$SO(10)_V \times SU(3)_H$ model

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The $SO(10)_V \times SU(3)_H$ model, in which the quark-mass hierarchy is induced through their mixing with horizontal superheavy fermions, is discussed. Fritzsch mass matrices are natural in our model, resulting from the mass matrices of horizontal superheavy up quarks and down quarks closely proportional to each other.

Recently, a new mechanism of quark- and lepton-mass generation via their mixing with hypothetical superheavy fermions was proposed for the model with $SU(5)_V \times SU(3)_H$ symmetry.[1] Although the minimal standard SU(5) grand unified model by Georgi and Glashow[2] is simple and beautiful, it is in disagreement with proton-decay experiments.[3] Using the standard analysis of the renormalization-group equations, it is obvious that this serious problem still exists in Ref. 1. On the other hand, the SO(10) grand unified model is in agreement with proton-decay experiments.[4] In the standard SO(10) model, each fermion generation is put in an irreducible **16** spinorial representation which is automatically anomaly-free. It is known that the following three generations exist, which repeatedly are put in only **16** spinorial representations in the ordinary SO(10) model:

$$(u_i,d_i,\nu_e,e),\quad (c_i,s_i,\nu_\mu,\mu),\quad (t_i(?),b_i,\nu_\tau,\tau)\ , \tag{1}$$

where $i=1,2,3$ is the color index. So it is hard to understand the replication of generations, fermion mass hierarchy, and the structure of weak mixing.

In this paper, we propose a model with $SO(10)_V \times SU(3)_H$ symmetry. Here SO(10) is a vertical grand unified group, $SU(3)_H$ is a local generation symmetry which unifies the generations [Eq. (1)] in a horizontal triplet. At the same mass scale M_H, the $SU(3)_H$ is breaking. To suppress the flavor-changing neutral currents,[5] generated by the horizontal gauge bosons, $M_H > 10^6$ GeV. In our model, the hierarchy between the masses of three generations [Eq. (1)] and the weak mixing angles are due to the spontaneous breaking of the $SU(3)_H$ symmetry. The quark mass matrices are obtained, which have the form suggested by Fritzsch.[6] The relations between the quark masses and mixing angles, which are derived from our model, are in good agreement with recent experiments. The value of the *CP*-violation parameter $\epsilon \sim s_2 s_3 \sin\delta$ is $\sim 10^{-3}$.

Now let the quarks and leptons of Eq. (1) be put in the representations $(16_V, \bar{3}_H)$ of $SO(10)_V \times SU(3)_H$ symmetry. After the reduction of 16_V^α under $[SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_x]_V$, we have

$$\begin{aligned}16_V^\alpha = {} & (3,2,1,(\tfrac{1}{24})^{1/2})^\alpha + (\bar{3},1,2,(-\tfrac{1}{24})^{1/2})^\alpha \\ & + (1,2,1,(-\tfrac{3}{8})^{1/2})^\alpha + (1,1,2,(\tfrac{3}{8})^{1/2})^\alpha\ ,\end{aligned} \tag{2}$$

where $\alpha=1,2,3$ are $SU(3)_H$ indices. At the grand unified mass scale M_x of $SO(10)_V$, we introduce $(\mathbf{45};1)$ Higgs representation, which decomposes under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_x \times SU(3)_H$ as follows:

$$\begin{aligned}(\mathbf{45};1) = {} & (8,1,0;1) + (3,2,2,-(\tfrac{1}{6})^{1/2};1) \\ & + (\bar{3},2,2,(\tfrac{1}{6})^{1/2};1) + (3,1,1,(\tfrac{2}{3})^{1/2};1) \\ & + (\bar{3},1,1,-(\tfrac{2}{3})^{1/2};1) + (1,3,1,0;1) \\ & + (1,1,3,0;1) + (1,1,1,0;1)\ .\end{aligned} \tag{3}$$

So the scheme for symmetry breaking is

$$SO(10)_V \times SU(3)_H \underset{M_x}{\overset{(45,1)}{\longrightarrow}} G = [SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_x]_V \times SU(3)_H \tag{4}$$

and the quarks and leptons cannot obtain masses without the breaking of $SU(3)_H$ symmetry.

Using the color index i, generation index α, isospin I_L, I_R, and hypercharge Y of G, we can explicitly indicate the particles as

$$\begin{aligned}&\begin{bmatrix}u_{i\alpha}\\ d_{i\alpha}\end{bmatrix}_L \equiv q_{Li\alpha}(\tfrac{1}{2},0,\tfrac{1}{3})\ ,\\ &\begin{bmatrix}u_i^\alpha\\ d_i^\alpha\end{bmatrix}_R \equiv q_{Ri}^\alpha(0,\tfrac{1}{2},\tfrac{1}{3})\ ,\\ &\begin{bmatrix}\nu_\alpha\\ e_\alpha\end{bmatrix}_L \equiv l_{L\alpha}(\tfrac{1}{2},0,-1)\ ,\\ &\begin{bmatrix}\nu^\alpha\\ e^\alpha\end{bmatrix}_R \equiv l_R^\alpha(0,\tfrac{1}{2},-1)\ .\end{aligned} \tag{5}$$

The numbers in the parentheses on the right-hand side in Eq. (5) are I_L, I_R and hypercharge Y. The electric charge of fermions is $Q = I_L{}^3 + I_R{}^3 + Y/2$.

Notice that the assignment of quarks and leptons [see Eq. (2) or (5)] has $SU(3)_H$ triangle anomalies. To cancel these anomalies, we need to introduce also the hypothetical superheavy horizontal quarks and leptons which put in the $(1,\bar{3})$ representations of $SO(10)_V\times SU(3)_H$ symmetry and the assignment of their hypercharge as

$$\begin{aligned}&U_{Ri\alpha}(0,0,\tfrac{4}{3}),\ \ D_{Ri\alpha}(0,0,-\tfrac{2}{3})\ ,\\&E_{R\alpha}(0,0,-2),\ \ N_{R\alpha}(0,0,0)\ .\end{aligned}\tag{6}$$

$U_{Ri\alpha}$ and $D_{Ri\alpha}$ are the superheavy *up quarks* and *down quarks*, respectively, and $E_{R\alpha}$ and $N_{R\alpha}$ are the superheavy *charged leptons* and *neutrinos*, respectively. For the superheavy quarks and leptons, the weak isospin $I_L{}^3=I_R{}^3=0$. The quantum numbers in the parentheses of Eqs. (6) are I_L, I_R, and Y, respectively.

To break the horizontal symmetry $SU(3)_H$, we introduce also the following Higgs scalars,[1] which are put in the representations $2(1,\bar{3})$ and $(1,6)$ of $SO(10)_V\times SU(3)_H$ symmetry, respectively, namely,

$$\xi^\alpha(1,\bar{3}),\ \ \eta^\alpha(1,\bar{3}),\ \ \chi_{\{\alpha\beta\}}(1,6)\ .$$

Their vacuum expectation values are

$$\langle\xi^\alpha\rangle=p\delta_3^\alpha,\ \ \langle\eta^\alpha\rangle=q\delta_1^\alpha,\ \ \langle\chi_{\{\alpha\beta\}}\rangle=r\delta_\alpha{}^3\delta_\beta{}^3\ .\tag{7}$$

We also need to break $[SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_x]_V$ down to the low-energy standard model $SU(3)_C\times SU(2)_L\times U(1)_Y$, which we can obtain by using a $(126,1)$ Higgs representation of $SO(10)_V\times SU(3)_H$ at mass scale M'. After the reduction of $(126,1)$ under $[SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_x]_V\times SU(3)_H$, we have

$$\begin{aligned}(126,1)=&(8,2,2,0;1)+(6,3,1,(\tfrac{1}{6})^{1/2};1)+(\bar{6},1,3,-(\tfrac{1}{6})^{1/2};1)+(3,3,1,-(\tfrac{1}{6})^{1/2};1)\\&+(3,2,2,(\tfrac{2}{3})^{1/2};1)+(\bar{3},2,2,-(\tfrac{2}{3})^{1/2};1)+(\bar{3},1,3,(\tfrac{1}{6})^{1/2};1)+(3,1,1,-(\tfrac{1}{6})^{1/2};1)\\&+(\bar{3},1,1,(\tfrac{1}{6})^{1/2};1)+(1,3,1-(\tfrac{3}{2})^{1/2};1)+(1,2,2,0;1)+(1,1,3,(\tfrac{3}{2})^{1/2};1)\ .\end{aligned}\tag{8}$$

Of course, we further need to break $SU(3)_C\times SU(2)_L\times U(1)_Y$ down to $SU(3)_C\times U(1)_{em}$. In fact, from the view of the subgroup $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_x\times SU(3)_H$, we can use the standard Higgs doublets [see Ref. 7] $\phi_L(1;0,\frac{1}{2},0;1)$ and $\phi_R(1;0,0,\frac{1}{2};1)$ which are embedded in the representations of $SO(10)_V\times SU(3)_H$. The vacuum expectation values are

$$\langle\phi_L\rangle=\begin{bmatrix}0\\v_L\end{bmatrix},\ \ \langle\phi_R\rangle=\begin{bmatrix}0\\v_R\end{bmatrix}\ .\tag{9}$$

To suppress the right charged current, we require $v_R\gg v_L$. But our model is a grand unified model with horizontal symmetry, which is different from that in Ref. 7. Here, v_R can be obtained by using renormalization-group equations of the evolution coupling constants in the $SO(10)_V\times SU(3)_H$ model:[8]

$$\begin{aligned}&\ln\left[\frac{M_x}{v_R}\right]=\frac{\dfrac{1}{e^2(\mu)}-\dfrac{8}{3g_3{}^2(\mu)}-2(\tfrac{5}{3}b_1+b_2^L-\tfrac{8}{3}b_3)\ln\left[\dfrac{v_R}{\mu}\right]}{2(b_2^L+b_2^R-\tfrac{8}{3}+\tfrac{2}{3}b_i^x)}\ ,\\&\sin^2\theta_W(\mu)=\tfrac{3}{8}-\tfrac{5}{4}e^2(\mu)\left[(\tfrac{3}{5}b_2^R+\tfrac{2}{5}b_1^x-b_2^L)\ln\left[\frac{M_x}{v_R}\right]+(b_1-b_1^L)\ln\left[\frac{v_R}{\mu}\right]\right]\ ,\end{aligned}\tag{10}$$

where

$$b_1=\frac{1}{16\pi^2}\frac{2}{3}f,\ \ b_2^L=b_2^R=\frac{1}{16\pi^2}(-\tfrac{22}{3}+\tfrac{2}{3}f),\ \ b_3=\frac{1}{16\pi^2}(-11+\tfrac{2}{3}f)\ ,$$

b_1^x is the coefficient of the $U(1)_x$ β function, and f is the number of quark flavors. Neglecting Higgs contributions, from Eqs. (1), (4), and (10), for $\sin^2\theta_W=0.21$, $v_r\simeq10^{10}$ GeV, $M_x\simeq10^{18}$ GeV. It is enough to suppress the right charged current and the life of proton decay is very long.

Now, from Eqs. (5)–(7) and (9), the Yukawa coupling allowed by the symmetry of the model is [notice the point is there are no bare mass terms for ordinary quarks and leptons]

$$\begin{aligned}&[(C_0\chi_{\{\alpha\beta\}}+C_1\xi_{[\alpha\beta]}+C_2\eta_{[\alpha\beta]})\bar{D}_{R\alpha}D_L^\beta+\text{H.c.}+(C_0'\chi_{\{\alpha\beta\}}+C_1'\xi_{[\alpha\beta]}+C_2'\eta_{[\alpha\beta]})\bar{U}_{R\alpha}U_L^\beta+\text{H.c.}]\\&\quad+(g\bar{g}_{L\alpha}D_{R\alpha}\phi_L+g_R\bar{q}_R^\alpha D_L^\alpha\phi_R+\text{H.c.})+[g_l'\bar{g}_{L\alpha}U_{R\alpha}(i\tau_2\phi_L^*)+g_R'\bar{q}_R^\alpha U_L^\alpha(i\tau_2\phi_R^*)+\text{H.c.}]\ .\end{aligned}\tag{11}$$

Inserting Eqs. (7) and (9) into Eq. (11), 6×6 quark mass matrices can be obtained, which have been discussed in Ref. 9, for the down quarks:

$$\begin{bmatrix} 0 & 0 & 0 & g_L v_L & 0 & 0 \\ 0 & 0 & 0 & 0 & g_L v_L & 0 \\ 0 & 0 & 0 & 0 & 0 & g_L v_L \\ g v_R & 0 & 0 & M_D & 0 & 0 \\ 0 & g v_R & 0 & 0 & -M_S & 0 \\ 0 & 0 & g v_R & 0 & 0 & M_B \end{bmatrix}, \tag{12}$$

where

$$\begin{bmatrix} M_D & 0 & 0 \\ 0 & M_S & 0 \\ 0 & 0 & M_B \end{bmatrix} = U_D \begin{bmatrix} C_0 r & C_1 p & 0 \\ C_1 p & 0 & C_2 q \\ 0 & C_2 q & 0 \end{bmatrix} U_D^\dagger . \tag{13}$$

From Eqs. (12) and (13) we see that the ordinary down-quark (d,s,b) mass arises by mixing with superheavy down-quark (D,S,B) mass. If $M_B \gg 1 g v_R \mid$ (Ref. 9), then

$$m_d = \frac{g_L v_L g_R v_R}{M_D} ,$$

$$m_s = \frac{g_L v_L g_R v_R}{M_S} , \tag{14}$$

$$m_b = \frac{g_L v_L g_R v_R}{M_B} .$$

Similarly, for the up quark

$$m_u = \frac{g_L' v_L g_R' v_R}{M_U} ,$$

$$m_c = \frac{g_L' v_L g_R' v_R}{M_C} , \tag{15}$$

$$m_t = \frac{g_L' v_L g_R' v_R}{M_T} ,$$

where

$$\begin{bmatrix} M_U & & \\ & M_C & \\ & & M_R \end{bmatrix} = U_u \begin{bmatrix} C_0' r & C_1' p & 0 \\ C_1' & 0 & C_2' q \\ 0 & C_2 q & 0 \end{bmatrix} U_u^\dagger . \tag{16}$$

It is interesting to note that the Fritzsch mass-matrix ansatz can be explained by using the mass matrices of superheavy horizontal up quark (U,C,T) and down quark (D,S,B) closely proportional to each other. The symmetric Fritzsch form for the quark mass matrices is given by

$$F^u = \begin{bmatrix} 0 & |f|e^{i\phi_1} & 0 \\ |f|e^{i\phi_1} & 0 & |k|e^{i\phi_2} \\ 0 & |k|e^{i\phi_1} & |l|e^{i\phi_3} \end{bmatrix} ,$$

$$F^d = \begin{bmatrix} 0 & |f'|e^{i\phi_1'} & 0 \\ |f'|e^{i\phi_1'} & 0 & |k'|e^{i\phi_2'} \\ 0 & |k'|e^{i\phi_1'} & |l'|e^{i\phi_3'} \end{bmatrix} ; \tag{17}$$

i.e., only the third generation gets a diagonal mass, but the first and second generation masses arise through mixings between neighboring generations. Fritzsch mass matrices are interesting because they explain the smallness of *KM* mixing angles in terms of the flavor mass hierarchy, and three weak mixing angles and one *CP*-violating phase can be predicted from the quark-mass eigenvalues and two linear combination of phase ϕ_1,ϕ_2,ϕ_3 and ϕ_1',ϕ_2',ϕ_3' in Eq. (17). How can we obtain the Fritzsch mass matrices in our model? From the structure of mass matrices in Eqs. (13) and (16), we see that up-superquark matrices have the same form as down-superquark matrices. So it is natural to assume that the up-superquark mass and down-superquark mass ratio in each generation should be the same:

$$\frac{M_D}{M_U} = \frac{M_S}{M_C}, \quad \frac{M_S}{M_C} = \frac{M_B}{M_T} . \tag{18}$$

From Eqs. (14), (15), and (18), we get, for the ordinary quarks,

$$\frac{m_u}{m_d} = \frac{m_c}{m_s} , \tag{19}$$

$$\frac{m_t}{m_b} = \frac{m_c}{m_s} ; \tag{20}$$

i.e., the ordinary charge $\frac{2}{3}$ and $-\frac{1}{3}$ quark mass ratio in each generation is generation independent. The known ordinary quark values[10] are

$$\begin{aligned} & m_u \simeq 5.1 \text{ MeV}, \quad m_c \simeq 1.35 \text{ GeV} , \\ & m_t = 30\text{--}50 \text{ GeV(?)}, \quad m_d \simeq 8.9 \text{ MeV} , \\ & m_s \simeq 175 \text{ MeV}, \quad m_b \simeq 5.3 \text{ GeV} . \end{aligned} \tag{21}$$

So Eq. (20) is a better approximation than Eq. (19). We need to correct the relations of Eq. (18). We expect that the correction of $M_S/M_C = M_B/M_T$ is smaller than that of $M_D/M_U = M_S/M_C$, for example,

$$\begin{aligned} & \left[\frac{M_D}{M_S}\right]^{1/2} - \left[\frac{M_U}{M_C}\right]^{1/2} = O(\Delta) , \\ & \left[\frac{M_S}{M_B}\right]^{1/2} - \left[\frac{M_C}{M_T}\right]^{1/2} = O(\Delta^2) , \end{aligned} \tag{22}$$

where Δ is a small correction, $\Delta \simeq 0.1$, then from Eqs. (14) and (15) we have

$$\begin{aligned} & \left[\frac{m_d}{m_s}\right]^{1/2} - \left[\frac{m_u}{m_c}\right]^{1/2} = O(\lambda(\Delta)) , \\ & \left[\frac{m_s}{m_b}\right]^{1/2} - \left[\frac{m_c}{m_t}\right]^{1/2} = O(\lambda^2(\Delta)) ; \end{aligned} \tag{23}$$

$\lambda(\Delta)$ is also a small correction. Now, we think of λ as the Wolfenstein parameter,[11] $\lambda = U_{us} \simeq 0.225$. It is noted that if there is no mixing at all between generations, Eqs. (23) still hold in this model. On the other hand, we can show that Eqs. (23) are consistent with the structure that is needed for the Fritzsch ansatz to yield a set of Koboyashi-Maskawa (KM) angles in close agreement with the observed pattern. Following Ref. 12, we define the real symmetric matrix F_r^a [see Eqs. (17)] as

$$F_r^a = \begin{bmatrix} 0 & |f|^a & 0 \\ |f|^a & 0 & |k|^a \\ 0 & |k|^a & |l|^a \end{bmatrix},$$

where $a = u,d$. Then using the suitable definition of phase matrices which represent the rephasing freedom of KM elements, the components of the KM matrix U_{ij} can be computed from quarks m_i and the phases. From the quark-mass hierarchy, Cheng and Li[12] obtain an approximate form, under the suitable phase choices and assumptions,

$$U_{us} \simeq \left[\frac{m_d}{m_s}\right]^{1/2} e^{i\phi}, \quad U_{cb} = \frac{m_d}{m_s},$$

$$\phi = (m_s m_u / m_c m_d)^{1/2}.$$

Thus we have

$$s_1 \simeq \left[\frac{m_d}{m_g}\right]^{1/2}, \quad s_2 \sim -\left[\frac{m_c}{m_t}\right]^{1/2}, \quad s_3 \simeq \left[\frac{m_s}{m_j}\right]^{1/2}$$

and the value of the *CP*-violation parameter $\epsilon \sim s_2 s_3$ is $\sim 10^{-3}$, which are in good agreement with the recent experiments.

Note added. A model incorporating a horizontal SU(2) gauge symmetry in addition to the standard $SU(2)_2 \times U(1)$ gauge symmetry was proposed by Ong.[13] A number of papers have appeared to study the concept of horizontal gauge symmetry.[14]

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