Low-energy phenomenology of a realistic composite model

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The low-energy limit of the strongly coupled standard model (Abbott-Farhi composite model) is analyzed. The effects of the excited W isotriplet and isoscalar bosons are investigated and compared with experimental data. As a result, constraints on parameters (masses, coupling constants, etc.) of these vector bosons are obtained. They are not severe enough (certain cancellations are possible) to exclude the model on experimental basis.

I. INTRODUCTION

In spite of overwhelming experimental confirmation of the standard electroweak model, the possibility that leptons, quarks, and weak intermediate bosons are composite objects has been given considerable examination.[1] One reason is historical: the assumption of compositeness has always been successful in attempts to extend our understanding of physical phenomena. The standard electroweak model rests on the assumption of the spontaneous breakdown of a gauge symmetry. On the other hand, one may entertain the idea that the electroweak force is not a fundamental one, but a residual force between composite particles composed of "preons" confined by some unbroken gauge-symmetry interaction. This possibility was examined by Abbott and Farhi.[2,3] The phenomenological consequences of their model are the subject of this paper.

The Lagrangian of the Abbott-Farhi model has the same form as the standard-model Lagrangian with the usual field content and quantum number assignments. However, the parameters determining the potential for the scalar field and the strength of the $SU(2)_L$ gauge interaction are different from their standard-model values so that no spontaneous symmetry breakdown occurs and the $SU(2)_L$ gauge interaction is confining at the weak-interaction scale. Thus, the model is essentially the confining phase of the standard model and from now on we call it the strongly coupled standard model (SCSM). The left-handed physical fermions are $SU(2)_L$-singlet bound states of the fundamental fermion (preon) and scalar. The right-handed physical fermions are pointlike. There is a spin-zero bound state, bilinear in scalar fields, which corresponds to the neutral Higgs boson in the standard model. Also, there is a triplet of spin-one bosons which are identified with $W^{\pm}$ and W^0. The term triplet in the previous sentence refers to a global SU(2) symmetry of the Lagrangian without electromagnetism and Yukawa couplings,[2] hereafter denoted as $SU(2)_W$. Of course, every composite particle can possibly have excited states.

From two left-handed fundamental fermions (or a fermion-antifermion pair) it is possible to form $SU(2)_L$-singlet states which transform according to (0,0), $(\frac{1}{2},\frac{1}{2})$, and (0,1) representations of the Lorentz group,[4] having no counterpart in the standard model. A current-current interaction is mediated by exchange of these bosons, thus changing the form of the standard-model effective four-fermion interaction.

A serious problem is the absence of dynamical calculations for composite models: hence, one cannot deduce the masses, coupling constants, and other physical parameters of quarks, leptons, and other bound states. However, one can use the experience from strong interactions and QCD to make some predictions about the low-energy particle spectrum and effective interactions.[5–7] Assuming that the vector-meson dominance is a good approximation, the form of the current-current effective interaction and some mass relations can be deduced. For example, if one assumes that the composite fermion electromagnetic form factor is saturated by a single pole coming from the W-like state, the standard-model four-fermion interaction and m_Z/m_W ratio are obtained.[5–7]

In Sec. II, in addition to the W-like states, we take into account the contribution of the $(\frac{1}{2},\frac{1}{2})$ isoscalar [with respect to the global $SU(2)_W$] bosons of the form $\bar{L}_a\gamma_\mu L^b$. L^a is the left-handed fundamental $SU(2)_L$ doublet fermion field with flavor index which includes color $a=1,\ldots,12$ for 3 generations. The isoscalar boson exchange modifies the standard-model neutral-current four-fermion interaction by introducing an isoscalar current term and also changing the coefficients of $J_L^{(3)}\cdot J_{\rm em}$ and $J_{\rm em}{}^2$, where $\mathbf{J}_L\equiv(J_L^{(1)},J_L^{(2)},J_L^{(3)})$ is the isovector and $J_{\rm em}$ is the electromagnetic current. In Sec. III we consider the effect of excited W-like states, which only change the coefficients of $J_L^{(3)}\cdot J_{\rm em}$ and $J_{\rm em}{}^2$. The general procedure of our analysis is given in Secs. II and III; essentially the same strategy is repeated in Sec. IV, where we include both the contribution of the excited W and also of the isoscalar bosons. The experimental bounds on the isoscalar current-current interaction place an upper bound on the effect of the isoscalar exchange and allow us to examine separately the contribution of the excited W bosons. Comparing the results with those of Sec. III where the isoscalar bosons are decoupled we obtain a whole set of possible parameter values. In the Conclusion we comment on the (0,0) bosons [the (0,1) bosons do not contribute to the low-energy current-current interaction] and the possible future tests of the model, which we consider not ruled out by presently available experimental results known to us. In Table I we give the expressions for the phenomenological parameters of the four-fermion interaction (according to the parametrization of Hung and Sakurai[8]) in terms of four parameters appearing in the low-energy current-current Lagrangian of the Abbott-Farhi model.

II. CONTRIBUTION AND MASS OF THE ISOSCALAR BOSONS

From the left-handed fundamental fermion fields it is possible to construct a global $SU(2)_W$-singlet vector field of the form $V_a^{\mu b}=\bar{L}_a\gamma^\mu L^b$ [L^a is the fundamental $SU(2)_L$-doublet field with flavor index $a=1,\ldots,12$] transforming according to the $(\frac{1}{2},\frac{1}{2})$ representation of the Lorentz group. By exchange of these vector bosons (and possibly their excited states) four-fermion effective interactions are mediated. Also, the flavor diagonal vector bosons mix with the photon, thus producing the physical mass eigenstates.

The contribution of isoscalar bosons has been studied by many authors.[6,7,9,10] In Ref. 10 an isoscalar boson which couples to the hypercharge current is considered and a lower limit on its mass is obtained. In the case of the SCSM a large multiplet of isoscalar bosons coupled to left-handed currents is expected; only one particular linear combination couples to the left hypercharge current. In Refs. 7 and 9 a similar multiplet of isoscalar bosons is introduced, but the underlying preon model and the physical content of the multiplet are different from the SCSM. Also, the author of Ref. 9 used a relation between the hypercharge current, third component of the isovector current, and the electromagnetic current which does not hold in our case (there is only a relation for left-handed parts of these currents). As a consequence, our low-energy Lagrangian is different.

In the following, we use the effective Lagrangian method combined with the Lagrangian formulation of the vector-meson dominance.[6,11,12] For a detailed discussion of the assumptions used, see Ref. 5. Assuming that the isovector and isoscalar parts of the electromagnetic form factor of physical fermions are saturated by single poles, coming from a state identified with $W^{(3)}$ and from the lightest isoscalar vector boson, we can write the effective Lagrangian density:

$$\mathscr{L}=\mathscr{L}_1+\mathscr{L}_2+\mathscr{L}_3\,, \tag{1}$$

where

$$\mathscr{L}_1=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-eF_\mu J^\mu_{\rm em}\,, \tag{2}$$

$$\mathscr{L}_2=-\tfrac{1}{4}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu}-\tfrac{1}{2}\lambda_W W^{(3)}_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}m_W{}^2\mathbf{W}_\mu\cdot\mathbf{W}^\mu-g_W\mathbf{W}_\mu\cdot\mathbf{J}^\mu_L\,, \tag{3}$$

$$\begin{aligned}\mathscr{L}_3=&-\tfrac{1}{4}\sum_a V^a_{a\mu\nu}V^{a\mu\nu}_a+\tfrac{1}{2}m_s{}^2\sum_a V^a_{a\mu}V^{\mu a}_a-\tfrac{1}{2}\lambda_s\sum_a y_a V^{\mu\nu a}_a F_{\mu\nu}-g_s\sqrt{2}\sum_a V^{\mu a}_a J^a_{La\mu}\\&-\tfrac{1}{4}\sum_{a\neq b}V^b_{a\mu\nu}V^{\mu\nu a}_b+\tfrac{1}{2}m_s{}^2\sum_{a\neq b}V^b_{a\mu}V^{\mu a}_b-g_s\sqrt{2}\sum_{a\neq b}V^a_{b\mu}J^{\mu b}_{La}\,,\end{aligned} \tag{4}$$

TABLE I. Phenomenological parameters characterizing four-fermion interactions as defined in Ref. 8, given as functions of parameters appearing in Lagrangian (11) for the Abbott-Farhi model, and in terms of $\sin^2\theta_W$ for the standard model.

	Abbott-Farhi model [Lagrangian (11)]	Standard model ($x^2=\sin^2\theta_W$)
α	$1-2x_0{}^2+2\xi\lambda_s{}^2$	$1-2x^2$
β	1	1
γ	$-\frac{2}{3}x_0{}^2+\xi+\frac{2}{3}\xi\lambda_s{}^2$	$-\frac{2}{3}x^2$
δ	ξ	0
g_v	$-\frac{1}{2}+2x_0{}^2+\frac{\xi}{2}-2\xi\lambda_s{}^2$	$-\frac{1}{2}+2x^2$
g_A	$-\frac{1}{2}+\frac{\xi}{2}$	$-\frac{1}{2}$
h_{vv}	$(-\frac{1}{2}+2x_0{}^2)^2+4c+\frac{\xi}{4}-2\xi\lambda_s{}^2+4\bar{y}\xi\lambda_s{}^4$	$(-\frac{1}{2}+2x^2)^2$
h_{AA}	$\frac{1}{4}+\frac{\xi}{4}$	$\frac{1}{4}$
h_{vA}	$\frac{1}{4}-x_0{}^2+\frac{\xi}{4}-\xi\lambda_s{}^2$	$\frac{1}{4}-x^2$
$\tilde{\alpha}$	$-1+2x_0{}^2+2\xi\lambda_s{}^2$	$-1+2x^2$
$\tilde{\beta}$	$-1+4x_0{}^2$	$-1+4x^2$
$\tilde{\gamma}$	$\frac{2}{3}x_0{}^2+\xi+\frac{2}{3}\xi\lambda_s{}^2$	$\frac{2}{3}x^2$
$\tilde{\delta}$	$\xi+\frac{4}{3}\xi\lambda_s{}^2$	0

and cubic and quartic self-couplings of W's and V's are omitted. F_μ is the electromagnetic potential, for any letter A:

$$A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu ,$$

$\mathbf{W}_\mu \equiv (W_\mu^{(1)}, W_\mu^{(2)}, W_\mu^{(3)})$ is the isotriplet vector field with mass m_W, $V_{\mu a}^b$ is the isoscalar vector field composed of fundamental fermion fields with flavor indices a and b. From the assumption that the isoscalar and isovector parts of the fermion electromagnetic form factors are saturated by single poles, using the asymptotic freedom of the $SU(2)_L$ interaction one obtains[5–7,9,10] $\lambda_W = e/g_W$ and $\lambda_s = e/g_s\sqrt{2}$. The electromagnetic current is

$$J_{\rm em}^\mu = \sum_a \bar{\psi}_a Q_a \gamma^\mu \psi^a , \qquad (5)$$

with ψ^a denoting quark or lepton fields and Q_a the corresponding charge matrix ($a = 1, \ldots, 12$); the left-handed isotriplet current is

$$\mathbf{J}_L^\mu = \sum_a \bar{\psi}_{La} \gamma^\mu \frac{\boldsymbol{\tau}}{2} \psi_L^a \qquad (6)$$

[$\boldsymbol{\tau} \equiv (\tau_1, \tau_2, \tau_3)$-Pauli matrices], while the left-handed isoscalar current is given as

$$J_{La}^{\mu b} = \tfrac{1}{2} \bar{\psi}_{La} \gamma^\mu \psi_L{}^b . \qquad (7)$$

y_a is the hypercharge of the doublet with flavor index a, $y_a = -1$ for leptons and $y_a = \frac{1}{3}$ for quarks.

$\mathscr{L}_3$ contains the contribution of isoscalar vector bosons which do not have a counterpart in the standard electroweak model. If we neglect this part of the Lagrangian, the standard-model result for the effective four-fermion interaction and W and Z masses can be derived.[5–7,10] In our discussion, we neglect the breaking of the global SU(12) (-flavor) symmetry by $SU(3) \times U(1)$ gauge interaction and Yukawa couplings, which lead to splittings of order $\alpha_3 G_F{}^{-1/2}$ and $m^2 G_F{}^{1/2}$, where α_3 is the strong coupling constant and m is a typical quark or lepton mass.

In $\mathscr{L}_3$ the contributions of flavor diagonal and nondiagonal bosons are separated, because only the flavor-diagonal vector bosons can mix with the photon. The contribution of excited states of the global $SU(2)_W$ triplet is considered in the next section.

The effective four-fermion interaction Lagrangian in the $q^2 \to 0$ limit can be obtained by standard procedure (see Refs. 5, 6, 9, 10, and 13):

$$\mathscr{L}_{\rm NC} = -\left[\frac{g_W{}^2}{2m_W{}^2} + \frac{g_s{}^2}{2m_s{}^2} \right] (J_L^{(3)})^2 + \frac{e^2}{m_W{}^2} J_L^{(3)} \cdot J_{\rm em} - \frac{e^4}{2g_W{}^2 m_W{}^2} J_{\rm em}{}^2 - \frac{g_s{}^2}{2m_s{}^2} J_L{}^2 + \frac{e^2}{m_s{}^2} J_{\rm em} \cdot \sum_a y_a J_{La}^a - \frac{e^4 \tilde{y}}{4g_s{}^2 m_s{}^2} J_{\rm em}{}^2 , \qquad (8)$$

where

$$J_l \equiv \sum_a J_{La}^a \qquad (9)$$

and we used a Fierz transformation to obtain

$$J_{La}^b \cdot J_{Lb}^a = \tfrac{1}{2} J_L{}^2 + \tfrac{1}{2} \mathbf{J}_L \cdot \mathbf{J}_L .$$

The above formula shows that isoscalar-boson exchange also mediates charged-current interaction of the form

$$-\frac{g_s{}^2}{2m_s{}^2} J_L^{(+)} J_L^{(-)} .$$

Adding this to the $W^\pm$-exchange contribution we have the identification

$$\frac{g_s{}^2}{2m_s{}^2} + \frac{g_W{}^2}{2m_W{}^2} = \frac{4G_F}{\sqrt{2}} . \qquad (10)$$

The neutral-current Lagrangian can then be written in the form

$$\mathscr{L}_{\rm NC} = -\frac{4G_F}{\sqrt{2}} \left[(J_L^{(3)} - x_0{}^2 J_{\rm em})^2 + c J_{\rm em}{}^2 + \xi \left[J_L{}^2 - 2\lambda_s{}^2 J_{\rm em} \cdot \sum_a y_a J_{La}^a + \lambda_s{}^4 \tilde{y} J_{\rm em}{}^2 \right] \right] , \qquad (11)$$

where

$$x_0{}^2 \equiv \frac{\lambda_W{}^2}{1+u} , \qquad (12a)$$

$$c \equiv \frac{\lambda_W{}^4 u}{(1+u)^2} , \qquad (12b)$$

$$\xi \equiv \frac{u}{1+u} , \qquad (12c)$$

with

$$u \equiv g_s{}^2 m_W{}^2 / g_W{}^2 m_s{}^2 . \qquad (12d)$$

To calculate the physical masses of vector bosons we have to take into account their mixing with photon.[5–7,10] A straightforward method is to calculate the photon propagator summing all the relevant diagrams. The procedure is simple because there are only two diagrams contributing to one-particle-irreducible vacuum polarization, corresponding to mixing with $W_\mu^{(3)}$ and the linear combination $\tilde{y}^{-1/2} \sum_a y_a V_{\mu a}^a$. The full photon propagator (up to gauge terms) is then given by

$$D_{\mu\nu}(q^2) = -\frac{i g_{\mu\nu}}{q^2} \frac{1}{1 - \lambda_W{}^2 \dfrac{q^2}{q^2 - m_W{}^2} - \tilde{y} \lambda_s{}^2 \dfrac{q^2}{q^2 - m_s{}^2}} . \qquad (13)$$

The smaller pole gives the mass of the Z (we suppose $m_s > m_W$), while the other gives the physical mass of the isoscalar boson. If both masses are to be real (no tachyon) the condition $1-\lambda_W{}^2-\tilde{y}\lambda_s{}^2>0$ must be satisfied, i.e., $\lambda_s{}^2<(1-\lambda_W{}^2)/\tilde{y}$.

Now we want to compare the predictions from Lagrangian (11) with experimental results. In our analysis, we need to reproduce coefficients of operators in the effective Lagrangian at the energy scale where intermediate bosons decouple (approximately at $m_W{}^2$). Yet, for example, physical parameters in (11) are actually measured in low-q^2 experiments.

The evolution of the leading-logarithmic corrections can be done in the framework of the effective-field-theory (EFT) approach,[14] and, in view of the existing experimental errors, we do not need any better accuracy. Note that composite models below the W mass have the same particle spectrum as the standard model. The only change is that the effective Lagrangian below m_W contains additional nonrenormalizable interactions; the ones not present in the perturbative phase. On the other hand, all these additional pieces are $O(\xi)$. The numerical analysis shows that the leading-logarithmic corrections in the standard phase are comparable or bigger than $O(\xi)$. Therefore, it is not necessary to consider mixings between operators from the standard part of the effective Lagrangian with those whose coefficients are $O(\xi)$. We conclude that for our purposes (comparison with experiments) g_W, λ_W, and e run as in the standard model, but ξ does not run at all.

The parameter $x_0{}^2$ plays the role of $\sin^2\theta_W$ of the standard model. At $q^2=m_W{}^2$ it can be obtained from the measurement of the W mass. Indeed, using (12a), (12d), and (10) we have

$$x_0{}^2=\frac{e^2\sqrt{2}}{8G_F m_W{}^2}\ . \tag{14}$$

The calculations in the standard model give[15] $e^2(m_W{}^2)/4\pi\simeq\frac{1}{128}$. On the other hand, G_F does not have logarithmic corrections. Using the UA2 Collaboration result (Ref. 16) $m_W=81.2\pm0.8\pm1.5$ GeV we obtain $x_0{}^2(m_W{}^2)=0.226\pm0.010$.

We can also use low-energy experiments to find the allowed regions of values of parameters ξ and $x_0{}^2$ at $q^2\approx0$. The experimentally allowed region in the $\xi-x_0{}^2$ plane is the shaded region in Fig. 1. The constraints are from the following experimental results (Ref. 17): $\delta=0.002\pm0.049$, $g_A^e=-0.494\pm0.026$, $g_V^e=-0.050\pm0.052$, $\alpha=0.533\pm0.037$ (for the definition of these parameters, see Ref. 8, in terms of parameters appearing in the effective Lagrangian they are given in Table I); and from polarized electron-deuteron scattering (Ref. 18):

$$\tilde{\alpha}+\tilde{\gamma}/3+(\tilde{\beta}+\tilde{\delta}/3)/4=-0.53\pm0.05\ .$$

From relations (12c) and (12d) we see that $\xi>0$ for this model; hence, we have $0.051>\xi>0$ and $0.240>x_0{}^2>0.215$ at 68% confidence limit (C.L.). We also took into account the restrictions on the parameter $\lambda_s{}^2$ coming from the absence of tachyon $\lambda_s{}^2<(1-\lambda_W{}^2)/\tilde{y}\le0.25$ for any λ_W. Using the constraint $\xi<0.051$ and (12c), we obtain $0<u<0.054$. From (12a)

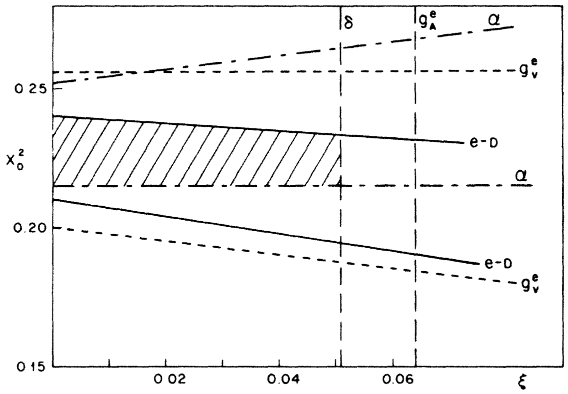

FIG. 1. Experimentally allowed region of the $x_0{}^2$-ξ plane for Lagrangian (11). The theoretical constraints $\xi>0$ and $\lambda_s{}^2<0.25$ are also used.

we now have $0.253>\lambda_W{}^2>0.215$; hence, the tachyon absence actually implies $\lambda_s{}^2<0.20$, i.e., $g_s{}^2/g_W{}^2=\lambda_W{}^2/2\lambda_s{}^2>0.54$. From this and the constraint on u we conclude that $m_s/m_W>3.2$, i.e., the mass of the isoscalar bosons which do not mix with the photon must be larger than 250 GeV (at 68% C.L.).

One can also expect that the color-octet isoscalar bosons will mix with gluons.[7,9] Assuming that the color form factors of quarks are saturated by the lowest poles we obtain

$$\lambda_c=\frac{g_c}{\sqrt{2}g_s}\ , \tag{15}$$

where λ_c is the gluon-boson mixing and g_c is the SU(3) coupling constant (at scale $\sim m_W$). No tachyon condition in this case implies $1-3\lambda_c{}^2>0$ and using the value for the strong coupling constant from Ref. 19,

$$g_c{}^2(m_W{}^2)/e^2(m_W{}^2)\approx12\ ,$$

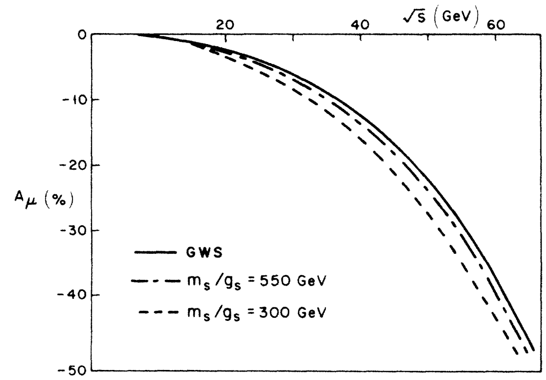

FIG. 2. Global asymmetry of the $e^+e^-\to\mu^+\mu^-$ process vs center-of-mass energy. The contribution of SU(2)$_W$-singlet bosons with mass m_s and coupling constant g_s increases $|A_\mu|$ with respect to the standard (GWS) model.

we obtain a rather strong limit: $m_s \gtrsim 700$ GeV. This result is so strong that it puts the whole procedure in question. At such high energies the isoscalar bosons may not be well separated from the rest of the exotic sector. Thus, using vector-meson dominance in the isoscalar channel is less reliable than in the isovector channel. We believe that a conservative estimate of the lower limit on the isoscalar mass is a couple of hundred GeV.

The isoscalar bosons $V^b_{\mu a}$ also contribute to $e^+e^- \to \mu^+\mu^-$ cross section and asymmetry. To analyze this process in the strong-coupling phase of the model it is necessary to diagonalize the effective current-current Lagrangian at arbitrary q^2. A convenient procedure is given in Ref. 6; hence, we will not give the details of the calculation. The part of the Lagrangian relevant for $e^+e^- \to \mu^+\mu^-$ can be written in the form

$$\mathscr{L} = \frac{1}{2}\frac{e^2}{q^2} J_{\rm em}{}^2 + e^2 \sum_i \bar{e}(\gamma_\nu v_i^e - \gamma_\nu \gamma_5 a_i^e) e \frac{F_i{}^2}{q^2 - M_i{}^2} \times \bar{\mu}(\gamma^\nu v_i^\mu - \gamma^\nu \gamma_5 a_i^\mu)\mu \,, \tag{16}$$

and the forward-backward asymmetry

$$A_\mu \equiv \frac{N(\theta < 90^\circ) - N(\theta > 90^\circ)}{N(\theta < 90^\circ) + N(\theta > 90^\circ)} \tag{17}$$

(N number of events) is given by[10]

$$A_\mu = \frac{3}{4}\frac{\chi}{\eta} \,, \tag{18}$$

where

$$\chi \equiv 2 \sum_i \frac{sF_i{}^2 a_i^e a_i^\mu}{s - M_i{}^2} + \sum_{i,j} \frac{s^2 F_i{}^2 F_j{}^2}{(s - M_i{}^2)(s - M_i{}^2)} (v_i^e a_j^e + a_i^e v_j^e)(v_i^\mu a_j^\mu + a_i^\mu v_j^\mu) \,, \tag{19}$$

$$\eta \equiv 1 + 2 \sum_i \frac{sF_i{}^2 v_i^e v_i^\mu}{s - M_i{}^2} + \sum_{i,j} \frac{s^2 F_i{}^2 F_j{}^2}{(s - M_i{}^2)(s - M_j{}^2)} (v_i^e v_j^e + a_i^e a_j^e)(v_i^\mu v_j^\mu + a_i^\mu a_j^\mu) \,, \tag{20}$$

and $\sqrt{s}$ is the center-of-mass energy ($s \equiv q^2$). The asymmetry A_μ versus s is plotted in Fig. 2 for some values of m_s/g_s. It was found numerically that in the energy range shown in Fig. 2 the asymmetry does depend only on ratio m_s/g_s and not on the mass and coupling constant of the isoscalar boson separately. Here we did not include the QED radiative corrections which are small in the considered energy range (they change A_μ by approximately $+1.5\%$).[20] Presently available experimental results[21] have rather large errors and we cannot obtain any reasonable bounds on m_s/g_s.

III. EXCITED W BOSONS

In Sec. II we have neglected the possible contributions to electromagnetic form factors and four-fermion interactions coming from exchange of the excited states of W bosons. Here, we examine a case where only two weak isotriplets: W bosons and their excited states W' (hereafter all parameters pertaining to the excited W' bosons are primed) mediate the weak interactions.

In literature, it is a standard approach to discuss the physical parameters of the W' boson using sum rules for the isovector channel of a given composite model.[10,13,22,23] Arguments such as the Q^2 duality (Ref. 22) or the asymptotic-freedom sum rules[23] put the mass of the W' above 400 GeV and give a relation of the form $\lambda'_W \approx \lambda_W(m_W/m'_W)$. However, Devyanin and Jaffe[24] have shown that in the SCSM the sum-rule analysis cannot give any information about the mass and couplings of the W' bosons. In our analysis we regard the physical parameters of W' as *a priori* unconstrained.

From the form of the charged-current interactions we get one normalization condition (see Refs. 5 and 13):

$$\frac{g_W{}^2}{m_W{}^2} + \frac{g'_W{}^2}{m'_W{}^2} = \frac{g_W{}^2}{m_W{}^2}(1 + r^2\mu^2) = \frac{8G_F}{\sqrt{2}} \,, \tag{21}$$

where $r = g'_W/g_W$ and $\mu = m_W/m'_W$. The low-energy four-fermion neutral-current Lagrangian can be written in the standard form:

$$\mathscr{L}_{\rm NC} = -\frac{4G_F}{\sqrt{2}}[(J_L^{(3)} - x_0^2 J_{\rm em})^2 + cJ_{\rm em}{}^2] \,, \tag{22}$$

where all the coefficients of the current-current operators are measured experimentally. This provides another set of normalization conditions:[13]

$$\frac{e\lambda_W g_W}{m_W{}^2} + \frac{e\lambda'_W g'_W}{m'_W{}^2} = \frac{e\lambda_W g_W}{m_W{}^2}(1 + kr\mu^2) = x_0{}^2\frac{8G_F}{\sqrt{2}} \,, \tag{23}$$

$$\frac{e^2\lambda_W{}^2}{m_W{}^2} + \frac{e^2\lambda'_W{}^2}{m'_W{}^2} = \frac{e^2\lambda_W{}^2}{m_W{}^2}(1 + k^2\mu^2) = (x_0{}^4 + c)\frac{8G_F}{\sqrt{2}} \,, \tag{24}$$

where $k = \lambda'_W/\lambda_W$. Assuming that the isovector part of the electromagnetic form factor is saturated by two poles coming from W and W', we get

$$e = g_W\lambda_W + g'_W\lambda'_W = g_W\lambda_W(1 + kr) \,. \tag{25}$$

We see that small values of r^2, k^2, and μ^2 (i.e., the decoupling of W') yield the physical parameters of the W bosons g_W, λ_W, m_W at their standard-model values.

To simplify the discussion one can eliminate g_W and

λ_W by considering

$$A \equiv \frac{c}{x_0{}^4} = \mu^2 \frac{(k-r)^2}{(1+rk\mu^2)^2} \tag{26}$$

and

$$B \equiv \frac{8G_F x_0{}^2}{\sqrt{2}e^2} m_W{}^2 - 1 = \frac{rk}{1+rk}(\mu^2 - 1) \ . \tag{27}$$

Since $B<0$ (for definiteness we take $r>0$, $k>0$) we see that in any composite model where effectively only two isotriplets (W and W') mediate the weak force, the mass of the lowest W boson has to be smaller than the one predicted by the standard model $\sqrt{2}e^2/8G_F x_0{}^2$.

The two equations (26) and (27) cannot yet give bounds on the possible values of r^2, k^2, and μ^2. As an additional independent constraint we have to compare the predicted mass of the Z boson with its observed value. The predicted mass of the Z can be obtained by solving

$$\frac{\lambda_W{}^2}{1-m_W{}^2/m_Z{}^2} + \frac{\lambda_W'{}^2}{1-m_W'{}^2/m_Z{}^2} = 1 \ . \tag{28}$$

We cannot transform (28) into a constraint on k^2, r^2, and μ^2 alone. But we can study an experimentally measured quantity

$$D \equiv m_Z{}^2/m_W{}^2 \ , \tag{29}$$

which in this model depends only on k^2, r^2, μ^2, and

$$\lambda_W{}^2 = (\lambda_W^{\rm GWS})^2 \frac{1+r^2\mu^2}{(1+kr)^2} \ , \tag{30a}$$

where

$$(\lambda_W^{\rm GWS})^2 = \frac{\sqrt{2}e^2}{8G_F m_W{}^2} = \left[\frac{38.65}{m_W}\right]^2 \tag{30b}$$

is the standard-model value of the γ-$W^{(3)}$ mixing at $q^2 = m_W{}^2$.

To solve (28) we have to find where the function

$$f(x) = \frac{\lambda_W{}^2}{1-m_W{}^2 x} + \frac{\lambda_W'{}^2}{1-m_W'{}^2 x}, \quad x \equiv \frac{1}{m_Z{}^2} \ , \tag{31}$$

is equal to 1. In general, $f(x)$ looks like in Fig. 3, from which we see that the model has a tachyon unless $f(0) = \lambda_W{}^2 + \lambda_W'{}^2 < 1$, and also that for any nonzero λ_W' (i.e., $k^2 > 0$) we can rule out the region

$$\mu^2 > \left[\frac{m_{W\rm expt}}{m_{Z\rm expt}}\right]^2 = 0.796 \tag{32}$$

[at 68% C.L., using the UA2 data].[16]

From the form of $f(x)$ we see that the second term in (31) is negative at $x = 1/m_Z{}^2$. It means that this additional term has the effect of diminishing the predicted value of m_Z (as compared to the case when W' decouples). On the other hand, when $\lambda_W{}^2$ increases (decreases) m_Z increases (decreases). From (30) we see that $\lambda_W{}^2$ can increase or decrease over its standard-model value, depending on what r^2, k^2, and μ^2 we choose.

The basic strategy of our analysis is the following. First, we determine which values of $A_{\rm expt}$, $B_{\rm expt}$, and $D_{\rm expt}$ are experimentally allowed. Then, we investigate which regions of the parameter space (r^2,k^2,μ^2) yield a

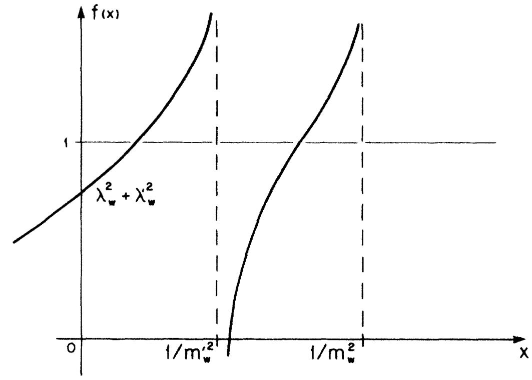

FIG. 3. The function $f(x)$ given by Eq. (31), in the case $\lambda_W{}^2 + \lambda_W'{}^2 < 1$.

model predicting the acceptable values of A, B, and D.

Since we are using experimental data that come from measurements at different energy scales, to make a discussion realistic, we must take into account radiative corrections. As discussed in Sec. II we can assume that g_W, λ_W, and e run as in the standard model, but μ^2, k^2, and r^2 do not run.

Thus, we can use the well-studied radiative corrections calculated in the standard model within any renormalization scheme.[25] They differ from the EFT calculations[26] by terms $O(\alpha G_F)$, not relevant in our discussion. With this in mind we adopt the following values for the parameters:

$$0 \le A_{\rm expt} = \frac{c}{x_0{}^4(\text{at low } q^2)} \le 0.18 \text{ at 68\% C.L.} \ , \tag{33a}$$

$$B_{\rm expt} = -0.044 \pm 0.082 \ , \tag{33b}$$

$$D_{\rm expt} = 1.30 \pm 0.04 \ , \tag{33c}$$

$$(\lambda_W^{\rm GWS})^2 = 0.227 \pm 0.013 \ . \tag{33d}$$

Here we have used the world averaged

$$x_0{}^2(q^2 = m_W{}^2) = 0.217 \pm 0.014$$

and

$$x_0{}^2 \ (\text{low } q^2) = 0.234 \pm 0.013 \pm 0.009$$

coming from the neutral-current experiments,[27] c bounded to be ≤ 0.01 (at 68% C.L.) by the e^+e^- experiments,[20] and the values of $(\lambda_W^{\rm GWS})^2$ and $D_{\rm expt}$ as measured by the UA2 Collaboration.[16] Note that because of the familiar form of the neutral-current Lagrangian (22) we could use the existing results of fits to experimental data to determine $x_0{}^2$ (low q^2), contrary to the case in Sec. II, where we had to determine the corresponding parameter $x_0{}^2$ fitting Lagrangian (11) to experimental data.

The origin of the parameter space ($r^2 = k^2 = \mu^2 = 0$) corresponds to the standard-model limit and in that case we have

$$A = 0 \ , \quad B = 0 \ , \quad D = 1.29 \pm 0.02 \ , \tag{34}$$

clearly within the experimentally allowed region (33). In fact, the same set (34) of the predicted values A, B, and D can be obtained in other limits where the excited sector is decoupled:

$$r^2 \to 0, \ \mu^2 \to 0, \ k^2 \text{ anything }, \tag{35a}$$

$$k^2 \to 0, \ \mu^2 \to 0, \ r^2 \text{ anything }, \tag{35b}$$

$$k^2 \to 0, \ r^2 \to 0, \ \mu^2 \text{ anything }. \tag{35c}$$

The examples (35a)–(35c) already show that there is no one-to-one correspondence between the points (r^2, k^2, and μ^2) and predictions (A, B, and D). In general, we can expect that there is not enough information in the low-q^2 measurements to put one comprehensive bound, restricting the possible values of r^2, μ^2, and k^2 all at the same time. For the same reason there is no hope of getting any single fit of the most probable values of r^2, k^2, and μ^2 evaluated from the experimental data. In this situation, the best we can do is to study the model along some sections of the r^2, k^2, μ^2 space.

To take into account the constraints from experimentally allowed values of A, B, and D simultaneously, we define a function

$$\chi^2 \equiv \left[\frac{A - A_{\text{expt}}}{\Delta A_{\text{expt}}}\right]^2 + \left[\frac{B - B_{\text{expt}}}{\Delta B_{\text{expt}}}\right]^2 + \left[\frac{D - D_{\text{expt}}}{\Delta D_{\text{expt}}}\right]^2 . \tag{36}$$

We put $A_{\text{expt}} = 0$ and from (33a) $\Delta A_{\text{expt}} = 0.18$. The other relevant values of parameters B and D are given by (33b) and (33c). χ^2 depends on r^2, k^2, μ^2, and λ_W^{GWS}. If for a given choice of r^2, k^2, μ^2 and $0.214 \le (\lambda_W^{\text{GWS}})^2 \le 0.240$ we have $\chi^2 < 1$, we conclude that experimental data do not exclude the model with specified parameters, at 1σ (68% C.L.) level. In Fig. 4 we show a 1σ bound on the possible values of k^2 and μ^2 at $r^2 = 0.5$, 0.8, 1, 1.5, and 1.9.

In a recent analysis,[28] the UA1 Collaboration quotes lower limits on masses of additional intermediate vector bosons. Looking at the neutrino-electron decay channel of W' at standard couplings (corresponding to $r = 1$ in our notation) they obtain $m_W' > 210$ GeV at 90% C.L. Our results for $r^2 = 1$ (see Fig. 4) give $m_W' > 170$ GeV at 68% C.L. The collider limit on the mass of the additional Z boson is even lower: $m_Z' > 160$ GeV at 90% C.L. Note that all of these results do not put a very strict limit on the masses of the excited sector.

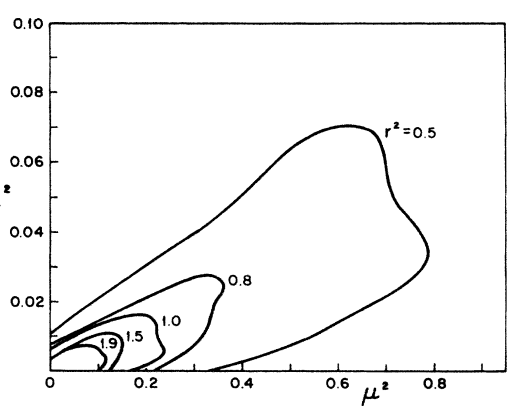

FIG. 4. Regions of the allowed parameter space (by criterion $\chi^2 < 1$), shown as sections for some fixed r^2, when the contribution of $SU(2)_W$-singlet bosons is neglected.

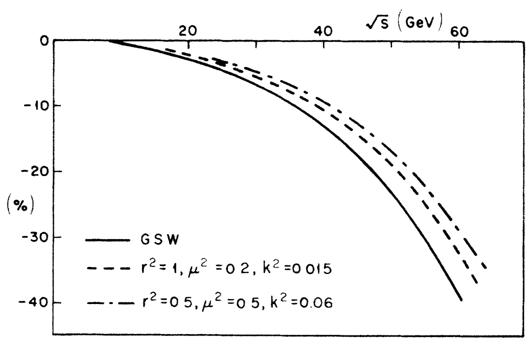

FIG. 5. Global asymmetry of the $e^+e^- \to \mu^+\mu^-$ process vs center-of-mass energy. The contribution of excited Z usually (but not for all values of r^2, k^2, and μ^2) decreases $|A_\mu|$ with respect to the standard-model (GWS) case.

The $e^+e^- \to \mu^+\mu^-$ asymmetry can be calculated by diagonalizing the four-fermion interaction Lagrangian for $q^2 > 0$ and then using expressions (18)–(20). The results for typical values of parameters r^2, k^2, and μ^2, are plotted in Fig. 5.

IV. EXCITED W AND ISOSCALAR BOSONS

Now we want to analyze the case when both isoscalar bosons and the excited isotriplet W' contribute to the low-q^2 phenomenology. The empirical form of the neutral-current interaction is the same as in Sec. II. But the coefficients of operators in Eq. (11) are now functions of the physical parameters of the W, W', and isoscalar bosons. The relations (23)–(25) hold also in this case, while (21) is replaced by

$$\frac{g_W{}^2}{m_W{}^2}(1 + r^2\mu^2) = \frac{8G_F}{\sqrt{2}}(1 - \xi) . \tag{37}$$

This means that now we have

$$A \equiv \frac{c}{x_0{}^4} = \frac{\mu^2(r-k)^2 + \xi(1 + rk\mu^2)^2}{(1 + rk\mu^2)^2(1-\xi)} \tag{38}$$

and

$$\lambda_W{}^2 = (\lambda_W^{\text{GWS}})^2 \frac{1 + r^2\mu^2}{(1 + kr)^2(1-\xi)} , \tag{39}$$

while the expression for B is unchanged. The experimental analysis of the Lagrangian (11) has been done in Sec. II and we may quote

$$x_0{}^2(\text{low } q^2) = 0.227 \pm 0.013 \tag{40}$$

and

$$\xi < 0.051 \text{ at } 68\% \text{ C.L.} \tag{41}$$

Using the EFT approach one can calculate the leading-logarithmic corrections. Using the results of Ref. 26 we get

$$x_0{}^2(q^2 \sim m_W{}^2)=0.216\pm0.015 \ . \tag{42}$$

As explained before, we do not need to discuss any scaling of ξ. These experimental data can be used to constrain parameters defined in Secs. II and III:

$$A_{\text{expt}} < 0.18 \text{ at } 68\% \text{ C.L.} \ , \tag{43a}$$

$$B_{\text{expt}} = -0.048\pm0086 \ , \tag{43b}$$

$$D_{\text{expt}} = 1.30\pm0.04 \ , \tag{43c}$$

$$\tilde{y}\lambda_s{}^2 m_s{}^2 > 8.0 m_W{}^2 \ . \tag{43d}$$

To calculate the predicted value of m_Z (and D in due course) in the present case when all three bosons $W^{(3)}$, $W'^{(3)}$, and the isoscalar couple to the photon we have to investigate the function

$$f(x)=\frac{\lambda_W{}^2}{1-m_W{}^2x}+\frac{\lambda'_W{}^2}{1-m'_W{}^2x}+\frac{\tilde{y}\lambda_s{}^2}{1-m_s{}^2x} \ . \tag{44}$$

To calculate $m_Z{}^2=1/x_Z$ we have to find the largest solution of $f(x_Z)=1$. We use the constraint (43d) together with the no tachyon constraint

$$\lambda_W{}^2+\lambda'_W{}^2+\tilde{y}\lambda_s{}^2 \le 1 \tag{45}$$

to eliminate λ_s and m_s from the analysis. The form of (44) shows that the coupling of the isoscalar boson to the photon has the largest effect on the mass of Z when both inequalities (43d) and (45) are saturated. Therefore, we want to investigate now what values of parameters of the excited W' boson are compatible with the experiment, under the condition that the isoscalar is coupled in the strongest possible way, i.e., when

$$\tilde{y}\lambda_s{}^2=1-\lambda_W{}^2-\lambda'_W{}^2 \ , \tag{46a}$$

$$m_s{}^2\tilde{y}\lambda_s{}^2=8.0 m_W{}^2 \ . \tag{46b}$$

From now on the procedure is the same as in Sec. III, i.e., a χ^2 is defined as in (36); r^2, k^2, and μ^2 have the same meaning as in the previous section and we use (43a)–(43c). D is calculated from (44) under the conditions (46).

As before, we give the results in the form of plots of allowed regions for which $\chi^2<1$ in the k^2-μ^2 plane for several fixed r^2. For $r^2=0.5$, 0.8, 1, 1.5, and 1.9 the results are plotted in Fig. 6.

In the final analysis one has to compare Fig. 4 with Fig. 6. The former corresponds to the case when $\lambda_s{}^2 \to 0$ and $m_s{}^2 \to \infty$. Since the values of $\lambda_s{}^2$ and $m_s{}^2$ must be somewhere between $\lambda_s{}^2=0$, $m_s{}^2=\infty$ and values given by (46), we may conclude that for a given r^2 any point (k^2,μ^2) that lies on the union of the two allowed regions (shown in Figs. 4 and 6), corresponds to a model in which both W' and isoscalar bosons mediate the weak force and whose low-q^2 phenomenology is compatible with experimental data. In particular, when we adopt the bounds

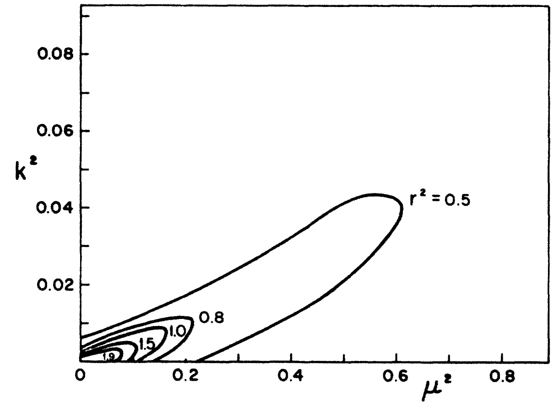

FIG. 6. Regions of the allowed parameter space (by criterion $\chi^2<1$), shown as sections for some fixed r^2, for the case when the $SU(2)_W$-singlet bosons are coupled in the strongest possible way.

from the analysis of vector-meson dominance in the color channel, the isoscalar bosons have a small influence on the low-q^2 phenomenology and do not affect the bounds on the W'; Fig. 4 better describes the allowed region.

V. CONCLUSION

We compared the low-energy phenomenology and the prediction for the ratio m_Z/m_W of the strongly coupled standard model, with experimental data. From the absence of experimental evidence for isoscalar currents[17] it is possible to restrict the ratio of the mass and coupling constant for isoscalar bosons: $m_s/g_s \gtrsim 550$ GeV. Under the assumption that the isoscalar part of electromagnetic form factors is saturated by one pole coming from the isoscalar boson, the absence of tachyon implies $g_s \gtrsim 0.45$, i.e., $m_s \gtrsim 250$ GeV. The same analysis applied to color form factors of quarks gives $g_s \gtrsim 1.3$ and $m_s \gtrsim 700$ GeV. If the contributions from other states in this channel are included, the constraints on g_s (and subsequently on m_s) go away.

On the other hand, from the low-energy experiments, and the m_Z/m_W ratio it is not possible to obtain any interesting lower limit on the excited W' mass. This is because we do not assume any functional relation between r^2, k^2, and μ^2. Of course, those relations must exist and if one understands better the dynamics of the underlying confining force it will be possible to derive them. The application of the sum-rule method to GWS and the confining phase of the standard model has been worked out in Ref. 24. Because of the absence of experimental data, the authors claim, it is not possible to deduce a definite relation connecting the parameters of the W and W' sectors. Thus, one may conclude, it is possible only to determine the experimentally allowed region in the r^2, k^2, μ^2 space. We have presented sections of this region at various $r^2=$const (Figs. 4 and 6). For our choices of r^2, the parameters of W' are more constrained with the isoscalar (Fig. 6) then without (Fig. 4). This follows from the fact that the effect of the isoscalar coupling on the predicted mass m_Z^2 goes in the same direction as the effect of the W' coupling. On the other hand, the experimental errors of the coefficients of the effective Lagrangian (22) and (11) are comparable. Therefore, if the isoscalar couples in the strongest possible way it squeezes out some previously allowed regions of the W' parameter space.

Our results may differ from the other authors who use sum-rule methods to deduce relations between r^2, k^2, and μ^2 (Refs. 10, 13, 29, 30). In general, we recover their results by studying the model along the corresponding sections in r^2, k^2, μ^2 space. On the other hand, in our case such a procedure is unjustified[24] so we decided to present the most general analysis.

We also calculated the $e^+e^-\to\mu^+\mu^-$ asymmetry for two cases: (1) taking into account only the isoscalar bosons, and (2) with the contribution of the excited W bosons only. For the energy range considered $[q^2<(60\text{ GeV})^2]$ in the first case $|A_\mu|$ increases with respect to its standard-model value, in accordance with expectations, because more than one weak boson contributes to the process. However, in the second case $|A_\mu|$ decreases, because the coupling of the W boson to fermions is changed. Hence, if we take into account both the excited W and the isoscalar bosons, the results are rather close to the standard-model values. From presently available results of A_μ measurements we cannot deduce any interesting bound about either the excited W or the isoscalar bosons, even if we take into account the contribution of only one of them.

In this paper, we did not consider the contribution of the Lorentz scalar bosons which can be formed from two fundamental fermions. These scalar bosons can mediate a low-energy current-current interaction of the form[4]

$$\mathscr{L}=-\frac{g^2}{2m^2}(\mathbf{J}_L\cdot\mathbf{J}_L-J_L{}^2)\ .$$

This means that the coefficient of $(-4G_F/\sqrt{2})J_L{}^2$ (denoted by ξ) can be either positive or negative and our bound on the isoscalar mass does not hold. The presence of another unknown parameter m/g makes it impossible to place a bound on m_s/g_s from low-energy experiments.

Experiments at high energies (on the TeV scale) should be able to distinguish between the strongly coupled and perturbative (Glashow-Weinberg-Salam) phases of the model. However, some experiments at presently available energies, for example precise measurements of the masses and especially of the widths of W and Z, would add valuable information. At this time, on the basis of presently available experimental results, we do not consider the realization of the confining phase in nature to be ruled out.

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