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# $\pi NN$ form factor in the Skyrme model

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The $\pi NN$ form factor for the Skyrme model is calculated using a semiclassical technique based on collective variables. The form-factor mass is found to be 582 MeV, which is in substantial disagreement with form factors obtained from one-boson-exchange-potential studies of $NN$ scattering.

The Skyrme model[1] was proposed 25 years ago as a phenomenology of hadronic physics. The model is highly unusual in that it contains no fundamental fermion degrees of freedom. Instead, the model relies on meson fields in the form of a nonlinear $\sigma$ model with additional higher-order derivative couplings. Baryons are interpreted as solitons with the topological winding number acting as the baryon number. Theoretical advances in the intervening years have renewed interest in the Skyrme model. These advances include 't Hooft's demonstration[2] that in the large-$N$ limit, QCD is equivalent to a theory of interacting mesons with a quartic coupling that scales as $1/N$ and Witten's observation that the properties of baryons and mesons scale with $N$ as though baryons were solitons.[3] More recently, Witten has shown that for more than two flavors, a Wess-Zumino term must be added to the nonlinear $\sigma$ model in order to reproduce the anomalies of QCD, and that this term induces an anomalous baryon current which is precisely the winding-number current identified by Skyrme.[4]

In the past several years there have been many studies of phenomenological aspects of the Skyrme model. Static properties of nucleons have been calculated[5-8] and reasonable agreement with experiment has been found (i.e., errors of about 30%). In order to calculate the static properties of physical states, the hedgehog classical solutions must be projected onto states of good angular momentum and isospin. A semiclassical projection scheme based on collective coordinates has been developed in Ref. 5. Inspired by this success in describing the static properties of nucleons, there have been a number of attempts to calculate dynamic properties of baryons. These include relatively inconclusive studies of the $NN$ interaction[9] as well as a reasonably successful description of the baryon resonances to about 3 GeV.[10] In addition, the electromagnetic form factors of nucleons have been calculated using a semiclassical approach.[11]

In this note, the $\pi NN$ form factor will be studied using a similar semiclassical approach. It is found that the form factor is much softer than estimates of the form factor obtained from one-boson-exchange-potential (OBEP) studies of $NN$ scattering.

The Lagrangian for the Skyrme model with massive pions[7] is

$$\mathcal{L}=\frac{F_\pi}{16}\mathrm{tr}(\partial_\mu U\,\partial^\mu U)+\frac{1}{32e^2}\mathrm{tr}\{[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]^2\}+\frac{m_\pi{}^2F_\pi{}^2}{8}\mathrm{tr}(U-2)\ , \tag{1}$$

where $U$ is an SU(2) matrix, $F_\pi$ is the pion decay constant, $e$ is a dimensionless coupling constant, and $m_\pi$ is the pion mass. Following Ref. 7, $F_\pi$ will be taken to be 108 MeV (substantially smaller than the experimental value of 186 MeV) and $e=4.84$. These values have been chosen to reproduce correctly the masses for the nucleon and $\Delta$. The matrix $U$ represents the pion and $\sigma$ fields, $U=2(\sigma+i\boldsymbol{\tau}\cdot\boldsymbol{\phi})/F_\pi$. Static solutions to the Euler-Lagrange equations associated with (1) can be found using the hedgehog ansatz, $U_0=\exp[if(r)\hat{\mathbf{r}}\cdot\boldsymbol{\tau}]$, which correlates isospin with spatial direction. The variational equation for the profile function $f$ is

$$f''=\frac{[2sc(1/\rho^2+4s^2/\rho^4-4f'^2/\rho^2)-2f'/\rho+\beta^2 s]}{1+8s^2}\ , \tag{2}$$

where the derivatives are given in terms of the dimensionless length variable $\rho=eF_\pi r$, $\beta$ is the dimensionless pion mass $m_\pi/eF_\pi$, and $s$ and $c$ are $\sin(f)$ and $\cos(f)$. The boundary conditions which yield a winding number (i.e., baryon number) of unity are $f(0)=\pi$ and $f(\infty)=0$.

Of course, degenerate solutions can be obtained from $U_0$ by translations and by rotations in either space or isospace, $U(\mathbf{r})=AU_0(\mathbf{r}-\mathbf{R})A^\dagger$. The simplest method for introducing collective degrees of freedom into the problem is to let $A$ and $\mathbf{R}$ be time dependent.[5,11] Thus, the ansatz

$$U(\mathbf{r},t)=A(t)U_0(\mathbf{r}-\mathbf{R}(t))A^\dagger(t) \tag{3}$$

is substituted into the Lagrangian and $\mathbf{R}$ and $A$ are treated as dynamical variables. Expressions for the momentum, spin, and isospin in terms of the collective variables can be obtained from the Noether currents: $\mathbf{P}=M\dot{\mathbf{R}}$, $s_i=-i\lambda\,\mathrm{tr}(\tau_i A^\dagger\dot{A})$, and $I_j=\lambda\,\mathrm{tr}(\tau_j A\dot{A}^\dagger)$, where the soliton mass $M$ and moment of inertia $\lambda$ are known functionals of the profile function[5-7] $f(\mathbf{r})$. The collective wave functions with good quantum numbers for $\mathbf{P}$, $\mathbf{s}^2=\mathbf{I}^2$, $s_3$, and $I_3$ are

$$|I=s,m_s,m_I;\mathbf{P}\rangle=(2\pi)^{-3/2}\exp(i\mathbf{P}\cdot\mathbf{R})\,|I=s,m_s,m_I\rangle \ , \tag{4}$$

where $|I=s,m_s,m_I\rangle$ are functions of $A$ and are given explicitly in Ref. 5.

Although this semiclassical collective formalism was used in Ref. 11 to calculate electromagnetic form factors up to $Q^2$ of several $\mathrm{GeV}^2$, it should be noted that the method breaks down for momenta of order of the nucleon mass. There are several ways to understand this. The simplest is to note that the collective Hamiltonian obtained by making a Legendre transformation of the collective Lagrangian is $H=M+\mathbf{P}^2/2M+\mathbf{s}^2/2\lambda$, which is clearly nonrelativistic. The nonrelativistic nature of the formalism can be understood in terms of the ansatz for the collective variables. The collective-coordinate method works only if the ansatz describes solutions to the full Euler-Lagrange equations. While $U_0(\mathbf{r}-\mathbf{R})$ is a solution for stationary $\mathbf{R}$, $U_0(\mathbf{r}-\mathbf{R}(t))$ is generally not. The form of the collective Hamiltonian guarantees that the self-consistent solutions for $\mathbf{R}(t)$ are $\mathbf{R}_0+\mathbf{V}t$, where $\mathbf{V}$ is time independent. If the Lagrangian had been Galilean invariant, $U_0(\mathbf{r}-\mathbf{R}_0-\mathbf{V}t)$ would clearly have been a solution to the full equations of motion. However, the Lagrangian is Lorentz invariant and therefore, $U_0(r-\mathbf{R}_0-\mathbf{V}t)$ is only a solution to first order in $\mathbf{V}$. The restriction to nonrelativistic momenta should not be surprising in view of the semiclassical nature of the collective coordinate procedure. Recall that in this semiclassical treatment $1/N$ plays the role of the coupling constant and the nucleon mass scales like $N$. Thus, for fixed momentum, the semiclassical or large-$N$ limit implies that the nucleon mass is much larger than the momentum and the nonrelativistic regime is appropriate.

The $\pi NN$ coupling constant and form factor can be obtained by study of the source for the pion field $\mathbf{j}$, which is defined by the equation of motion for the pion, $(\Box+m_\pi^2)\boldsymbol{\phi}=\mathbf{j}$. The strategy for calculating $\pi NN$ couplings for the Skyrme model is to compare the pion field in a nucleon state of an "old-fashioned" Lagrangian with fundamental pion and nucleon degrees of freedom with the pion field in a nucleon state in the Skyrme model.[5] The old-fashioned Lagrangian is

$$\mathcal{L}=\tfrac{1}{2}\partial_\mu\boldsymbol{\phi}\cdot\partial^\mu\boldsymbol{\phi}+\tfrac{1}{2}m_\pi^2\boldsymbol{\phi}^2+ig_{\pi NN}(\bar\psi\boldsymbol{\tau}\gamma_5\psi)\cdot\boldsymbol{\phi} \ .$$

The matrix element of the third component of the pion-source current evaluated between two proton spin-up states, one with momentum $\mathbf{P}$ and the other with momentum $\mathbf{P}'$, can be easily obtained with this Lagrangian. It is convenient to work in the Breit frame ($\mathbf{P}'=-\mathbf{P}$), since in this frame the energy transfer is zero, and thus, the invariant four-momentum transfer is given simply by the three-momentum transfer. Because the present treatment of the Skyrme model is nonrelativistic, it is sufficient to consider the nonrelativistic reduction of the matrix element of the source. In the Breit frame, it is given by

$$\langle p\uparrow;\mathbf{P}'|j^3(\mathbf{r})|p\uparrow;P\rangle=ig_{\pi NN}q_3/2M(2\pi)^3\exp(i\mathbf{q}\cdot\mathbf{r}) \ , \tag{5}$$

where $\mathbf{q}=\mathbf{P}-\mathbf{P}'$, and $M$ is the mass of the nucleon.

In the Skyrme model the third component of the pion field is given by

$$\phi^3(r)=-\frac{iF_\pi}{4}\,\mathrm{tr}[\tau_3AU_0(\mathbf{r}-\mathbf{R})A^\dagger] \ .$$

$\phi^3(\mathbf{r})$, evaluated between the collective wave functions for spin-up protons with momenta of $\mathbf{P}$ and $\mathbf{P}'$ in the Breit frame, yields

$$\frac{F_\pi}{6(2\pi)^3}\exp(i\mathbf{q}\cdot\mathbf{r})\int d^3R\,\exp(-i\mathbf{q}\cdot\mathbf{R})\sin[f(R)]\hat{\mathbf{R}}_3 \ .$$

The matrix element for the pion source is obtained by left multiplying this expression with the operator $(\Box+m_\pi^2)$,

$$\langle p\uparrow;\mathbf{P}'|(\Box+m_\pi^2)\phi^3(r)|p\uparrow;\mathbf{P}\rangle=\frac{F_\pi}{6(2\pi)^3}\exp(i\mathbf{q}\cdot\mathbf{r})\int d^3R\,(q^2+m_\pi^2)\exp(-i\mathbf{q}\cdot\mathbf{R})\sin[f(R)]\hat{\mathbf{R}}_3 \ , \tag{6}$$

where $q$ is the magnitude of the three-momentum transfer. The pion source can be reexpressed as

$$\langle p\uparrow;\mathbf{P}'|j^3(r)|p\uparrow;\mathbf{P}\rangle=(2\pi)^{-3}\exp(i\mathbf{q}\cdot\mathbf{r}-iq_0t)\int d^3R\,\exp(-i\mathbf{q}\cdot\mathbf{R})J(R)\hat{\mathbf{R}}_3 \ , \tag{7}$$

where

$$J(r)\hat{\mathbf{r}}_3=\frac{F_\pi}{6}(-\nabla^2+m_\pi^2)\sin(f)\hat{\mathbf{r}}_3=\frac{F_\pi}{6}\left[-\cos(f)f''+\sin(f)f'^2-\frac{2}{r}\cos(f)f'+\frac{2}{r^2}\sin(f)+m_\pi^2\sin(f)\right]\hat{\mathbf{r}}_3 \ .$$

The equation of motion for $f$, Eq. (2), can be used to replace $f''$, which reduces $J(r)$ to a functional of $f$ and $f'$ only. The $\pi NN$ coupling is obtained by comparing Eqs. (5) and (7). With the aid of the Fourier-Bessel decomposition one finds

$$g_{\pi NN}=\frac{8\pi M}{q}\int dr\,r^2J(r)j_1(qr) \ , \tag{8}$$

where $j_1$ is a spherical Bessel function. From Lorentz invariance, $g_{\pi NN}$ must only be a function of $Q^2=\mathbf{q}\cdot\mathbf{q}-q_0^2$ which in the Breit frame is given by $q^2$.

The expression for $g_{\pi NN}$ in Eq. (8) was derived for $Q^2>0$. However, by analytic continuation one can evaluate $g_{\pi NN}$ for $Q^2<0$. It is convenient to express the pion-nucleon coupling as a coupling constant at the pion pole, $Q^2=-m_\pi^2$, times a form factor:

$$g_{\pi NN}(Q^2)=g_{\pi NN}(Q^2=-m_\pi^2)f_{\pi NN}(Q^2) \ . \tag{9}$$

The value for $g_{\pi NN}(Q^2=-m_\pi^2)$ obtained from numerical evaluation of Eq. (8) is 11.9. The form factor $f_{\pi NN}$ is plotted in Fig. 1.

It should be noted that the value for $g_{\pi NN}$ at the pion

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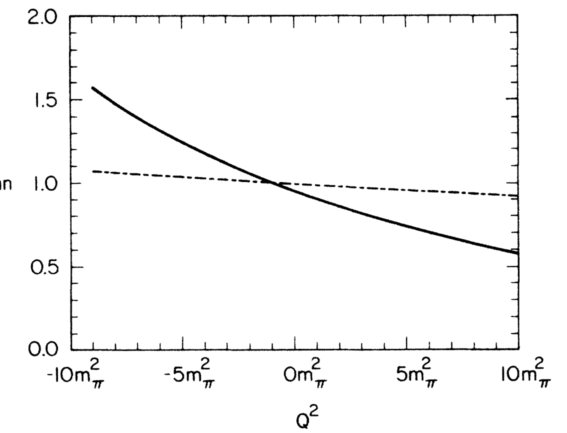


FIG. 1. The $\pi NN$ form factor. The solid line is the form factor for the Skyrme model calculated using the collective coordinate method. The dot-dashed line is the form factor used in Ref. 13 to fit $NN$ scattering using an OBEP potential.

pole, obtained from integration of the pion source in Eq. (8), agrees numerically with the value of $g_{\pi NN}$ calculated in Ref. 7 via evaluation of the coefficient of the pion tail. It is easy to show that the form of $g_{\pi NN}(Q^2=-m_\pi^2)$ from the pion source can be reexpressed in terms of the amplitude of the pion tail. Consider Eq. (6) in the limit $Q^2 \to -m_\pi^2$. Then, it is clear that the only contributions to the matrix element of the pion source will be from the part of the Fourier integral

$$\int d^3R \exp(-i\mathbf{q}\cdot\mathbf{R}) \sin[f(R)]\hat{\mathbf{R}} \ ,$$

which diverges with a pole of the form $1/(q^2+m_\pi^2)$. For large $R$, $\sin[f(r)]$ asymptotes to

$$g(r)=c\exp(-m_\pi r)/r(m_\pi+1/r) \ ,$$

where $c$ is the amplitude of the pion tail. Writing $\sin[f(r)]$ as $\{\sin[f(r)]-g(r)\}+g(r)$, one sees that the Fourier integral of the term in brackets is convergent and hence does not contribute. In contrast, the Fourier integral of the $g(r)$ term, which can be done analytically, diverges with a $1/(q^2+m_\pi^2)$ pole; the residue is proportional to the pion tail amplitude $c$. Using this expression for the matrix element of the pion source for $Q^2=-m_\pi^2$, one finds

$$g_{\pi NN}(Q^2=-m_\pi^2)=8\pi cMF_\pi/6 \ , \tag{10}$$

which is the expression used in Ref. 7 to find $g_{\pi NN}$.

Another special value of momentum transfer is $Q^2=0$. Since the Lagrangian in Eq. (1) satisfies the PCAC (partial conservation of axial-vector current) condition $\partial_\mu A_i^\mu=\phi_i$, the Goldberger-Treiman[12] relation between $g_A$ and $g_{\pi NN}(Q^2=0)$ should be satisfied. Thus, if the semiclassical methods for calculating $g_A$ and $g_{\pi NN}$ are consistent, one will find

$$g_{\pi NN}(Q^2=0)=2g_A M/F_\pi \ . \tag{11}$$

The value of $g_A$ obtained using the semiclassical projection method of Ref. 5 is 0.65,[7] which implies a value of $g_{\pi NN}(Q^2=0)$ of 11.3. Evaluation of Eq. (8) gives precisely this value, which indicates that the semiclassical method is consistent.

As noted above, one can analytically continue the expression in Eq. (8) to $Q^2<0$. However, at $Q^2<-9m_\pi^2$, the expression breaks down, giving a divergent result for the form factor. Mathematically, this can be seen from the asymptotic form for the pion source which for large $r$ can be shown to go as $m_\pi^3\exp(-3m_\pi r)/r^3$. For $q^2<0$, the magnitude of the spherical Bessel function $j_1(qr)$ goes like $\exp(|q|r)/|q|r$ for large $r$. Thus, one sees that for $q^2<-9m_\pi^2$ the integrand in Eq. (8) diverges exponentially with $r$. Physically, the nonanalytic behavior of $f_{\pi NN}(Q^2)$ at $Q^2=-9m_\pi^2$ is the branch cut associated with the threshold for the emission of three pions.

It is clear that this semiclassical method for calculating the $\pi NN$ form factor has certain reasonable formal properties. It correctly associates $g_{\pi NN}(Q^2=0)$ with the Goldberger-Treiman result and $g_{\pi NN}(Q^2=-m_\pi^2)$ with the amplitude of the pion tail. Moreover, a branch cut for three-pion emission is automatically predicted. The question remains, however, as to how well the Skyrme model reproduces the $\pi NN$ form factor found in nature. Unfortunately, this form factor is not directly measurable and can only be extracted from the experimental data with the aid of theoretical constructs. Analysis of $NN$ scattering using OBEP potentials is perhaps the most simple way of getting at the $\pi NN$ form factor. Holinde[13] studied such scattering using a form factor parametrized by

$$f_{\pi NN}(Q^2)=(\Lambda^2-m_\pi^2)/(\Lambda^2+Q^2) \ , \tag{12}$$

where $\Lambda$ is the form-factor mass. The best fit to $NN$ scattering phase shifts was found for $\Lambda=1530$ MeV. It is significant to note that although this determination of the form-factor mass is not unambiguous, a significantly smaller mass ($\Lambda \lesssim 1000$ MeV) is exlcuded. In Fig. 1 the parametrized form factor from Eq. (12) with $\Lambda=1530$ MeV is plotted along with the calculated form factor for the Skyrmion. One sees that they are strikingly different; the form factor determined from OBEP studies of scattering is significantly harder (i.e., it has a significantly larger form-factor mass). One can quantify this by defining a form-factor mass for the Skyrme-model $\pi NN$ form factor according to

$$\Lambda^2+m_\pi^2=-\left(\left.\frac{\partial f_{\pi NN}}{\partial Q^2}\right|_{Q^2=-m_\pi^2}\right)^{-1} . \tag{13}$$

The form-factor mass for the Skyrme model is 582 MeV which is far too light.

Of course the determination of the form factor from studies of OBEP potentials may not be completely reliable. It should be noted, however, that the conclusion that the $\pi NN$ form factor is very hard is supported by a study of $P$-wave $\pi N$ scattering by Wei and Banerjee.[14] Using a theory based on the Low expansion they found the $P_{33}$ (i.e., $\pi N\Delta$) form factor is quite hard; a form-factor mass of greater than $10m_\pi$ was required to fit the data. The $\pi NN$ form factor could not be determined with this approach. The analysis in this paper for the $\pi NN$ form factor can be repeated for the $\pi N\Delta$ form factor. One finds that $f_{\pi NN}$ and $f_{\pi N\Delta}$ are the same if one neglects kinematical factors relating to the $N$-$\Delta$ mass difference (which are $1/N$ effects). Thus, the large $\pi N\Delta$ form-factor mass determined in Ref. 14 is seen to be consistent with the OBEP results and inconsistent with the semiclassical results from the Skyrme model.

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