Brief Reports

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Correction to the width of heavy Higgs bosons: An addendum to "Radiative decay of heavy Higgs bosons"

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We determine the width for radiative decay of heavy Higgs bosons $H\to W^+W^-\gamma$ for hard photons as a function of the Higgs-boson mass and the photon-energy cutoff, and correct the result of a previous calculation.

If the Higgs boson is sufficiently heavy it will decay predominantly into $W^\pm$ or Z^0 gauge bosons. The width for $H\to W^+W^-$ is[1]

$$\Gamma(H\to W^+W^-)=\frac{\alpha m_H^3}{16\sin^2\theta_W M_W^2}\left(1-\frac{4M_W^2}{M_H^2}\right)^{1/2}\times\left(1-\frac{4M_W^2}{m_H^2}+12\frac{M_W^4}{m_H^4}\right), \quad (1)$$

and thus grows as m_H^3 as the Higgs-boson mass gets large. The rapid increase of the width with increasing Higgs-boson mass is a manifestation of the enhanced coupling of the Higgs boson to longitudinal W bosons. It is this enhanced coupling which forbids us from performing perturbative calculations involving Higgs bosons heavier than about 1 TeV.[1,2]

Recently, it was suggested by one of us[3] that the radiative decay $H\to W^+W^-\gamma$ with hard photons is a significant fraction of the principal decay mode $H\to W^+W^-$, and that this was relevant to the breakdown of perturbation theory for heavy Higgs bosons. In this paper we recalculate the radiative decay $H\to W^+W^-\gamma$ and find that it is only a few percent of $H\to W^+W^-$, as is typical of radiative processes. We also obtain a semianalytic result for the branching ratio of the process $H\to W^+W^-\gamma$ relative to $H\to W^+W^-$. As in Ref. 3, we cut the photon energy off to avoid the infrared singularity. The complementary calculation of the radiative corrections to (1) from soft and virtual photons has been performed elsewhere.[4]

We assume that the $WW\gamma$ vertex is that of the standard model including the anomalous magnetic moment of unity. Then the differential decay width corresponding to the diagrams shown in Fig. 1 is

$$\begin{aligned}\frac{d\Gamma(H\to W^+W^-\gamma)}{dx\,dx_1}=\frac{\alpha^2 m_H}{16\pi\sin^2\theta_W}\Bigg[&\left(2+\frac{(p_1\cdot p_2)^2}{M_W^4}\right)\left(\frac{2p_1\cdot p_2 M_W^2}{p_1\cdot k\,p_2\cdot k}-\frac{M_W^4}{(p_1\cdot k)^2}-\frac{M_W^4}{(p_2\cdot k)^2}\right)\\&+\frac{1}{(p_1\cdot k)^2}[(p_2\cdot k)^2-2p_2\cdot k\,p_1\cdot p_2]+\frac{1}{(p_2\cdot k)^2}[(p_1\cdot k)^2-2p_1\cdot k\,p_1\cdot p_2]\\&+\frac{2}{p_1\cdot k}\left(p_2\cdot k-p_1\cdot p_2+2\frac{(p_1\cdot p_2)^2}{M_W^2}+\frac{p_1\cdot p_2 p_2\cdot k}{M_W^2}\right)\\&+\frac{2}{p_2\cdot k}\left(p_1\cdot k-p_1\cdot p_2+2\frac{(p_1\cdot p_2)^2}{M_W^2}+\frac{p_1\cdot p_2 p_1\cdot k}{M_W^2}\right)+4\frac{p_1\cdot p_2}{M_W^2}+2\Bigg], \quad (2)\end{aligned}$$

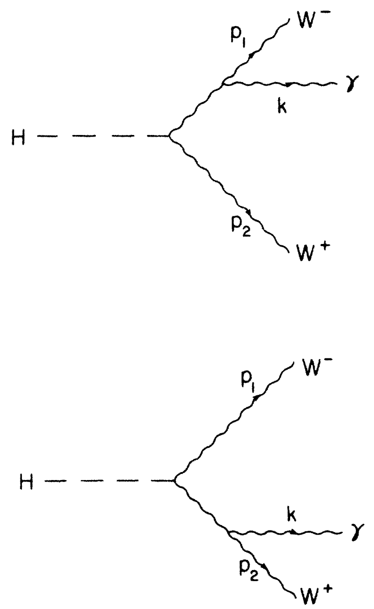

FIG. 1. Feynman diagrams for the decay $H \to W^+W^-\gamma$.

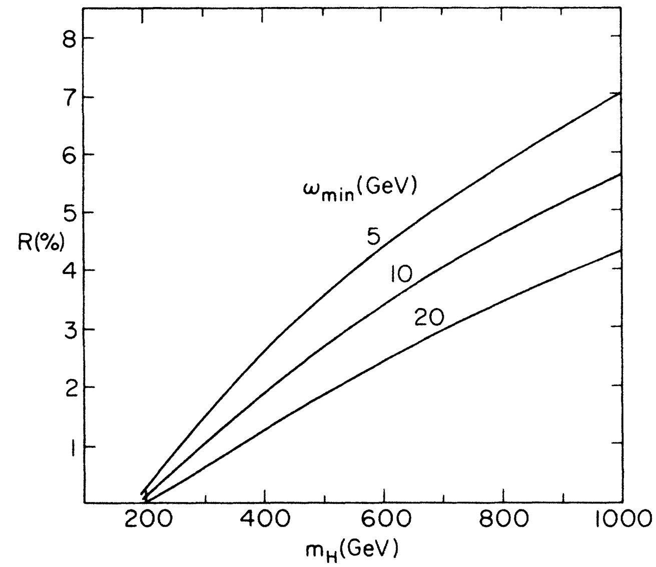

FIG. 2. $R \equiv \Gamma(H \to W^+W^-\gamma)/\Gamma(H \to W^+W^-)$ as a function of m_H for various photon-energy cutoffs. We used a W mass of 82 GeV and took the fine-structure constant to be $\frac{1}{128}$.

where x_1 is $2E_1/m_H$, x is $2\omega/m_H$, and ω is the energy of the photon. In terms of x_1 and x the dot products in (2) are

$$p_1 \cdot k = \frac{m_H^2}{2}(x_1 + x - 1) , \tag{3a}$$

$$p_2 \cdot k = \frac{m_H^2}{2}(1 - x_1) , \tag{3b}$$

$$p_1 \cdot p_2 = \frac{m_H^2}{2}(1 - x - \tfrac{1}{2}r^2) . \tag{3c}$$

In terms of $r \equiv 2M_W/m_H$ and $y \equiv 2\omega_{\min}/m_H$, where $\omega_{\min}$ is the photon-energy cutoff, the limits on the integrals are

$$\int_y^{1-r^2} dx \int_{x_1^-}^{x_1^+} dx_1 , \tag{4}$$

with

$$x_1^{\pm} = 1 - \tfrac{1}{2}x \pm \tfrac{1}{2}\frac{x}{1-x}[(1-x)(1-x-r^2)]^{1/2} . \tag{5}$$

Substituting (3) into (2), doing the x_1 integral using (4) and (5), and dividing by (1) leaves

$$R \equiv \Gamma(H \to W^+W^-\gamma)/\Gamma(H \to W^+W^-)$$
$$= \frac{2\alpha}{\pi}\frac{1}{(1-r^2)^{1/2}}\int_y^{1-r^2} dx \left[\frac{1}{x}(1 - \tfrac{1}{2}r^2 - x)\ln\left(\frac{1-x+[(1-x)(1-x-r^2)]^{1/2}}{1-x-[(1-x)(1-x-r^2)]^{1/2}} \right) + \left(\frac{8x}{3r^4-4r^2+4} - \frac{1}{x} \right)[(1-x)(1-x-r^2)]^{1/2} \right] . \tag{6}$$

Values of R, for a few values of $\omega_{\min}$, are shown in Fig. 2.

All of the integrals in (6) can be done analytically except for

$$I = \int_y^{1-r^2} dx \frac{1}{x}\ln\left(\frac{1-x+[(1-x)(1-x-r^2)]^{1/2}}{1-x-[(1-x)(1-x-r^2)]^{1/2}} \right) . \tag{7}$$

The mean-value theorem allows us to write I as

$$I = \ln\left(\frac{1-y+[(1-y)(1-y-r^2)]^{1/2}}{1-y-[(1-y)(1-y-r^2)]^{1/2}} \right) \ln\left(\frac{y+h(1-y-r^2)}{y} \right) , \tag{8}$$

where h is a function of y and r^2 which is greater than zero and less than one. Using (8), (6) becomes

$$R=\frac{2\alpha}{\pi}\frac{1}{(1-r^2)^{1/2}}\left\{\left[(1-\tfrac{1}{2}r^2)\ln\left[\frac{y+h(1-y-r^2)}{y}\right]+y-\frac{2r^4-r^6}{6r^4-8r^2+8}\right]\right.$$
$$\times\ln\left[\frac{2-2y-r^2+2[(1-y)(1-y-r^2)]^{1/2}}{r^2}\right]-(1-r^2)^{1/2}$$
$$\times\ln\left[\frac{2-2y-2r^2+r^2y+2(1-r^2)^{1/2}[(1-y)(1-y-r^2)]^{1/2}}{yr^2}\right]$$
$$\left.+\left[\frac{7}{3}+\frac{2}{3}\frac{y(2-4y-r^2)}{3r^4-4r^2+4}\right][(1-y)(1-y-r^2)]^{1/2}\right\}. \quad (9)$$

Holding h fixed at 0.7 gives an approximate expression for R which is accurate to within 8% for all $\omega_{\min}$ between 0.1 and 50 GeV and all m_H between 300 and 1000 GeV.

As a result of this calculation, we clearly see that the result found by one of us in Ref. 3 is in error, since the values of the branching ratio R found here range from only 4.3% to 7% at $m_H=1$ TeV, rather than 20%–40%. Our values for R are shown graphically in Fig. 2 as a function of m_H for three values of $\omega_{\min}$. The numerical values from (6) and our analytic expression (9) both show the logarithmic dependence on $\omega_{\min}$ that we expect intuitively. No analytic expression such as our (9) [or (6)] is given in Ref. 3.

In conclusion, note that the radiative decay width for a heavy Higgs boson does increase with the Higgs-boson mass. As can be seen from (9) this happens because the width contains a product of two logarithms whose arguments depend on the mass of the Higgs boson; one from the infrared divergence and one from what would be a collinear divergence if the W were massless. Nevertheless, the magnitude of the radiative decay width is not large but is, as one expects, a few percent of the total width.

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