Effects of core motion on static properties of the nucleon

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The effects of core motion in the nucleon are calculated in a simple model in which the translation-invariant Hamiltonian describes a pion field interacting with a nucleon core. The ground state of the system is shown to be a self-consistent state in which the pion field binds the core, which, in turn, acts as an effective nonstatic source of the pion field. The form factor of the nonstatic source is softer than the intrinsic form factor of the corresponding Hamiltonian without core motion. The effects of core motion on the nucleon magnetic moments and charge radii are presented for the case that the core properties are computed in the bag model. It is found that for a given set of observed properties, the bag with core motion is smaller than the corresponding bag without core motion.

I. INTRODUCTION

It is now widely believed that the nucleon has a core or confinement region inhabited by quarks and gluons, whose behavior is governed by quantum chromodynamics, and that outside the core the interaction between nucleons can be approximately described in terms of meson exchange. One consequence of this picture has been a renewed interest in various models of nucleon-meson systems. The Hamiltonian for a nonrelativistic nucleon core with field operator $\tilde{\psi}(\mathbf{p})$ interacting linearly with a pion field whose annihilation operator $a_\lambda(\mathbf{k})$ is

$$H=\int \tilde{\psi}^\dagger(\mathbf{p})\frac{p^2}{2M}\tilde{\psi}(\mathbf{p})d\mathbf{p}+\int \omega(k)a_\lambda^\dagger(\mathbf{k})a_\lambda(\mathbf{k})d\mathbf{k} - \int \frac{a_\lambda(\mathbf{k})+a_\lambda^\dagger(-\mathbf{k})}{[16\pi^3\omega(k)]^{1/2}}\delta(\mathbf{p}-\mathbf{q}-\mathbf{k})\tilde{\psi}^\dagger(\mathbf{p})\tau_\lambda\boldsymbol{\sigma}\cdot\mathbf{J}_\pi\left(\mathbf{k},\frac{\mathbf{p}+\mathbf{q}}{2}\right)\tilde{\psi}(\mathbf{q})d\mathbf{p}\,d\mathbf{q}\,d\mathbf{k}\ , \tag{1}$$

where the summation convention is used for the isospin index λ, and M is the mass of the nucleon core; m will be used for the pion mass. The first term in H is the kinetic energy of the nucleon core, the second is the usual pion field energy, and the third is the Yukawa interaction of the pion field with the nucleon core. The aim in studying Hamiltonians like (1) is to learn something about the pion current form factor $\mathbf{J}_\pi(\mathbf{k},\mathbf{K})$ that describes the (effective) interaction of the nucleon core with the pion field. It is assumed that $\mathbf{J}_\pi$ is such that all the integrals that would occur in the corresponding perturbation theory are finite.

In chiral bag (CB) models[1,2] that use confinement and chiral symmetry to derive various forms of the pion current form factor $\mathbf{J}_\pi$, this form factor depends on the radius of the bag. In treating the interaction of the bag or core with the pion field in these models, the static approximation was used for the core; that is, the core was assumed fixed in space; fixing the core affects both the core kinetic energy term, which is neglected, and the Yukawa interaction term, which takes a simpler form, so that the static-model (SM) Hamiltonian was taken to be

$$H_{\rm SM}=\int \omega(k)a_\lambda^\dagger(\mathbf{k})a_\lambda(\mathbf{k})d\mathbf{k} - \int \frac{a_\lambda(\mathbf{k})+a_\lambda^\dagger(-\mathbf{k})}{[16\pi^3\omega(k)]^{1/2}}\tau_\lambda\boldsymbol{\sigma}\cdot\mathbf{J}_{\rm SM}(\mathbf{k})d\mathbf{k}\ , \tag{2}$$

and the assumption was made that

$$\mathbf{J}_{\rm SM}(\mathbf{k})=\mathbf{J}_{\rm CB}(\mathbf{k},0)\ , \tag{3}$$

where $\mathbf{J}_{\rm CB}$ is the form of the pion current form factor $\mathbf{J}_\pi$ in the particular chiral bag model under consideration. Comparison of the nucleon static properties computed from the Hamiltonian of Eq. (2) with observed data has been used[1,2] to determine a best value of the bag radius that appears as a parameter in $\mathbf{J}_{\rm CB}$.

However, the core is not really static; it necessarily moves within the meson field that it generates. The purpose of this paper is to assess the effect of core motion on the static properties of the nucleon as computed with the Hamiltonian of (1), which is here assumed to be an appropriate effective Hamiltonian for the nonstatic core interacting with the pion field, effective in the sense that it does not contain the fundamental quark and gluon degrees of freedom explicitly. When the core motion is treated in the simplest approximation, it is shown below that the Hamiltonian of (1) reduces to a form that is similar to the static-model Hamiltonian of (2), with an interaction form factor that takes the core motion into account. In addition to the terms in (2) the reduction leads to a term that describes the kinetic energy of the core motion. Moreover, the interaction form factor in this approximation [see (6) and (7)] depends on the wave function of the core. In particular, the relationship between the "intrinsic" pion current form factor $J_\pi(\mathbf{k},(\mathbf{p}+\mathbf{q})/2)$ and the effective pion current form factor $\mathbf{J}_{\rm SM}(\mathbf{k})$, which will be called the "nonstatic form factor," is no longer (3), but depends on the wave function $f(\mathbf{r})$ of the core. In the special case that $\mathbf{J}_\pi(\mathbf{k},(\mathbf{p}+\mathbf{q})/2)$ is independent of its second argument, it is not surprising that the nonstatic form factor is

$$\mathbf{J}_{\rm SM}(\mathbf{k})=\mathbf{J}_\pi(\mathbf{k})\tilde{\rho}(\mathbf{k}) , \tag{4}$$

where $\tilde{\rho}(\mathbf{k})$ is the Fourier transform of the core probability density $|f(\mathbf{r})|^2$; this form is evidently consistent with the form (3) for the case of a static core for which $\tilde{\rho}(\mathbf{k})$ is 1. Since the comparison with static nucleon properties is made in terms of $\mathbf{J}_{\rm SM}$, it follows immediately that the motion of the core must be considered before conclusions can be drawn about $\mathbf{J}_\pi$. Of course, if the motion of the core is small, so that $\tilde{\rho}(\mathbf{k})$ is nearly equal to 1, then $\mathbf{J}_{\rm SM}$ is nearly equal to $\mathbf{J}_\pi$, and the core motion can safely be ignored.

Various types of core motion could be considered. Particularly simple is the self-consistent motion of the core within the pion field that it generates. One result of the following analysis is the demonstration that such a self-consistent motion of the core actually is a consequence of the Hamiltonian (1); that is, there is a self-consistent bound state of the system consisting of the nucleon core with its pion field. Roughly speaking, the core motion in this state is analogous to the motion of the proton in the ground state of the hydrogen atom. In addition to showing the existence of the self-consistent bound state, the quantitative effects of this self-consistent motion on the computed static properties of the nucleon are calculated for the form of the intrinsic pion current form factor,

$$\mathbf{J}_\pi(\mathbf{k},\mathbf{K})=(4\pi\gamma)^{1/2}\frac{\mathbf{k}}{m}\frac{3j_1(kR)}{kR} , \tag{5}$$

that has been used in computations that do not take core motion into account; here R is the radius of the bag or core and γ is the (unrenormalized) pion-core coupling that is usually written $f^2/4\pi$. It should be emphasized here that the aim of the computations is to obtain a quantitative estimate of the size of the effects of core motion, rather than to make any special claim for the special sort of core motion that is implied by the self-consistent state of the bag with its pion field. The self-consistent core motion defined above is a special form of core motion that should at least give a reasonable estimate of the magnitude and trend of the effects of core motion on static nucleon properties without the necessity of introducing additional parameters to specify the core motion. Other meson fields will also affect the motion of the core; the specific quantitative effects will vary with the dynamical origin of the core motion in the particular model used, but the self-consistent wave function gives a simple picture of the trend of the effects of core motion.

Section II considers the self-consistent core motion, and shows that such a motion gives a variational nucleon ground state for the Hamiltonian. In Sec. III the general outline of the computations of static nucleon properties is presented. Section IV describes the results of the computations, and Sec. V contains some remarks on the computations. A summary of the paper is given in Sec. VI.

II. SELF-CONSISTENT CORE MOTION

In order to test for the existence of a self-consistent state of the moving core in its pion field, a single-mode approximation is used for the motion of the core; the core is assumed to be in a $0s$ state (zero nodes) with radial wave function $f(r)$, whose Fourier transform is $\tilde{f}(k)$; in the subspace in which the core is restricted to be in this $0s$ state, the Hamiltonian of (1) is effectively H_{0s}:

$$H_{0s}=T_F+\int\omega(k)a_\lambda^\dagger(\mathbf{k})a_\lambda(\mathbf{k})d\mathbf{k} -\tau_\lambda\sigma_i\int\frac{a_\lambda(\mathbf{k})+a_\lambda^\dagger(-\mathbf{k})}{[16\pi^3\omega(k)]^{1/2}}\frac{k_i}{m}\tilde{u}(k)d\mathbf{k} , \tag{6}$$

where

$$T_F=\int\frac{p^2}{2M}|\tilde{f}(p)|^2d\mathbf{p} ,$$
$$\tilde{u}(k)=\int\delta(\mathbf{p}-\mathbf{q}-\mathbf{k})\times\tilde{f}^*(p)\frac{m\mathbf{k}\cdot\mathbf{J}_\pi(\mathbf{k},(\mathbf{p}+\mathbf{q})/2)}{k^2}\tilde{f}(q)d\mathbf{p}\,d\mathbf{q} . \tag{7}$$

The standard methods for solving the pion-nucleon static-model Hamiltonian can be applied to the pion part of H_{0s}, that is, $H_{0s}-T_F$ (Refs. 2–5). The nonstatic form factor, which is the current form factor in this effective static-model Hamiltonian, is $(\mathbf{k}/m)\tilde{u}(k)$. It has been shown[2,4] that an excellent approximation to the ground-state energy of the effective static-model Hamiltonian is obtained by taking just one p-wave mode $\phi_i(\mathbf{k})$ (three substates, $i=x,y,z$) and neglecting the part of $a_\lambda(\mathbf{k})$ that is orthogonal to this mode. Then the pion field annihilation operator takes the form

$$a_\lambda(\mathbf{k})=A_{\lambda i}\phi_i(\mathbf{k}) ,$$
$$\phi_i(\mathbf{k})=\frac{k_i\tilde{u}(k)}{Gm[16\pi^3\omega^3(k)]^{1/2}} , \tag{8}$$
$$G^2=\frac{1}{12\pi^2m^2}\int_0^\infty\frac{k^4\tilde{u}^2(k)}{\omega^3(k)}dk ,$$

where the special form of the p-wave mode that is used here has been shown to be the appropriate one in Ref. 6. The summation convention is now used also for the p-wave index i. The p-wave modes are normalized, and $A_{\lambda i}$ is the annihilation operator for a meson in the isospin substate λ and p-wave substate i. The internal Hamiltonian, that is, the Hamiltonian for the system where both the core and the pion field are restricted to the internal modes only,[7] is obtained by substituting (8) into H_{0s}, so that

$$H_{\rm int}=T_F+VH_A\ ,$$
$$H_A=\frac{1}{G^2}[A^\dagger_{\lambda i}A_{\lambda i}-G\tau_\lambda\sigma_i(A^\dagger_{\lambda i}+A_{\lambda i})]\ , \quad (9)$$
$$V=\frac{1}{12\pi^2m^2}\int_0^\infty\frac{k^4\tilde{u}^2(k)}{\omega^2(k)}dk\ .$$

Let the ground-state energy of H_A be $\epsilon(G)$; the even function $\epsilon(G)$ has long been known for G small and for G large, where it takes the values -9 and -3, respectively.[3] Reference 5 describes the results of a recent computation of $\epsilon(G)$ for weak and intermediate values of the coupling strength G, while the strong-coupling results are given in Ref. 2. The quantities T_F, V, G, and $\epsilon(G)$ are all functionals of the 0s momentum-space wave function $\tilde{f}(q)$; the particular function $\tilde{f}$ that minimizes the functional $E\{f\}$, where

$$E\{f\}=T_F+V\epsilon(G) \quad (10)$$

is the ground-state wave function for the core in what will be called the internal-field approximation. In the present work, the energies and wave functions have been computed as in Ref. 5. Some details from that work are reproduced in the Appendix.

In order to obtain some insight into the behavior of $E\{f\}$, it is useful to consider the case that $f(r)$ is a Gaussian or exponential function that depends on a single parameter S that represents the size of the wave function; then the energy functional $E\{f\}$ becomes simply a function $E(S)$ of S. The parameter S will be referred to in the following as the "core-motion parameter."

As an example, the current form factor of (5) will be used to illustrate the behavior of $E(S)$. Figure 1 shows the function $E(S)$ for the case that the bag radius R is 0.4 fm and the unrenormalized coupling constant γ is 0.4, with $f(r)$ chosen to be the exponential $c\exp(-r/S)$. At small values of the core-motion parameter S the kinetic energy term dominates, and the total energy varies as S^{-2}. For this particular value of R, the pion field interaction with the core gives a negative contribution to the total energy that exceeds the positive kinetic energy for intermediate values of S. When S becomes large, the core density decreases so that the interaction energy becomes less negative. The minimum in the curve of $E(S)$ in Fig. 1 demonstrates the existence of a self-consistent state in which the core executes a motion within the π-meson field that it generates. This state is the analog for the non-Abelian current operator $\tau_\lambda\boldsymbol{\sigma}\cdot\mathbf{J}_\pi$ of the ground state of the Hamiltonian of the polaron,[8,9] which has an Abelian current operator. States of this type, in which a fermion or fermions and the generated Bose field form a self-consistent bound state, have been considered for the case in which the current is Abelian and the Bose field is nonlinear in Refs. 10.

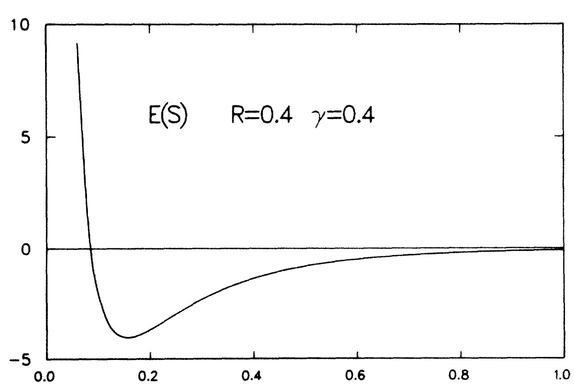

FIG. 1. Energy $E(S)$ of the self-consistent state as a function of the core-motion parameter S; the values of the bag radius R and the unrenormalized coupling constant γ are fixed at the indicated values.

III. COMPUTATION OF STATIC PROPERTIES OF THE NUCLEON

The Hamiltonian of (1) with the current form factor of (5) contains the parameters M, m, γ, and R. The meson mass m was taken to be the charged pion mass. For the purpose of obtaining a quantitative estimate of the size of the effects of core motion on static nucleon properties, the core mass M was taken to be the nucleon mass. At some point, this should be corrected for the effective mass of the pion field around the core; the size of this effective mass is estimated in Sec. V. With M and m fixed, the remaining parameters are R and γ. The parameter γ was fixed by requiring that the renormalized Yukawa pion-nucleon coupling constant $\gamma_{\rm ren}$ be 0.08. Reference 5 gives details of the computation of $\gamma_{\rm ren}$. Thus, the only free parameter in the computations was the bag radius R.

In the computations, the 0s core wave function $f(r)$ was expanded in associated Laguerre polynomials with variable exponential falloff S; up to five such polynomials were used in this expansion. It turned out that the overlap of the function f with the first term in the expansion was better than 0.99 for the best value of S.

For given R and γ, the core wave function was varied to minimize the energy (10) in the internal-field approximation, and the corresponding renormalized coupling constant $\gamma_{\rm ren}$ was computed. Then γ was varied, for fixed R, until $\gamma_{\rm ren}$ was 0.08. Thus γ can be considered to be a function of R, $\gamma=\gamma(R)$. The values of $\epsilon(G)$ were taken from Ref. 5, and should be reasonably accurate for weak and intermediate coupling; in particular, the values of $\epsilon(G)$ should be valid as long as the coupling constant G does not become so large that the number of virtual pions in the state vector is greater than 6. From R and $\gamma(R)$, the approximate ground-state eigenvector was determined in the internal-field approximation by using the methods of Ref. 5. The pion field contributions to charge radii and magnetic moments are computed from the pion charge and magnetic moment densities, which, in turn, are expectation values of the corresponding pion field current operators. The pion charge density and magnetic-moment density operators are

$$\rho_3(\mathbf{r})=e\epsilon_{3\mu\nu}\phi_\mu\dot{\phi}_\nu\ ,$$
$$\boldsymbol{\mu}_3(\mathbf{r})=\tfrac{1}{2}\mathbf{r}\times\mathbf{j}_3(\mathbf{r})\ , \quad (11)$$
$$\mathbf{j}_\lambda(\mathbf{r})=-\frac{e}{2}\epsilon_{\lambda\mu\nu}\phi_\mu(\mathbf{r})\overleftrightarrow{\partial}\phi_\nu(\mathbf{r})\ ,$$

where $\phi_\lambda(\mathbf{r})$ and $\dot{\phi}_\lambda(\mathbf{r})$ are the pion field operators in configuration space that are related to the annihilation operators $a_\lambda(\mathbf{k})$ by

$$\phi_\lambda(\mathbf{r})=\int \frac{a_\lambda(\mathbf{k})+a_\lambda^\dagger(-\mathbf{k})}{[16\pi^3\omega(k)]^{1/2}}e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{k}\ ,$$
$$\dot{\phi}_\lambda(\mathbf{r})=\int \frac{a_\lambda(\mathbf{k})+a_\lambda^\dagger(-\mathbf{k})}{[(16\pi^3\omega(k)]^{1/2}}[-i\omega(k)]e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{k}\ , \quad (12)$$

where $a_\lambda(\mathbf{k})$ is given in (8). The charge density of the system is given by

$$\rho_{\text{ch}}(\mathbf{r})=\rho_\pi(\mathbf{r})+\int \rho(\mathbf{r}-\mathbf{r}')\rho_{\text{bag}}(\mathbf{r}')d\mathbf{r}'\ , \quad (13)$$

where ρ_π is the pion charge density of Ref. 3 in the single-mode approximation, ρ is the core probability density $|f(\mathbf{r})|^2$, and ρ_{bag} is the bag charge density used in Ref. 2, which has isoscalar and isovector parts. Then it follows directly that the mean-square charge radius operator is the sum of three contributions, one from the pion charge density, one from the core motion, and one from the internal charge density of the core, as given by the bag model, namely,

$$\langle r^2\rangle=\langle r^2\rangle_\pi+\langle r^2\rangle_S+\langle r^2\rangle_V\tau_3\ ,$$
$$\langle r^2\rangle_S=\langle r^2\rangle_V=\tfrac{1}{2}\left[\lambda R^2+\int r^2|f(\mathbf{r})|^2 d\mathbf{r}\right]\ , \quad (14)$$

where $\lambda=0.57$ is taken from the bag-model calculations of Ref. 2, τ_3 is the isospin of the bag that acts on the state-vector components of Appendix A, and $\langle r^2\rangle_\pi$ is the pion contribution in the single-mode approximation. The quarks are taken to have mass zero and to be in the lowest s state. The mean-square charge radius is the expectation value of the operator of (14) in the nucleon ground state described in the Appendix. The core motion makes no direct contribution to the magnetic moment of the nucleon, since it is an s wave; it makes an indirect contribution by affecting the nucleon state vector, and, hence, the pion contribution to the magnetic moment. The bag contribution to the magnetic moment was computed as for a static bag, as in Ref. 2.

IV. RESULTS OF COMPUTATIONS

Figure 2 shows the computed core-motion parameter S as a function of the bag radius R, and Fig. 3 shows the dependence of the normalized coupling constant G and of the expectation value n_π of the pion-number operator on R. For small values of R, the pion-core coupling constant is large, the pion field binds the core tightly, and the core-motion parameter is small. As R increases, the pion-core coupling decreases, and the core-motion parameter increases. At a value of R of about 0.65 fm, the self-consistent state ceases to be bound in this approximation.

The results for the proton and neutron magnetic moments are presented in Fig. 4 and the charge radii squared in Fig. 5. In the case of the magnetic moments, for bag radii in the range $0.4<R<0.6$ the largest contribution, 90–60 %, respectively, comes from the pion degrees of freedom. The inclusion of core motion decreases the magnitudes of the proton and neutron moments. This is simply a consequence of the reduced core density which im-

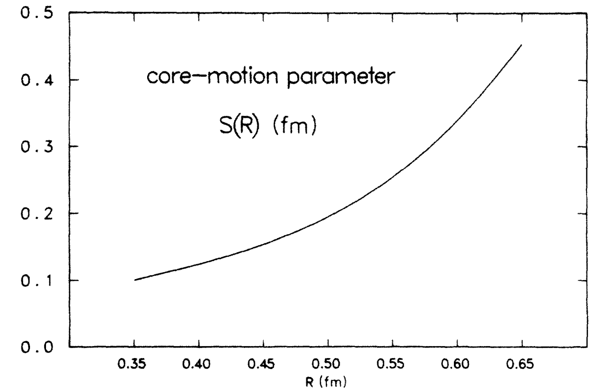

FIG. 2. The core-motion parameter $S(R)$ as a function of the bag radius R; the renormalized coupling constant γ_R is 0.08.

plies a decrease in the pion field strength and therefore a smaller pion electric current. For the range of bag radii specified above, the effects for the proton are roughly 20% while for the neutron the range is 20–30 %. Note that the isoscalar contribution, which comes from the quarks, is small in this range since it is proportional to the bag radius.

For the charge radii the situation is somewhat different, in that the pion contribution dominates for small bag radii when the core-motion parameter S is small, but the

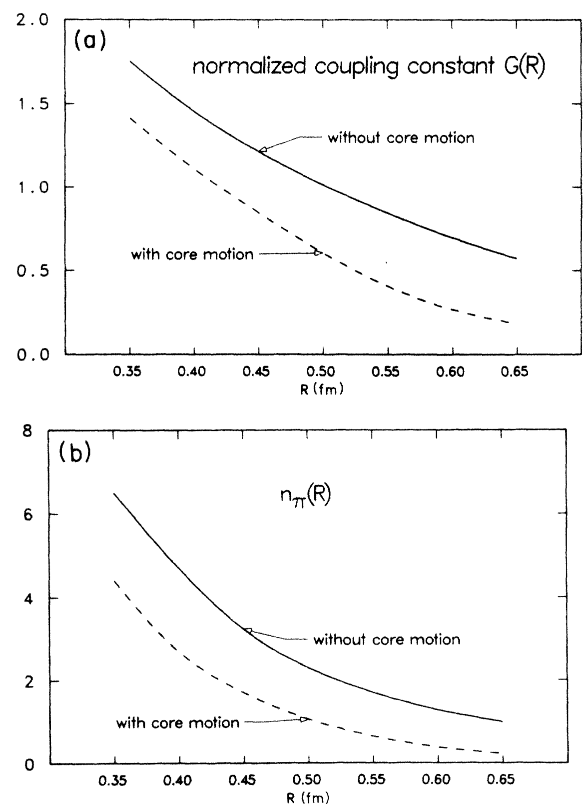

FIG. 3. (a) The normalized coupling constant $G(R)$ [see Eq. (8)] as a function of R with $\gamma_R=0.08$. (b) The expectation value of the pion-number operator as a function of R with $\gamma_R=0.08$.

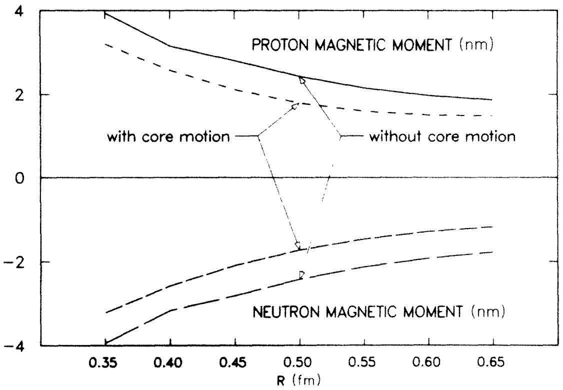

FIG. 4. Computed values of the nucleon magnetic moments, demonstrating the effect of core motion.

core-motion contribution dominates for large radii when the core-motion parameter S is large. For the proton, the motion of the core tends to spread out the charge distribution of the core and this spreading is most pronounced when the core is less localized, i.e., when the coupling is weak or the bag radius is large. For bag radii such that $0.4 < R < 0.6$ the proton charge radius squared increases by 3—20% due to motion of the positively charged core. (The components of the proton wave function that have neutral core make no contribution to the charge radius.) The situation for the neutron is somewhat different. It should be kept in mind that in this model the dressed neutron, roughly speaking, is composed of a positively charged core surrounded by a negatively charged pion cloud and a neutral core surrounded by a neutral pion cloud. The neutral core contribution to the charge radius is zero for equal-mass up and down quarks. However, the motion of the charged core makes a positive contribution which tends to cancel the negative contribution from the pion degrees of freedom. When the core motion increases, the positive contribution to the squared charge radius also increases. Hence, the magnitude of the neutron charge radius decreases when core motion is included. For the range of bag radii specified above the magnitude of the neutron charge radius squared decreases by 7—70% due to core motion.

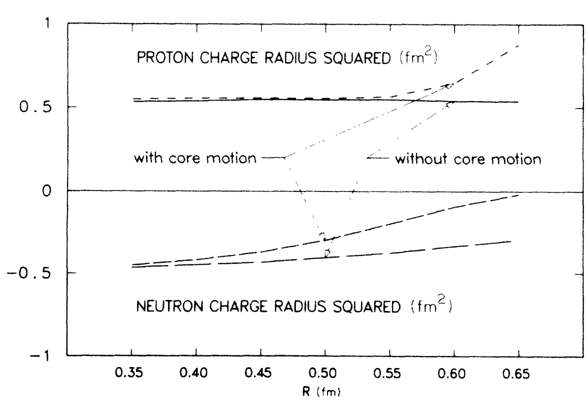

FIG. 5. Computed values of the nucleon charge radii, demonstrating the effect of core motion.

Roughly speaking, the numerical results are not to be trusted for small bag radii, R less than about 0.4 fm, where the calculations are limited by the accuracy of the intermediate-coupling approximation to the dressed nucleon state vector. As was shown in Fig. 3(a), for small values of R, the pion-core coupling constant G gets large, and the number of pions n_π in the state vector also grows. The accuracy of the methods used in our computations has not been proven for large numbers of virtual pions. Both the present computations and strong-coupling calculations without core motion[2] show that the number n_π of pions in the dressed nucleon state is roughly five when $R = 0.3$ fm; in terms of the methods used in Ref. 5, this is a large number of pions. In order to obtain a better approximation to the pion part of the state vector, the number of pions included in the intermediate-coupling calculations must be increased. Although the numerical values are unreliable for $R < 0.4$ fm, we believe that the trends shown by the curves in the figures are correct even in the region near 0.35 fm.

For large bag radii, R greater than about 0.6 fm, the coupling is weak, and the approximations used in the description of the core motion become inaccurate. As in the case of the polaron, translation invariance becomes important when the coupling is weak, and the single s-wave mode approximation used here is no longer adequate. This situation could be improved by including p-wave modes, as in the work of Lee and Pines[9] on the polaron. Again, we believe that the trends shown in our computations are correct, even near 0.65 fm.

Overall, the figures show that in models described by the Hamiltonian of (1) in which a quark bag is coupled to pion degrees of freedom, the effects due to core motion on the static properties of the proton and neutron are substantial. The relation between the intrinsic form factor J_π and the nonstatic form factor is such that its extent is greater than that of the intrinsic form factor. This is especially clear in the curves of Fig. 5 for the charge radii. In other words, for a given set of observed nucleon properties, the bag with core motion is smaller than the corresponding bag without core motion.

V. REMARKS

(i) In common with bag models that neglect core motion and treat pion field interactions with a bag core in terms of an equivalent static model, the present calculation ignores the effects of the pion field on the motion of the quarks in the bag.

(ii) The model Hamiltonian defined by (1) is translationally invariant; however the single core mode (in this case s wave) approximation breaks translational symmetry. As in the case of the polaron[9] this situation could be improved by including more modes of the core, specifically the three p-wave modes which become important when the core is less tightly bound. In the calculations presented above, this happens when the bag radius is large. The inclusion of p-wave modes would also allow for the possibility of describing odd-parity baryons. In such an approach both the core and the pions form a shell structure which is determined self-consistently.

(iii) The effective mass of the self-consistent ground state can be computed by methods that have been developed previously for such systems;[11] the result is that the core contributes an amount M to the effective mass and the pion field contribution is

$$\frac{n_\pi}{18\pi^2 m^2 G^2}\int_0^\infty \frac{k^6\tilde{u}^2(k)}{\omega^4(k)}dk\ , \tag{15}$$

where n_π is the expectation value of the pion-number operator. Figure 6 shows how the pion-field contribution m_π to the nucleon mass varies with R in the particular case that the core motion is of the self-consistent character used in our computations. The mass parameter M in the Hamiltonian should be less than the nucleon mass by an amount equal to the pion-field contribution to the nucleon mass; a fully self-consistent computation would take this effect into account. It is easy to see that this will tend to increase the core-motion parameter S.

(iv) For these estimates of the effects of core motion, the excited states of the quarks in the core or bag have not been included. Of course, a realistic computation that included all important physical effects would have to deal with the contributions of the Δ isobar as well as other influential excited states.

(v) No nonlinear terms in the pion field have been included in the Hamiltonian of (1). The composite nature of the pion does imply that such nonlinear terms should probably be included in effective Hamiltonians for pion interaction. In the present calculation aimed at computing the effects of core motion, nonlinear terms would probably be an unnecessary complication.

(vi) The electromagnetic current operators that have been used for the pion field are consistent with the Hamiltonian. On the other hand, the electromagnetic current operators for the core have been taken in rudimentary form. This simple treatment is adequate for the purpose of calculating the effects of core motion, but a careful study of nucleon parameters would need to treat the physics of the electromagnetic interaction of the core more carefully.

(vii) The most general form of the intrinsic pion current form factor, $J_\pi(\mathbf{k},(\mathbf{p}+\mathbf{q})/2)$, corresponds to a pion-core interaction that is nonlocal in configuration space in two ways: (a) the pion-field operator is at a different point from the core density and (b) the core density with which the pion field interacts is nonlocal in the operators ψ and $\psi^\dagger$. The form factor in (5) is independent of its second argument, which implies that the nonlocality of type (b) does not occur, and the core density is local. We believe that our results are not sensitive to this limitation on the form of $\mathbf{J}_\pi$.

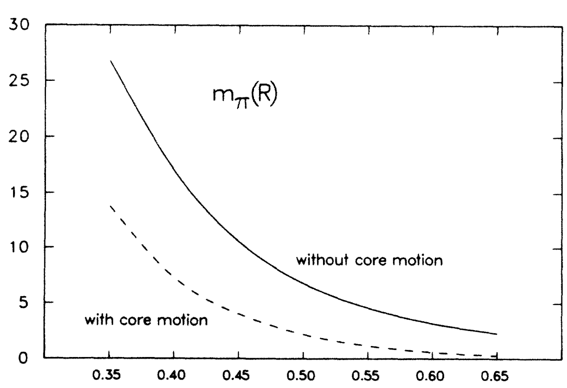

FIG. 6. The pion-field contribution to the nucleon effective mass computed according to Eq. (15), measured in units of m.

VI. SUMMARY

A simple translation-invariant Hamiltonian that describes a nucleon core interacting with a pion field has been used to study the effects of nucleon core motion on magnetic moments and mean-square charge radii of the nucleon as computed in bag models that contain pion-field interaction. The pion-core interaction was taken to be the one used in calculations in the little-bag and cloudy-bag models.[1] For sufficiently strong coupling of the pion field to the core (sufficiently small bag radii), the nucleon ground state of the Hamiltonian was shown to be a self-consistent state in which the core generates the pion field and is in turn bound in that pion field. The magnetic moments and charge radii were computed for the self-consistent state and compared with the corresponding moments and radii computed without core motion. The main effect of the core motion is to soften the corresponding static-model form factor of the core-pion interaction; that is, this nonstatic form factor for the bag with core motion is larger in configuration space than the intrinsic form factor that describes the pion-core interaction in the Hamiltonian.

ACKNOWLEDGMENT

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APPENDIX

Some of the details of the wave function are reproduced here from Refs. 5 and 12. The basis states are coherent-pion-pair states, each of which is based on an invariant-pair-free (IPF) state with a fixed number of pions. An IPF state $|\,n,\alpha\rangle$ with n pions satisfies the relations

$$\begin{aligned} A\cdot A\,|\,n,\alpha\rangle &= 0\ ,\\ A^\dagger\cdot A\,|\,n,\alpha\rangle &= n\,|\,n,\alpha\rangle\ . \end{aligned} \tag{A1}$$

The nucleon state in the present case has components with $T_\pi = L_\pi = 0$ and with $T_\pi = L_\pi = 1$. As was discussed in Refs. 5 and 12, there are 12 such IPF states with up to six pions in a single p-wave mode. The coherent-pair state with coherence parameter y based on the state $|\,n,\alpha\rangle$ is given by

$$|\,n,\alpha,y\rangle = g_{2n+9}(yA^\dagger\cdot A^\dagger)\,|\,n,\alpha\rangle\ , \tag{A2}$$

where $g_\nu(x)$ is the pair coherence function

$$g_\nu(x) = \sum_{m=0}^{\infty}\frac{(\nu-2)!!}{2^m m!(\nu+2m-2)!!}x^\mu\ . \tag{A3}$$

TABLE III. Same as Table II but for model F_1.

	$\rho=1.01$		$\rho=1.02$	
	$x_{\min}$	$M_2^{\min}$ (GeV)	$x_{\min}$	$M_2^{\min}$ (GeV)
Non-SUSY				
$M_F=M_W$	0.92	126.9	0.66	111.4
$M_F=10M_W$	0.90	127.8	0.64	111.9
SUSY				
$M_F=M_W$	0.99	123.7	0.73	109.6
$M_F=10M_W$	0.95	125.3	0.69	110.5

Non-SUSY:

$$x_W=0.218 ,$$

$$U=28.09 ,$$

$$\alpha_u{}^{-1}=41.32,\ \lambda=\begin{cases}0.185, & M_F=M_W,\\ 0.188, & M_F=10M_W,\end{cases}$$

$$\alpha_F{}^{-1}(M_F)=1022-129X;$$

SUSY: (13)

$$x_W=0.258 ,$$

$$U=28.94 ,$$

$$\alpha_u{}^{-1}=23.81,\ \lambda=\begin{cases}0.175, & M_F=M_W,\\ 0.180, & M_F=10M_W,\end{cases}$$

$$\alpha_F{}^{-1}(M_F)=12\,875-313X .$$

These results are similar to pattern C discussed earlier. Using the notation of Eq. (9) we obtain $A=1, B=34+200x$ ($x\equiv v_s{}^2/v_d{}^2$) with the results shown in Table III. We also obtain the limits

$$\left|\frac{\lambda^2}{16}\frac{M_1{}^2}{M_2{}^2}X_L^\nu\right|\lesssim 2.4\times 10^{-2} ,$$

$$\sin^2\phi\lesssim 4.8\times 10^{-2} . \tag{14}$$

Clearly, we must have M_2 somewhat larger than the limit obtained from the ρ-parameter analysis to suppress the ratio $M_1{}^2/M_2{}^2$ appearing in (14).

Pattern F_2 is somewhat similar to pattern D described earlier in I. We find

$$T_L=n_1+n_2+n_5+4n_7=3+4n_7 ,$$

$$T_Y=n_1+n_2+n_5+6n_7=3+6n_7 , \tag{15}$$

$$T_{F_2}=64n_1+144n_2+200n_3+784n_5+800n_6+384n_7 =1992+384n_7.$$

TABLE IV. Same as Table III but for model F_2.

	$\rho=1.01$		$\rho=1.02$	
	$x_{\min}$	$M_2^{\min}$ (GeV)	$x_{\min}$	$M_2^{\min}$ (GeV)
Non-SUSY				
$M_F=M_W$	9.93	350.5	5.12	257.5
$M_F=10M_W$	9.90	356.0	5.10	261.3
SUSY				
$M_F=M_W$	10.13	312.3	5.33	231.4
$M_F=10M_W$	10.07	322.4	5.27	238.2

Using (15) and taking $n_7=0$ we find that

Non-SUSY:

$$x_W=0.222 ,$$

$$U=27.68 ,$$

$$\alpha_u{}^{-1}=32.09+X/\pi,\ \lambda=\begin{cases}0.181, & M_F=M_W,\\ 0.184, & M_F=10M_W,\end{cases}$$

$$\alpha_F{}^{-1}(M_F)=10\,826-155X ;$$

SUSY: (16)

$$x_W=0.276 ,$$

$$U=26.53 ,$$

$$\alpha_u{}^{-1}=2.99+3X/2\pi,\ \lambda=\begin{cases}0.162, & M_F=M_W,\\ 0.165, & M_F=10M_W,\end{cases}$$

$$\alpha_F{}^{-1}(M_F)=16\,090-470X .$$

We then obtain $A=4$, $B=(248+500x)/3$ with $x=v_s{}^2/v_d{}^2$ using (16) and the results obtained in Table IV. In addition to this we also obtain the limits

$$\left|\frac{\lambda^2}{16}\frac{M_1{}^2}{M_2{}^2}X_L^\nu\right|\lesssim 4.3\times 10^{-3} ,$$

$$\sin^2\phi\lesssim 3.8\times 10^{-3} , \tag{17}$$

both of which are reasonably small.

We now turn to a discussion of the Z_2 total width and leptonic branching ratio. Neglecting terms of order $m_f{}^2/M_2{}^2$ the decay rate $Z_2\to f\bar{f}$ is given by

$$\Gamma(Z_2\to \bar{f}f)=N_c\frac{\alpha_x M_2}{6}(X_L{}^2+X_R{}^2) , \tag{18}$$

where Table V gives the values of X_L, X_R for all the fermions in the **27** representation. With X_L, X_R determined

TABLE V. X_L and X_R quantum numbers of all the **27** E_6 fermions for models E, F_1, and F_2.

	E		F_1		F_2	
	X_L	X_R	X_L	X_R	X_L	X_R
ν_L	8	0	−6	0	4	0
e_L	8	0	−6	0	4	0
d_R	0	−8	0	6	0	4
u_L	4	0	2	0	2	0
d_L	4	0	2	0	2	0
u_R	0	−4	0	−2	0	−2
e_R	0	−4	0	−2	0	−2
ν_R	0	0	0	0	0	−10
N_L	−12	0	4	0	−6	0
E_L	12	0	4	0	−6	0
D_R	0	12	0	−4	0	6
N_R	0	8	0	4	0	4
E_R	0	8	0	4	0	4
D_L	−8	0	−4	0	−4	0
ν_L''	20	0	10	0	0	0