect, rather than the scalar-vector-tensor repulsion of Refs. 1 and 2.

## II. A QUASI-RIEMANNIAN THEORY IN FOUR DIMENSIONS

In the context of unified theories in $D>4$ dimensions, Weinberg has suggested the possibility of considering generalized theories of gravity, called quasi-Riemannian,[3] which are covariant under general coordinate transformations, but have a local symmetry group $G_T$ other than the $D$-dimensional Lorentz group $SO(1,D-1)$. The new tangent-space group is required to be of the form $G_T=SO(1,n-1)\times G_T'$, where $4\leq n\leq D$ and $G_T' \subseteq SO(D-n)$. The geometrical structure of these generalized theories has been recently investigated in Refs. 4 and 5.

Taking seriously the possibility that the Lorentz group may not be the local gauge symmetry of gravity even in four dimensions, we consider in this paper a quasi-Riemannian theory for $D=4$ with $G_T=SO(3)$ as an effective theory to discuss some gravitational consequence of an eventual four-dimensional breaking of the Lorentz gauge symmetry, assuming (as suggested also by Zee[6]) that local rotational invariance continues to hold.

According to the procedure recently developed in Ref. 4, such a theory can be formulated in terms of the components of the original Lorentz connection $\Omega^{ab}=\Omega^{ab}_{\mu}dx^{\mu}$ and of the anholonomic basis $V^a=V^a_{\mu}dx^{\mu}$, which satisfy

$$dV^a+\Omega^a{}_b\wedge V^b=0 \tag{1}$$

(conventions: greek letters denote four-dimensional world indices and latin letters are tangent-space indices; however, $a,b,c,d,\ldots$ run from 1 to 4, while $i,j,k,\ldots$ run from 1 to 3). With the choice $G_T=SO(3)$, the $V^i$ and $V^4$ components of $V^a$ transform, respectively, as an SO(3) vector and scalar, and the SO(3,1) connection $\Omega$ decomposes into the SO(3) connection $\omega^{ik}=\Omega^{ik}$ and the one-form $\varpi^{i4}=\Omega^{i4}$ which transforms covariantly under SO(3). Therefore the Lorentz curvature two-form

$$R^{ab}(\Omega)=d\Omega^{ab}+\Omega^a{}_c\wedge\Omega^{cb} \tag{2}$$

may be decomposed as

$$\begin{aligned} R^{ik}(\Omega)&=R^{ik}(\omega)+\varpi^i{}_4\wedge\varpi^{4k}\,,\\ R^{i4}(\Omega)&=D(\omega)\varpi^{i4}\,, \end{aligned} \tag{3}$$

where $R^{ik}(\omega)$ is the SO(3) curvature two-form

$$R^{ik}(\omega)=d\omega^{ik}+\omega^i{}_j\wedge\omega^{jk}\,, \tag{4}$$

and $D(\omega)$ is the SO(3) exterior covariant derivative

$$D(\omega)\varpi^{i4}=d\varpi^{i4}+\omega^i{}_j\wedge\varpi^{j4}\,. \tag{5}$$

 

Following then Ref. 4, the most general $G_T$-invariant action, which does not involve more than second derivative of the vierbeins, for $G_T = \mathrm{SO}(3)$ is given by

$$S = -\int d^4x V[R_{ik}{}^{ik}(\omega) + c_1\bar{\omega}_{[ik]4}\bar{\omega}^{[ik]4} + c_2\bar{\omega}_{\{ik\}4}\bar{\omega}^{\{ik\}4} + c_3\bar{\omega}_i{}^i{}_4\bar{\omega}_k{}^{k4} + c_4\bar{\omega}_4{}^4{}_i\bar{\omega}_4{}^{4i}] , \quad (6)$$

where $R_{ik}{}^{ik}(\omega)$ is the SO(3) scalar curvature, and

$$\bar{\omega}_{[ik]4} = \tfrac{1}{2}(\bar{\omega}_{ik4} - \bar{\omega}_{ki4}) ,$$
$$\bar{\omega}_{\{ik\}4} = \tfrac{1}{2}(\bar{\omega}_{ik4} + \bar{\omega}_{ki4}) - \tfrac{1}{3}\delta_{ik}\bar{\omega}_j{}^j{}_4 \quad (7)$$

(we use the notations of Ref. 4).

This action depends on four arbitrary parameters $c_1, \ldots, c_4$, which are, of course, model dependent (for simplicity, we have neglected the possibility of adding also a cosmological constant term, inessential for our considerations); in particular, the SO(3,1) scalar curvature $R_{ab}{}^{ab}(\Omega)$, corresponding to the standard Einstein action, is obtained in the limit in which $c_1 = 1$, $c_2 = -1$, $c_3 = \frac{2}{3}$, and $c_4 = 0$, and only for these particular values of the parameters the local SO(3,1) symmetry is restored.

In order to show the possibility of obtaining repulsive gravitational interactions as a consequence of the SO(3,1) breaking, we will consider here only a "minimal" deviation from the local Lorentz symmetry,[7,8] which can be represented in terms of only one dimensionless parameter $\alpha$ by putting

$$c_1 = -c_2 = \tfrac{3}{2}c_3 = 1 - \alpha, \quad c_4 = 0 . \quad (8)$$

With this choice, the action (6) can be rewritten in compact form as

$$S = \tfrac{1}{2}\int [R^{ab}(\Omega)\epsilon_{abcd} - \alpha R^{4b}(\Omega)\epsilon_{4bcd}] \wedge V^c \wedge V^d \quad (9)$$

and it can be shown[7] that this one-parameter model of gravity with broken Lorentz gauge symmetry can be related to the "simplest and most predictive" model of Lorentz noninvariance considered by Nielsen and Picek.[9,10]

Performing the variation of the action (12) one finds that, in the absence of spinning sources, the connection $\Omega$ is torsionless, in agreement with Eq. (1), and the gravitational field equations in vacuum become

$$R^{ab}(\Omega) \wedge V^c\epsilon_{abcd} = \alpha R^{4b}(\Omega) \wedge V^c\epsilon_{4bcd} . \quad (10)$$

Solving these generalized Einstein equations in the case of a static and spherically symmetric source of mass $m$, one obtains (in polar coordinates) the line element[7]

$$ds^2 = (1-2m/r)^\gamma dt^2 - (1-2m/r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) , \quad (11)$$

where $\gamma$ is related to the parameter $\alpha$, which characterizes the strength of the SO(3,1) breaking, by

$$\gamma = (1-\alpha/2)^{-1} . \quad (12)$$

Of course the usual Schwarzschild solution is recovered in the limit $\alpha \to 0$, that is $\gamma \to 1$.

Finally, comparing the predictions of this theory with the results of the Solar System tests of general relativity (perihelion precession, radar-echo delay, etc.) it is easy to show that no disagreement is found with the experimental data, provided that[11]

$$|\alpha| \lesssim 4\times 10^{-3} . \quad (13)$$

## III. RADIAL MOTION IN THE FIELD OF A STATIC CENTRAL SOURCE

In the context of a quasi-Riemannian model of gravitational Lorentz noninvariance, the Bianchi identities and the vacuum field equations imply that a test body, in an external gravitational field, can be consistently associated to a covariantly conserved current (covariance here is to be understood with respect to general coordinate transformations). For example, in the particular case we are considering, rewriting Eq. (10) in the explicit form

$$R_\alpha{}^\beta - \tfrac{1}{2}\delta_\alpha{}^\beta R = -\frac{\alpha}{2}(R_\mu{}^\nu\delta_\alpha{}^\beta + R_{\mu\alpha}{}^{\nu\beta} - R_\alpha{}^\nu\delta_\mu{}^\beta)V_\nu{}^4V^\mu{}_4 \quad (14)$$

(where $R_{\mu\nu\alpha}{}^\beta$ is the usual Riemann curvature tensor, and $R_\alpha{}^\beta = R_{\alpha\mu}{}^{\mu\beta}$) one can easily verify that, for the solution (14), the Riemann-covariant divergence of the right-hand side of Eq. (14) is vanishing, in agreement with the contracted Bianchi identity.

Therefore, assuming that the local SO(3,1) symmetry is broken only in the pure gravity part of the action, according to the hypothesis of minimal deviation from Lorentz invariance, it follows that the motion of unpolarized test bodies in a vacuum is still geodesic [this is no longer true, in general, in the case of nonvacuum solutions of the quasi-Riemannian field equations, unless the local SO(3,1) symmetry is suitably broken also in the matter Lagrangian, and a fine-tuning of the additional parameters is performed to arrange a vanishing covariant divergence of the full Lorentz noninvariant part of the field equations[12]].

If the SO(3,1) gauge symmetry of gravity is minimally broken down to SO(3) according to Eq. (9), a test particle in the external field of a static central source follows then the geodesics of the metric (11). Considering in particular a radial motion ($\dot\varphi = \dot\theta = 0$), we obtain from (11) the two geodesic equations

$$\dot t(1-2m/r)^\gamma = k , \quad (15)$$

$$\dot r^2 = k^2(1-2m/r)^{1-\gamma} - (1-2m/r) , \quad (16)$$

where a dot denotes differentiation with respect to the proper time $s$, and $k^2 \geq 1$ is an integration constant related to the value of the particle velocity at infinity, $v_\infty$, by $k^2 = (1-v_\infty{}^2)^{-1}$. To discuss the classical behavior of a particle falling radially according to these equations, we remember that the regions of space penetrable for a given total energy are characterized by $\dot r^2 > 0$ [imaginary velocities would correspond to forbidden zones where the effective gravitational potential, defined according to Eq. (16), is higher than the total energy of the particle]. Obviously in general relativity ($\gamma = 1$) all the values of $r$ from 0 to $+\infty$ are allowed. In our case we find that the requirement $\dot{r}^2 > 0$ is satisfied for all $k$ (i.e., for every given total energy at infinity) in the region $r > 2m$ provided that $\gamma > 0$, and in the region $r < 2m$ provided that $(1-2m/r)^\gamma < 0$. As we are considering a very small deviation from Lorentz symmetry [see Eqs. (12) and (13)], then $\gamma > 0$ and the motion outside the Schwarzschild sphere $r = 2m$ is always allowed, while inside is allowed if $\gamma$ is given by the ratio of two odd numbers. If the condition $(1-2m/r)^\gamma < 0$ for $r < 2m$ is not satisfied, then the interior region becomes classically impenetrable[13] (a similar situation occurs also in the context of Rosen's bimetric theory of gravity[14]).

In this case, a physically acceptable motion describing a particle bouncing back from the Schwarzschild surface must be characterized by $\dot{r} = 0$ at $r = 2m$ (continuity of the velocity at the inversion point), and this is obtained, according to Eq. (16), if $\gamma < 1$. Considering then the radial acceleration, which can be obtained differentiating Eq. (16),

$$\ddot{r} = -\frac{m}{r^2}[1-k^2(1-\gamma)(1-2m/r)^{-\gamma}] , \qquad (17)$$

we find that, for $0 < \gamma < 1$, a test particle is acted on by a gravitational force which, outside the Schwarzschild radius $(r > 2m)$, is attractive $(\ddot{r} < 0)$ only if $k^2(1-\gamma) < 1$, and only for values of $r$ such that $r > r_0$, where

$$r_0 = 2m\{1-[k^2(1-\gamma)]^{1/\gamma}\}^{-1} . \qquad (18)$$

Therefore if the modification of the Schwarzschild field, induced by a breaking of the local SO(3,1) symmetry and parametrized by $\gamma$, is subject to the constraint $0 < \gamma < 1$, the effective gravitational force is everywhere repulsive (in the region $r > 2m$) for particles with sufficiently high kinetic energy at infinity, so that $k^2 > (1-\gamma)^{-1}$, while, if this last condition is not satisfied, the force is repulsive only inside the radius $r_0$. In both cases, the force becomes infinitely repulsive at the Schwarzschild radius $r = 2m$, irrespective of how small the breaking of Lorentz symmetry (that is the deviation of $\gamma$ from 1) is.

## IV. CONCLUSION

If the gravitational interaction in four dimensions is described by a theory which is not locally Lorentz invariant, then the sign of the force experienced by a test body, in the field of a static and spherically symmetric source, may depend on the kinetic energy of the body at infinity and on the distance from the central source.

Experimental constraints on the values of the parameters which characterize the breaking of the SO(3,1) gauge symmetry, obtained from the Solar System tests of general relativity, imply that the gravitational force may change sign only immediately close to the Schwarzschild radius, see Eqs. (18) and (13) [unless one considers ultrarelativistic test particles, such that $k^2(1-\gamma) \to 1$]. Therefore a gravitational repulsion of macroscopic matter could eventually become observable only in the case of extremely collapsed astronomical objects.

Nevertheless, it seems interesting to point out that a deviation, *even infinitesimal,* from the local Lorentz symmetry, may produce repulsive forces so much strong as to render impenetrable, at least classically, the interior of the Schwarzschild sphere.

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