Electromagnetic form factors and static properties of the nucleon in a relativistic potential model of independent quarks with chiral symmetry

N. Barik and B. K. Dash*
Department of Physics, Utkal University, Bhubaneswar 751 004, Orissa, India
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Nucleon charge and magnetic form factors $G_{E,M}^{p,n}(q^2)$ have been presented in a quark model with an equally mixed scalar and vector potential in harmonic form taking the pionic contributions into account. The static properties such as the magnetic moment, charge radius, and axial-vector coupling constant in the neutron-β-decay process are shown to be in excellent agreement with the corresponding experimental values. The role of the finite extension of the quark-pion vertex in determining the charge radius and magnetic moment due to the pion cloud surrounding the nucleons has been studied.

I. INTRODUCTION

In chiral quark models, nucleons are described as an assembly of confined nonstrange quarks surrounded by a pion cloud. The existence of the pion-cloud coupling to the bare nucleon arises out of the compelling necessity to preserve the chiral symmetry in the flavor-SU(2) sector, which is believed to be an excellent symmetry of strong interactions. The charged pion cloud, surrounding the bare nucleon, obviously contributes to the total electromagnetic current of the nucleon and thereby plays an important role in determining the magnetic moments and charge radii of the proton and neutron.

Taking into account the pionic contributions, the electromagnetic properties of nucleons have been calculated in chiral bag models[1–6] as well as in some relativistic confining potential[7–9] models. Of all the chiral bag models, the cloudy bag model (CBM), though more phenomenological in nature, has been quite successful in this respect. But it is not entirely free from any objections because of its static spherical bag boundary to which it owes much of its success and simplicity. This is because of the fact that it is difficult to believe that the spherical bag boundary remains static and unperturbed by the creation of a pion. Furthermore, in any bag model, to restore chiral symmetry, it is essential to introduce an additional pion field in the region exterior only to the spherical bag boundary. On the other hand, the exclusion of pions from the interior of the static bag, for a number of reasons, may not be correct and reasonable. Although the CBM is correct in not excluding explicitly the pions from the bag volume, the very inclusion of pions in the interior region is more or less *ad hoc*. Above all, the formation of a bag or its properties or anything like that has not yet been derived, strictly speaking, from any fundamental theory. There are only some suggestive arguments leading one to believe that the formation of a bag may not be far from the truth. Hence the bag essentially provides a phenomenological description of the nonperturbative gluon interaction including gluon self-coupling.

In view of this fact, the motivation of our work is mainly to replace the bag by some alternative phenomenological potential $U(r)$, representing in the same way as in the bag model the nonperturbative gluon interactions. The chiral potential models,[7–9] which are comparatively more straightforward, are no doubt attempts in this direction. The term in Lagrangian density for quarks corresponding to the effective scalar potential being chirally odd through all space requires the introduction of an additional pionic component everywhere in order to preserve chiral symmetry. The effective potential of individual quarks in such models, which is basically due to the interaction of quarks with the gluon field, may be thought of as being mediated in a self-consistent manner through Nambu—Jona-Lasinio-type models,[10] by some kind of instanton-induced effective quark-quark contact interaction with position-dependent coupling strength. The position-dependent coupling strength, supposed to be determined by the multigluon mechanism, is impossible to calculate from first principles, although it is believed to be small at the origin and increases rapidly towards the hadron surface. Therefore, one needs to introduce the effective potential for individual quarks in a phenomenological manner to seek *a posteriori* justification for finding its conformity with the supposed qualitative behavior of the position-dependent coupling strength in the contact interaction.

However, with no theoretical prejudice in favor of any particular mechanism for generating confinement of individual quarks, we prefer to work in an alternative but similar scheme with a purely phenomenological individual quark potential $U(r)$ in the equally mixed scalar-vector harmonic form to represent the nonperturbative gluon interaction, including gluon self-coupling the same way as for the bag. Nevertheless in our choice of $U(r)=\frac{1}{2}(1+\gamma^0)V(r)$, we have been guided by the usual aesthetic compulsion of providing simplicity and tractability for the model. Such a potential model has been used in our earlier works[11] for a reasonable prediction of the core contribution to the magnetic moments of the octet baryon and charge radius of the proton as well as the weak electric and magnetic form factors for semileptonic

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baryon decays. Then incorporating chiral symmetry in the SU(2)-flavor sector in the usual manner, we have obtained the mass spectrum[12] of the octet baryons and also estimated the quark-pion coupling constant,[12] consistent with those extracted from experimental vector-meson decay width ratios by Suzuki and Bhaduri.[12] Therefore, in the present work we employ such a chiral potential model to study the electromagnetic properties of nucleons by taking into account the pionic contributions in the usual manner, together with the spurious center-of-mass correction at appropriate stages. This model with a harmonic form in particular for the scalar-vector mixed potential turns out to be quite simple and tractable in these respects, yielding very satisfactory results for the electromagnetic properties of the nucleons.

We present in Sec. II the potential model with chiral symmetry in the u-d-flavor sector which leads to the nucleon-pion coupling and the axial form factor. We also give an account of the renormalization of the nucleon-pion coupling constant. In Sec. III we discuss the electromagnetic form factors for nucleons which in their turn lead to the estimation of the static quantities such as the charge radius and magnetic moment of the nucleons, which are found to be in reasonable agreement with the corresponding experimental data.

II. POTENTIAL MODEL WITH CHIRAL SYMMETRY

In this section we briefly outline the framework of the model incorporating chiral symmetry with quark-pion interaction term in the Lagrangian density taken in a linear form. For completeness we first mention the static properties of the nucleon core as obtained in the model with the prescription for taking into account the center-of-mass corrections at appropriate stages. Then, with the assumption that the resulting hadronic states do not contain large multipion components, pionic corrections to the electromagnetic properties of the nucleon can be calculated in the usual perturbative expansion approach.

A. Potential model

We start with the assumption that the nonstrange quarks in a nucleon core move independently in an average effective potential taken in the form

$$U(r)=\tfrac{1}{2}(\zeta+\gamma^0)V(r) ,$$

with

$$V(r)=ar^2+V_0, \quad a>0 , \tag{2.1}$$

and obey the Dirac equation derivable from a Lagrangian density

$$\mathcal{L}_q^0=\frac{i}{2}\bar{q}(x)\gamma^\mu\overleftrightarrow{\partial}_\mu q(x)-\bar{q}(x)[U(r)+m_q]q(x) . \tag{2.2}$$

Under a global infinitesimal chiral transformation

$$q(x)\to q(x)-i\gamma^5\left[\frac{\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}}{2}\right]q(x) , \tag{2.3}$$

the axial-vector current of the quarks is not conserved, as the scalar term

$$\left[\frac{\zeta}{2}V(r)+m_q\right]\bar{q}(x)q(x)=G(r)\bar{q}q$$

in $\mathcal{L}_q^0(x)$ is chirally odd. The vector part of the potential poses no problem in this respect. To restore chiral symmetry, we introduce, in the usual manner, a zero-mass elementary isovector pion field $\boldsymbol{\varphi}$ with linearized interaction Lagrangian density:

$$\mathcal{L}_I=\frac{i}{f_\pi}G(r)\bar{q}(x)\gamma^5(\boldsymbol{\tau}\cdot\boldsymbol{\varphi})q(x) , \tag{2.4}$$

where $f_\pi\simeq 93$ MeV is the phenomenological pion-decay constant. Then the four-divergence of the total axial-vector current $\mathbf{A}^\mu(x)$, due to quarks and pions as well, vanishes. However, introducing a pion field of small but finite mass $m_\pi\simeq 140$ MeV, one may have

$$\partial_\mu\mathbf{A}^\mu(x)=-f_\pi m_\pi{}^2\boldsymbol{\varphi}(x) , \tag{2.5}$$

yielding the usual PCAC (partial conserved axial-vector current) relation. Then the chiral-symmetric Lagrangian density for nucleons with a bare-quark core surrounded by a pionic cloud becomes

$$\mathcal{L}=\mathcal{L}_q^0(x)+\mathcal{L}_\pi^0(x)+\mathcal{L}_I(x) ,$$

when

$$\mathcal{L}_\pi^0=\tfrac{1}{2}(\partial_\mu\boldsymbol{\varphi})^2-m_\pi{}^2\boldsymbol{\varphi}^2 . \tag{2.6}$$

Without $\mathcal{L}_I$, which is dictated by chiral symmetry, the model would describe bare nucleon states and free pions. First of all, neglecting the pion coupling with the quarks, we can study the bare nucleon in terms of its individual quarks obeying the Dirac equation obtainable from the Lagrangian density $\mathcal{L}_q^0$. The spatial orbits $q(\mathbf{r})$ of the nonstrange individual quarks satisfy the equation

$$[\gamma^0E_q-\boldsymbol{\gamma}\cdot\mathbf{p}-m_q-U(r)]q(\mathbf{r})=0 , \tag{2.7a}$$

which, after absorbing the constant V_0 part of the potential $U(r)$ appropriately in E_q and m_q, becomes

$$[\gamma^0E_q'-\boldsymbol{\gamma}\cdot\mathbf{p}-m_q'-U'(r)]q(r)=0 . \tag{2.7b}$$

Here,

$$E_q'=(E_q-V_0/2),\ m_q'=(m_q+\zeta V_0/2) ,$$

and $U'(r)=\frac{1}{2}(\zeta+\gamma^0)ar^2$. Then, considering the $(1S_{1/2})^3$ configuration only for the ground state of the nucleons, the spatial orbits $q(\mathbf{r})$ can be written as

$$q(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} ig(r)/r \\ \boldsymbol{\sigma}\cdot\hat{\mathbf{r}}f(r)/r\end{bmatrix} . \tag{2.8}$$

But with $\zeta\neq 1$, the solutions for $g(r)$ and $f(r)$ are not quite straightforward. Therefore, we prefer to take $\zeta=1$, which yields with $\lambda_q=(E_q'+m_q')$ and $r_0=(a\lambda_q)^{-1/4}$ the reduced radial parts of the upper and lower components in a simple and straightforward manner as

$$g(r)=N_q(r/r_0)\exp(-r^2/2r_0^2)\ ,$$
$$f(r)=-\frac{N_q}{\lambda_q r_0}(r/r_0)^2\exp(-r^2/2r_0^2)\ . \tag{2.9}$$

Here E_q is the ground-state ($1S_{1/2}$) individual quark-binding energy obtainable from the energy eigenvalue condition

$$\left[\frac{\lambda_q}{a}\right]^{1/2}(E_q'-m_q')=3\ , \tag{2.10}$$

and N_q is the overall normalization factor satisfying the relation

$$\frac{N_q{}^2\sqrt{\pi}r_0}{8\lambda_q}=1/(3E_q'+m_q')\ . \tag{2.11}$$

Now calculating in the usual manner, certain quantities of the bare nucleon such as the magnetic moment μ_p^0, charge radius $\langle r^2\rangle_p^0$, and the axial-vector coupling constant g_A^0 for neutron-β decay can be obtained at this stage as

$$\mu_p^0=\frac{4M_p}{(3E_q'+m_q')}\mu_{N}\ ,$$
$$\langle r^2\rangle_p^0=\frac{(11E_q'+m_q')}{(3E_q'+m_q')(E_q'^2-m_q'^2)}\ , \tag{2.12}$$
$$g_A^0=\tfrac{5}{9}(5E_q'+7m_q')/(3E_q'+m_q')\ .$$

We have found that with

$$(a,m_q')\equiv(2.273\ \text{fm}^{-3},10\ \text{MeV})\ , \tag{2.13}$$

the eigenvalue condition (2.10) yields $E_q'=540$ MeV, which in turn with $V_0=-137.5$ gives the N and Δ masses correctly,[12] after the appropriate corrections such as those due to c.m. motion, the pionic contribution, and one-gluon-exchange interaction are accounted for. This also provides an order-of-magnitude prediction for the quark-core contributions to bare-nucleon properties as

$$(\mu_p^0,(\langle r^2\rangle_p^0)^{1/2},g_A^0)\equiv(2.3\mu_{N},0.85\ \text{fm},0.944)\ , \tag{2.14}$$

which seems to be quite reasonable in view of the possible corrections (c.m. and pionic) involved. A detailed account of the quark-core contribution to the static properties of baryons with c.m. correction has been reported elsewhere,[13] where we have adopted the prescriptions followed by Wong[13] and other workers[14] for the c.m. corrections. However, for completeness, we briefly mention this procedure for taking into account the c.m. correction.

The static three-quark baryon-core state with the core center at $\mathbf{x}$ can be decomposed into components $\varphi(\mathbf{P})$ of plane-wave momentum eigenstates as

$$|\,3q,\mathbf{x}\rangle=\int\frac{d^3\mathbf{P}}{W(P)}\exp(i\mathbf{P}\cdot\mathbf{x})\phi(\mathbf{P})\,|\,B(\mathbf{P})\rangle\ , \tag{2.15}$$

where the momentum eigenstates $|\,B(\mathbf{P})\rangle$ of the baryon core B are normalized usually as

$$\langle B(\mathbf{P}')\,|\,B(\mathbf{P})\rangle=(2\pi)^3\delta(\mathbf{P}-\mathbf{P}')W(P)\ ,$$

with

$$W(P)=(M_B{}^2+\mathbf{P}^2)^{1/2}/M_B\ . \tag{2.16}$$

The momentum profile function $\phi(\mathbf{P})$ can be obtained from (2.15) and (2.16) as

$$\phi^2(\mathbf{P})=\frac{W(P)}{(2\pi)^3}\tilde{I}(\mathbf{P})\ , \tag{2.17}$$

where

$$\tilde{I}(P)=\frac{1}{(2\pi)^3}\int d^3\mathbf{r}\exp(-i\mathbf{P}\cdot\mathbf{r})\langle 3q,0\,|\,3q,\mathbf{r}\rangle \tag{2.18}$$

is the Fourier transform of the Hill-Wheeler overlap function $I(\mathbf{r})=\langle 3q,0\,|\,3q,\mathbf{r}\rangle$, which for the three $1S_{1/2}$ quarks in the present model comes out as

$$I(\mathbf{r})=[(1-cr^2/r_{0q}{}^2)\exp(-r^2/4r_{0q}{}^2)]^3\ , \tag{2.19}$$

when $c=(E_q'-m_q')/6(3E_q'+m_q')$. This result, along with (2.15), can be used to calculate expectation values of any function $F(\mathbf{P})$ as

$$\langle F(\mathbf{P})\rangle=\langle 3q,0\,|\,F(\mathbf{P})\,|\,3q,0\rangle=\int d^3\mathbf{P}\,\tilde{I}(\mathbf{P})F(\mathbf{P})\ . \tag{2.20}$$

If $E_B(=\sum_q E_q)$ and M_B denote, respectively, the relativistic energy and mass of the quark core of the baryon state under consideration, with its c.m. momentum $\mathbf{P}_B$, then one can obtain[13] according to (2.20), $\langle\mathbf{P}_B{}^2\rangle=\sum_q\langle p^2\rangle_q$, with $\langle p^2\rangle_q$ as the average of the square of the individual quark momenta with respect to the corresponding single-quark state. $\langle p^2\rangle_q$ in this model can be obtained as

$$\langle p^2\rangle_q=\frac{(11E_q'+m_q')}{6(3E_q'+m_q')}(E_q'^2-m_q'^2)\ . \tag{2.21}$$

This also can similarly enable one to evaluate certain quantities such as $\delta_B=\langle M_B/E_B\rangle$, $\delta_B{}^2=\langle M_B^2/E_B^2\rangle$, and $\langle R^2\rangle$ with $R=(\sum_q E_q r_q/\sum_q E_q)$, in terms of which the center-of-mass effects can be estimated following the prescriptions of Ref. 14. According to this, one can obtain the corrected values of μ_B, $\langle r^2\rangle_B$, and $g_A(B)$ as

$$\mu_B'=\left[3\mu_B^0+Q_B\frac{M_P}{M_B}(1-\delta_B)\right]\Big/(1+\delta_B+\delta_B{}^2)\ ,$$
$$\langle r^2\rangle_B'=(3\langle r^2\rangle_B^0-3Q_B\langle R^2\rangle]/(2+\delta_B{}^2)\ , \tag{2.22}$$
$$g_A'(B)=g_A^0(B)/(1-\tfrac{1}{3}\langle\mathbf{P}_B{}^2\rangle/M_B{}^2)\ ,$$

where Q_B is the total charge of the baryon core. It can be shown that to a good approximation $\delta_B=\frac{1}{2}(1+\delta_B{}^2)$ and for nucleons in particular $\langle R^2\rangle\simeq\frac{1}{3}\langle r^2\rangle_q$. Here, $\langle r^2\rangle_q$, the mean-square radius of each nonstrange quark with respect to the corresponding single quark state, is obtained in this model as

$$\langle r^2\rangle_q=\frac{3}{2}\frac{(11E_q'+m_q')}{(3E_q'+m_q')(E_q'^2-m_q'^2)}\ . \tag{2.23}$$

Then, particularly for nucleons, one can express (2.22) as

$$\mu_N'=\frac{2\mu_N^0}{(1+\delta_N{}^2)}+\frac{Q_N}{3}\frac{M_P}{M_N}\frac{(1-\delta_N{}^2)}{(1+\delta_N{}^2)}\ , \tag{2.24}$$

$$\langle r^2 \rangle'_N = \frac{3\langle r^2 \rangle_N^0 - Q_N \langle r^2 \rangle_{q=u}}{(2+\delta_N{}^2)} , \tag{2.25}$$

$$g'_A = g_A^0 / (1 - \tfrac{1}{3} \langle \mathbf{P}_N{}^2 \rangle / M_N{}^2) . \tag{2.26}$$

These relations give, with the calculated value of $\delta_B{}^2 = 0.7338$,

$$[\mu'_P, \langle r^2 \rangle'_P{}^{1/2}, g'_A] \equiv (2.71\mu_N, 0.732 \text{ fm}, 1.182) . \tag{2.27}$$

However, if one actually takes into account the coupling of pions to the bare nucleon, the core contributions to the static properties as obtained in Eqs. (2.12) and (2.14) before including c.m. corrections, will in fact get modified because of the so-called wave-function renormalization and vertex dressing. This aspect will be taken up in the following section. In any case, the results obtained in Eqs. (2.14) and (2.27) provide a reasonable preliminary estimate justifying the suitability of this simple phenomenological model with chiral symmetry for the study of electromagnetic properties of nucleons.

B. Pion-nucleon coupling and the axial form factor

As a next step, we now consider the pion-Lagrangian density $\mathscr{L}_\pi = (\mathscr{L}_\pi^0 + \mathscr{L}_I)$, including the pion-quark interaction term $\mathscr{L}_I$, to obtain another preliminary estimate of the pion-nucleon coupling constant.

The field equation for the pion field $\varphi_j(x)$ can be obtained from $\mathscr{L}_\pi$ as

$$(\Box + m_\pi{}^2)\varphi_j(x) = -\frac{i}{f_\pi} G(r) \bar{q}(x) \gamma^5 \tau_j q(x) . \tag{2.28}$$

The right-hand side of Eq. (2.28) is the source function for the quark-pion coupling. Then the coupling of pions to quarks in the nucleon can be given by the source function:

$$J_j^5(x) = \sum_q -\frac{i}{f_\pi} G(r) \bar{q}(x) \gamma^5 \tau_j q(x) . \tag{2.29}$$

For a static source $J_j^5(\mathbf{r})$, one can introduce the pion-nucleon form factor $G_{NN\pi}(q^2)$ as

$$i G_{NN\pi}(q^2) \langle \sigma^N \cdot \mathbf{q} \tau_j^N \rangle = 2M_N \left\langle N \left| \int d^3\mathbf{r} \exp(i\mathbf{q}\cdot\mathbf{r}) J_j^5(\mathbf{r}) \right| N \right\rangle . \tag{2.30}$$

Here, σ^N and τ^N refer to nucleon-spin and isospin operators, to be taken between nucleon states. If one does not take into account the recoil effect, then for the three quarks in $1S_{1/2}$ orbits, as given in Eqs. (2.8) and (2.9), one easily obtains

$$G_{NN\pi}(q^2) = \frac{M_N}{f_\pi} g_A^0 u(q) , \tag{2.31}$$

with

$$u(q) = \left[1 - \frac{3}{2} \frac{1}{\lambda_q (5E'_q + 7m'_q)} \mathbf{q}^2 \right] \exp\left[-\frac{\mathbf{q}^2 r_0{}^2}{4} \right] , \tag{2.32}$$

and g_A^0 is as given in Eq. (2.12). The q^2 dependence of $u(q)$ is shown in Fig. 1.

It is worthwhile at this stage to obtain the nucleon axial form factor which is related to the generalized axial-vector current

$$A_j^\mu(x) = \sum_q \bar{q}(x) \gamma^\mu \gamma^5 \frac{\tau_j}{2} q(x) + f_\pi \partial^\mu \varphi_j(x) . \tag{2.33}$$

But since the pion-quark coupling is linear in the pion field $\varphi_j(x)$ and the pionic part of the axial-vector current is proportional to $\partial^\mu \varphi_j(x)$, there will be no contribution[8] to the axial form factor $G_A(q^2)$ from $f_\pi \partial^\mu \varphi_j(x)$ if the pion field is a continuous function. Then $G_A(q^2)$ which is determined essentially by the quark core alone provides simply a measure of the spin distribution of the quark core. It can be defined in the Breit frame with $q^2 \ll 4M_N{}^2$ as

$$G_A(q^2) \langle \sigma^N \tau_j^N / 2 \rangle = \left\langle N \left| \int d^3\mathbf{r} \exp(i\mathbf{q}\cdot\mathbf{r}) \mathbf{A}_j(\mathbf{r}) \right| N \right\rangle , \tag{2.34}$$

where $\mathbf{A}(\mathbf{r})$ is the space component of the quark axial-vector current only in $A_j^\mu(x)$. Once again, if one does not take into account the c.m. motion, then with the three quarks in $1S_{1/2}$ orbits, one can easily obtain

$$G_A(q^2) = g_A^0 u(q) , \tag{2.35}$$

where $u(q)$ is the same as that given in Eq. (2.32). Thus the axial constant $g_A^0 = G_A(0)$.

Now, on comparing Eqs. (2.31) and (2.35), one obtains immediately (for $\mathbf{q}^2 \ll 4M_N{}^2$) the usual relation between the pion-nucleon form factor with the axial form factor:

$$G_{NN\pi}(q^2) = \frac{M_N}{f_\pi} G_A(q^2) . \tag{2.36}$$

Then the pseudoscalar pion-nucleon coupling constant $G_{NN\pi}$, which is defined at $q^2 = m_\pi{}^2$, is obtained as

$$G_{NN\pi} = \frac{M_N}{f_\pi} g_A^0 u(q^2 = m_\pi{}^2) , \tag{2.37}$$

which leads, with c.m. correction in g_A^0, to the $NN\pi$ coupling constant $G_{NN\pi}{}^2/4\pi \simeq 13.025$, as compared to the experimental value 14.1. The pseudovector $NN\pi$ coupling

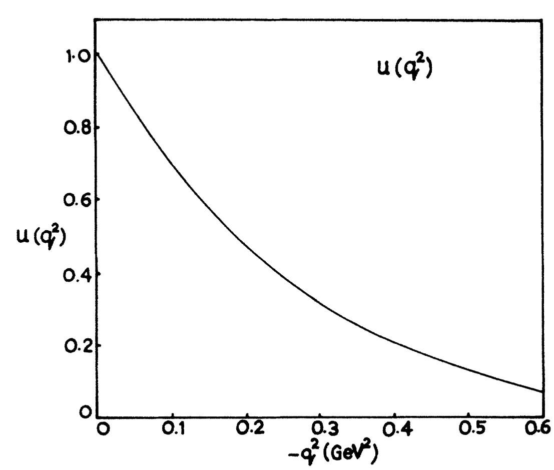

FIG. 1. q^2 dependence of the vertex form factor $u(q)$.

constant $f_{NN\pi}$ can be computed from the usual relationship $\sqrt{4\pi}(f_{NN\pi}/m_\pi)=G_{NN\pi}/2M_N$ to give $f_{NN\pi}\simeq 0.269$, as against the standard value 0.283.

C. Pionic perturbation and renormalization of $f_{NN\pi}$

The pion-coupling effects can be studied in low-order perturbation theory following the usual Hamiltonian technique as in CBM by projecting again the quark Hamiltonian on to the nonexotic baryon subspace of color-singlet baryons such as N, Δ, R (the Roper resonance), etc., as

$$\widetilde{H}=\widetilde{H}_0+\widetilde{H}_\pi+\widetilde{H}_I^\pi , \qquad (2.38)$$

when

$$\widetilde{H}_0=\sum_B |B\rangle\langle B| M_B^0=\sum_B \hat{b}_B^\dagger \hat{b}_B M_B^0 , \qquad (2.39)$$

with the color-singlet nonexotic baryon states $|B\rangle$ as the eigenstates of the Hamiltonian $\widetilde{H}^0$ [obtained from $\mathscr{L}_q^0(x)$ in a canonical way] with masses M_B^0. Here, $\hat{b}_B^\dagger$ creates a three-quark baryon state with quantum numbers of N, Δ, etc. Similarly, with $\hat{a}_{k_j}$ and $\hat{a}_{k_j}^\dagger$ as the pion destruction and creation operators and $w_k=(\mathbf{k}^2+m_\pi{}^2)^{1/2}$ as the pion energy, the Hamiltonian for the quantized free pion field $\varphi_j(x)$ becomes

$$\widetilde{H}_\pi=\sum_j \int d^3\mathbf{k}\, w_k \hat{a}_{k_j}^\dagger \hat{a}_{k_j} . \qquad (2.40)$$

Finally, the interaction Hamiltonian corresponding to $\mathscr{L}_I(x)$ in the already mentioned nonexotic baryon subspace can be written in the form

$$\widetilde{H}_I^\pi=-\frac{i}{f_\pi}\sum_{B,B'}\int d^3\mathbf{r}\, G(r)\left\langle B'\left|\sum_q \bar{q}(\mathbf{r})(\boldsymbol{\tau}\cdot\boldsymbol{\varphi})\gamma^5 q(\mathbf{r})\right|B\right\rangle \hat{b}_{B'}^\dagger \hat{b}_B , \qquad (2.41)$$

which, on using the pion-field expansion, becomes

$$\widetilde{H}_I^\pi=-\frac{1}{(2\pi)^{3/2}}\sum_{B,B',j}\int d^3\mathbf{k}[V_j^{BB'}(\mathbf{k})\hat{b}_{B'}^\dagger \hat{b}_B \hat{a}_{k_j}+\text{H.c.}] . \qquad (2.42)$$

Here, H.c. denotes the Hermitian conjugate and $V_j^{BB'}(k)$, the baryon-pion absorption vertex function in the point pion approximation, is given by

$$V_j^{BB'}(\mathbf{k})=-\frac{i}{f_\pi}(2w_k)^{-1/2}\int d^3\mathbf{r}\, G(r)\exp(i\mathbf{k}\cdot\mathbf{r})\left\langle B'\left|\sum_q \bar{q}(\mathbf{r})\gamma^5 q(r)\tau_j\right|B\right\rangle . \qquad (2.43)$$

Assuming that for the $BB'\pi$ vertex, the spatial orbits of all the quarks in the initial and final baryon state are the same $1S_{1/2}$, one can use Eqs. (2.8) and (2.9) to obtain

$$V_j^{BB'}(\mathbf{k})=\frac{i}{f_\pi}(2w_k)^{-1/2}\frac{N_q{}^2\sqrt{\pi}k^{-3/2}}{\sqrt{2}\lambda_q r_{0q}{}^4}I(k) \times\left\langle B'\left|\sum_q(\boldsymbol{\sigma}_q\cdot\mathbf{k})\tau_j\right|B\right\rangle , \qquad (2.44)$$

where

$$I(k)=2\int_0^\infty dr\, r^{5/2}G(r)J_{3/2}(kr)\exp(-r^2/r_{0q}{}^2) . \qquad (2.45)$$

Now, using the standard integral result for $I(k)$ and the axial-vector coupling constant $g_A(B)$ in the present model, the expression for $V_j^{BB'}(\mathbf{k})$ can be simplified further. As for example, considering the $NN\pi$-vertex function $V_j^{NN}(\mathbf{k})$, we get

$$V_j^{NN}(\mathbf{k})=\frac{i}{2f_\pi}(2w_k)^{-1/2}g_A(N)ku(k)(\boldsymbol{\sigma}^{NN}\cdot\hat{\mathbf{k}})\tau_j^{NN} , \qquad (2.46)$$

when $u(k)$ is the same as that given in Eq. (2.32). Finally, using the familiar Goldberger-Treiman relation $\sqrt{4\pi}(f_{NN\pi}/m_\pi)=g_A(N)/2f_\pi$, one gets

$$V_j^{NN}(\mathbf{k})=i\sqrt{4\pi}\left[\frac{f_{NN\pi}}{m_\pi}\right]\frac{ku(k)}{(2w_k)^{1/2}}(\boldsymbol{\sigma}^{NN}\cdot\hat{\mathbf{k}})\tau_j^{NN} . \qquad (2.47)$$

In the same manner, the general baryon-pion vertex function can be written as

$$V_j^{BB'}(\mathbf{k})=i\sqrt{4\pi}\left[\frac{f_{BB'\pi}}{m_\pi}\right]\frac{ku(k)}{(2w_k)^{1/2}}(\boldsymbol{\sigma}^{BB'}\cdot\hat{\mathbf{k}})\tau_j^{BB'} . \qquad (2.48)$$

The bare pseudovector coupling constants $f_{BB'\pi}$ are taken in the SU(6) ratio as $f_{NN\pi}:f_{\Delta N\pi}:f_{\Delta\Delta\pi}=1:\sqrt{72/25}:\frac{1}{5}$. Now, with the vertex function $V_j^{BB'}(\mathbf{k})$ at hand, it is possible to calculate the pionic effects such as vertex modification, wave-function renormalization, and pionic self-energy of baryons in a perturbative manner.

The pionic self-energy of the baryons can be obtained with the help of the single-loop self-energy diagram (Fig. 2) from second-order perturbation theory as

$$\Sigma_B(E)=\sum_k\sum_{B'}\frac{V_j^{*BB'}(\mathbf{k})V_j^{BB'}(\mathbf{k})}{(E-w_k-M_{B'}^0)} , \qquad (2.49)$$

when $\sum_k\equiv\sum_j\int d^3\mathbf{k}/(2\pi)^3$. The physical baryon state $|\widetilde{B}\rangle$ may be distinguished from the bare-quark—core baryon state $|B\rangle$ by writing it up to the one-pion level as

$$|\widetilde{B}\rangle=Z_B{}^{1/2}|B\rangle+\sum_{B'}C_{B'\pi}|B'\pi\rangle , \qquad (2.50)$$

where B' are the appropriate baryon intermediate states, and Z_B and $|C_{B'\pi}|^2$ are the probabilities of finding the

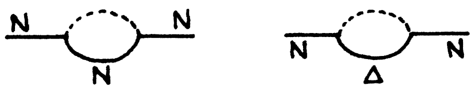

FIG. 2. Lowest-order nucleon self-energy due to quark-pion coupling.

states $|B\rangle$ and $|B'\pi\rangle$, respectively, in the physical baryon state. It is worthwhile to point out here that if all possible corrections such as gluonic and pionic corrections applicable to the quark-core quantities are treated independently, as though they were of the same order of magnitude, then at the quark-core level (N,Δ), $(\Lambda,\Sigma,\Sigma^*)$, and (Ξ,Ξ^*) would be separately mass degenerate. Considering in particular the case of a nucleon, the appropriate intermediate states like N and Δ can then be treated on mass shell with $M_N^0 = M_\Delta^0$, so that the physical nucleon state is written as

$$|\tilde{N}\rangle = Z_N{}^{1/2}|N\rangle + C_{N\pi}|N\pi\rangle + C_{\Delta\pi}|\Delta\pi\rangle\ , \qquad (2.51)$$

with

$$\epsilon_{N\pi} = |C_{N\pi}|^2 = 3I_{\pi 1} f_{NN\pi}{}^2$$

and

$$\epsilon_{\Delta\pi} = |C_{\Delta\pi}|^2 = \tfrac{96}{25} I_{\pi 1} f_{NN\pi}{}^2\ .$$

Here,

$$I_{\pi 1} = \frac{1}{\pi m_\pi{}^2}\int_0^\infty dk \frac{k^4 u^2(k)}{w_k{}^3} = \frac{1}{\pi m_\pi{}^2}(I_4 - 2Ar_0{}^2 I_6 + A^2 r_0{}^4 I_8)\ , \qquad (2.52)$$

when $A = (E_u' - m_u')/2(5E_u' + 7m_u')$ and the reduced integrals

$$I_{2n} = \int_0^\infty dk \frac{k^{2n}}{w_k{}^3}\exp(-r_0{}^2 k^2/2)$$

evaluated in a convergent series form can be written as

$$I_{2n} = \frac{\exp(z)}{2\alpha^{(n-1)}}\sum_{m=0}^{\infty}\begin{bmatrix} n-\frac{1}{2} \\ m \end{bmatrix}(-z)^m \Gamma(n-m-1,z)\ , \qquad (2.53)$$

where, $\alpha = r_0{}^2/2$, $z = \alpha m_\pi{}^2$, and $\binom{n-1/2}{m}$ are the binomial coefficients. Then evaluating $I_{\pi 1}$ as $I_{\pi 1} = 0.7247$, one can obtain the bare-nucleon probability Z_N in a physical nucleon state $|\tilde{N}\rangle$ from the normalization requirement as

$$Z_N = 1 - (\epsilon_{N\pi} + \epsilon_{\Delta\pi}) = (1 - \tfrac{171}{25} I_{\pi 1} f_{NN\pi}{}^2)\ . \qquad (2.54)$$

It is worthwhile to point out that because of the quark-pion coupling effecting wave-function renormalization and also vertex modification, the pion-nucleon coupling constant $f_{NN\pi}$ is supposed to get renormalized. In a theory without antinucleons and therefore with no renormalization of the pion propagator the renormalized $NN\pi$ coupling constant is usually given by

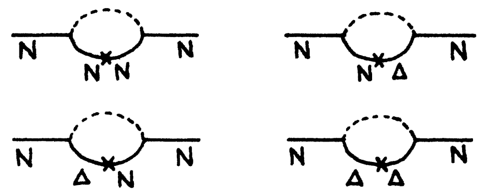

FIG. 3. Lowest-order contribution to the vertex renormalization of the nucleon axial-vector coupling constant.

$$f_{NN\pi}^{(r)} = Z_N Z_1{}^{-1} f_{NN\pi}\ . \qquad (2.55)$$

The wave-function renormalization factor Z_N given by Eq. (2.54) reduces $f_{NN\pi}^{(r)}$ from its bare value $f_{NN\pi}$, whereas the dressing of the vertex described by Z_1 tends to increase this value. Figure 3 shows the first-order dressing of the vertex which yields,[15] with $\epsilon_{N\Delta\pi} = \frac{32}{5} f_{NN\pi}{}^2 I_{\pi 1}$,

$$Z_1{}^{-1} = (1 + \tfrac{1}{9}\epsilon_{N\pi} + \tfrac{5}{9}\epsilon_{\Delta\pi} + \tfrac{8}{15}\epsilon_{N\Delta\pi}) = (1 + \tfrac{147}{25} I_{\pi 1} f_{NN\pi}{}^2)\ . \qquad (2.56)$$

Then, with the bare value $f_{NN\pi} = 0.269$ obtained earlier, one obtains the renormalized $NN\pi$ coupling constant as $f_{NN\pi}^{(r)} \simeq 0.23$. Thus it is shown that within the present model, the renormalizations of the pion coupling to bare nucleons is not only finite but small, as is the case in CBM. Therefore, a perturbation approach, in taking into account the pion-cloud effects, seems quite justified in the study of electromagnetic properties of nucleons, in the present framework of a chiral potential model.

III. ELECTROMAGNETIC PROPERTIES OF NUCLEONS

Now, with the convergence property of model at hand, one can use a perturbative approach to include explicitly the role of the pion cloud in studying the electromagnetic properties of the nucleons. For that, we first of all give a brief outline of the electromagnetic form factors of the nucleons.

For a given nucleon current $J^\mu(x)$, the Dirac-Pauli nucleon form factors are written in the form

$$\langle N(P_f)|J^\mu(0)|N(P_i)\rangle = U_{s'}(P_f)\left[F_1(q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}F_2(q^2)\right]U_s(P_i)\ , \qquad (3.1)$$

when $F_{1,2}(q^2) = F_{1,2}^S(q^2) + \tau_3 F_{1,2}^V(q^2)$ and $U_s(P)$ are the nucleon spinors with P_i and P_f as the ingoing and outgoing nucleon four-momenta, such that $P_{i,f}{}^2 = M_N{}^2$. Here, q_μ is the four-momentum transfer with $q^2 = (q_0{}^2, -\mathbf{q}^2)$. In the Breit frame, with $-\mathbf{P}_i = \mathbf{P}_f = \mathbf{q}/2$ and $q_0 = 0$, one can have

$$\langle N(\mathbf{q}/2)|J^0(0)|N(-\mathbf{q}/2)\rangle = (1+\eta)^{-1/2}\chi_{s'}^\dagger \chi_s[G_E^S(q^2) + \tau_3 G_E^V(q^2)]\ ,$$

$$\langle N(\mathbf{q}/2)|\mathbf{J}(0)|N(-\mathbf{q}/2)\rangle = (1+\eta)^{-1/2}\chi_{s'}^\dagger \frac{i\sigma x \mathbf{q}}{2M_N}\chi_s[G_M^S(q^2) + \tau_3 G_M^V(q^2)]\ , \qquad (3.2)$$

where $\tau_3=\pm 1$ for the proton and neutron, respectively, and the Sach's form factors $G(q^2)$ with $\eta=\mathbf{q}^2/4M_N{}^2$ are

$$G_E^{S,V}(q^2)=F_1^{S,V}(q^2)-\eta F_2^{S,V}(q^2) ,\quad G_M^{S,V}(q^2)=F_1^{S,V}(q^2)+F_2^{S,V}(q^2) . \tag{3.3}$$

In the static limit ($M_N\to\infty$), if we define

$$\lim_{M_N\to\infty}\langle N(\mathbf{q}/2)\,|\,J^\mu(0)\,|\,N(-\mathbf{q}/2)\rangle =\langle N(s',\lambda')\,|\,J^\mu(-\mathbf{q}^2)\,|\,N(s,\lambda)\rangle , \tag{3.4}$$

where $|N(s,\lambda)\rangle$ is the nucleon spin-isospin states, we can obtain

$$\langle N(s',\lambda')\,|\,[G_E^S(q^2)+\tau_3 G_E^V(q^2)]\,|\,N(s,\lambda)\rangle =(1+\eta)^{1/2}\langle N(s',\lambda')\,|\,J^0(-\mathbf{q}^2)\,|\,N(s,\lambda)\rangle ,$$
$$\langle N(s'\lambda')\,|\,\frac{i\sigma\times\mathbf{q}}{2M_N}[G_M^S(q^2)+\tau_3 G_M^V(q^2)]\,|\,N(s,\lambda)\rangle =(1+\eta)^{1/2}\langle N(s'\lambda')\,|\,\mathbf{J}(-\mathbf{q}^2)\,|\,N(s,\lambda)\rangle . \tag{3.5}$$

For computation of the theoretical values of the nucleon form factors, it is useful to define the Fourier transform $J^\mu(-\mathbf{q}^2)$ as

$$J^\mu(-\mathbf{q}^2)=\int d^3\mathbf{r}\,J^\mu(\mathbf{r})\exp(i\mathbf{q}\cdot\mathbf{r}) , \tag{3.6}$$

where the nucleon current considered as the sum of the quark-core current and the pion current due to chiral quark-pion coupling is given as

$$J^\mu(\mathbf{r})=J_c^\mu(\mathbf{r})+J_\pi^\mu(\mathbf{r}) ,$$

with

$$J_c^\mu(\mathbf{r})=\sum_{q=1}^{3} e_q\bar{q}(\mathbf{r})\gamma^\mu q(\mathbf{r}) ,\quad J_\pi^\mu(x)=e\epsilon_{3ij}\varphi_i(x)\partial^\mu\varphi_j(x) . \tag{3.7}$$

However, with the usual expansions of the free pion field $\varphi_i(x)$, $J_\pi^\mu(x)$ can be simplified to obtain the pionic charge and current densities in terms of the pion creation and destruction operators, as given in Ref. 5. It is worthwhile to point out here that in calculating the nucleon form factors from Eqs. (3.5) through Eqs. (3.6) and (3.7), one must use the physical nucleon state $|\tilde{N}\rangle$ as given in Eq. (2.51) for the nucleon spin-isospin state $|N(s,\lambda)\rangle$.

A. Charge form factor $G_E(q^2)$ and nucleon charge radius

The nucleon charge form factor, in general, for the proton and neutron can be given as

$$G_E^N(q^2)=G_{E,c}^N(q^2)+G_{E,\pi}^N(q^2) , \tag{3.8}$$

where $G_{E,c}^N(q^2)$ is the contribution of the quark-core current and $G_{E,\pi}^N(q^2)$ is that due to the pionic current.

1. *Core contribution $G_{E,c}^N(q^2)$*

First of all, we proceed to obtain the core contribution $G_{E,c}^N(q^2)$ to the nucleon charge form factor which arises because of the virtual photon coupling to the quarks in the bare-nucleon core, together with processes corresponding to vertex and self-energy corrections [Fig. 4(a)].

Now, according to Eq. (2.51), the physical proton and neutron states, written in the perturbative expansion form, up to the one-pion level, would be

$$|\tilde{p}\rangle=Z_N{}^{1/2}|p\rangle+C_{N\pi}(\sqrt{2/3}\,|n\pi^+\rangle-\sqrt{1/3}\,|p\pi^0\rangle)+C_{\Delta\pi}(\sqrt{1/2}\,|\Delta^{++}\pi^-\rangle-\sqrt{1/3}\,|\Delta^+\pi^0\rangle+\sqrt{1/6}\,|\Delta^0\pi^+\rangle) ,$$
$$|\tilde{n}\rangle=Z_N{}^{1/2}|n\rangle+C_{N\pi}(\sqrt{1/3}\,|n\pi^0\rangle-\sqrt{2/3}\,|p\pi^-\rangle)+C_{\Delta\pi}(\sqrt{1/2}\,|\Delta^-\pi^+\rangle+\sqrt{1/6}\,|\Delta^+\pi^-\rangle-\sqrt{1/3}\,|\Delta^0\pi^0\rangle) . \tag{3.9}$$

For the charge-neutral bare three-quark states such as $|n\rangle$ and $|\Delta^0\rangle$, the expectation value of $J_c^0(-\mathbf{q}^2)$ would be zero, leading specifically to no charge distribution for the neutron in the lowest order. However, for a charged bare three-quark state such as $|p\rangle$, $|\Delta^{++}\rangle$, or $|\Delta^-\rangle$, etc., the expectation value of $J_c^0(-\mathbf{q}^2)$ would be nonzero. If we denote this as $nG_{E,c}^{0p}(q^2)$, where n gives the total charge of the three-quark state as an integral multiple of proton charge, then $G_{E,c}^{0p}(q^2)$ in the present model, using the quark spatial orbits as given in Eqs. (2.8) and (2.9), would be

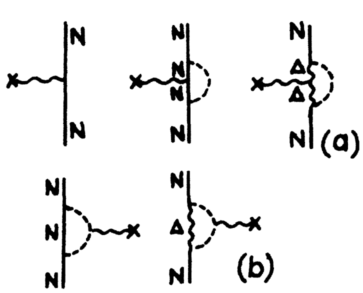

FIG. 4. Relevant diagrams contributing to the electric charge form factors for nucleons due to (a) photon-quark interaction, (b) photon-pion interaction.

$$G_{E,c}^{0p}(q^2)=(1+\eta)^{1/2}\left[1-\frac{1}{2}\frac{\mathbf{q}^2}{(E_q'+m_q')(3E_q'+m_q')}\right]\times\exp(-\mathbf{q}^2r_0{}^2/4) . \tag{3.10}$$

Then from (3.8), using the physical proton and neutron states as given explicitly in (3.9), one can obtain the core contribution to the charge form factor for proton and neutron, respectively, as

$$G_{E,c}^p(q^2)=(Z_N+\tfrac{1}{3}\epsilon_{N\pi}+\tfrac{4}{3}\epsilon_{\Delta\pi})G_{E,c}^{0p}(q^2) ,\quad G_{E,c}^n(q^2)=(\tfrac{2}{3}\epsilon_{N\pi}-\tfrac{1}{3}\epsilon_{\Delta\pi})G_{E,c}^{0p}(q^2) . \tag{3.11}$$

Now using $\epsilon_{N\pi}$ and $\epsilon_{\Delta\pi}$ from Sec. II, we obtain

$$G^p_{E,c}(q^2)=(Z_N+\tfrac{153}{25}f^2_{NN\pi}I_{\pi 1})G^{0p}_{E,c}(q^2)\ ,$$
$$G^n_{E,c}(q^2)=\tfrac{18}{25}f^2_{NN\pi}I_{\pi 1}G^{0p}_{E,c}(q^2)\ . \tag{3.12}$$

Thus, it is evident that the charge distribution of the neutron in the core in this model is a first-order effect of pion coupling arising directly from the $|p\pi^-\rangle$, $|\Delta^+\pi^-\rangle$, and $|\Delta^-\pi^+\rangle$ components.

2. *Pionic contribution* $G^N_{E,\pi}(q^2)$

Finally, to calculate the pionic contribution to the charge distribution, we first regard the pions as pointlike objects. Later on, the strong-binding effects in the isovector $\pi\pi$ channel and the effect of the finite pion size will be incorporated phenomenologically in terms of the pion form factor and an additional multiplying cutoff factor, respectively.

Since pions contribute only to the isovector form factors, one can define effectively the pion charge-density operator by

$$j^0_\pi(\mathbf{r})=\langle\tilde N\,|\,J^0_\pi(\mathbf{r})\,|\,\tilde N\rangle\ . \tag{3.13}$$

This leads to two nonzero terms depicted in Fig. 4(b) which can be written according to the relevant intermediate stages as

$$j^0_\pi(\mathbf{r})=j^0_{\pi,N}(\mathbf{r})+j^0_{\pi,\Delta}(\mathbf{r})\ . \tag{3.14}$$

Now the Fourier transform of expressions in Eq. (3.14) would yield $G^V_{E,N\pi}(q^2)$ and $G^V_{E,\Delta\pi}(q^2)$. However, for simplicity we first of all calculate these quantities in the $\mathbf{q}^2\rightarrow 0$ limit, so that

$$G^V_{E,N_\Delta,\pi}(0)=\lim_{\mathbf{q}^2\rightarrow 0}(1+\eta)^{1/2}\int d^3\mathbf{r}\exp(i\mathbf{q}\cdot\mathbf{r})\tau_3 j^0_{\pi,N_\Delta}(\mathbf{r})\ , \tag{3.15}$$

when according to Ref. 15, with degenerate N and Δ states,

$$G^V_{E,N\pi}(0)=\tau_3 2ef_{NN\pi}{}^2I_{\pi 1}\ ,$$
$$G^V_{E,\Delta\pi}(0)=-\tau_3\tfrac{32}{25}ef_{NN\pi}{}^2I_{\pi 1}\ . \tag{3.16}$$

Then, $G^V_{E,\pi}(0)=G^V_{E,N\pi}(0)+G^V_{E,\Delta\pi}(0)$ in the units of e is

$$G^V_{E,\pi}(0)=\tau_3\tfrac{18}{25}f_{NN\pi}{}^2I_{\pi 1}\ . \tag{3.17}$$

Here, only terms of second order in $1/f_\pi$ have been kept. Now, in view of the fact that a pion may have finite size and $\pi\pi$ pairs in the isovector vector channel undergo strong interaction, we can introduce the entire q^2 dependence of the pionic charge form factor in a phenomenological manner, through additional multiplying factors like the pion form factor $F_\pi(q^2)=(1-\frac{1}{6}\langle r^2\rangle_\pi\mathbf{q}^2)$ with $\langle r^2\rangle_\pi^{1/2}=0.78$ fm (experimental value) and a cutoff factor[16,17] $C(q)=\exp(-\mathbf{q}^2\Lambda^2/4)$, with the size parameter $\Lambda^2\simeq 2$ fm^2 corresponding to the effective pion radius[16] $r_\pi\simeq 0.4$ fm. Then we have

$$G^N_{E,\pi}(q^2)\simeq F_\pi(q^2)C(q)G^V_{E,\pi}(0)$$
$$=\tau_3(1-\tfrac{1}{6}\langle r^2\rangle_\pi\mathbf{q}^2)\tfrac{18}{25}f_{NN\pi}{}^2I_{\pi 1}\exp\left[-\frac{\mathbf{q}^2\Lambda^2}{4}\right]\ , \tag{3.18}$$

where $\tau_3=+1$ for the proton and -1 for the neutron.

Hence, the total charge form factor for the proton and neutron, respectively, can be given as

$$G^{p,n}_E(q^2)=G^{p,n}_{E,c}(q^2)+G^{p,n}_{E,\pi}(q^2)\ . \tag{3.19}$$

Now, using Eqs. (3.12) and (3.18) in (3.19), it is straightforward to observe that the total charge of the physical nucleon $Q_{p,n}=G^{p,n}_E(0)$ comes out correctly. The exact cancellation between Eqs. (3.12) and (3.18) at $q^2=0$ guarantees the total neutron charge Q_n to be zero. On the other hand, the core charge $Q_{p,c}=G^p_{E,c}(0)$ and the pion-cloud charge $Q_{p,\pi}=G^p_{E,\pi}(0)$ add up exactly to give $Q_p=1$, implying thereby the conservation of charge. In Figs. 5 and 6, respectively, we have shown the q^2 dependence of $G^p_E(q^2)$ and $G^n_E(q^2)$ as calculated from Eq. (3.19) in comparison with those obtained in MIT-bag-model calculations. We observe that the overall agreement with the experimental data[18] is reasonably good, with discrepancies more prominent for the higher q^2 region only. This may be due to the fact that possible recoil corrections have not been taken care of in our calculation. One should further note that the dressing of the quark core by a pion cloud in this work has effect only on the isovector form factor $G^V_E\simeq\frac{1}{2}(G^p_E-G^n_E)$. Modification of isoscalar form factor $G^S_E=\frac{1}{2}(G^p_E+G^n_E)$ requires a description of photon coupling to at least three-pion intermediate states. In that case, a small correction to G^S_E would affect G^n_E much more than G^p_E. Therefore, apart from a qualitative comparison of G^n_E with the experimental data, we do not attach greater quantitative significance to the results for the neutron form factor in particular.

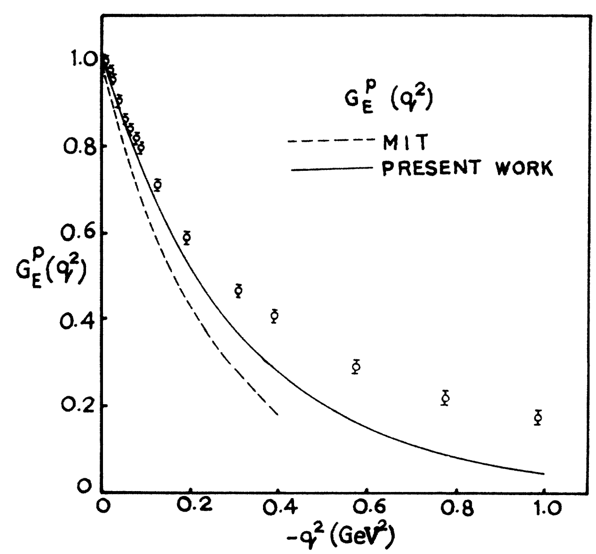

FIG. 5. Electric charge form factor $G^p_E(q^2)$ for a proton, in comparison with experiment and MIT work.

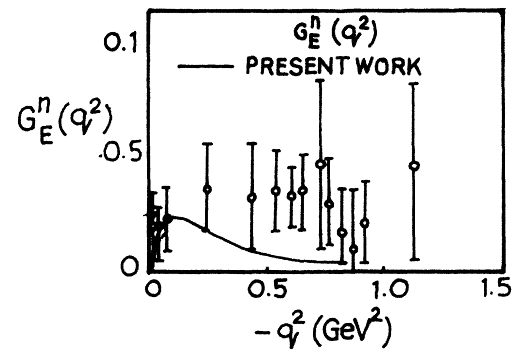

FIG. 6. Electric charge form factor $G_E^n(q^2)$ for a neutron, in comparison with experment.

3. *Nucleon charge radius*

The nucleon charge radii, however, can be computed from $G_E^p(q^2)$ and $G_E^n(q^2)$ in the usual manner as

$$\langle r^2\rangle_N = -6\left.\frac{\partial G_E^N(q^2)}{\partial \mathbf{q}^2}\right|_{\mathbf{q}^2=0} . \tag{3.20}$$

But in order to incorporate the effect of c.m. motion in the core radius only, while neglecting it for the pionic component, we separately calculate the radius of the nucleon core coming from $G_{E,c}^N(q^2)$ and the correction factor coming from $G_{E,\pi}^N(q^2)$. This yields

$$\langle r^2\rangle_{p,c} = (Z_N + \tfrac{153}{25} f_{NN\pi}{}^2 I_{\pi 1})\langle r^2\rangle_{p,c}^0 ,$$

$$\langle r^2\rangle_{n,c} = \tfrac{18}{25} f_{NN\pi}{}^2 I_{\pi 1}\langle r^2\rangle_{p,c}^0 ,$$

when

$$\langle r^2\rangle_{p,c}^0 = \left[\frac{-3}{4M_N{}^2} + \frac{3}{2}\frac{(11E_q' + m_q')}{(E_q'^2 - m_q'^2)(3E_q' + m_q')}\right] \tag{3.21}$$

and

$$\delta\langle r^2\rangle_{N,\pi} = \tau_3 \tfrac{18}{25} f_{NN\pi}{}^2 I_{\pi 1}(\langle r^2\rangle_\pi + \tfrac{3}{2}\Lambda^2) .$$

Now applying the c.m. correction to the core radius $\langle r^2\rangle_{p,c}$ and $\langle r^2\rangle_{n,c}$ according to Eq. (2.25), we obtain

$$\langle r^2\rangle_N = \langle r^2\rangle_{N,c}' + \delta\langle r^2\rangle_{N,\pi} . \tag{3.22}$$

Carrying out the calculation, we find the charge radius for the proton $\langle r^2\rangle_p{}^{1/2} \simeq 0.79$ fm and that for the neutron $\langle r^2\rangle_n{}^{1/2} \simeq -0.344$ fm. These results are presented in Table I, in comparison with the corresponding experimental values; this comparison shows excellent agreement.

B. Magnetic form factor $G_M(q^2)$ and nucleon magnetic moments

Using Eqs. (3.6) and (3.7) in (3.5), the magnetic form factor $G_M^N(q^2)$ can, in principle, be derived in the model, when for the nucleon spin-isospin state $|N(s,\lambda)\rangle$ one must take the physical nucleon state $|\tilde{N}\rangle$. Since the nucleon current density has a core part and a pionic part, the magnetic form factor, as a result can be written accordingly as

$$G_M^N(q^2) = G_{M,c}^N(q^2) + G_{M,\pi}^N(q^2) . \tag{3.23}$$

1. *Core contribution $G_{M,c}^N(q^2)$*

Core contribution to the nucleon magnetic form factor is essentially due to the photon-quark interactions, as depicted in Fig. 7(a). But unlike the charge density operator, the current operator can cause transition between the nucleon and Δ states. First of all, we can calculate the zeroth-order term in $G_{M,c}^N(q^2)$ corresponding to the bare three-quark component of $|\tilde{N}\rangle$. This leads to

$$G_{M,c}^{0p}(q^2) = \mu_p^0 \left[1 + \frac{\mathbf{q}^2}{4M_N{}^2}\right]^{1/2} \exp(-\mathbf{q}^2 r_0{}^2/4) ,$$

$$G_{M,c}^{0n}(q^2) = -\frac{2\mu_p^0}{3}\left[1 + \frac{\mathbf{q}^2}{4M_N{}^2}\right]^{1/2} \exp(-\mathbf{q}^2 r_0{}^2/4) , \tag{3.24}$$

when

$$\mu_p^0 = \frac{4M_p}{(3E_q' + m_q')}\mu_N .$$

However, because of the vertex correction and wave-function renormalization, the bare-quark—core contribution to the magnetic form factor would get modified,

TABLE I. Static electromagnetic properties of the nucleon.

Quantity	Present work	Expt.	CBM values
$\langle r^2\rangle_p{}^{1/2}$	0.79 fm	0.85±0.02 fm	0.83 fm
$\langle r^2\rangle_n{}^{1/2}$	−0.344 fm	−0.341 fm	−0.36 fm
μ_p	$2.730\mu_N$	$2.7928\mu_N$	$2.6\mu_N$
μ_n	$-1.975\mu_N$	$-1.913\mu_N$	$-2.01\mu_N$
g_A	1.182	1.255±0.006	1.33

With $(a, V_0, r_0, m_u' = m_d') \equiv (2.273\ \text{fm}^{-3},\ -137.5\ \text{MeV},\ 0.63\ \text{fm},\ 10\ \text{MeV})$
$(I_{\pi 1}, I_{\pi 2}) \equiv (0.724\,664\,3,\ 1.902)$
$(\delta_N{}^2, \Lambda^2) \equiv (0.7338,\ 2\ \text{fm}^2)$

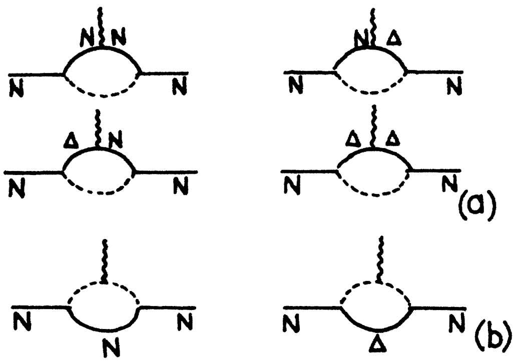

FIG. 7. Relevant diagrams contributing to the magnetic form factors for nucleons due to (a) photon-quark interaction, and (b) photon-pion interaction.

which according to Ref. 15, becomes

$$G^p_{M,c}(q^2)=Z_N(1+\tfrac{1}{27}\epsilon_{N\pi}+\tfrac{20}{27}\epsilon_{\Delta\pi}+\tfrac{4}{9}\epsilon_{N\Delta\pi})G^{0p}_{M,c}(q^2)\ ,$$
$$G^n_{M,c}(q^2)=Z_N(1+\tfrac{2}{9}\epsilon_{N\pi}+\tfrac{5}{18}\epsilon_{\Delta\pi}+\tfrac{2}{3}\epsilon_{N\Delta\pi})G^{0n}_{M,c}(q^2)\ . \tag{3.25}$$

2. Pionic contribution $G^N_{M,\pi}(q^2)$

Next we calculate the pionic contribution to the magnetic form factor; we define the pion current operator:

$$\mathbf{j}_\pi(\mathbf{r})=\langle\tilde{N}\,|\,\mathbf{j}_\pi(\mathbf{r})\,|\,\tilde{N}\rangle=\mathbf{j}_{\pi,N}(r)+\mathbf{j}_{\pi,\Delta}(\mathbf{r})\ , \tag{3.26}$$

where $\mathbf{j}_{\pi,N}(r)$ and $\mathbf{j}_{\pi,\Delta}(\mathbf{r})$ are the contribution of the pionic current with bare nucleon and Δ intermediate states shown in Fig. 7(b). In analogy with Eq. (3.5), we define the pionic contribution to the magnetic form factor as

$$G^{VN}_{N,\pi}(q^2)=\tau_3[G^V_{M,N\pi}(q^2)+G^V_{M,\Delta\pi}(q^2)]\ . \tag{3.27}$$

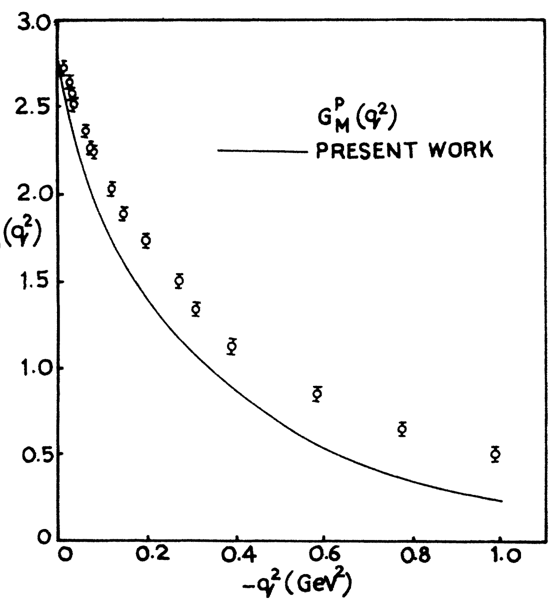

FIG. 8. Magnetic form factor $G^p_M(q^2)$ for a proton, in comparison with experiment.

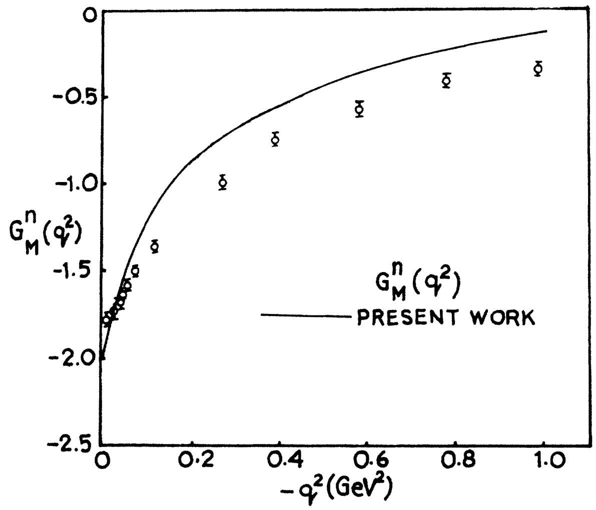

FIG. 9. Magnetic form factor $G^n_M(q^2)$ for a neutron, in comparison with experiment.

According to Ref. 15, with degenerate N and Δ states, it is easy to obtain, in the limit $q^2\to 0$,

$$G^N_{M,\pi}(0)=\tau_3\tfrac{88}{25}f_{NN\pi}{}^2 I_{\pi 2}\ ,$$

with

$$I_{\pi 2}=\frac{M_p}{\pi m_\pi{}^2}\int_0^\infty dk\,\frac{k^4u^2(k)}{w_k{}^4}$$
$$=\frac{M_p}{\pi m_\pi{}^2}(I'_4-2Ar_0{}^2I'_6+A^2r_0{}^4I'_8)\ . \tag{3.28}$$

The reduced integrals

$$I'_{2n}=\int_0^\infty dk\,k^{2n}\exp(-\alpha\mathbf{k}^2)/w_k{}^4$$

can be evaluated in a closed form as

$$I'_{2n}=\frac{\Gamma(n+\frac{1}{2})}{2\alpha^{n-3/2}}(z)^{n-3/2}\psi(n+\tfrac{1}{2},n-\tfrac{1}{2},z)\ , \tag{3.29}$$

when $z=\alpha m_\pi{}^2=r_0{}^2m_\pi{}^2/2$ and $\psi(a,b,z)$ are the Kummer's confluent hypergeometric functions of the second kind.[19] The $I_{\pi 2}$ can be evaluated as $I_{\pi 2}=1.902$ MeV, enabling the explicit calculation of $G^N_{M,\pi}(0)$. As done before, if we choose to introduce the entire q^2 dependence in a phenomenological manner through the same additional multiplying factors $C(q)F_\pi(q^2)$ as used in case of $G^N_{E,\pi}(q^2)$, we obtain

$$G^N_{M,\pi}(q^2)=F_\pi(q^2)C(q)G^N_{M,\pi}(0)$$
$$=\tau_3\tfrac{88}{25}f_{NN\pi}{}^2I_{\pi 2}(1-\tfrac{1}{6}\langle r^2\rangle_\pi\mathbf{q}^2)\exp\left[-\frac{\mathbf{q}^2\Lambda^2}{4}\right]\ . \tag{3.30}$$

Then we can compute the q^2 dependence of $G^N_M(q^2)$ without taking into account the recoil corrections. The results so obtained for $G^p_M(q^2)$ and $G^n_M(q^2)$ have been presented in Figs. 8 and 9, respectively, in comparison with the experimental data, which shows very good agreement.

3. *Nucleon magnetic moment*

The magnetic moments μ_p and μ_n of the proton and neutron, respectively, can now be computed from $G_M^p(q^2)$ and $G_M^n(q^2)$ at the $q^2 \to 0$ limit so as to give, in nuclear magneton units,

$$\mu_{p(n)} = \mu_{p(n),c} + \delta\mu_{N,\pi} , \tag{3.31}$$

when from Eqs. (3.24) and (3.25) we have the core contributions

$$\begin{aligned} \mu_{p,c} &= Z_N(1+\tfrac{29}{5} f_{NN\pi}{}^2 I_{\pi 1})\mu^0_{p,c} , \\ \mu_{n,c} &= Z_N(1+6 f_{NN\pi}{}^2 I_{\pi 1})\mu^0_{n,c} . \end{aligned} \tag{3.32}$$

However, if we apply the c.m. corrections to the core contributions to the magnetic moment only, according to the procedure adopted through Eq. (2.24), then we can obtain $\mu'_{N,c}$. Finally, the correction to the nucleon magnetic moment due to the pion cloud is obtained from Eq. (3.29) as

$$\delta\mu_{N,\pi} = \tau_3 \tfrac{88}{25} f_{NN\pi}{}^2 I_{\pi 2} , \tag{3.33}$$

which is positive for the proton and negative for the neutron. Hence,

$$\mu_{p(n)} = \mu'_{p(n),c} + \delta\mu_{N,\pi} . \tag{3.34}$$

On calculation, one finds that $|\delta\mu_{N,\pi}| = 0.5356\mu_{\mathcal{N}}$, whereas the core contributions after c.m. correction are $\mu'_{p,c} = 2.1923\mu_{\mathcal{N}}$ and $\mu'_{n,c} = -1.44\mu_{\mathcal{N}}$. Thus, applying the pionic corrections to the core values, we obtain $\mu_p = 2.73$ $\mu_{\mathcal{N}}$ and $\mu_n = -1.975\mu_{\mathcal{N}}$, which are very consistent with the experimental values.

IV. SUMMARY AND CONCLUSION

We now summarize the results we obtained and the observations made in studying the electromagnetic properties of the nucleons by incorporating chiral symmetry to a simple phenomenological potential model with the individual quark potential in an equally mixed scalar-vector harmonic form. Although the tools used in the perturbative approach with a linearized pion-quark interaction are quite well known, the calculations involved in considering the pion-cloud effects become more straightforward and tractable in the model, yielding very satisfactory results for the electromagnetic properties of the nucleons.

We have treated all possible corrections (pionic, gluonic, and those due to c.m. motion) to the bare-nucleon values independently, assuming that they are of the same order of magnitude. In that case, $|N\rangle$ and $|\Delta\rangle$ can be treated as mass-degenerate bare states. In such a simplistic approach the integral expressions involved in the perturbative calculation of the self-energy, vertex modification, and wave-function renormalization turn out to be simple and can be evaluated as shown in the text. The values of these relevant integrals are summarized in Table I, along with the results obtained for the charge radii, magnetic moments, and the coupling constants. In all these calculations we have used the standard value of $f_{NN\pi} = 0.283$. The results obtained are shown to agree remarkably well with the corresponding experimental values.

The nucleon form factors such as $G_A(q^2)$, $G_E^N(q^2)$, and $G_M^N(q^2)$ have been studied, neglecting the recoil corrections. The contribution of the bare-nucleon core to the charge form factor $G_E^N(q^2)$ and the magnetic form factor $G_M^N(q^2)$, in view of the pion-quark coupling giving rise to vertex dressing and wave-function renormalization, gets modified keeping its q^2 dependence in tact. However, obtaining the q^2 dependence of the corresponding pionic parts in $G_E^N(q^2)$ and $G_M^N(q^2)$ is not so straightforward. In view of the fact that the pionic contributions to the isovector part of the charge and magnetic form factors in this model are significantly small, we evaluate them from the theoretical expressions only in the limit $q^2 \to 0$. Then its overall q^2 dependence is provided in a phenomenological manner, considering the finite size of the pion and also the strong-binding effects in the isovector $\pi\pi$ channel. The results for $G_E^p(q^2)$, $G_E^n(q^2)$, $G_M^p(q^2)$, and $G_M^n(q^2)$ are shown in Figs. 5, 6, 8, and 9, respectively. The overall behavior of $G_E^p(q^2)$, $G_E^n(q^2)$, $G_M^p(q^2)$, and $G_M^n(q^2)$ is found to be in good qualitative agreement with the experimental data. Inclusion of recoil correction and the short-distance one-gluon-exchange effects not considered here might improve the agreement.

Thus we conclude that such a chiral quark model, with an equally mixed scalar and vector-harmonic potential for individual quarks incorporating pion-quark interaction in a linearized form, provides, in the usual perturbative approach, a reasonable and consistent description of the electromagnetic properties of nucleons. The pion-cloud effects, although not very significant in this model, give an overall result very much in agreement with the experiment. The charge radii, magnetic moment, and the axial-vector coupling constant in neutron-β decay, after pionic and c.m. corrections, are in remarkable agreement with the experimental values. In view of the simplicity of the model the results are quite encouraging.

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*Present address: Dhenkanal College, Dhenkanal, Orissa, India.

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